

## NONLOCAL DEFLECTION OF MICROTUBULES UNDER POINT LOAD

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### Abstract

*Bending of microtubules due to a point load has been investigated by using Euler Bernoulli beam theory. The governing equations are derived based on Hamilton's principle. The size effect is taken into consideration using the Eringen's nonlocal elasticity theory. Some parametric results have been presented for nonlocal beam.*

**Keywords:** Nonlocal beam, microtubules, Dirac-Delta function.

### 1. Introduction

MTs are proteins which are organized in a network that is interconnected with microfilaments and intermediate filaments to form the cytoskeleton structures [8]. The main investigation areas are determining material, physical, chemical properties [7]-[16]-[17].

In nano and micro-sized structures which the size effect is important, for instance carbon nanotubes, microtubules, Nano electromechanical systems and micro electromechanical systems, and mathematical modeling of cancer cells etc., the nonlocal elasticity theory (Eringen) is commonly used. Applying first the nonlocal elasticity theories to nanotechnology is by Peddieson et al [2]. Then, many researchers have been interested in the static and dynamic analysis of the CNTs [9]-[10-14]-[18-25]. The mechanical response of microtubules is first investigated by Gao and Lei [15]. Currently is still done a lot of work on this subject [4-6], [26-38].

### 2. Nonlocal Elasticity

According to the nonlocal elasticity theory of Eringen [1], the stress at any reference point is effecting the whole body which not depends only on the strains at this point but also on strains at all points of the body. This definition of the Eringen's nonlocal elasticity is based on the atomic theory of lattice dynamics, and some experimental observations on phonon dispersion. The simplified version of the Eringen nonlocal elasticity theory is as followed,

$$[1 - (e_0 a)^2 \nabla^2] \sigma_{kl} = \tau_{kl} \quad (1)$$

where  $e_0$  is a material constant, and  $a$  is the internal characteristic lengths, respectively. The specific form of the Eq. (1) for beams [11]-[12]

$$\left[ 1 - (e_0 a)^2 \frac{\partial^2}{\partial x^2} \right] \sigma_{xx} = E \varepsilon_{xx} \quad (2)$$

### 3. Bending analysis under point load

A typical structure of protein microtubules and continuum model subjected to point load are shown in Figs.(1) and (2). In order to calculate the deflection of microtubules, Nonlocal Euler-Bernoulli Beam theory will be used. For modeling,  $L$  is the length of microtubules,  $R_{out}$  and  $R_{in}$  outer and inner radius,  $t$  thickness,  $E$  Young's modulus,  $e_0 a$  nonlocal parameter.

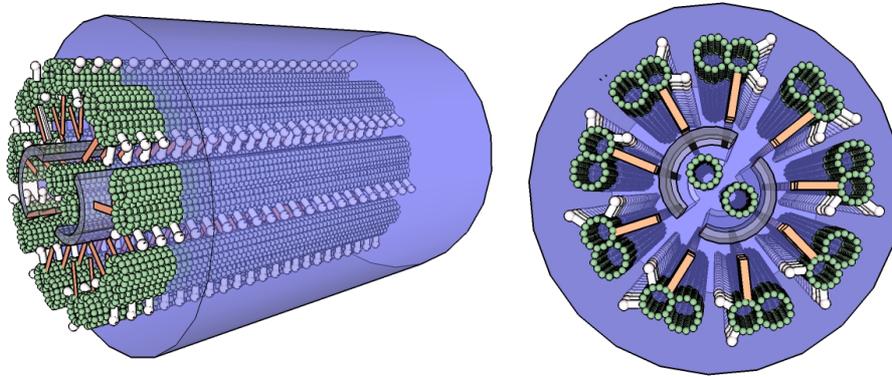


Fig. 1. A typical protein microtubules

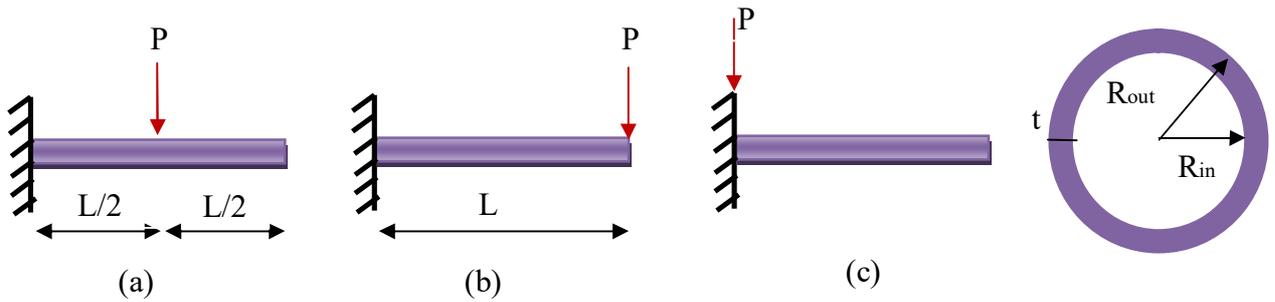


Fig. 2. Continuum model of microtubules with variable loadings

In case of the effect of the point bending moment or point load is applied on structure, Dirac- $\delta$  function can be used. Dirac- $\delta$  function is generally used for mathematical modeling of the loads which are applied for very short time. Heaviside Step Function is used to express the effects which start and continue at any time. There are a derivative relationship between Dirac- $\delta$  function and Heaviside Step Function. [13-14]

$$\delta(x - x_0) = \frac{dH(x - x_0)}{dx} \quad (3)$$

The definition of Dirac- $\delta$  function and Heaviside Step Function for specific value range is as follows [13-14]:

$$\delta(x-x_0) = \begin{cases} \infty, & x = x_0 \\ 0, & x \neq x_0 \end{cases} \quad H(x-x_0) = \begin{cases} 1, & x \geq x_0 \\ 0, & x < x_0 \end{cases} \quad (4)$$

