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Q-Curvature Tensor on f-Kenmotsu 3-Manifolds

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Abstract

The object of the present paper is to consider f-Kenmotsu 3-manifolds fulfilling certain curvature conditions on Q-curvature tensor with the Schouten-van Kampen connection. Certain consequences of Q-curvature tensor on such manifolds bearing Ricci soliton in perspective of Schouten-van Kampen association are likewise displayed. In the last segment, examples are given.

1. Introduction

Let $\vec{M}$ be a $(2n+1)$-dimensional almost contact manifold with an almost contact metric structure $(\breve{\phi},\xi,\eta,g)$ [1]. We denote by $\vec{\Omega}$, the fundamental 2-form of $\vec{M}$ i.e., $\vec{\Omega}(\vec{X},\vec{Y})=g(\vec{X},\breve{\phi}\vec{Y})$, $\vec{X},\vec{Y}\in\chi(\vec{M})$, where $\chi(\vec{M})$ being the Lie algebra of the differentiable vector fields on $\vec{M}$. Furthermore, we recall the following definitions [1,2].
The manifold $\vec{M}$ and its structure $(\breve{\phi},\xi,\eta,g)$ is said to be:
(i) normal if the almost complex structure defined on the product manifold $\vec{M}\times\Re$ is integrable (equivalently $[\breve{\phi},\breve{\phi}]+2d\eta\otimes\xi=0$),
(ii) almost cosymplectic if $d\eta=0$ and $d\breve{\phi}=0$,
(iii) cosymplectic if it is normal and almost cosymplectic (equivalently, $\vec{\nabla}\breve{\phi}=0$, $\vec{\nabla}$ being covariant differentiation with respect to the Levi-Civita connection).
Olszak and Rosca [3] contemplated normal locally conformal almost cosymplectic manifold and gave the geometric translation of f-Kenmotsu manifolds and its curvature tensors. Among others, they proved that a Riccisymmetric f-Kenmotsu manifold is an Einstein manifold.
The Schouten-van Kampen connection is quite possibly the most widely recognized connection acclimated to two or three necessary allocations on a differentiable manifold conceding with a relative connection [4,5]. Solov'ev has investigated hyperdistributions in Riemannian manifolds using the Schouten-van Kampen connection [6,7]. From that point, Olszak has contemplated the Schouten-van Kampen connection with an almost contact metric structure [8]. He has depicted a few classes of almost contact metric manifolds bearing the Schouten-van Kampen connection and closed some particular curvature properties of this connection on such manifolds.
Let $\vec{M}$ be a $(2n+1)$-dimensional Riemannian manifold. On the off chance that there exists a balanced correspondence between each facilitate neighborhood of $\vec{M}$ and an area in Euclidean space with the end goal that any geodesic of the Riemannian manifold compares to a straight line in the Euclidean space, at that point $\vec{M}$ is supposed to be locally projectively flat. For $n\geq 1$, $\vec{M}$ is locally projectively flat if and just if the notable projective curvature tensor P vanishes. Truth be told, P is projectively flat (i. e., P=0) if and just if the manifold is of consistent curvature [9]. ξ-conformally flat K-contact manifolds have been concentrated by Zhen et al. [10]. Yildiz et al. [11] considered f-Kenmotsu 3-manifolds with the Schouten-van Kampen connection and demonstrated that such manifold is consistently ξ-projectively flat. The projective curvature tensor is characterized by [12]:

$$P(\vec{X},\vec{Y})\vec{Z}=\vec{R}(\vec{X},\vec{Y})\vec{Z}-\frac{1}{2n}\{\vec{Ric}(\vec{Y},\vec{Z})\vec{X}-\vec{Ric}(\vec{X},\vec{Z})\vec{Y}\}, \tag{1.1}$$

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where $\vec{Ric}$ is the Ricci tensor on $\vec{M}$.
A change in a $(2n+1)$-dimensional Reimannian manifold $\vec{M}$, which changes each geodesic circle of $\vec{M}$ into a geodesic circle of $\vec{M}$, is supposed to be a concircular change [13, 14]. A concircular change is consistently a conformal change [13]. It means a geodesic circle by a bend in $\vec{M}$ whose first curvature is steady and second arch is indistinguishably zero. Subsequently the geometry of concircular change is a speculation of intrusive geometry as in the difference in measurement is more broad than incited by a circle safeguarding diffeomorphism. A significant invariant of concircular transformation is the concircular curvature tensor C, characterized by [14]

$$C(\vec{X},\vec{Y})\vec{Z} = \vec{R}(\vec{X},\vec{Y})\vec{Z} - \frac{\vec{scal}}{2n(2n+1)}\{g(\vec{Y},\vec{Z})\vec{X} - g(\vec{X},\vec{Z})\vec{Y}\}, \tag{1.2}$$

for all $\vec{X},\vec{Y},\vec{Z} \in \chi(\vec{M})$, where $\vec{R}$ is the Reimannian curvature tensor and $\vec{scal}$ is the scalar curvature with respect to the Levi-Civita connection. An $(2n+1)$-dimensional Riemannian manifold $(\vec{M}^n, g)$, the Q-curvature tensor is defined as [15]

$$Q(\vec{X},\vec{Y})\vec{Z} = \vec{R}(\vec{X},\vec{Y})\vec{Z} - \frac{\breve{\psi}}{2n}\{g(\vec{Y},\vec{Z})\vec{X} - g(\vec{X},\vec{Z})\vec{Y}\}, \tag{1.3}$$

where $\breve{\psi}$ is an arbitrary scalar function. If $\breve{\psi}$=$\frac{\vec{scal}}{(2n+1)}$, then Q- curvature tensor reduces to concircular curvature tensor. Mantica and Suh [15] have studied pseudo-Q-symmetric Riemannian manifolds.
In a Riemannian manifold $(\vec{M},g)$, the metric g is called a Ricci soliton if [16]

$$\frac{1}{2}\mathfrak{L}_{\vec{V}}g + \vec{Ric} + \lambda g = 0, \tag{1.4}$$

where $\mathfrak{L}$ is the Lie derivative, $\vec{Ric}$ the Ricci tensor, $\vec{V}$ a complete vector field on $\vec{M}$ and λ is a constant. Compact Ricci solitons are the fixed points of the Ricci flow $\frac{\partial}{\partial t}g$=-2$\vec{Ric}$ projected from the space of metrics onto its quotient modulo diffeomorphisms and scalings, and often arise as blow-up limits for the Ricci flow on compact manifolds. The Ricci soliton is said to be shrinking, steady and expanding if λ is negative, zero and positive respectively. A Ricci soliton with $\vec{V}$=0 is reduced to Einstein equation. During the last two decades, the geometry of Ricci solitons have been light up by the several mathematicians [17–19]. It has became more important after Perelman applied Ricci solitons to solve the long standing Poincare conjecture posed in 1904.
Our paper is structured as follows: After the introduction. In section 2 we recall the fundamental results of the Schouten-van Kampen connection and f-Kenmotsu 3-manifolds. In the portion 3 we review the thought of Ricci solition on f-Kenmotsu 3-manifolds with the Schouten-van Kampen connection. In segment 4 we study ξ-Q flat f-Kenmotsu 3-manifolds with the Schouten-van Kampen connection. We demonstrate the some results on f-Kenmotsu 3-manifolds with the Schouten-van Kampen connection under the condition $\widetilde{Q}\cdot\widetilde{Ric}$=0, $\widetilde{Q}\cdot\widetilde{R}$=0, $\widetilde{Q}\cdot\widetilde{P}$=0, $\widetilde{Q}(\xi,\vec{X})\cdot\widetilde{Q}$=0 and $((\xi\wedge_{\widetilde{Ric}}\vec{X})\cdot\widetilde{Q})$=0 in the sections 5-9, respectively. In the last segment, we give the examples.

