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Coordinate Transformation for Sector and Annular Sector Shaped Graphene Sheets on Silicone Matrix

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Abstract:

In the present manuscript, we developed a systematic formulation for some type graphene sheets having annular sector, sector shaped or curvilinear side graphene located on a silicone matrix via nonlocal elasticity theory for numerical solution. An eight-node curvilinear element is used for transformation of the governing equation of motion of annular sector graphene from physical region to computational region in conjunctions with the thin plate theory. Silicone matrix is modeled by using the Winkler-Pasternak elastic foundations. The formulation is usefully for different shaped graphene sheets.

Keywords: Scale effect, silicone matrix, sector graphene, discrete singular convolution, vibration.

1. Introduction

After invitation of carbon nanotubes, micro scaled mechanical systems had been widely used in microcomputers, biomedical, micro electromechanical purposes and modern industries. For examples; thin films, nano-sheet resonators, biomedical devices, nano electro mechanical applications, micro props, paddle-like resonators, atomic force microscopy, mechanical actuators and nano sensors. Graphene based structures have been also widely used in micro-electro-mechanical systems (MEMS) for high frequency and high sensitive purposes for example molecular gas detectors, solar cells, integrated circuits and nano ribbons due to their ultra mechanical, thermal, optical and electrical properties [1-6]. Mechanical properties are widely investigated of the graphene sheets by researchers [7-12] in the past ten years.

It is known that the analysis based on the classical elasticity theory does not take into consider the internal length scale effect of nanostructure. To introduce the size effect to the governing equations, material length scale parameters must be taken into account. Atomistic simulation model or hybrid atomistic-continuum model are computationally expensive. So, some higher-order continuum theories have been proposed by this time. In the early of 1970s, nonlocal elasticity theory is proposed by Eringen [13] for modeling of the length-scale problems in continuum mechanics. By this time, this theory is widely used by researchers for modeling of micro- or nano-scaled structures [14-28]. In the literature, mechanical characteristics of rectangular and circular nano/micro plates have been investigated via nonlocal elasticity. The effects of elastic matrix on frequency had been investigated just for rectangular and circular micro plates or graphene, by this time. In the present study, however, free vibration analysis of micro-scaled annular sector and sector graphene resting on an elastic matrix is firstly investigated using the geometric transformation based on the nonlocal continuum theory in conjunction with the discrete singular convolution method.

2. Discrete singular convolution (DSC)

The method of discrete singular convolution has recently been proposed for engineering and mathematical physics problems by Wei [29] in 1999 via theory of distributions. After this, Wei [30,31] first introduced this method for solving mechanical problems. By this time, a variety of structural mechanics problems have been analyzed using the method of DSC [32-40] in successfully. In the present paper, details of the DSC method are not given in detail; interested readers may refer to the works of [41-53]. Consider a distribution, T and $\eta(t)$ as an element of the space of test function. A singular convolution can be defined by [32]

$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx\,, \tag{1}$$

where $T(t-x)$ is a singular kernel. For example, singular kernels of delta type [33]

$$T(x) = \delta^{(n)}(x)\,; \quad (\mathrm{n} = 0,1,2,...,). \tag{2}$$

Kernel $T(x) = \delta(x)$ is important for interpolation of surfaces and curves, and $T(x) = \delta^{(n)}(x)$ for n>1 are essential for numerically solving differential equations. With a sufficiently smooth approximation, it is more effective to consider a discrete singular convolution [34]

$$F_{\alpha}(t) = \sum_{k} T_{\alpha}(t - x_k) f(x_k)\,, \tag{3}$$

where F_{α} (t) is an approximation to $F(t)$ and $\{x_k\}$ is an appropriate set of discrete points on which the DSC is well defined [32-35]. Note that, the original test function $\eta(x)$ has been replaced by f(x). This new discrete expression is suitable for computer realization. The mathematical property or requirement of f(x) is determined by the approximate kernel T_{α}. Recently, the use of some new kernels and regularizer such as delta regularized was proposed to solve applied mechanics problem. The researchers is generally used the regularized delta Shannon kernel by this time [30-37]. The Shannon's kernel is regularized as [34]

$$\delta_{\Delta,\sigma}(x - x_k) = \frac{\sin[(\pi/\Delta)(x - x_k)]}{(\pi/\Delta)(x - x_k)} \exp\left[-\frac{(x - x_k)^2}{2\sigma^2}\right];\ \sigma>0. \tag{4}$$

where Δ is the grid spacing. It is also known that the truncation error is very small due to the use of the Gaussian regularizer, the above formulation given by Eq. (4) is practical, and has an essentially compact support for numerical interpolation. Equation (4) can also be used to provide discrete approximations to the singular convolution kernels of the delta type [35]

$$f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta_{\Delta}(x - x_k) f(x_k)\ , \tag{5}$$

where $\delta_\Delta(x-x_k)=\Delta\delta_\alpha(x-x_k)$ and superscript (n) denotes the n*th*-order derivative. The $2M+1$ is the computational bandwidth which is centred around x, and is usually smaller than the whole computational domain. In the DSC method, the function $f(x)$ and its derivatives with respect to the x coordinate at a grid point x_i are approximated by a linear sum of discrete values $f(x_k)$ in a narrow bandwidth [$x-x_M$, $x+x_M$]. This can be expressed as [36]

