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IMPROVED ROUTH-PADÉ APPROXIMANTS USING VECTOR EVALUATED GENETIC ALGORITHM TO UNSTABLE SYSTEMS

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Abstract

This note describes a novel approach to Routh-Padé approximation problem relating to the construction of reduced-order approximants for continuous-time unstable systems. In this method, stability and the first r time-moments/Markov-parameters are preserved as well as the errors between a set of subsequent time-moments/Markov-parameters of the system and those of the model are minimized. For the solution of this problem a method using the concept of Pareto-optimality is proposed. Pareto-optimal curve is the solution of Multi-objective Optimization problem. Evolutionary Algorithm such as real parameter Genetic Algorithm is used to get Pareto-optimal curve. The search area for GA is very wide and it usually converges to a point near global optima.

Keywords: Model reduction, Padé approximation, Routh criterion, Pareto-optimal solutions, VEGA.

1. Introduction

There are many situations, for example, those related with missile and aircraft control systems [7,26] and chemical process control [18] where the system to be controlled is inherently unstable and of relatively high dynamic complexity. A low degree reduced model is often searched for, so that an analog or digital simulation of the system is possible. Therefore, problem of model reduction for unstable systems has received considerable attention. Several authors Hutton and Friedland [19], Shamash [46], Krishnamurthy and Sheshadri [26] have proposed methods for deriving simplified models for high-order unstable systems. The reported methods deal with simplification of unstable systems in frequency domain.

Here, an alternative approach of model reduction of a class of unstable systems is proposed. In proposed method, systems dynamics is split into stable and unstable parts. The unstable part is retained unchanged in the reduced dimensional model while the simplification is carried out only on the stable part. The simplification procedure of Hutton [19] converts the unstable system into a stable one by shift of the imaginary axis. In this procedurehe amount of shift is arbitrary and leads to nonunique lower dimensional models. A large number of time-domain and frequency-domain system simplification techniques have been developed to suit different requirement. Amongst them, a frequency domain method is Padé approximation in which 2r terms of the power series expansion (time moments) of the high-order (nth-order) transfer function $G_n(s)$ are fully retained in low-order (rth-order) model $G_r(s)$. The Padé approximation does not guarantee the stability of the reduced-order model. To overcome the problem of stability, several stable reduction methods such as Routh approximation [19,38,46], the Hurwitz polynomial approximation [1], the stability equation method [8] and the method using Michailov stability criterion [49] have been proposed. The Routh approximation [19,38,46] has the drawback of matching only the first r time moments $(t_1, t_2, ..., t_r)$ of $G_n(s)$ to the respective time moments $(\hat{t}_1, \hat{t}_2, ..., \hat{t}_r)$ of $G_r(s)$ (in recent years the extension of Routh approximation techniques [19,38,46] to interval systems has attracted the attention of many researchers [4,5,16,18,24]. Later Shamash [47] considered the effect of including some Markov parameters $(M_1, M_2, ...)$ along with time moments, which is generally essential to ensure both initial and steady state response approximation. However, the technique of [6,26,35,45] is again confined to matching of only r terms (a time moments and b Markov parameters, where $a + b = r$). Several variants of Routh approximation were subsequently reported [6,26,35,45]; however, they again remain confined to only r terms matching for the purpose of preserving stability, a task which can be achieved arbitrarily [43,44]. Note that infinite numbers of stable models can be constructed if the objective is to match only r terms [43]. Thus, the basic problem is to match or near match a few terms in excess of r terms while preserving stability [2,3]. Some attempt was made previously [41] to partially solve this problem. Singh [41] suggested a technique based on the successive variances of the model. The method [41] requires the determination of the stability region in terms of the free parameters. Other closely related problems have also received attention [1,9-15,17,20-25,27-37,44-46]. Recently, geometric programming based (computer-oriented) methods [39,40] for the solution of the Routh-Padé approximation problem are presented. In these methods [39,40], Geometric programming based computer-aided methods have been reported recently first r time moments/Markov parameters are fully retained and the sum of the weighted squares of errors between a set of subsequent time moments/Markov parameters of the system and those of the model are minimized while preserving stability. These methods [39,40] have the drawback that the question of finding some means (free of hit and trial) of deciding the values of the number of time moments/Markov parameters (say m) to be matched or near-matched and the weights to correspond to assured substantial improvement in system approximation as well as the question of establishing the existence of such values are left unresolved.

In this note, a nonlinear programming based (computer-oriented) method for the solution of Routh-Padé approximation problem is presented. The method is essentially a multi-objective optimization procedure in which not only stability is preserved and the first r terms of the power series expansion of $G_n(s)$ are fully retained but also the errors between a set of subsequent time moments/Markov parameters of the system and those of the model are minimized. This alleviates the problem of finding m and weights. The applicability of proposed method is shown by means of numerical example. The search area for GA is very wide and it usually converges to a point near global optima [16]. Though Pareto-optimality, which is a key step in the present technique, is well known to the best of author's knowledge, this is the first instance of explicitly showing its usefulness for obtaining reduced-order models for unstable systems.