2. Preliminaries

Let $\vec{M}$ be a real $(2n+1)$-dimensional differentiable manifold endowed with an almost contact structure $(\breve{\phi},\xi,\eta,g)$ satisfying

$$\breve{\phi}^2 = I - \eta\otimes\xi, \quad \eta(\xi) = 1, \quad \breve{\phi}\xi = 0, \quad \eta\circ\breve{\phi} = 0, \quad \eta(\vec{X}) = g(\vec{X},\xi), \tag{2.1}$$

and

$$g(\breve{\phi}\vec{X},\breve{\phi}\vec{Y}) = g(\vec{X},\vec{Y}) - \eta(\vec{X})\eta(\vec{Y}), \tag{2.2}$$

for any vector fields $\vec{X},\vec{Y} \in \chi(\vec{M})$, where I is the identity of the tangent bundle $T\vec{M}$, $\breve{\phi}$ is a tensor field of $(1,1)$-type, η is a 1-form, ξ is a vector field and g is a metric tensor of $\vec{M}$. We say that $(\breve{\phi},\xi,\eta,g)$ is a f-Kenmotsu manifold [20, 21] if the covariant differentiation of $\breve{\phi}$ satisfies

$$(\nabla_{\vec{X}}\breve{\phi})\vec{Y} = f\{g(\breve{\phi}\vec{X},\vec{Y})\xi - \eta(\vec{Y})\breve{\phi}\vec{X}\}, \tag{2.3}$$

where $f \in C^\infty(\vec{M})$ such that $df\wedge\eta = 0$. If $f = \alpha(\neq 0) =$ constant, then the manifold $(\vec{M},g)$ is an α-Kenmotsu manifold [21]. Kenmotsu manifold is an example of f-Kenmotsu manifold with f=1 [22, 23]. If f=0, then the manifold $(\vec{M},g)$ reduces to cosymplectic [21]. An f-Kenmotsu manifold is said to be regular if $f^2+\dot{f}\neq 0$, where $\dot{f} = \xi f$. For an f-Kenmotsu manifold from (2.3) it follows that

$$\nabla_{\vec{X}}\xi = f\{\vec{X} - \eta(\vec{X})\xi\}. \tag{2.4}$$

The condition $df\wedge\eta = 0$ holds if dim $\vec{M} \geq 5$. In general this relation does not hold if dim $\vec{M}$=3 [23]. It is well-known that in a Riemannian 3-manifold.

$$\vec{R}(\vec{X},\vec{Y})\vec{Z} = g(\vec{Y},\vec{Z})\vec{Q}\vec{X} - g(\vec{X},\vec{Z})\vec{Q}\vec{Y} + \vec{Ric}(\vec{Y},\vec{Z})\vec{X} - \vec{Ric}(\vec{X},\vec{Z})\vec{Y} - \tfrac{\vec{scal}}{2}\{g(\vec{Y},\vec{Z})\vec{X} - g(\vec{X},\vec{Z})\vec{Y}\}. \tag{2.5}$$

In a f-Kenmotsu 3-manifold, we have [3].

$$\vec{R}(\vec{X},\vec{Y})\vec{Z} = (\tfrac{\vec{scal}}{2} + 2f^2 + 2\dot{f})(\vec{X}\wedge\vec{Y})\vec{Z} - (\tfrac{\vec{scal}}{2} + 3f^2 + 3\dot{f})\{\eta(\vec{X})(\xi\wedge\vec{Y})\vec{Z} + \eta(\vec{Y})(\vec{X}\wedge\xi)\vec{Z}\}, \tag{2.6}$$

$$\vec{Ric}(\vec{X},\vec{Y}) = (\frac{\vec{scal}}{2} + f^2 + \dot{f})g(\vec{X},\vec{Y}) - (\frac{\vec{scal}}{2} + 3f^2 + 3\dot{f})\eta(\vec{X})\eta(\vec{Y}), \tag{2.7}$$

where $\vec{scal}$ is the scalar curvature of $\vec{M}$. From (2.6) and (2.7) we obtain

$$\vec{R}(\vec{X},\vec{Y})\xi = -(f^2+\dot{f})[\eta(\vec{Y})\vec{X}-\eta(\vec{X})\vec{Y}], \tag{2.8}$$

$$\vec{Ric}(\vec{X},\xi) = -2(f^2+\dot{f})\,\eta(\vec{X}), \tag{2.9}$$

$$\vec{Ric}(\xi,\xi) = -2(f^2+\dot{f}), \tag{2.10}$$

$$\vec{Q}\xi = -2(f^2+\dot{f})\xi, \tag{2.11}$$

for any vector fields $\vec{X},\vec{Y}$ on $\vec{M}$.
On the other hand $\vec{H}$ and $\vec{V}$ are two complementary, orthogonal distributions on $\vec{M}$ such that dim$\vec{H}$=$n-1$, dim$\vec{V}$=1, and the distribution $\vec{V}$ is non-null. Thus $T\vec{M}$=$\vec{H}\oplus\vec{V}$, $\vec{H}\cap\vec{V}$=$\{0\}$ and $\vec{H}\perp\vec{V}$. Assume that ξ is a unit vector field and η is a linear form such that $\eta(\xi)$=1, $g(\xi,\xi)$=ε=±1 and

$$\vec{H} = \ker\eta, \quad \vec{V} = \text{span}\{\xi\}. \tag{2.12}$$

For any $X\in T\vec{M}$, by $\vec{X}^h$ and $\vec{X}^v$ we denote the projections of $\vec{X}$ onto $\vec{H}$ and $\vec{V}$, respectively. Thus, we have $\vec{X}=\vec{X}^h+\vec{X}^v$ with

$$\vec{X}^h = \vec{X}-\eta(\vec{X})\xi, \quad \vec{X}^v = \eta(\vec{X})\xi. \tag{2.13}$$

The Schouten-van Kampen connection $\widetilde{\nabla}$ associated to the Levi-Civita connection $\vec{\nabla}$ and adapted to the pair of the distributions $(\vec{H},\vec{V})$ is defined by [5]

$$\widetilde{\nabla}_{\vec{X}}\vec{Y} = (\vec{\nabla}_{\vec{X}}\vec{Y}^h)^h + (\vec{\nabla}_{\vec{X}}\vec{Y}^v)^v. \tag{2.14}$$

From (2.13), we compute

$$(\vec{\nabla}_{\vec{X}}\vec{Y}^h)^h = \vec{\nabla}_{\vec{X}}\vec{Y}-\eta(\vec{\nabla}_{\vec{X}}\vec{Y})\xi-\eta(\vec{Y})\vec{\nabla}_{\vec{X}}\xi, \tag{2.15}$$

$$(\vec{\nabla}_{\vec{X}}\vec{Y}^v)^v = \eta(\vec{\nabla}_{\vec{X}}\vec{Y})\xi+\eta(\vec{\nabla}_{\vec{X}}\vec{Y})\xi, \tag{2.16}$$

which enables us to express the Schouten-van Kampen connection with help of the Levi-Civita connection in the following way [6]

$$\widetilde{\nabla}_{\vec{X}}\vec{Y} = \vec{\nabla}_{\vec{X}}\vec{Y}-\eta(\vec{Y})\vec{\nabla}_{\vec{X}}\xi+(\vec{\nabla}_{\vec{X}}\eta)(\vec{Y})\xi. \tag{2.17}$$

In view of the Schouten-van Kampen connection (2.17), many properties of some geometric objects connected with the distributions $\vec{H},\vec{V}$ can be characterized [6,7]. For example $\widetilde{\nabla}g=0, \widetilde{\nabla}\xi=0, \widetilde{\nabla}\eta=0$.