$$\left.\frac{d^n f(x)}{d\,x^n}\right|_{x=x_i} = f^{(n)}(x) \approx \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(n)}(x_i-x_k)\,f(x_k)\,; \quad (n=0,1,2,...,). \tag{6}$$

where superscript n denotes the nth-order derivative with respect to x. The x_k is a set of discrete sampling points centred around the point x, σ is a regularization parameter, Δ is the grid spacing, and $2M+1$ is the computational bandwidth which is usually smaller than the size of the computational domain [36,37]. For example, the second order derivative at $x=x_i$ of the DSC kernels for directly given [38]

$$\delta_{\Delta,\sigma}^{(2)}(x-x_j)=\frac{d^2}{d\,x^2}\Big[\ \delta_{\Delta,\sigma}(x-x_j)\Big]\Big|_{x=x_i}, \tag{7}$$

The discretized forms of Eq. (7) can then be expressed as

$$f^{(2)}(x)=\left.\frac{d^2 f}{d\,x^2}\right|_{x=x_i} \approx \sum_{k=-M}^{M} \delta_{\Delta,\sigma}^{(2)}(k\Delta x_N)\,f_{\,i+k,\,j}\,. \tag{8}$$

3. Geometric mapping

By using the transformation rule, a non-rectangular physical domain (Figs. 1-7) can be easily transformed into a normalized computational domain via geometric mapping. This technique has been widely used in the finite elements and differential quadrature methods by this time. In order to transformation from physical domain to computational domain, let consider an eight-node curvilinear quadrilateral domain as shown in Fig. 1(a). Thus, the following equations are used for the coordinate transformation [43,44]

$$x=\sum_{i=1}^{8}\Psi_i(\xi,\eta)\,x_i\ , \qquad y=\sum_{i=1}^{8}\Psi_i(\xi,\eta)\,y_i \tag{9,10}$$

Hence, first-order, and second order derivatives of a function are given via chain rule

$$\begin{Bmatrix} u_x \\ u_y \end{Bmatrix} = [J_{11}]^{-1}\begin{Bmatrix} u_\xi \\ u_\eta \end{Bmatrix} \tag{11}$$

$$\begin{Bmatrix} u_{xx} \\ u_{yy} \\ 2u_{xy} \end{Bmatrix} = [J_{22}]^{-1}\begin{Bmatrix} u_{\xi\xi} \\ u_{\eta\eta} \\ u_{\xi\eta} \end{Bmatrix} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\begin{Bmatrix} u_\xi \\ u_\eta \end{Bmatrix} \tag{12}$$

where ξ_i and η_i are the coordinates of Node i in the ξ-η plane, and J_{ij} are the elements of the Jacobian matrix. These are expressed as follows;

$$[J_{11}] = \begin{bmatrix} x_\xi & y_\xi \\ x_\eta & y_\eta \end{bmatrix}, \qquad [J_{21}] = \begin{bmatrix} x_{\xi\xi} & y_{\xi\xi} \\ x_{\eta\eta} & y_{\eta\eta} \\ x_{\xi\eta} & y_{\xi\eta} \end{bmatrix} \tag{13,14}$$

$$[J_{22}] = \begin{bmatrix} {x_\xi}^2 & {y_\xi}^2 & x_\xi y_\xi \\ {x_\eta}^2 & {y_\eta}^2 & x_\eta y_\eta \\ x_\xi x_\eta & y_\xi y_\eta & \frac{1}{2}(x_\xi y_\eta + x_\eta y_\xi) \end{bmatrix}. \tag{15}$$

Thus, an arbitrary-shaped quadrilateral plate may be represented by the mapping of a square plate defined in terms of its natural coordinates in different numerical applications [43-48]. Shape functions for related points are given as follows

$$\Psi_i(\xi,\eta) = \frac{1}{4}(1+\xi\xi_i)(1+\eta\eta_i)(\xi\xi_i + \eta\eta_i - 1) \quad \text{for} \quad i\text{=1,3,5,7} \tag{16}$$

$$\Psi_i(\xi,\eta) = \frac{1}{4}(1-\xi^2)(1+\eta\eta_i) \quad \text{for} \quad i\text{=2,6} \tag{17}$$

$$\Psi_i(\xi,\eta) = \frac{1}{4}(1+\xi\xi_i)(1-\eta^2) \quad \text{for} \quad i\text{=4,8} \tag{18}$$