This paper is organized as follows. In Sec. 2 we briefly review the results of [39,40]. The improvement is presented in Sec. 3 and numerical example is given in Sec. 4. Finally paper is concluded in Sec. 5.

2. Brief Review of Existing Results

Consider a single-input-single-output system described by the transfer function

$$G_n(s) = \frac{a_1 s^{n-1} + a_2 s^{n-2} + \ldots + a_n}{s^n + b_1 s^{n-1} + \ldots + b_n} \qquad (1)$$

$$= t_1 + t_2 s + \ldots + t_n s^{n-1} + \ldots \qquad (2)$$

(expansion around $s = 0$)

$$= M_1 s^{-1} + M_2 s^{-2} + \ldots + M_n s^{-n} + \ldots \qquad (3)$$

(expansion around $s = \infty$)

The problem is to determine its stable reduced-order (*r*th-order) approximant

$$G_r(s) = \frac{\hat{a}_1 s^{r-1} + \hat{a}_2 s^{r-2} + \ldots + \hat{a}_r}{s^r + \hat{b}_1 s^{r-1} + \ldots + \hat{b}_r} \qquad (4)$$

$$= \hat{t}_1 + \hat{t}_2 s + \ldots + \hat{t}_r s^{r-1} + \ldots \qquad (5)$$

$$= \hat{M}_1 s^{-1} + \hat{M}_2 s^{-2} + \ldots + \hat{M}_r s^{-r} + \ldots \qquad (6)$$

A. Formulation of the objective function

The formulation of the multiobjective optimization problem will be explained for *r* being even. Formulation for r being odd can be done in a similar way. It is easy to verify that for r even, the following equations hold true:

$$\left.\begin{aligned} \hat{a}_{r+1-i} &= \sum_{j=1}^{i} \hat{t}_j \hat{b}_{r-i+j} \\ \hat{a}_i &= \sum_{j=1}^{i} \hat{M}_j \hat{b}_{i-j} \end{aligned}\right\} \quad i = 1, \ldots, \frac{r}{2} \qquad (7)$$

$$\left.\begin{aligned} \hat{t}_i &= \Big(-\sum_{j=1}^{i-1} \hat{t}_j \hat{b}_{r-i+j} + \sum_{j=1}^{r+1-i} \hat{M}_j \hat{b}_{r+1-i-j}\Big)\hat{b}_r^{-1} \\ \hat{M}_i &= \sum_{j=1}^{r+1-i} \hat{t}_j \hat{b}_{i-1+j} - \sum_{j=1}^{i-1} \hat{M}_j \hat{b}_{i-j} \end{aligned}\right\} \quad i = \frac{r}{2}+1, \frac{r}{2}+2, \ldots, r \qquad (8)$$

$(\hat{b}_0 = 1;\ \hat{b}_i = 0 \text{ for } i \notin \{0, \ldots, r\};\ \hat{t}_i, \hat{M}_i = 0 \text{ for } i < 1)$.

We seek a stable model for which *r* equations given by

$$\left.\begin{aligned} \hat{t}_i - t_i &= 0 \\ \hat{M}_i - M_i &= 0 \end{aligned}\right\} \quad i = 1,\ldots,\frac{r}{2} \tag{9}$$

are satisfied ,which implies, from (7),

$$\left.\begin{aligned} \hat{a}_{r+1-i} &= \sum_{j=1}^{i} t_j \hat{b}_{r-i+j} \\ \hat{a}_i &= \sum_{j=1}^{i} M_j \hat{b}_{i-j} \end{aligned}\right\} \quad i = 1,\ldots,\frac{r}{2}. \tag{10}$$

There exist an infinite number of stable models for which (10) is satisfied [18]. This arbitrariness in stability preservation is exploited in [33] by minimizing the sum of the weighted squares of errors.

To find the improved model, VEGA [16] is used to generate Pareto-optimal solutions by minimizing objective functions $Z^{t}_{\frac{r}{2}+i}, Z^{M}_{\frac{r}{2}+i}$ given by

$$\left.\begin{aligned} z^{t}_{\frac{r}{2}+i} &= \Bigl(1 - \frac{\hat{t}_{\frac{r}{2}+i}}{t_{\frac{r}{2}+i}}\Bigr)^2 \\ z^{M}_{\frac{r}{2}+i} &= \Bigl(1 - \frac{\hat{M}_{\frac{r}{2}+i}}{M_{\frac{r}{2}+i}}\Bigr)^2 \end{aligned}\right\} \quad i = 1,\ldots,\frac{r}{2} \tag{11}$$