Proposition 2.1 ([24]). *Let $\vec{M}$ be a f-Kenmotsu 3-manifold with the Schouten-van Kampen connection $\widetilde{\nabla}$ we have*

$$\widetilde{\nabla}_{\vec{X}}\vec{Y} = \vec{\nabla}_{\vec{X}}\vec{Y}+f\{g(\vec{X},\vec{Y})\xi-\eta(\vec{Y})\vec{X}\}. \tag{2.18}$$

$$\widetilde{R}(\vec{X},\vec{Y})\vec{Z} = \vec{R}(\vec{X},\vec{Y})\vec{Z}+f^2\{g(\vec{Y},\vec{Z})\vec{X}-g(\vec{X},\vec{Z})\vec{Y}\}+\dot{f}\{g(\vec{Y},\vec{Z})\eta(\vec{X})\xi-g(\vec{X},\vec{Z})\eta(\vec{Y})\xi+\eta(\vec{Y})\eta(\vec{Z})\vec{X}-\eta(\vec{X})\eta(\vec{Z})\vec{Y}\}. \tag{2.19}$$

$$\widetilde{Ric}(\vec{Y},\vec{Z}) = \vec{Ric}(\vec{Y},\vec{Z})+(2f^2+\dot{f})g(\vec{Y},\vec{Z})+\dot{f}\eta(\vec{Y})\eta(\vec{Z}), \tag{2.20}$$

$$\ddot{Q}\vec{X} = \vec{Q}\vec{X}+(2f^2+\dot{f})\vec{X}+\dot{f}\eta(\vec{X})\xi, \tag{2.21}$$

$$\widetilde{scal} = \vec{scal}+6f^2+4\dot{f}, \tag{2.22}$$

where $\widetilde{R}$, $\vec{R}$, $\widetilde{Ric}$, $\vec{Ric}$, $\ddot{Q}$, $\vec{Q}$ and $\widetilde{scal}$, $\vec{scal}$ are consider as the Riemann curvature, Ricci tensors, Ricci operators and the scalar curvatures of the connection $\widetilde{\nabla}$ and $\vec{\nabla}$ respectively.

3. Ricci Soliton on f-Kenmotsu 3-Manifold with the Schouten-Van Kampen Connection

In this section, we study the nature of Ricci soliton on f-Kenmotsu 3-manifold with respect to the Schouten-van Kampen connection $\widetilde{\nabla}$. Let $(\vec{M}^3,\phi,\xi,\eta,g)$ be a f-Kenmotsu 3-manifold with the Schouten-van Kampen connection, since $\widetilde{\nabla}g$=0 and $\widetilde{T}\neq 0$ then from [25], we have

$$(\widetilde{\mathfrak{L}}_{\vec{V}}g)(\vec{X},\vec{Y})=g(\widetilde{\nabla}_{\vec{X}}\vec{V},\vec{Y})+g(\vec{X},\widetilde{\nabla}_{\vec{Y}}\vec{V})=(\mathfrak{L}_{\vec{V}}g)(\vec{X},\vec{Y}), \tag{3.1}$$

where $\widetilde{\mathfrak{L}}$ denotes the Lie derivative on the manifold with respect to the Schouten-van Kampen connection. Thus from (1.4) we can write

$$(\widetilde{\mathfrak{L}}_{\vec{V}}g+2\widetilde{Ric}+2\lambda g)(\vec{X},\vec{Y})=0, \tag{3.2}$$

that is

$$g(\widetilde{\nabla}_{\vec{X}}\vec{V},\vec{Y})+g(\vec{X},\widetilde{\nabla}_{\vec{Y}}\vec{V})+2\widetilde{Ric}(\vec{X},\vec{Y})+2\lambda g(\vec{X},\vec{Y})=0, \tag{3.3}$$

Putting $\vec{V}$=ξ in (3.3) and using (2.4) we obtain

$$\widetilde{Ric}(\vec{X},\vec{Y})=-(\lambda+f)g(\vec{X},\vec{Y})+f\eta(\vec{X})\eta(\vec{Y}) \tag{3.4}$$

In view of (2.20) and (3.4), we get

$$\vec{Ric}(\vec{X},\vec{Y})=-(\dot{f}+2f^2+f+\lambda)g(\vec{X},\vec{Y})+(-\dot{f}+f)\eta(\vec{X})\eta(\vec{Y}) \tag{3.5}$$

Thus we can state the following:

Proposition 3.1. *A f-Kenmotsu 3-manifold with respect to the Schouten-van Kampen connection $\widetilde{\nabla}$ admitting Ricci soliton then the manifold is an η-Einstein manifold with the Schouten-van Kampen connection $\widetilde{\nabla}$ and Levi-Civita connection $\vec{\nabla}$.*

Proposition 3.2. *A Ricci soliton on an f-Kenmotsu 3-manifold with respect to the Schouten-van Kampen connection $\widetilde{\nabla}$ is always steady.*

Also from (3.4), we get

$$\widetilde{scal}=-2f-3\lambda. \tag{3.6}$$

In view of (2.22) and (3.6), one can easily bring out that

$$\lambda=-\frac{1}{3}(\vec{scal}+6f^2+4\dot{f}+2f). \tag{3.7}$$

We have the following:

Proposition 3.3. *A Ricci soliton on f-Kenmotsu 3-manifold with the Schouten-van Kampen connection $\widetilde{\nabla}$ is an expanding, steady or shrinking according as $\vec{scal}<-6f^2-4\dot{f}-2f$, $\vec{scal}$=$-6f^2-4\dot{f}-2f$ or $\vec{scal}>-6f^2-4\dot{f}-2f$.*

Proposition 3.4. *A Ricci soliton on α-Kenmotsu 3-manifold with the Schouten-van Kampen connection $\widetilde{\nabla}$ is an expanding, steady or shrinking according as $\vec{scal}<-6\alpha^2-2\alpha$, $\vec{scal}=-6\alpha^2-2\alpha$ or $\vec{scal}>-6\alpha^2-2\alpha$.*

Proposition 3.5. *A Ricci soliton on cosymplectic 3-manifold with respect to the Schouten-van Kampen connection $\widetilde{\nabla}$ is an expanding, steady or shrinking according as $\vec{scal}<0$, $\vec{scal}=0$ or $\vec{scal}>0$.*