4. Nonlocal elasticity theory

After the invention of carbon nanotubes, scaled-based continuum approaches are being popular in modeling of the micro or nano sized structures. As stated by Eringen [13] the linear theory of nonlocal elasticity leads to a set of integro-partial differential equations for the displacements field for homogeneous, isotropic bodies. According to the nonlocal elasticity theory of Eringen's, the stress at any reference point in the body depends not only on the strains at this point but also on strains at all points of the body. This definition of the Eringen's nonlocal elasticity is based on the atomic theory of lattice dynamics and some experimental observations on phonon dispersion. In this theory, the fundamental equations involve spatial integrals which represent weighted averages of the contributions of related strain tensor at the related point in the body. For homogenous and isotropic elastic solids, the linear theory of nonlocal elasticity is described by the following equations [13]:

$$\sigma_{kl,l} + \rho(f_l - \frac{\partial^2 u_l}{\partial t^2}) = 0, \tag{19}$$

$$\sigma_{kl}(x) = \int_V \alpha(|x - x'|, \chi)\tau_{kl}(x')dV(x'), \tag{20}$$

$$\tau_{kl}(x') = \lambda\varepsilon_{mm}(x')\delta_{kl} + 2\mu\varepsilon_{kl}(x'), \tag{21}$$

$$\varepsilon_{kl}(x') = \frac{1}{2}\left(\frac{\partial u_k(x')}{\partial x'_l} + \frac{\partial u_l(x')}{\partial x'_k}\right), \tag{22}$$

where σ_{kl} is the nonlocal stress tensor, ρ is the mass density of the body, f_l is the body (or applied) force density, u_l is the displacement vector at a reference point x in the body, $\tau_{kl}(x')$ is the classical (Cauchy) or local stress tensor at any point x' in the body, $\varepsilon_{kl}(x')$ is the linear strain tensor at point x' in the body, t is denoted the time, V is the volume occupied by the elastic body, $\alpha|x-x'|$ is the distance in Euclidean form, λ and μ are the Lame constants. The non-local kernel $\alpha|x-x'|$ defines as the impact of the strain at the point x' on the stress at the point x in the elastic body. The value of χ depends on the ratio ($e_0 a/l$) which is material constant. The value a depends on the internal (granular distance, lattice parameter, distance between C-C bonds as molecular diameters) and external characteristics lengths (crack length or wave length) and e_0 is a constant appropriate to each material for adjusting the model to match reliable results by experiments or some other theories. If $\alpha|x|$ takes on a Green function of a linear differential operator given as [13]

$$\hbar\alpha(|x'-x|) = \delta(|x'-x|) \tag{23}$$

the nonlocal constitutive relation given by Eq.(20) is reduced to the differential equation

$$\hbar\sigma_{kl} = \tau_{kl} \tag{24}$$

Furthermore the integro-partial differential equation given by Eq. (19) is also reduced to the following partial differential equation

$$\tau_{kl,l} + \hbar(f_l - \rho\ddot{u}_k) = 0 \tag{25}$$

Eringen (3) proposed a nonlocal model for this linear differential operator given as

$$\hbar = 1 - (e_0 a)^2\nabla^2 = 0 \tag{26}$$

where ∇^2 is the Laplacian. Consequently, the constitutive relations can be written as

$$\left[1 - (e_0 a)^2\nabla^2\right]\sigma_{kl} = \tau_{kl} \tag{27}$$

5. Fundamental equations

Let us consider a single graphene in Cartesian coordinate systems. The displacement fields are given as

$$\bar{u}(x,y,z,t) = u(x,y,t) - z\frac{\partial w}{\partial x} \tag{28}$$

$$\bar{v}(x,y,z,t) = v(x,y,t) - z\frac{\partial w}{\partial y} \tag{29}$$

$$\bar{w}(x,y,z,t) = w(x,y,t) \tag{30}$$

where u, v and w are the displacement functions of the middle surface of the graphene. The strain components are then given below:

$$\varepsilon_x = \frac{\partial u}{\partial x} - z\frac{\partial^2 w}{\partial x^2} \tag{31a}$$

$$\varepsilon_y = \frac{\partial v}{\partial x} - z\frac{\partial^2 w}{\partial y^2} \tag{31b}$$

$$\varepsilon_z = 0 \tag{31c}$$

$$\gamma_{xy} = \frac{1}{2}\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) - z\frac{\partial^2 w}{\partial x \partial y} \tag{32a}$$

$$\gamma_{xz} = 0\,,\ \gamma_{yz} = 0 \tag{32b, 32c}$$

by using the Eq. (27), the non-local constitutive equations can be written as

$$\left[1-(e_0 a)^2\nabla^2\right]\sigma^{nl}{}_{kl} = \sigma_{kl} \tag{33}$$

$$\left[1-(e_0 a)^2\nabla^2\right]N^{nl}_{kl} = N_{kl} \tag{34}$$

$$\left[1-(e_0 a)^2\nabla^2\right]M^{nl}_{kl} = M_{kl} \tag{35}$$

Similarly, the stress-strain relations write

$$\sigma_{xx} - (e_0 a)^2\left[\frac{\partial^2\sigma_{xx}}{\partial x^2} + \frac{\partial^2\sigma_{xx}}{\partial y^2}\right] = \frac{E}{1-\nu^2}\varepsilon_{xx} + \nu\frac{E}{1-\nu^2}\varepsilon_{yy} \tag{36a}$$

$$\sigma_{yy} - (e_0 a)^2\left[\frac{\partial^2\sigma_{yy}}{\partial x^2} + \frac{\partial^2\sigma_{yy}}{\partial y^2}\right] = \nu\frac{E}{1-\nu^2}\varepsilon_{xx} + \frac{E}{1-\nu^2}\varepsilon_{yy} \tag{36b}$$

$$\tau_{xy} - (e_0 a)^2\left[\frac{\partial^2\tau_{xy}}{\partial x^2} + \frac{\partial^2\tau_{xy}}{\partial y^2}\right] = 2G\gamma_{xy} \tag{36c}$$