Using (8) subject to (9), (11) can be expressed as

$$\left.\begin{aligned} z^{t}_{\frac{r}{2}+i} &= \Bigl(1 - \frac{\hat{t}_{\frac{r}{2}+i}}{t_{\frac{r}{2}+i}}\Bigr)^2 = f(\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_r) \\ z^{M}_{\frac{r}{2}+i} &= \Bigl(1 - \frac{\hat{M}_{\frac{r}{2}+i}}{M_{\frac{r}{2}+i}}\Bigr)^2 = f(\hat{b}_1, \hat{b}_2, \ldots, \hat{b}_r) \end{aligned}\right\} \quad i = 1,\ldots,\frac{r}{2} \tag{12}$$

B. Formulation of the stability constraints

Now following [37], the denominator polynomial of (4) can be expressed as

$$\begin{aligned} &s^r + \hat{a}_1 s^{r-1} + (\hat{a}_2 + \hat{a}_3 + \ldots + \hat{a}_r) s^{r-2} + \hat{a}_1(\hat{a}_3 + \hat{a}_4 + \ldots + \hat{a}_r) s^{r-3} \\ &\quad + [\,\hat{a}_2(\hat{a}_4 + \hat{a}_5 + \ldots + \hat{a}_r) + \hat{a}_3(\hat{a}_5 + \hat{a}_6 + \ldots + \hat{a}_r) + \hat{a}_4(\hat{a}_6 + \hat{a}_7 + \ldots + \hat{a}_r) + \ldots \end{aligned}$$

$$+\hat{a}_{r-2}\hat{a}_r\,]\,s^{r-4}+\ldots+\hat{a}_{1+q}\hat{a}_{3+q}\ldots\hat{a}_{r-2}\hat{a}_r \qquad (13)$$

which is constructed by taking the coefficients of the first two rows of the Routh array with the elements of its first column given [37] by

$$1, \hat{a}_1, \hat{a}_2, \hat{a}_1\hat{a}_3, \hat{a}_2\hat{a}_4, \hat{a}_1\hat{a}_3\hat{a}_5, \ldots, \hat{a}_{1+q}\hat{a}_{3+q}\ldots\hat{a}_{r-2}\hat{a}_r \qquad (14)$$

where q =1 for r even and q =0 for r odd. By setting

$$\begin{aligned} &\hat{a}_1\hat{b}_1^{-1} = 1\,, \quad (\hat{a}_2 + \hat{a}_3+\ldots+\hat{a}_r)\hat{b}_2^{-1} = 1, \\ &\hat{a}_1(\hat{a}_3 + \hat{a}_4+\ldots+\hat{a}_r)\hat{b}_3^{-1} = 1, \ldots, (\hat{a}_{1+q}\hat{a}_{3+q}\ldots\hat{a}_{r-2}\hat{a}_r)\hat{b}_r^{-1} = 1 \end{aligned} \qquad (15)$$

(15) is matched with the denominator polynomial of the model in (4), namely, with

$$\hat{b}_r + \hat{b}_{r-1}s+\ldots+\hat{b}_1 s^{r-1} + s^r \qquad (16)$$

and the necessary and the sufficient condition that all the roots of (16) be strictly in the left half plane is [37]

$$\hat{a}_1 > 0, \hat{a}_2 > 0, \ldots, \hat{a}_r > 0 \qquad (17a)$$

which, of course, implies

$$\hat{b}_1 > 0, \hat{b}_2 > 0, \ldots, \hat{b}_r > 0\,. \qquad (17b)$$

Note that, for a given $r, \hat{b}_i$, $i = 1, \ldots, r$, can easily be expressed in terms of $\hat{a}_i$, $i = 1, \ldots, r$, by constructing an inverse Routh array (i.e., with the element of its first column given by (14)) in a manner analogous to [37].Thus, pertaining to r=4, (15) becomes

$$\hat{b}_1 = \hat{a}_1,\ \hat{b}_2 = \hat{a}_3 + \hat{a}_3 + \hat{a}_4,\ \hat{b}_3 = \hat{a}_1(\hat{a}_3 + \hat{a}_4)\,,\ \hat{b}_4 = \hat{a}_2\hat{a}_4\,. \qquad (18)$$

3. Application of VEGA

Now, the problem is to minimize (12), satisfying (17a). The vector evaluated genetic algorithm (VEGA) [16] is proposed herein for solving the above stated problem. VEGA is the simplest possible multi-objective GA [16] and is straightforward extension of a single-objective extension of multi-objective optimization. Since a number of objectives (say Q) have to be handled, GA population is divided at every generation into Q equal subpopulations randomly. Each subpopulation is assigned a fitness value based on different objective function.

After each solution is assigned a fitness value, the selection operator restricted among solutions of each subpopulation, is applied until the complete subpopulation is filled [16]. The following VEGA procedure is used [16].