In [24], Yildiz et al. demonstrated that f-Kenmotsu 3-manifold is projectively flat with respect to the Schouten-van Kampen connection if and only if $\vec{M}$ is a Ricci-flat manifold with respect to the Schouten-van Kampen connection $\widetilde{\nabla}$. Therefore in perspective on this outcome and utilizing (3.4) we express the following:

Corollary 3.6. *A Ricci soliton on a projectively flat f-Kenmotsu 3-manifold with respect to the Schouten-van Kampen connection $\widetilde{\nabla}$ is always steady.*

With the help of Theorem 6.1. of [24] and (3.4) we have the following:

Corollary 3.7. *A Ricci soliton on a conharmonically flat f-Kenmotsu 3-manifold with respect to the Schouten-van Kampen connection $\widetilde{\nabla}$ is always steady.*

4. ξ-$\widetilde{Q}$ Flat f-Kenmotsu 3-Manifold with the Schouten-Van Kampen Connection

In this section, we consider ξ-$\widetilde{Q}$ flat f-Kenmotsu 3-manifold admitting the Schouten-van Kampen connection $\widetilde{\nabla}$. Now we state the following definitions and result:

Definition 4.1. *A f-Kenmotsu 3-manifold is said to be ξ-$\widetilde{Q}$ flat if $\widetilde{Q}(\vec{X},\vec{Y})\xi=0$ on $\vec{M}$.*

Theorem 4.2. *A f-Kenmotsu 3-manifold with the Schouten-van Kampen connection $\widetilde{\nabla}$ is ξ-$\widetilde{Q}$ flat if and only if ψ=0.*

Proof. From (1.3) we have

$$\widetilde{Q}(\vec{X},\vec{Y})\xi=\widetilde{R}(\vec{X},\vec{Y})\xi-\frac{\psi}{2}[\eta(\vec{Y})\vec{X}-\eta(\vec{X})\vec{Y}], \tag{4.1}$$

for any for any vector fields $\vec{X}$ and $\vec{Y}\in\chi(\vec{M})$. With the help of (2.6) and (2.19), equation (4.1) reduces

$$\widetilde{Q}(\vec{X},\vec{Y})\xi=-\frac{\psi}{2}[\eta(\vec{Y})\vec{X}-\eta(\vec{X})\vec{Y}]. \tag{4.2}$$

This completes the proof. □

If ψ=$\frac{\vec{scal}}{3}$ then Q-curvature tensor reduces to concircular curvature tensor. Thus keeping in mind Theorem 4.2 and making use of (1.2) we obtain the followings:

Corollary 4.3. *A f-Kenmotsu 3-manifold with the Schouten-van Kampen connection $\widetilde{\nabla}$ is ξ-concircularly flat if and only if the scalar curvature of the manifold is zero.*

Corollary 4.4. *A ξ-concircularly flat complete Einstein f-Kenmotsu 3-manifold is Ricci flat.*

Corollary 4.5. *A Ricci soliton on ξ-concircularly flat complete Einstein f-Kenmotsu 3-manifold is always steady.*

If $0\neq f$=constant (we assume f=α) then $\dot{f}$=0. Thus we state the followings:

Corollary 4.6. *An α-Kenmotsu 3-manifold with the Schouten-van Kampen connection $\widetilde{\nabla}$ is ξ-$\widetilde{Q}$ flat if and only if ψ=0.*

Corollary 4.7. *In a ξ-$\widetilde{Q}$ flat α-Kenmotsu 3- manifold with the Schouten-van Kampen connection $\widetilde{\nabla}$ the Q-curvature tensor is equal to the Reimannian curvature tensor.*

Corollary 4.8. *In a ξ-$\widetilde{Q}$ flat α-Kenmotsu 3-manifold with the Schouten-van Kampen connection $\widetilde{\nabla}$ the concircular curvature tensor is equal to the Reimannian curvature tensor.*

Corollary 4.9. *A Ricci soliton on ξ-concircularly flat α-Kenmotsu 3-manifold with respect to the Schouten-van Kampen connection $\widetilde{\nabla}$ is always shrinking.*

5. f-Kenmotsu 3-Manifolds Satisfying $\widetilde{Q}\cdot\widetilde{Ric}$=0 with the Schouten-Van Kampen Connection

In this section we restrict our study to f-Kenmotsu 3-manifolds satisfying $\widetilde{Q}\cdot\widetilde{Ric}$=0 with the Schouten-van Kampen connection $\widetilde{\nabla}$. We conclude the following:

Theorem 5.1. *A f-Kenmotsu 3-manifolds with the Schouten-van Kampen connection $\widetilde{\nabla}$ satisfying $\widetilde{Q}\cdot\widetilde{Ric}$=0, then ether Q-curvature tensor is equal to the Riemannian curvature or the manifold is an η-Einstein manifold.*

Proof. Let $\vec{M}$ satisfies the condition $\widetilde{Q}(\xi,\vec{X})\cdot\widetilde{Ric}$=0. So it implies that

$$\widetilde{Ric}(\widetilde{Q}(\xi,\vec{X})\vec{Y},\vec{Z})+\widetilde{Ric}(\vec{Y},\widetilde{Q}(\xi,\vec{X})\vec{Z})=0, \tag{5.1}$$

for any $\vec{X},\vec{Y},\vec{Z}$ on $\vec{M}$. Using (1.3), (2.6) and (2.19) in (5.1), we have

$$\tfrac{\psi}{2}\left\{g(\vec{X},\vec{Y})\widetilde{Ric}(\xi,\vec{Z})-\widetilde{Ric}(\vec{X},\vec{Z})\eta(\vec{Y})+g(\vec{X},\vec{Z})\widetilde{Ric}(\xi,\vec{Y})-\widetilde{Ric}(\vec{X},\vec{Y})\eta(\vec{Z})\right\}=0. \tag{5.2}$$

For $\vec{Z}$=ξ and keeping in mind (2.9) and (2.20), we obtain

$$\psi\widetilde{Ric}(\vec{X},\vec{Y})=0, \tag{5.3}$$

which implies that either ψ=0, or $\widetilde{Ric}(\vec{X},\vec{Y})$=0. Thus we have:
Case (i) In particular, if ψ=0, and $\widetilde{Ric}(\vec{X},\vec{Y})\neq 0$ then from (1.3) we get $Q(\vec{X},\vec{Y})\vec{Z}=\vec{R}(\vec{X},\vec{Y})\vec{Z}$.
Case (ii) Also if $\psi\neq 0$ and $\widetilde{Ric}(\vec{X},\vec{Y})$=0, then from (2.20), the manifold is an η-Einstein manifold. This completes the proof. □

Again, if ψ=$\frac{\vec{scal}}{3}$ then Q-curvature tensor reduces to concircular curvature tensor. So from Theorem 5.1 and making use of (1.2), we can mention the following:

Corollary 5.2. *A f-Kenmotsu 3-manifolds with the Schouten-van Kampen connection $\widetilde{\nabla}$ satisfying $\widetilde{C}\cdot\widetilde{Ric}$=0 then either Q-curvature tensor is equal to concircular curvature tensor or the manifold is an η-Einstein manifold.*

Also, if $0\neq f$=constant (we assume f=α), then $\dot{f}$=0. Thus we state the followings:

Corollary 5.3. *A f-Kenmotsu 3-manifolds satisfying $\widetilde{Q}\cdot\widetilde{Ric}$=0 with the Schouten-van Kampen connection $\widetilde{\nabla}$ then ether the Q-curvature tensor is equal to the Riemannian curvature or the manifold is an η-Einstein manifold.*

Corollary 5.4. *An α-Kenmotsu 3-manifolds with the Schouten-van Kampen connection $\widetilde{\nabla}$ satisfying $\widetilde{C}\cdot\widetilde{Ric}$=0 then either Q-curvature tensor reduces to concircular curvature tensor or the manifold is an η-Einstein manifold.*

Again, in view of (5.3) and (3.4), we have the followings:

Corollary 5.5. *A Ricci soliton on f-Kenmotsu 3-manifolds with the Schouten-van Kampen connection $\widetilde{\nabla}$ satisfying $\widetilde{Q}\cdot\widetilde{Ric}$=0, then either the soliton is steady or Q-curvature tensor is equal to the Remannian curvature tensor.*

Corollary 5.6. *A Ricci soliton on f-Kenmotsu 3-manifold with the Schouten-van Kampen connection $\widetilde{\nabla}$ satisfying $\widetilde{C}\cdot\widetilde{Ric}$=0, then either the soliton is steady or concircular curvature tensor is equal to the Remannian curvature tensor.*

6. f-Kenmotsu 3-Manifolds Satisfying $\widetilde{Q}\cdot\widetilde{R}$=0 with the Schouten-Van Kampen Connection

At this stage we consider f-Kenmotsu 3-manifolds with the Schouten-van Kampen connection $\widetilde{\nabla}$ satisfying $\widetilde{Q}\cdot\widetilde{R}$=0 . Therefore we illustrate the following:

Theorem 6.1. *A f-Kenmotsu 3-manifolds satisfying $\widetilde{Q}\cdot\widetilde{R}$=0 with the Schouten-van Kampen connection $\widetilde{\nabla}$ then either Q-curvature tensor is equal to the Riemannian curvature, or it has the sectional curvature $-(f^2+\dot{f})$.*

Proof. Suppose that f-Kenmotsu 3-manifolds with the Schouten-van Kampen connection $\widetilde{\nabla}$ satisfying

$$\widetilde{Q}(\xi,\vec{X})\widetilde{R}(\vec{Y},\vec{Z})\vec{U}=0. \tag{6.1}$$

Equation (6.1) can be written as

$$\widetilde{Q}(\xi,\vec{X})\widetilde{R}(\vec{Y},\vec{Z})\vec{U}-\widetilde{R}(\widetilde{Q}(\xi,\vec{X})\vec{Y},\vec{Z})\vec{U}-\widetilde{R}(\vec{Y},\widetilde{Q}(\xi,\vec{X})\vec{Z})\vec{U}-\widetilde{R}(\vec{Y},\vec{Z})\widetilde{Q}(\xi,\vec{X})\vec{U}=0, \tag{6.2}$$

for any vector fields $\vec{X}$, $\vec{Y}$, $\vec{Z}$ and $\vec{U}$ on $\vec{M}$. Using (1.3), (2.6) and (2.19) in (6.2), we obtain

$$\tfrac{\breve{\psi}}{2}[-g(\vec{X},\widetilde{R}(\vec{Y},\vec{Z})\vec{U})\xi+\eta(\widetilde{R}(\vec{Y},\vec{Z})\vec{U})-\eta(\vec{Y})\widetilde{R}(\vec{X},\vec{Z})\vec{U}-\eta(\vec{Z})\widetilde{R}(\vec{Y},\vec{X})\vec{U}-\eta(\vec{U})\widetilde{R}(\vec{Y},\vec{Z})\vec{X}]=0. \tag{6.3}$$

Taking the inner product with ξ of (6.3) and using (2.19) we get

$$\tfrac{\breve{\psi}}{2}[g(\vec{X},\vec{R}(\vec{Y},\vec{Z})\vec{U}+(f^2+\dot{f})\{g(\vec{Z},\vec{U})g(\vec{X},\vec{Y})-g(\vec{Y},\vec{U})g(\vec{X},\vec{Z})\}+\dot{f}\{g(\vec{X},\vec{Y})\eta(\vec{Z})\eta(\vec{U})-g(\vec{X},\vec{Z})\eta(\vec{Y})\eta(\vec{U})\}]=0. \tag{6.4}$$

It follows that either $\breve{\psi}$=0, or it has the sectional curvature $-(f^2+\dot{f})$.
This completes the proof. □

In particular, if $\breve{\psi}=\frac{\vec{scal}}{3}$ then Q-curvature tensor reduces to concircular curvature tensor. Therefore in view of the first result of the above Theorem 6.1 and making use of (1.2), we can mention the following:

Corollary 6.2. *If a f-Kenmotsu 3-manifolds with the Schouten-van Kampen connection $\widetilde{\nabla}$ satisfying $\widetilde{C}\cdot\widetilde{R}$=0 then either concircular curvature tensor is equal to the Riemannian curvature or it has the sectional curvature $-(f^2+\dot{f})$.*

Also with the help of (3.7) and Theorem 6.1, we conclude that:

Corollary 6.3. *If a f-Kenmotsu 3-manifolds with the Schouten-van Kampen connection $\widetilde{\nabla}$ satisfying $\widetilde{C}\cdot\widetilde{R}$=0 then either Ricci soliton is shrinking or it has the sectional curvature $-(f^2+\dot{f})$.*

If $0\neq f$=constant (we assume $f=\alpha$), then $\dot{f}$=0. Thus we state the followings:

Corollary 6.4. *If an α-Kenmotsu 3-manifolds with the Schouten-van Kampen connection $\widetilde{\nabla}$ satisfying $\widetilde{C}\cdot\widetilde{R}$=0 then either concircular curvature tensor is equal to the Riemannian curvature or it has the sectional curvature α^2.*

Corollary 6.5. *If an α-Kenmotsu 3-manifolds with the Schouten-van Kampen connection $\widetilde{\nabla}$ satisfying $\widetilde{C}\cdot\widetilde{R}$=0 then either Ricci soliton is shrinking or it has the sectional curvature α^2.*

7. f-Kenmotsu 3-Manifolds Satisfying $\widetilde{Q}\cdot\widetilde{P}$=0 with the Schouten-Van Kampen Connection

We consider f-Kenmotsu 3-manifolds with the Schouten-van Kampen connection $\widetilde{\nabla}$ satisfying the condition $\widetilde{Q}\cdot\widetilde{P}$=0. Then we have:

Theorem 7.1. *A f-Kenmotsu 3-manifolds satisfying $\widetilde{Q}\cdot\widetilde{P}$=0 with the Schouten-van Kampen connection $\widetilde{\nabla}$ is either the Q-curvature tensor is equal to the Riemannian curvature or it has the sectional curvature $\frac{1}{2}(\frac{\vec{scal}}{2}+f^2+2\dot{f})$.*