Stress resultants of the micro graphene can be given as

$$N_x = \int_{-h/2}^{h/2}\sigma_x dz\ ,\ N_y = \int_{-h/2}^{h/2}\sigma_y dz\,,\ N_{xy} = \int_{-h/2}^{h/2}\tau_{xy} dz \tag{37a}$$

$$M_x = \int_{-h/2}^{h/2} \sigma_x z dz\,,\; M_y = \int_{-h/2}^{h/2} \sigma_y z dz\,,\; M_{xy} = \int_{-h/2}^{h/2} \tau_{xy} z dz \qquad (37b)$$

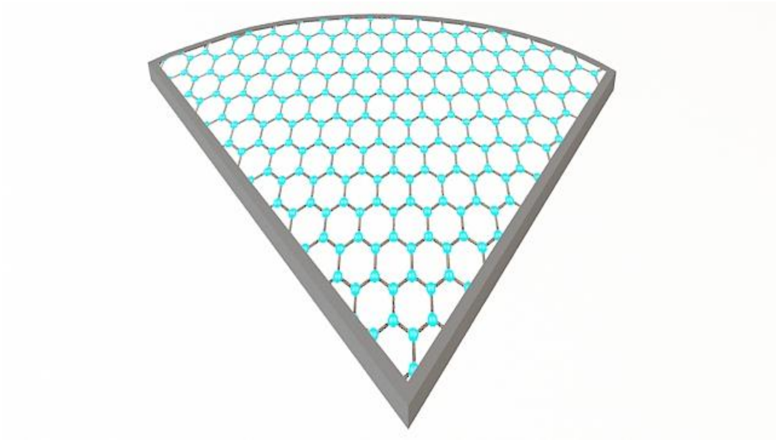


Fig. 1 Sector shaped graphene

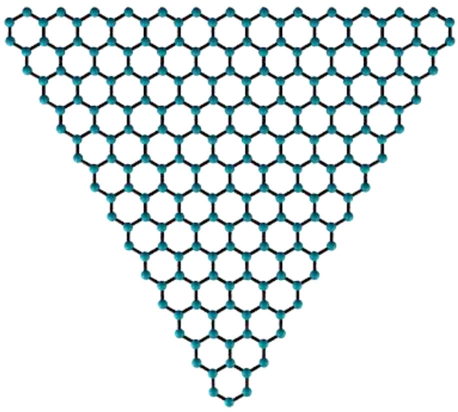


Fig. 2 Triangle shaped graphene

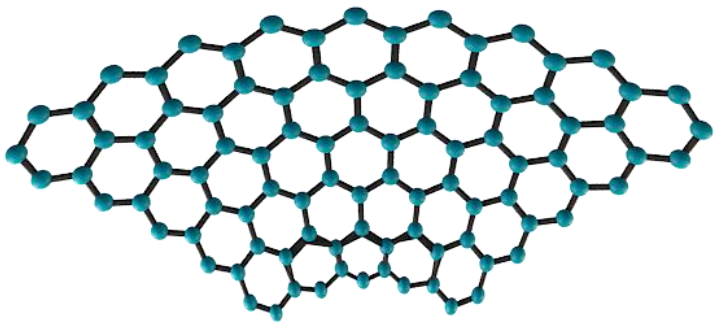


Fig. 3 Annular sector shaped graphene

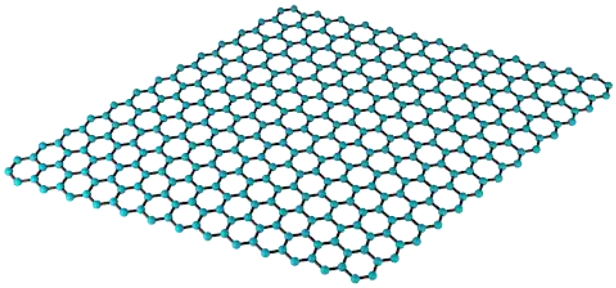
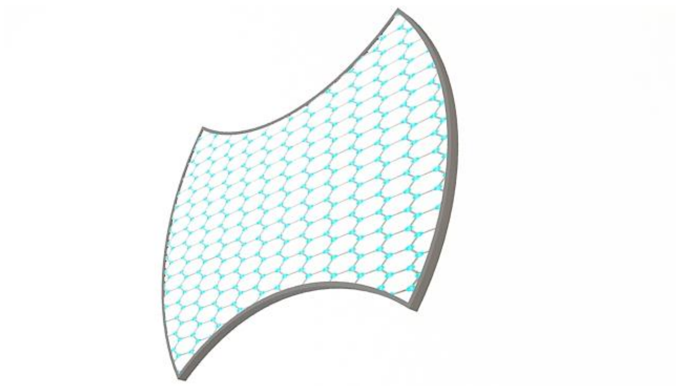