***Step* 1:** Set, for population size N, an objective function counter *i* = 1 and define $x = N/Q$

***Step* 2:** For all solution, $j = 1 + (i-1) * x$ to $j = i * x$, assign fitness as: $Z(\hat{\mathbf{b}}^{(j)}) = z_i(\hat{\mathbf{b}}^{(j)})$.

***Step* 3:** Perform proportionate selection on all x solutions to create a mating pool P_i.

***Step* 4:** If $i = Q$, go to *Step* 5. Otherwise, increment *i* by one and go to *Step* 2.

***Step* 5:** Combine all mating pools together: $P = \bigcup_{i=1}^{Q} P_i$. Perform crossover and mutation on *P* to create a new population [16].

In this VEGA, linear crossover operator is used. It creates three solutions, $0.5(\hat{b}_i^{(1,t)} + \hat{b}_i^{(2,t)})$, $(1.5\hat{b}_i^{(1,t)} - 0.5\hat{b}_i^{(2,t)})$, $(-0.5\hat{b}_i^{(1,t)} + 1.5\hat{b}_i^{(2,t)})$ from two parent solutions $\hat{b}_i^{(1,t)}$ and $\hat{b}_i^{(2,t)}$ at generation *t*, with the best two solutions being chosen as offspring. For performing mutation, random mutation is used. Instead of creating a solution from the entire search space, a solution in the vicinity of parent solution with a uniform probability distribution is chosen: $y_i^{(1,t+1)} = \hat{b}_i^{(1,t)} + (r_i - 0.5)\Delta_i$ where r_i is a random number in [0,1].

Here, an alternative approach of model reduction of a class of unstable systems is proposed. In proposed method, systems dynamics is split into stable and unstable parts. The unstable part is retained unchanged in the reduced dimensional model while the simplification is carried out only on the stable part .

Consider an unstable higher order system given by:

$$G(s) = \frac{x(s)}{y(s)z(s)}$$

$$= \frac{1}{y(s)}\left[D(s) + \frac{a_1 s^{n-1} + a_2 s^{n-2} + \cdots + a_n}{s^n + b_1 s^{n-1} + \cdots + b_n}\right] \qquad (19)$$

From (19), one finds:

$$G(s) = \frac{1}{y(s)}\left[D(s) + G_n(s)\right]$$

$$G(s) = \frac{1}{y(s)}\left[D(s) + (t_1 s + t_2 s + t_3 s^3 + \cdots)\right] \qquad (20)$$

(expansion of $G_n(s)$ around $s = 0$)

$$= \frac{1}{y(s)}\left[D(s) + (M_1 s^{-1} + M_2 s^{-2} + \cdots)\right] \qquad (21)$$

(expansion of $G_n(s)$ around $s = \infty$)

where $y(s)$ and $z(s)$ are unstable and stable poles, $D(s)$ is the quotient which obtained while making $G_n(s)$ to be strictly proper, $G_n(s)$ is stable transfer function of the system when unstable poles are segregated.
Assume that a reduced order model of the form

$$\hat{G}(s) = \frac{\hat{x}(s)}{y(s)\hat{z}(s)}$$

$$= \frac{1}{y(s)}\left[D(s) + \frac{\hat{a}_1 s^{r-1} + \hat{a}_2 s^{r-2} + \cdots + \hat{a}_r}{s^r + \hat{b}_1 s^{r-1} + \cdots + \hat{b}_r}\right] \tag{22}$$

is to be constructed.
From (22), one finds:

$$\hat{G}(s) = \frac{1}{y(s)}\left[D(s) + \hat{G}_r(s)\right] \tag{23}$$

$$G(s) = \frac{1}{y(s)}\left[D(s) + (\hat{t}_1 s + \hat{t}_2 s + \hat{t}_3 s^3 + \cdots)\right] \tag{24}$$

$$= \frac{1}{y(s)}\left[D(s) + (\hat{M}_1 s^{-1} + \hat{M}_2 s^{-2} + \cdots)\right] \tag{25}$$

where $r < n$, where $\hat{G}_r(s)$ is stable transfer function of the reduced-order mode $\hat{G}\ (s)$ when stsble poles are segregated.

The performance of the algorithm is verified by application to the following numerical example.

4. Example

Consider a system reported by Rao et. al. [38]

$$G(s) = \frac{2s^4 + 2s^3 + s^2 + 3s + 6}{s^2(s^3 + 7s^2 + 14s + 7)} \tag{26}$$

$$= \frac{1}{s^2}\left[(2s - 12) + \frac{57s^2 + 155s + 102}{(s^3 + 7s^2 + 14s + 8)}\right] \tag{27}$$

From (27) one has,

$$= \frac{1}{s^2}\left[(2s - 12) + G_3(s)\right] \tag{28}$$

Consider a stable third-order system

$$G_3(s) = \frac{57s^2 + 155s + 102}{(s^3 + 7s^2 + 14s + 8)} \tag{29}$$

$$(t_1 = 12.75, \quad t_2 = -2.9375, \quad t_3 = 1.1094$$

$$M_1 = 57, \quad M_2 = -244 \quad M_3 = 1012 \quad)$$

.