Proof. The condition $\widetilde{Q}(\xi,\vec{X})\widetilde{P}=0$ reflect that

$$(\widetilde{Q}(\xi,\vec{X})\widetilde{P})(\vec{Y},\vec{Z})\vec{U})=\widetilde{Q}(\xi,\vec{X})\widetilde{P}(\vec{Y},\vec{Z})\vec{U}-\widetilde{P}(\widetilde{Q}(\xi,\vec{X})\vec{Y},\vec{Z})\vec{U}-\widetilde{P}(\vec{Y},\widetilde{Q}(\xi,\vec{X})\vec{Z})\vec{U}-\widetilde{P}(\vec{Y},\vec{Z})\widetilde{Q}(\xi,\vec{X})\vec{U}=0, \tag{7.1}$$

for any vector fields $\vec{X}$, $\vec{Y}$, $\vec{Z}$ and $\vec{U}$ on $\vec{M}$. On the other hand from (1.3), we have

$$\widetilde{Q}(\xi,\vec{X})\widetilde{P}(\vec{Y},\vec{Z})\vec{U}=-\frac{\breve{\psi}}{2}\{g(\vec{X},\widetilde{P}(\vec{Y},\vec{Z})\vec{U})\xi-\eta(\widetilde{P}(\vec{Y},\vec{Z})\vec{U})\vec{X}\}, \tag{7.2}$$

$$\widetilde{P}(\widetilde{Q}(\xi,\vec{X})\vec{Y},\vec{Z})\vec{U}=-\frac{\breve{\psi}}{2}\{g(\vec{X},\vec{Y})\widetilde{P}(\xi,\vec{Y})\vec{Z}-\eta(\vec{Y})\widetilde{P}(\vec{X},\vec{Z})\vec{U}\}, \tag{7.3}$$

$$\widetilde{P}(\vec{Y},\widetilde{Q}(\xi,\vec{X})\vec{Z},\vec{U})=-\frac{\breve{\psi}}{2}\{g(\vec{X},\vec{Z})\widetilde{P}(\vec{Y},\xi)\vec{U}-\eta(\vec{Z})\widetilde{P}(\vec{Y},\vec{X})\vec{U}\}, \tag{7.4}$$

$$\widetilde{P}(\vec{Y},\vec{Z},\widetilde{Q}(\xi,\vec{X})\vec{U})=-\frac{\breve{\psi}}{2}\{g(\vec{X},\vec{U})\widetilde{P}(\vec{Y},\vec{Z})\xi-\eta(\vec{U})\widetilde{P}(\vec{Y},\vec{Z})\vec{X}\}. \tag{7.5}$$

Using (7.2), (7.3), (7.4) and (7.5) in (7.1), we get

$$\frac{\breve{\psi}}{2}\{-g(\vec{X},\widetilde{P}(\vec{Y},\vec{Z})\vec{U})\xi+\eta(\widetilde{P}(\vec{Y},\vec{Z})\vec{U})\vec{X}+g(\vec{X},\vec{Y})\widetilde{P}(\xi,\vec{Y})\vec{Z}-\eta(\vec{Y})\widetilde{P}(\vec{X},\vec{Z})\vec{U}+g(\vec{X},\vec{Z})\widetilde{P}(\vec{Y},\xi)\vec{U}-\eta(\vec{Z})\widetilde{P}(\vec{Y},\vec{X})\vec{U}$$
$$+g(\vec{X},\vec{U})\widetilde{P}(\vec{Y},\vec{Z})\xi-\eta(\vec{U})\widetilde{P}(\vec{Y},\vec{Z})\vec{X}\}=0. \tag{7.6}$$

Taking the inner product of (7.6) with ξ and using (1.1), (2.6), (2.8) and (2.19), which implies

$$\frac{\breve{\psi}}{2}\{g(\vec{X},\vec{R}(\vec{Y},\vec{Z})\vec{U})-\frac{1}{2}(\frac{s\vec{cal}}{2}+f^2+2\dot{f})(g(\vec{X},\vec{Y})g(\vec{Z},\vec{U})-g(\vec{X},\vec{Z})g(\vec{Y},\vec{U}))\}=0. \tag{7.7}$$

It is clear that either $\breve{\psi}$=0, or it has the sectional curvature $\frac{1}{2}(\frac{s\vec{cal}}{2}+f^2+2\dot{f})$. □

This leads to the proof of the Theorem 7.1.

For $\breve{\psi}=\frac{s\vec{cal}}{3}$ then Q-curvature tensor reduces to concircular curvature tensor. Therefore in view of the first result of the above Theorem 7.1 and use of (1.2), we can mention the following:

Corollary 7.2. *A f-Kenmotsu 3-manifolds with the Schouten-van Kampen connection $\widetilde{\nabla}$ satisfying $\widetilde{C}\cdot\widetilde{Ric}$=0 then either concircular curvature tensor is equal to the Remannian curvature tensor or it has the sectional curvature $\frac{1}{2}(f^2+2\dot{f})$.*

Again from Corollary 7.2, and (3.7), we have the following:

Corollary 7.3. *A f-Kenmotsu 3-manifolds with the Schouten-van Kampen connection $\widetilde{\nabla}$ satisfying $\widetilde{C}\cdot\widetilde{Ric}$=0 then either Ricci soliton is shrinking or it has the sectional curvature $\frac{1}{2}(f^2+2\dot{f})$.*

If $0\neq f$=constant (we assume $f=\alpha$), then $\dot{f}$=0. Thus we state the followings:

Corollary 7.4. *An α-Kenmotsu 3-manifolds with the Schouten-van Kampen connection $\widetilde{\nabla}$ satisfying $\widetilde{C}\cdot\widetilde{Ric}$=0 then either concircular curvature tensor is equal to the Remannian curvature tensor or it has the sectional curvature $\frac{\alpha^2}{2}$.*

Corollary 7.5. *An α-Kenmotsu 3-manifolds with the Schouten-van Kampen connection $\widetilde{\nabla}$ satisfying $\widetilde{C}\cdot\widetilde{Ric}$=0 then either Ricci soliton is shrinking or it has the sectional curvature $\frac{\alpha^2}{2}$.*

8. f-Kenmotsu 3-Manifolds Satisfying $\widetilde{Q}(\xi,\vec{X})\cdot\widetilde{Q}$=0 with the Schouten-Van Kampen Connection

In this section we study f-Kenmotsu 3-manifolds with the Schouten-van Kampen connection $\widetilde{\nabla}$ satisfying $\widetilde{Q}(\xi,\vec{X})\cdot\widetilde{Q}$=0 . We have the following:

Theorem 8.1. *A f-Kenmotsu 3-manifolds satisfying $\widetilde{Q}(\xi,\vec{X})\cdot\widetilde{Q}$=0 with the Schouten-van Kampen connection $\widetilde{\nabla}$ then either the Q-curvature tensor is equal to the Riemannian curvature or it has the sectional curvature -$(f^2+\dot{f})$.*

Proof. The condition $(\widetilde{Q}(\xi,\vec{X})\cdot\widetilde{Q})(\vec{Y},\vec{Z})\vec{U}$=0 implies that

$$\widetilde{Q}(\xi,\vec{X})\widetilde{Q}(\vec{Y},\vec{Z})\vec{U}-\widetilde{Q}(\widetilde{Q}(\xi,\vec{X})\vec{Y},\vec{Z})\vec{U}-\widetilde{Q}(\vec{Y},\widetilde{Q}(\xi,\vec{X})\vec{Z})\vec{U}-\widetilde{Q}(\vec{Y},\vec{Z})\widetilde{Q}(\xi,\vec{X})\vec{U}=0, \tag{8.1}$$

for any vector fields $\vec{X}$,$\vec{Y}$,$\vec{Z}$ and $\vec{U}$ on $\vec{M}$.