Fig. 4 Square shaped graphene



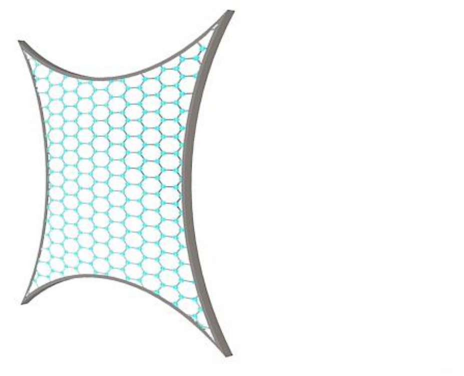


Fig. 5 Graphene with curvilinear coordinates

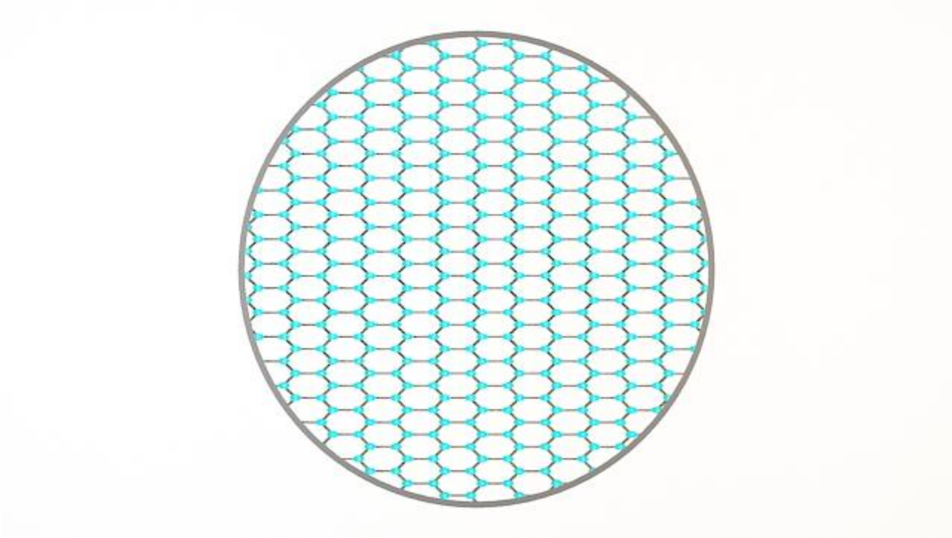


Fig. 6 Circular shaped graphene

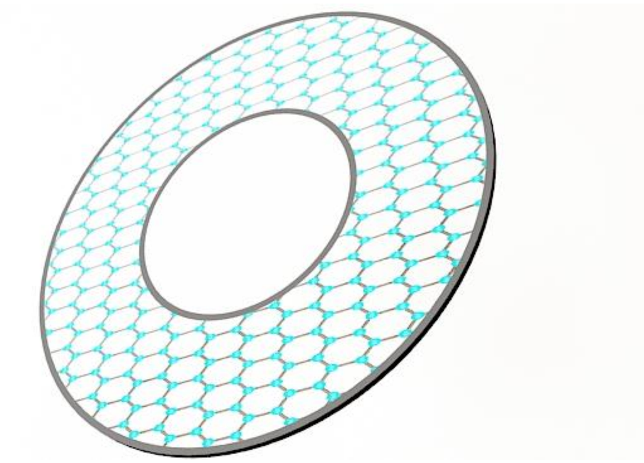


Fig. 7 Annular shaped graphene

The equations of motion for isotropic plate via Hamilton's principle are given

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=\rho h\frac{\partial^2 u}{\partial t^2} \tag{38}$$

$$\frac{\partial N_y}{\partial y}+\frac{\partial N_{xy}}{\partial x}=\rho h\frac{\partial^2 v}{\partial t^2} \tag{39}$$

$$\frac{\partial^2 M_x}{\partial x^2}+2\frac{\partial^2 M_{xy}}{\partial x\partial y}+\frac{\partial^2 M_y}{\partial y^2}+q(x)=\rho h\frac{\partial^2 w}{\partial t^2}+kw-k_g w\nabla^2(w) \tag{40}$$

By using the Eqs. (33-35) and Eqs. (36-40), the nonlocal forces can be obtained as

$$N_x - (e_0 a)^2 \left[\frac{\partial^2 N_x}{\partial x^2} + \frac{\partial^2 N_x}{\partial y^2} \right] = \frac{Eh}{(1-\nu^2)} \frac{\partial u}{\partial x} + \nu \frac{Eh}{(1-\nu^2)} \frac{\partial v}{\partial y} \tag{41}$$

$$N_y - (e_0 a)^2 \left[\frac{\partial^2 N_y}{\partial x^2} + \frac{\partial^2 N_y}{\partial y^2} \right] = \nu \frac{Eh}{(1-\nu^2)} \frac{\partial u}{\partial x} + \frac{Eh}{(1-\nu^2)} \frac{\partial v}{\partial y} \tag{42}$$

$$N_{xy} - (e_0 a)^2 \left[\frac{\partial^2 N_{xy}}{\partial x^2} + \frac{\partial^2 N_{xy}}{\partial y^2} \right] = Gh \left(\frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \right) \tag{43}$$