From (19) and (28) together with (31):

$$\hat{G}(s) = \frac{1}{s^2}\left[(2s-12) + \frac{\hat{a}_1 s^2 + \hat{a}_2}{s^2 + \hat{b}_1 s + \hat{b}_2}\right] \tag{30}$$

$$\hat{G}(s) = \frac{1}{s^2}\left[(2s-12) + G_2(s)\right] \tag{31}$$

Suppose a second-order approximant (r=2) is required. The approximant can systematically be arrived at by following the steps given below.

Step **1** From the requirement of the first ***r*** terms matching with p=1 (see (9)), one has

$$\left.\begin{array}{l} \hat{a}_2 = t_1 \hat{b}_2 \\ \hat{a}_1 = M_1 \end{array}\right\} \Rightarrow \left\{\begin{array}{l} \hat{a}_2 = 12.75\hat{b}_2 \\ \hat{a}_1 = 57. \end{array}\right. \tag{32}$$

Step **2** From (8) together with (9), one obtains

$$\left.\begin{array}{lcl} \hat{t}_2 & = & (-\hat{b}_1 + 57)\hat{b}_2^{-1} \\ \hat{M}_2 & = & \hat{b}_2 - 57\hat{b}_1 \end{array}\right\} \tag{33}$$

while (15) takes the following form

$$\left.\begin{array}{lcl} \hat{b}_1 & = & \hat{d}_1 \\ \hat{b}_2 & = & \hat{d}_2 \end{array}\right\} \tag{34}$$

***Step* 3** Using (34), objective function (12) and (17a) take the following forms respectively (for *m*=1)

$$\left.\begin{aligned} z_2^t &= (1-\frac{\hat{t}_2}{t_2})^2 \\ z_2^M &= (1-\frac{\hat{M}_2}{M_2})^2 \end{aligned}\right\} \qquad (35)$$

$$\hat{d}_1 > 0, \hat{d}_2 > 0. \qquad (36)$$

***Step* 4** The problem is to minimize (35) subject to (36). Following VEGA [16] parameters has been used to obtain the optimal values of $\hat{d}_1$ and $\hat{d}_2$.

$$\left.\begin{array}{ll} \text{Population size} & : 6 \\ \text{Selection} & : \text{Roulette - wheel selection operator} \\ \text{Crossover} & : \text{Linear crossover (Elite preserving)} \\ \text{Mutation} & : \text{Random mutation } (\Delta_i = 0.1) \end{array}\right\} \qquad (37)$$

can be obtained by Pareto-Optimality and V.E.G.A. For the following population of initial conditions, the population after crossover and mutation operators are shown in following table:

Table 1. Results for population

Sl. No.	Initial Population		Population after Selection Operator		Population after Crossover & Mutation Operator		Assigned Fitness Value
	b_1	b_2	b_1	b_2	b_1	b_2	
1.	4.8178	0.9203	4.8178	0.9203	4.805422	0.871356	457.001776
2.	4.744668	0.9203	4.744668	0.9203	4.695724	0.871356	459.437195
3.	5.743400	0.9013	5.743400	0.9013	4.656056	0.876006	455.456726
4.	4.7426 00	0.9234	4.7426 00	0.9234	4.693656	0.874456	0.043756
5.	4.887000	0.9023	4.887000	0.9023	4.676956	0.885506	0.041648
6.	4.776 000	0.9013	4.776 000	0.9013	4.727056	0.852356	0.048127

Applying Pareto-Optimality and V.E.G.A., algorithm converges to the following optimal solution and reduced-order model is following:

$$\hat{b}_1 = 4.676956, \qquad \hat{b}_2 = 0.885506$$

$$G_2(s) = \frac{57s + 11.2902015}{(s^2 + 4.676956s + 0.885506)} \qquad (38)$$

$$(\quad \hat{t}_1 = 12.75, \qquad \hat{t}_2 = 59.088306, \qquad \hat{t}_3 = 313.2146$$

$$\hat{M}_1 = 57, \qquad \hat{M}_2 = -265.700958, \qquad \hat{M}_3 = 1192.197754 \quad)$$

Thus unstable model is identified as:

$$\hat{G}(s) = \frac{1}{s^2}\left[(2s-12) + \hat{G}_2(s)\right]$$

$$\hat{G}(s) = \frac{1}{s^2}\left[(2s-12) + \frac{57s \ \ +11.2902015}{(s^2 + 4.676956s + 0.885506)}\right] \tag{39}$$

$$\hat{G}(s) = \frac{2s^3 + 21.353912s^2 + 57.894484s + 10.626072}{s^2(s^2 + 4.676956s + 0.885506)} \tag{40}$$