The general equation of nonlocal Euler-Bernoulli Beam under uniform distributed and point load is as followed:

$$-EI \frac{d^4 w}{dx^4} - \mu \left[ \frac{d^2 q}{dx^2} + \sum_{i=1}^m P_i \delta''(x-a_i) \right] = -q - \sum_{i=1}^m P_i \delta(x-a_i) \quad (5)$$

Where  $P_i$  is point load at  $i$ ,  $a_i$  is the location of the loads at  $i$ ,  $q$  is the uniform distributed loads on the structure. The deflection of the beam which is under  $P$  point load on any ' $l$ ' location ( $q=0$ );

$$EIw = -\mu P(x-l)H(x-l) + c_1 \frac{x^3}{6} + c_2 \frac{x^2}{2} + c_3 x + c_4 + P \frac{(x-l)^3}{6} H(x-l) \quad (6)$$

### 3.1. Case study for cantilever beam

The boundary conditions for this case are,

$$\text{at } x=0: w = \frac{dw}{dx} = 0 \quad \text{at } x=L: V = M = 0. \quad (7)$$

Using boundary conditions, the deflection is given by, [13-14]

$$w(x) = \frac{1}{EI} \left( -\mu P(x-l)H(x-l) + P \frac{(x-l)^3}{6} H(x-l) - \frac{Px^3}{6} + \frac{Plx^2}{2} \right) \quad (8)$$

The deflection occurring at  $x=l$ ,  $w(l) = \frac{Pl^3}{3EI}$

## 4. Numerical Examples

In this study, the beam deflection under point load applied to various locations is investigated. Some of the results which are showing the deflection are presented in Figure (3). This result in ratio of deflection shows the local deflection up to nonlocal deflection.  $w(x)/(PL^3/EI)$  is used for non-dimensional deflection. Nonlocal parameter is used  $e_0 a/L$ . The parameters used in this study are: the length to average radius ( $L/R_{avg}$ )=10; the elasticity modulus is  $E=0.1$  Gpa; point load is  $P=1$  Nn; the moment of inertia is  $I=\pi t R_{avg}^3$ .

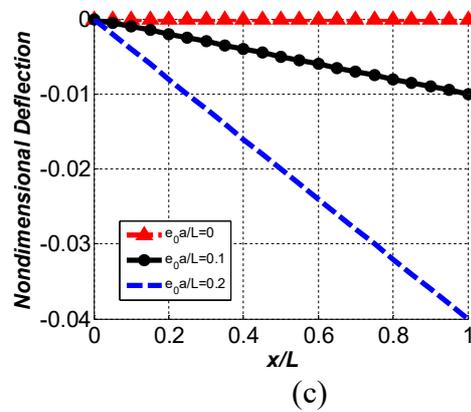
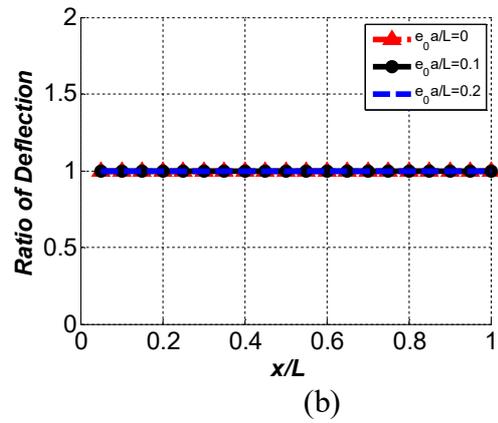
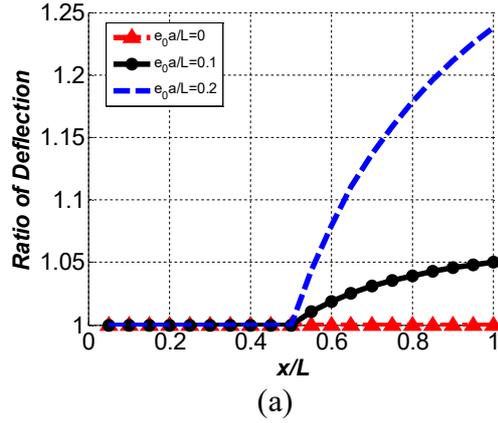


Fig.3. The effect of the small scale parameter on the non-dimensional length with respect to the ratio of deflection and non-dimensional deflection for cantilever beam. The load in (a) is at the middle of beam, in (b) at the  $x/L=1$  and in (c) on the support.

The influences of the small scale parameter on the non-dimensional length against the deflection ratio and non-dimensional deflection for cantilever microtubules subjected to a concentrated load are illustrated in Figs. 3(a, b, c), respectively. As seen in Figs. 2 (a,b,c), the location of the applied point load is considered as at the middle, at the free end and at the fixed end of the microtubules, respectively. Figs. (2a) and (2b) show that the effect of nonlocal parameters after the location of point loads. This situation has been also mentioned a

previous work by Wang and Liew [14]. An interesting behavior is observed in Fig.(3c) that an unexpected displacement occurred at the support due to the loading in this point. In case of the load is on the support, there will be no deflection on the beam according to local elasticity theory but according to nonlocal elasticity theory the beam can capable to deflection shown in Fig. (3c). This situation can be explained by the aforementioned definition of the nonlocal elasticity theory.

## 5. Concluding remarks

Bending analysis of microtubules is the investigated for cantilever case. Present equations from literature are used in a new perspective. The deflection is occurred in case the load on the support for nonlocal elasticity.

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