Similarly, moment resultants and shear forces are obtained as

$$M_x - (e_0 a)^2 \left[\frac{\partial^2 M_x}{\partial x^2} + \frac{\partial^2 M_x}{\partial y^2} \right] = -\frac{Eh^3}{12(1-\nu^2)} \left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \right) \tag{44}$$

$$M_y - (e_0 a)^2 \left[\frac{\partial^2 M_y}{\partial x^2} + \frac{\partial^2 M_y}{\partial y^2} \right] = -\frac{Eh^3}{12(1-\nu^2)} \left(\frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial x^2} \right) \tag{45}$$

$$M_{xy} - (e_0 a)^2 \left[\frac{\partial^2 M_{xy}}{\partial x^2} + \frac{\partial^2 M_{xy}}{\partial y^2} \right] = -\frac{Eh^3}{12(1+\nu)} \left(\frac{\partial^2 w}{\partial x \partial y} \right) \tag{46}$$

By using the Eqs. (48-50) and Eq. (44), governing equation for free vibration of a graphene sheet on elastic matrix via nonlocal thin plate theory can be given by

$$\begin{aligned} & D\left[\frac{\partial^4 w(x,y,t)}{\partial x^4} + 2\frac{\partial^4 w(x,y,t)}{\partial x^2 \partial y^2} + \frac{\partial^4 w(x,y,t)}{\partial y^4} \right] + \rho h \frac{\partial^2 w(x,y,t)}{\partial t^2} \\ & - (e_0 a)^2 \rho h \left(\frac{\partial^4 w(x,y,t)}{\partial x^2 \partial t^2} + \frac{\partial^4 w(x,y,t)}{\partial y^2 \partial t^2} \right) \\ & + kw(x,y,t) - (e_0 a)^2 k \left(\frac{\partial^2 w(x,y,t)}{\partial x^2} + \frac{\partial^2 w(x,y,t)}{\partial y^2} \right) \\ & - k_g \left(\frac{\partial^2 w(x,y,t)}{\partial x^2} + \frac{\partial^2 w(x,y,t)}{\partial y^2} \right) \\ & + (e_0 a)^2 k_g \left(\frac{\partial^4 w(x,y,t)}{\partial x^4} + 2\frac{\partial^4 w(x,y,t)}{\partial x^2 \partial y^2} + \frac{\partial^4 w(x,y,t)}{\partial y^4} \right) = 0 \end{aligned} \tag{47}$$

We assume a harmonic solution for free vibration in the form

$$w(x,y,t) = W(x,y) e^{i\omega t} \tag{48}$$

Substituting the Eq. (48) into the governing equation of motion (Eq.47) can be expressed

$$D\left[\frac{\partial^4 W(x,y)}{\partial x^4}+2\frac{\partial^4 W(x,y)}{\partial x^2\partial y^2}+\frac{\partial^4 W(x,y)}{\partial y^4}\right]-\rho h\omega^2\frac{\partial^2 W(x,y)}{\partial x^2}$$

$$+(e_0a)^2\rho h\left(\frac{\partial^2 W(x,y)}{\partial x^2}+\frac{\partial^2 W(x,y)}{\partial y^2}\right)$$

$$+kw(x,y)-(e_0a)^2k\left(\frac{\partial^2 W(x,y)}{\partial x^2}+\frac{\partial^2 W(x,y)}{\partial y^2}\right)$$

$$-k_g\left(\frac{\partial^2 W(x,y)}{\partial x^2}+\frac{\partial^2 W(x,y)}{\partial y^2}\right)$$

$$+(e_0a)^2k_g\left(\frac{\partial^4 W(x,y)}{\partial x^4}+2\frac{\partial^4 W(x,y)}{\partial x^2\partial y^2}+\frac{\partial^4 W(x,y)}{\partial y^4}\right)=0 \tag{49}$$

Eq. (49) can also be written in compact form as

$$D\nabla^2\nabla^2W(x,y)-\rho h\omega^2W''(x,y)+(e_0a)^2\rho h\omega^2\left[\nabla^2W(x,y)\right]$$

$$+kW(x,y)-(e_0a)^2k\nabla^2W(x,y)-k_g\nabla^2W(x,y)+(e_0a)^2k_g\nabla^2\left[\nabla^2W(x,y)\right]=0 \tag{50}$$

where D is the coefficient of bending rigidity for plate, h is the plate thickness, ρ is the density, x and y are the midplane Cartesian coordinates. Consider the following differential operators before discretizing the governing differential equations

$$\Re=\frac{\partial^2 W(x,y)}{\partial x^2}\ \text{and}\ S=\frac{\partial^2 W(x,y)}{\partial y^2} \tag{51,52}$$

Thus, the fourth-order derivatives can be given in terms of the second order derivatives, that is,

$$\frac{\partial^4 W(x,y)}{\partial x^4}=\frac{\partial^2}{\partial x^2}\left[\Re\right] \tag{53a}$$

$$\frac{\partial^4 W(x,y)}{\partial y^4}=\frac{\partial^2}{\partial y^2}\left[S\right] \tag{53b}$$

$$\frac{\partial^4 W(x,y)}{\partial x^2\partial y^2}=\frac{\partial^2}{\partial x^2}\left[\frac{\partial^2 W(x,y)}{\partial y^2}\right]=\frac{\partial^2}{\partial x^2}\left[S\right] \tag{53c}$$