For comparison, the models as obtained by techniques reported by Krishnamurthy and Sheshadri [27] and Rao et. al. [38]:

$$\hat{G}(s) = \frac{90s^3 - 344s^2 + 7s + 42}{s^2(45s^2 + 98s + 56)} \tag{41}$$

$$\hat{G}(s) = \frac{12.5s^3 - 4s^2 + 21s + 42}{s^2(45s^2 + 98s + 56)} \tag{42}$$

Step and impulse responses of (26), (40), (41) and (42) are plotted in Fig. 1 and Fig. 2. It is clearly seen that the response of approximation (40) is almost identical to that of (26) while (41) and (42) show deviation.

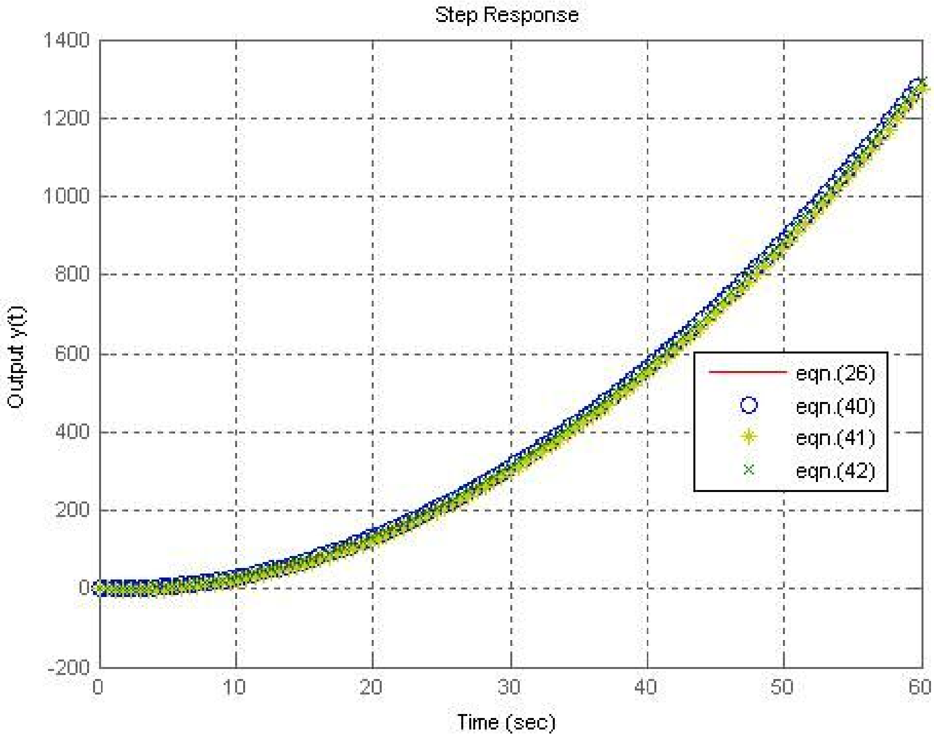

Fig. 1 Step responses of original system and its reduced order models

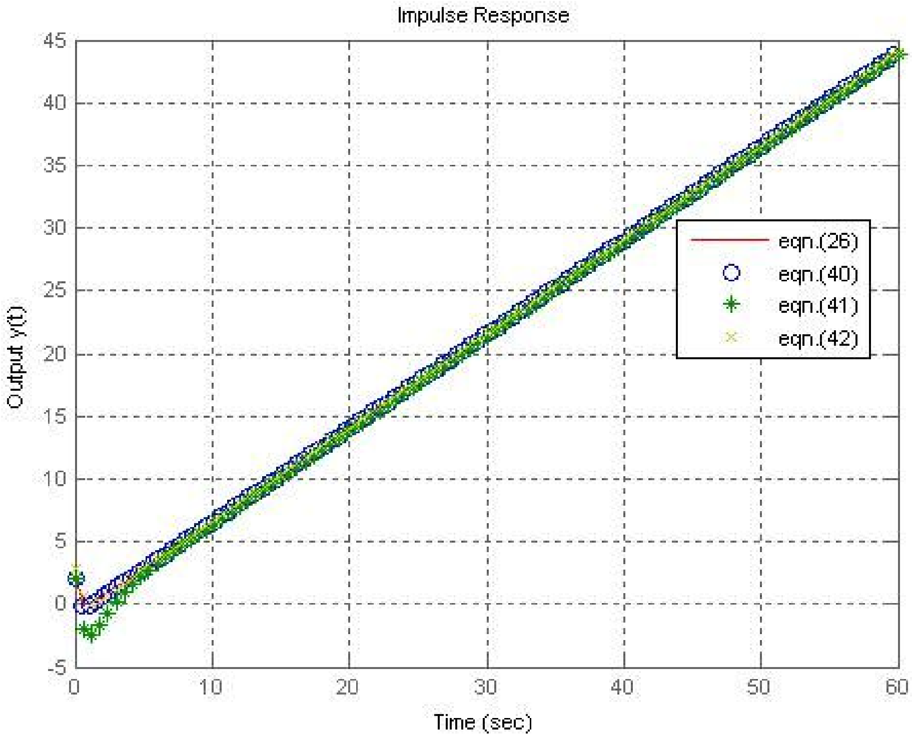

Fig. 2 Impulse responses of original system and its reduced order models

5. Conclusions

In this note, problem of finding Routh-Padé approximants has been viewed as a multi-objective optimization problem for unstable systems. An evolutionary algorithm, Vector Evaluated Genetic Algorithm (VEGA) by name, is used to solve the multi-objective optimization problem. This algorithm gives Pareto-optimal solutions (non-dominated set of solutions) and the best possible solution is selected. It is shown that, using Pareto-Optimality and V.E.G.A., the denominator of the model can be chosen so as to minimize errors between the $(r+1)$th and the subsequent time-moments and Markov-parameters of the model and the corresponding time-moments and Markov-parameters of the system while preserving stability. Having obtained the denominator in this manner, the numerator parameters can be determined in the usual manner, namely, by fully retaining the first r time-moments/Markov-parameters of the system. The present approach, therefore, leads to an improved approximant for unstable systems.

References