After the transformation process, the following form can be given for the first- and second-order derivatives respectively

$$\frac{\partial W}{\partial x} = [J_{11}]^{-1} \frac{\partial W}{\partial \xi} \tag{54a}$$

$$\frac{\partial W}{\partial y} = [J_{11}]^{-1} \frac{\partial W}{\partial \eta} \tag{54b}$$

$$\frac{\partial^2 W}{\partial x^2} = [J_{22}]^{-1} \frac{\partial^2 W}{\partial \xi^2} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1} \frac{\partial W}{\partial \xi} \tag{54c}$$

$$\frac{\partial^2 W}{\partial y^2} = [J_{22}]^{-1} \frac{\partial^2 W}{\partial \eta^2} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1} \frac{\partial W}{\partial \eta} \tag{54d}$$

Similarly, the fourth-order derivatives

$$\frac{\partial^4 W}{\partial x^4} = \frac{\partial^2 \Re}{\partial x^2} = [J_{22}]^{-1} \frac{\partial^2 \Re}{\partial \xi^2} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1} \frac{\partial \Re}{\partial \xi} \tag{55a}$$

$$\frac{\partial^4 W}{\partial y^4} = \frac{\partial^2 S}{\partial y^2} = [J_{22}]^{-1} \frac{\partial^2 S}{\partial \eta^2} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1} \frac{\partial S}{\partial \eta} \tag{55b}$$

$$\frac{\partial^4 W}{\partial x^2 \partial y^2} = \frac{\partial^2 S}{\partial x^2} = [J_{22}]^{-1} \frac{\partial^2 S}{\partial \xi^2} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1} \frac{\partial S}{\partial \xi} \tag{55c}$$

Using the differential operators in Eqs. (53-55), the governing equation, i.e., Eq. (50), takes the following form

$$\begin{aligned} &\frac{\partial^2 \Re}{\partial x^2} + 2\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} - \rho h \omega^2 \Re + (e_0 a)^2 \omega^2 \rho h [\Re + S] \\ &+ kW(x,y) - (e_0 a)^2 k[\Re + S] - k_g[\Re + S] + (e_0 a)^2 k_g \left[\frac{\partial^2 \Re}{\partial x^2} + 2\frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} \right] = 0 \end{aligned} \tag{56}$$

Employing the transformation rule, the governing equation (49) becomes,

$$\begin{aligned} &[J_{22}]^{-1} \frac{\partial^2 \Re}{\partial \xi^2} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1} \frac{\partial \Re}{\partial \xi} + 2\left([J_{22}]^{-1} \frac{\partial^2 S}{\partial \xi^2} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1} \frac{\partial S}{\partial \xi} \right) \\ &+ \left([J_{22}]^{-1} \frac{\partial^2 S}{\partial \eta^2} - [J_{22}]^{-1}[J_{21}][J_{11}]^{-1} \frac{\partial S}{\partial \eta} \right) \end{aligned}$$

$$
\begin{aligned}
& -\rho h\omega^2\left([J_{22}]^{-1}\frac{\partial^2 W}{\partial\xi^2}-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\frac{\partial W}{\partial\xi}\right) \\
& +(e_0a)^2\rho h\omega^2\left\{\left([J_{22}]^{-1}\frac{\partial^2 W}{\partial\xi^2}-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\frac{\partial W}{\partial\xi}\right)\right. \\
& \left.+\left([J_{22}]^{-1}\frac{\partial^2 W}{\partial\eta^2}-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\frac{\partial W}{\partial\eta}\right)\right\} \\
& +kW-(e_0a)^2k\left\{[J_{22}]^{-1}\frac{\partial^2 W}{\partial\xi^2}-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\frac{\partial W}{\partial\xi}\right. \\
& \left.+[J_{22}]^{-1}\frac{\partial^2 W}{\partial\eta^2}-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\frac{\partial W}{\partial\eta}\right\} \\
& -k_g\left\{[J_{22}]^{-1}\frac{\partial^2 W}{\partial\xi^2}-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\frac{\partial W}{\partial\xi}\right. \\
& \left.+[J_{22}]^{-1}\frac{\partial^2 W}{\partial\eta^2}-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\frac{\partial W}{\partial\eta}\right\} \\
& +k_g(e_0a)^2\left\{[J_{22}]^{-1}\frac{\partial^2 \Re}{\partial\xi^2}-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\frac{\partial \Re}{\partial\xi}\right. \\
& +2\left([J_{22}]^{-1}\frac{\partial^2 S}{\partial\xi^2}-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\frac{\partial S}{\partial\xi}\right) \\
& \left.+\left([J_{22}]^{-1}\frac{\partial^2 S}{\partial\eta^2}-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\frac{\partial S}{\partial\eta}\right)\right\}=0 \qquad (57)
\end{aligned}
$$