[1] Appiah, R.K, Pade methods of Hurwitz polynomial approximation with application to linear system reduction, *Int. J.Control.* 29: 39-48, 1979.

[2] Ashoor, N. and Singh, V., A note on low order modeling, *IEEE Trans. Automat. Contr.* 27, 1124-1126, 1982.

[3] Ashoor, N. and Singh, V., Remarks on system simplification under consideration of time response, *Electron. Lett.,* 18, 496-497, 1982.

[4] Bandyopadhyay, B. and Ismail, O and Gorez, R., Routh-Pade Approximation for Interval systems, *IEEE Trans. Automat. Contr.,* 39(12),1994

[5] Bandyopadhyay, B. and Upadhye, A. and Ismail O., γ-δ Routh approximation for interval systems, *IEEE Trans. Automat. Contr.,* 42(12), 1997.

[6] Bistritz, Y. and Shaked, U., Stable linear systems simplification via Pade approximations to Hurwitz polynomials, *Trans. ASME J. Dyn. Syst. Meas.Control.,* 103, 279-284, 1981.

[7] Blakelock , J.H., Automatic control of aircraft and missiles, *John wiley Inc. New york* , 226-235, 1965.

[8] Chen, T.C.and Chang, C.Y.and Han, K.W., Stable reduced-order Pade approximants using stability equation method, *Electron. Lett.,* 16, 345-346,1980.

[9] Choo, Y. Improvement to modified Routh approximation method., *Electron. Lett.* , 35, 606-607. 1999.

[10] Choo, Y., Improvement to modified Routh approximation method (correction), *Electron. Lett.,* 1119, 1999.

[11] Choo, Y., Direct method for obtaining modified Routh approximants., *Electron. Lett.,* 35, 627–1628 1999.

[12] Choo, Y., Improved bilinear Routh approximation method for discrete time systems, *Trans. ASME J. Dyn. Syst. Meas. Control.,* 123, 125–127, 2001.

[13] Choo, Y., Equivalence of bilinear Routh and Schwarz approximation methods for discrete-time systems, Electron. Lett., 38, 761–762, 2002.