DSC rules from Eq. (6) in Eq. (65), one obtains the DSC analog of the governing equations as

$$
\begin{aligned}
& [J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta\xi)\,\Re_{kj}\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta\xi)\,\Re_{kj}\right] \\
& +2\left[[J_{22}]^{-1}\left(\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta\xi)\,S_{kj}\right)-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left(\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta\xi)\,S_{kj}\right)\right] \\
& +\left[[J_{22}]^{-1}\left(\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta\eta)\,S_{ik}\right)-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left(\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta\eta)\,S_{ik}\right)\right] \\
& -\rho h\omega^2\left\{[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta\xi)W_{kj}\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta\xi)W_{kj}\right]\right\} \\
& +(e_0a)^2\rho h\omega^2\left\{[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta\xi)W_{kj}\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta\xi)W_{kj}\right]\right. \\
& \left.+[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta\eta)W_{ik}\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta\eta)W_{ik}\right]\right\} \\
& +kW_{ij}-k(e_0a)^2\rho h\omega^2\left\{[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta^{(2)}_{\Delta,\sigma}(k\Delta\xi)W_{kj}\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta^{(1)}_{\Delta,\sigma}(k\Delta\xi)W_{kj}\right]\right.
\end{aligned}
$$

$$
\begin{aligned}
&+[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta\eta)W_{ik}\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta\eta)W_{ik}\right]\Bigg\}\\
&+k_g(e_0a)^2[J_{22}]^{-1}\Bigg\{\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta\xi)\Re_{kj}\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta\xi)\Re_{kj}\right]\\
&+2\left[[J_{22}]^{-1}\left(\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta\xi)S_{kj}\right)-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left(\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta\xi)S_{kj}\right)\right]\\
&+\left[[J_{22}]^{-1}\left(\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta\eta)S_{ik}\right)-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left(\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta\eta)S_{ik}\right)\right]=0\Bigg\}
\end{aligned}
\tag{58}
$$

For convenience and simplicity, the following new variables are introduced [49]

$$\Im(W_{\xi\eta})=(k\Delta\xi)\Re_{kj}^2+2(k\Delta\xi)S_{kj}^2+(k\Delta\eta)S_{kj}^2 \tag{59}$$

$$\Xi(W_{\xi\eta})=(k\Delta\xi)\Re_{kj}+2(k\Delta\xi)S_{kj}+(k\Delta\eta)S_{ik} \tag{60}$$

Such that the governing equations of annular sector graphene on elastic matrix for free vibration can be expressed

$$
\begin{aligned}
&[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}\Im(W_{\xi\eta})\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}\Xi(W_{\xi\eta})\right]\\
&-\rho h\omega^2\Bigg\{[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta\xi)W_{kj}\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta\xi)W_{kj}\right]\Bigg\}\\
&+(e_0a)^2\rho h\omega^2\Bigg\{[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta\xi)W_{kj}\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta\xi)W_{kj}\right]\\
&+[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta\eta)W_{ik}\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta\eta)W_{ik}\right]\Bigg\}\\
&+kW_{ij}-k(e_0a)^2\rho h\omega^2\Bigg\{[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta\xi)W_{kj}\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta\xi)W_{kj}\right]\\
&+[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta\eta)W_{ik}\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta\eta)W_{ik}\right]\Bigg\}\\
&+k_g(e_0a)^2\Bigg\{[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}\Im(W_{\xi\eta})\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}\Xi(W_{\xi\eta})\right]=0\Bigg\}
\end{aligned}
\tag{61}
$$

To obtain the discretized form of Eq. (61) in its natural coordinate, we use the following form

$$\nabla^4(W_{\xi\eta})=\nabla^2\nabla^2(W_{\xi\eta}) \tag{62}$$

Substituting the first and last line of Eq. (61) into Eq. (62) the governing equation can now be given by

$$
\begin{aligned}
&[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}\Im(W_{\xi\eta})\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}\Xi(W_{\xi\eta})\right]\\
&\times[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}\Im(W_{\xi\eta})\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}\Xi(W_{\xi\eta})\right]\\
&-\rho h\omega^{2}\left\{[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta\xi)W_{kj}\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta\xi)W_{kj}\right]\right\}\\
&+(e_0a)^{2}\rho h\omega^{2}\left\{[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta\xi)W_{kj}\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta\xi)W_{kj}\right]\right.\\
&\left.+[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta\eta)W_{ik}\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta\eta)W_{ik}\right]\right\}\\
&+kW_{ij}-k(e_0a)^{2}\rho h\omega^{2}\left\{[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta\xi)W_{kj}\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta\xi)W_{kj}\right]\right.\\
&\left.+[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}(k\Delta\eta)W_{ik}\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}(k\Delta\eta)W_{ik}\right]\right\}\\
&+k_g(e_0a)^{2}\left\{[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}\Im(W_{\xi\eta})\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}\Xi(W_{\xi\eta})\right]\right\}\\
&\times\left\{[J_{22}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(2)}\Im(W_{\xi\eta})\right]-[J_{22}]^{-1}[J_{21}][J_{11}]^{-1}\left[\sum_{k=-M}^{M}\delta_{\Delta,\sigma}^{(1)}\Xi(W_{\xi\eta})\right]\right\}=0 \qquad (63)
\end{aligned}
$$

7. Concluding remarks

The formulation for free vibration of micro or nano-scaled non-rectangular graphene has been presented by the nonlocal elasticity theory. The method of discrete singular convolution is used for numerical simulation and geometric transformation. The elastic matrix under the graphene is modeled via Winkler-Pasternak two-parameter elastic foundations. A general formulation has obtained for graphene including the curvilinear coordinates. So, the resulting equation is capable to obtain frequency for rectangular, square, circular, annular and sector shaped graphene under size effect.

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