[14] Choo, Y. and Dongmin, K., SISO Continuous System Reduction via impulse response Gramian by iterative formulae, *Trans. ASME J. Dyn. Syst. Meas. Contr.,* 128, 391-393, 2006.
[15] Dolgin, Y. and Zeheb, E., On Routh-Pade model reduction of interval systems, *IEEE Trans. Autom. Control,* 48, 1610–1612,2003.
[16] Deb., K., Multi-objective Optimization using Evolutionary Algorithm., *New York, John Wiley and Sons Ltd.,* 2002.
[17] Douglas, J.M., *Process dynamics and control vol.2 Control system synthesis,prentice Hall New Jersey 1972 chapter 7.*
[18] Sastry, G.V.K.R. and Rao, G.R. and Rao, P.M. Large scale interval system modelling using Routh approximants, Electron. Lett., 36, 768–769, 2000.
[19] Hutton, M.F. and Friedland, B., Routh approximations for reducing order of linear time-invariant systems, IEEE Trans. Automat. Contr., AC-20, 329- 337, 1975.
[20] Hsieh, C.S and Hwang, C., Model reduction of continuous-time system using a modified Routh-approximation method, IEE Proc. D. Control Theory Appl., 136, 151–156 1989.
[21] Householder, A.S., *The numerical treatment of single non-linear equation, McGraw-Hill Book Co., New -Newyork 1970.*
[22] Hwang, C. and Hwang, J.H. and Guo, T.Y., Multifrequency Routh Approximants for linear systems, *IEE Proc. Control Theory Applicat.* , 142, 351-358, 1975.
[23] Hwang, C.Y. and Lee, Y.C., A new family of Routh approximants. Circuits Syst. Signal Process. , 16, 1-25, 1997.
[24] Hwang, C. and Yang, S.F., Comments on the computation of interval Routh approximants., *IEEE Trans. Autom. Control.,* 44, 1782–1787, 1999.
[25] Kelley, K.J., Aircraft manoeuvre optimization by reduced order approximation control and dynamic system (Ed:C.T. Leondes) , *Academic Press London*, 132-174, 1973.
[26] Krishnamurthy, V. and Sheshadri, V., Model reduction using Routh Stability criterion, IEEE Trans. Autom. Control. 1978; 23: 729-731.
[27] Krishnamurthy V. and V.Sheshadri V A simple and direct method of reduction order of linear systems using routh approximation in frequency domain IEEE Trans. Autom. Control. 21, 797-799, 1976.
[28] Luss, R.and Jaakola, T., Direct search and systematic reduction of size of search region. *AICHE J.*, 19, 760-766, 1973.
[29] Lucas, T.N., The bilinear method: a new stability-preserving order reduction approach. *Proc. Inst. Mech. Engg. I. J. Syst. Contr. Engg.,* 216, 429-436, 2002.
[30] Lucas, T.N., Constrained optimal Pade model reduction., ASME J .Dyna. Syst. Meas. Control, 119, 685-690, 1997.
[31] Lee, Y.C. and Hwang, C.and Hwang J.H., Model-reduction of SISO systems by Routh expansion and balancing method., J. Franklin Inst., 331B, 367–380, 1994.
[32] Lucas, T.N., 1988. Scaled impulse energy approximation for model reduction, *IEEE Trans.Automat. Contr.,* 133, 791-793, 1998.
[33] Manigandan T., Devarajan N. and Svanandam S.N. Design of PID controller using reduced order model, *Academic Open Internet Journal,* 15, 1-15, 2005.
[34] Pal, J., State reduced-order Padé approximants using Routh-Hurwitz array, *Electron. Lett.* , 15, 25-26, 1979.
[35] Parks, P.C, A new proof of the Routh-Hurwitz criterion using the second method of Lyapunov, *Proc. Camb. Philos. Soc.*, 694-702, 1962.

[36] Pal, J., Improved Pade approximants using stability equation method, *Electron. Lett.*, 19, 426-427, 1983.
[37] Puri, V. and Lam, D.P. , Stable model reduction by impulse response error minimization using Michailov criterion and Pade´ approximation, *Trans. ASME J. Dyn. Syst. Meas.Control*, 110, 389–394, 1988.
[38] Rao, A.S.and Lamba, S.S. and Rao, S.V., Routh-approximant time domain reduced-order modeling for single-input single-output systems, *IEE Proc. Control Theory Appl.*, 125,1059- 1063, 1978.
[39] Singh, V., Obtaining Routh-Pade approximants using Luss-Jaakola algorithm, *IEE Proc. Part I,* 152.. 129-132, 2005.
[40] Singh, V. and Dinesh Chandra, and Kar, H., Improved Routh-Pade approximants: A computer –aided approach., *IEEE Trans. Automat. Contr.,* 49, 292-295, 2004 .
[41] Singh V. Stable approximants for stable systems: A new approach, *Proc. IEEE.*, 69, 1155-1156, 1981.
[42] Singh, V., Improved stable Approximants using the Routh array, *IEEE Trans. Automat. Contr.*, AC-26, 581-582, 1981.
[43] Singh V. Remarks on system simplification using the Routh stability array. Proc. IEEE.1981; 69: 662, 1981.
[44] Singh, V., Nonuniqueness of model reduction using the Routh approach, *IEEE Trans. Autom. Control*, 24, 650-651, 1979.
[45] Shamash, Y., Truncation method of reduction: a viable alternative, *Electron. Lett.* , 17, 97-98, 1981.
[46] Shamash, Y., Model reduction usng the Routh stability criterion and the Pade approximation technique, *Int. J. Control* ,21, 475-484, 1975.
[47] Shamash, Y., Stable biased reduced-order models using the Routh method of Reduction, *Int. J. Syst. Sci.,* 11, 641-654, 1980.
[48] Wan, B.W., Linear model reduction using Michailov criterion and Pade approximation techniques, *Int. J.Control*, 3, 1073-1089, 1981.