In view of (2.6) and (2.19), equation (1.3) reduces to

$$\widetilde{Q}(\vec{Y},\vec{Z})\vec{U}=\left\{\frac{s\vec{cal}}{2}+3f^2+2\dot{f}-\frac{\breve{\psi}}{2}\right\}[g(\vec{Z},\vec{U})\vec{Y}-g(\vec{Y},\vec{U})\vec{Z}]$$
$$-\left\{\frac{s\vec{cal}}{2}+3f^2+2\dot{f}\right\}[g(\vec{Z},\vec{U})\eta(\vec{Y})\xi-g(\vec{Y},\vec{U})\eta(\vec{Z})\xi+\eta(\vec{Z})\eta(\vec{U})\vec{Y}-\eta(\vec{Y})\eta(\vec{U})\vec{Z}]. \tag{8.2}$$

Then we have

$$\widetilde{Q}(\xi,\vec{Z})\vec{U}=-\frac{\breve{\psi}}{2}[g(\vec{Z},\vec{U})\xi-\eta(\vec{U})\vec{Z}], \tag{8.3}$$

$$\widetilde{Q}(\xi,\vec{X})\widetilde{Q}(\vec{Y},\vec{Z})\vec{U}=-\frac{\breve{\psi}}{2}[g(\vec{X},\widetilde{Q}(\vec{Y},\vec{Z})\vec{U}))\xi-\eta(\widetilde{Q}(\vec{Y},\vec{Z})\vec{U})\vec{X}], \tag{8.4}$$

$$\widetilde{Q}(\widetilde{Q}(\xi,\vec{X})(\vec{Y},\vec{Z})\vec{U}=-\frac{\breve{\psi}}{2}[g(\vec{X},\vec{Y})\widetilde{Q}(\xi,\vec{Z})\vec{U})-\eta(\vec{Y})\widetilde{Q}(\vec{X},\vec{Z})\vec{U}], \tag{8.5}$$

$$\widetilde{Q}(\vec{Y},\widetilde{Q}(\xi,\vec{X})\vec{Z})\vec{U}=-\frac{\breve{\psi}}{2}[g(\vec{X},\vec{Z})\widetilde{Q}(\vec{Y},\xi)\vec{U}-\eta(\vec{Z})\widetilde{Q}(\vec{Y},\vec{X})\vec{U}], \tag{8.6}$$

$$\widetilde{Q}(\vec{Y},\vec{Z})\widetilde{Q}(\xi,\vec{X})\vec{U}=-\frac{\breve{\psi}}{2}[g(\vec{X},\vec{U})\widetilde{Q}(\vec{Y},\vec{Z})\xi-\eta(\vec{U})\widetilde{Q}(\vec{Y},\vec{Z})\vec{X}]. \tag{8.7}$$

Using (8.4), (8.5), (8.6) and (8.7) in (8.1), we get

$$\frac{\breve{\psi}}{2}[-g(\vec{X},\widetilde{Q}(\vec{Y},\vec{Z})\vec{U}))\xi+\eta(\widetilde{Q}(\vec{Y},\vec{Z})\vec{U})\vec{X}+g(\vec{X},\vec{Y})\widetilde{Q}(\xi,\vec{Z})\vec{U})-\eta(\vec{Y})\widetilde{Q}(\vec{X},\vec{Z})\vec{U}+g(\vec{X},\vec{Z})\widetilde{Q}(\vec{Y},\xi)\vec{U}-\eta(\vec{Z})\widetilde{Q}(\vec{Y},\vec{X})\vec{U} \\ +g(\vec{X},\vec{U})\widetilde{Q}(\vec{Y},\vec{Z})\xi-\eta(\vec{U})\widetilde{Q}(\vec{Y},\vec{Z})\vec{X}]=0. \tag{8.8}$$

Taking the inner product of (8.8) with ξ, and using (8.2) and (8.3) we obtain

$$\frac{\breve{\psi}}{2}[g(\vec{X},\vec{R}(\vec{Y},\vec{Z})\vec{U})+(f^2+\dot{f})[g(\vec{X},\vec{Y})g(\vec{Z},\vec{Y})-g(\vec{X},\vec{Z})g(\vec{Y},\vec{U})]=0. \tag{8.9}$$

This implies that either $\breve{\psi}$=0, or it has the sectional curvature -$(f^2+\dot{f})$.
If $\breve{\psi}$=0, then from (1.3) we get $Q(\vec{X},\vec{Y})\vec{Z}=\vec{R}(\vec{X},\vec{Y})\vec{Z}$. This complete the proof. □

Further if $\breve{\psi}=\frac{\vec{scal}}{3}$ then Q-curvature tensor reduces to concircular curvature tensor. Therefore in view of Theorem 8.1 and use of (1.2), we have the followings:

Corollary 8.2. *A f-Kenmotsu 3-manifolds satisfying $\widetilde{C}(\xi,\vec{X})\cdot\widetilde{C}$=0 with the Schouten-van Kampen connection $\widetilde{\nabla}$ then either the concircular curvature tensor is equal to the Riemannian curvature or it has the sectional curvature -$(f^2+\dot{f})$.*

Corollary 8.3. *A f-Kenmotsu 3-manifolds satisfying $\widetilde{C}(\xi,\vec{X})\cdot\widetilde{C}$=0 with the Schouten-van Kampen connection $\widetilde{\nabla}$ then either Ricci soltion is shrinking or it has the sectional curvature -$(f^2+\dot{f})$.*

If $0\neq f$=constant (we assume $f=\alpha$), then $\dot{f}$=0. Therefore, we have:

Corollary 8.4. *An α-Kenmotsu 3-manifolds satisfying $\widetilde{C}(\xi,\vec{X})\cdot\widetilde{C}$=0 with the Schouten-van Kampen connection $\widetilde{\nabla}$ then either the concircular curvature tensor is equal to the Riemannian curvature or it has the sectional curvature -α^2.*

Corollary 8.5. *An α-Kenmotsu 3-manifolds satisfying $\widetilde{C}(\xi,\vec{X})\cdot\widetilde{C}$=0 with the Schouten-van Kampen connection $\widetilde{\nabla}$ then either Ricci soltion is shrinking or it has the sectional curvature -α^2.*

9. f-Kenmotsu 3-Manifolds Bearing Ricci Soliton Satisfying $((\xi\wedge_{\widetilde{Ric}}\vec{X})\cdot\widetilde{Q})$=0 with the Schouten-Van Kampen Connection

In this segment we study f-Kenmotsu 3-manifolds bearing Ricci soliton satisfying $((\xi\wedge_{\widetilde{Ric}}\vec{X})\cdot\widetilde{Q})$=0 with the Schouten-van Kampen connection $\widetilde{\nabla}$. Therefore, we have the following:

Theorem 9.1. *A f-Kenmotsu 3-manifolds bearing Ricci soliton satisfying $((\xi\wedge_{\widetilde{Ric}}\vec{X})\cdot\widetilde{Q})$=0 with the Schouten-van Kampen connection $\widetilde{\nabla}$ then either Q-curvature tensor is equal to the Riemannian curvature or soliton is steady.*

Proof. The condition $((\xi\wedge_{\widetilde{Ric}}\vec{X})\cdot\widetilde{Q})(\vec{Y},\vec{Z})\vec{U}$=0 implies that

$$\widetilde{Ric}(\vec{X},\widetilde{Q}(\vec{Y},\vec{Z})\vec{U})\xi-\widetilde{Ric}(\xi,\widetilde{Q}(\vec{Y},\vec{Z})\vec{U})\vec{X}-\widetilde{Ric}(\vec{X},\vec{Y})\widetilde{Q}(\xi,\vec{Z})\vec{U} \\ +\widetilde{Ric}(\xi,\vec{Y})\widetilde{Q}(\vec{X},\vec{Z})\vec{U}-\widetilde{Ric}(\vec{X},\vec{Z})\widetilde{Q}(\vec{Y},\xi)\vec{U}+\widetilde{Ric}(\xi,\vec{Z})\widetilde{Q}(\vec{Y},\vec{X})\vec{U} \\ -\widetilde{Ric}(\vec{X},\vec{U})\widetilde{Q}(\vec{Y},\vec{Z})\xi+\widetilde{Ric}(\xi,\vec{U})\widetilde{Q}(\vec{Y},\vec{Z})\vec{X}=0. \tag{9.1}$$

Using (3.4) in (9.1), we get

$$-\lambda g(\vec{X},\widetilde{Q}(\vec{Y},\vec{Z})\vec{U})\xi+\lambda\eta(\widetilde{Q}(\vec{Y},\vec{Z})\vec{U})\vec{X}+\lambda g(\vec{X},\vec{Y})\widetilde{Q}(\xi,\vec{Z})\vec{U} \\ -\lambda\eta(\vec{Y})\widetilde{Q}(\vec{X},\vec{Z})\vec{U}+\lambda g(\vec{X},\vec{Z})\widetilde{Q}(\vec{Y},\xi)\vec{U}-\lambda\eta(\vec{Z})\widetilde{Q}(\vec{Y},\vec{X})\vec{U} \\ +\lambda g(\vec{X},\vec{U})\widetilde{Q}(\vec{Y},\vec{Z},\xi)-\lambda\eta(\vec{U})\widetilde{Q}(\vec{Y},\vec{Z})\vec{X}=0. \tag{9.2}$$

Taking the inner product of (9.2) with ξ and using (8.2) that implies

$$\left\{(\frac{\vec{scal}}{2}+3f^2+2\dot{f}-\frac{\breve{\psi}}{2}\right\}[-\lambda g(\vec{Z},\vec{U})g(\vec{X},\vec{Y})+3\lambda g(\vec{Y},\vec{U})g(\vec{X},\vec{Z})+3\lambda g(\vec{Z},\vec{U})\eta(\vec{Y}) \\ -3\lambda g(\vec{Y},\vec{U})\eta(\vec{Z})]-\left\{\frac{\vec{scal}}{2}+3f^2+2\dot{f}\right\}[-3\lambda g(\vec{Z},\vec{U})\eta(\vec{X})\eta(\vec{Y})+3\lambda g(\vec{Y},\vec{U})\eta(\vec{X})\eta(\vec{Z}) \\ -3\lambda g(\vec{X},\vec{Y})\eta(\vec{Z})\eta(\vec{U})+3\lambda g(\vec{X},\vec{Z})\eta(\vec{Y})\eta(\vec{U})+3\lambda g(\vec{Z},\vec{U})\eta(\vec{Y}) \\ -3\lambda g(\vec{Y},\vec{U})\eta(\vec{Z})]+\frac{\breve{\psi}}{2}[-3\lambda g(\vec{X},\vec{Y})g(\vec{Z},\vec{U})+3\lambda g(\vec{X},\vec{Y})\eta(\vec{Z})\eta(\vec{U}) \\ +3\lambda g(\vec{Z},\vec{U})\eta(\vec{X})\eta(\vec{Y})-3\lambda g(\vec{X},\vec{U})\eta(\vec{Z})\eta(\vec{Y})-3\lambda g(\vec{X},\vec{Z})\eta(\vec{Y})\eta(\vec{U}) \\ +3\lambda g(\vec{X},\vec{Z})g(\vec{Y},\vec{U})+3\lambda g(\vec{X},\vec{U})\eta(\vec{Y})\eta(\vec{Z})-3\lambda g(\vec{Y},\vec{U})\eta(\vec{Y})\eta(\vec{X}) \\ +3\lambda g(\vec{X},\vec{Z})\eta(\vec{Y})\eta(\vec{U})-3\lambda g(\vec{X},\vec{Y})\eta(\vec{Z})\eta(\vec{U})]=0. \tag{9.3}$$

For fix $\vec{U}=\xi$ in (9.3) and on simplification, we get

$$3\lambda\breve{\psi}[g(\vec{X},\vec{Z})\eta(\vec{Y})-g(\vec{X},\vec{Y})\eta(\vec{Z})]=0. \tag{9.4}$$

This implies that either λ=0, or $\breve{\psi}$=0. If λ=0, and $\breve{\psi}\neq 0$, then the Ricci soliton is steady. Whereas if $\lambda\neq 0$ and $\breve{\psi}$=0, so from (1.3), we obtain $Q(\vec{X},\vec{Y})\vec{Z}=\vec{R}(\vec{X},\vec{Y})\vec{Z}$. This complete the proof. □

As per consequence if $\breve{\psi}=\frac{\vec{scal}}{3}$ then Q-curvature tensor reduces to concircular curvature tensor. Therefore in view of Theorem 9.1 and use of (1.2), we have the following:

Corollary 9.2. *A f-Kenmotsu 3-manifolds bearing Ricci soliton satisfying $((\xi\wedge_{\widetilde{Ric}}\vec{X})\cdot\widetilde{C})$=0 with the Schouten-van Kampen connection $\widetilde{\nabla}$ then either concircular curvature tensor is equal to the Riemannian curvature or Ricci soliton is steady.*

10. Examples

Example 10.1. *We consider the* 3*-dimensional manifold* $\vec{M} = \{(u,v,w) \in \Re^3, w \neq 0\}$*, where* (u,v,w) *are the standard coordinate in* $\Re^3$*. Let* $(\vec{e}_1,\vec{e}_2,\vec{e}_3)$ *be linearly independent vector fields at each point of* $\vec{M}$*, given by*

$$\vec{e}_1 = \frac{1}{w}\frac{\partial}{\partial u}, \quad \vec{e}_2 = \frac{1}{w}\frac{\partial}{\partial v}, \quad \vec{e}_3 = -\frac{\partial}{\partial w}$$

are linearly independent at each point of $\vec{M}$*. Let g be the Riemannian metric defined*

$$g(\vec{e}_1,\vec{e}_2) = g(\vec{e}_2,\vec{e}_3) = g(\vec{e}_1,\vec{e}_3) = 0, \quad g(\vec{e}_1,\vec{e}_1) = g(\vec{e}_2,\vec{e}_2) = g(\vec{e}_3,\vec{e}_3) = 1.$$

and given by

$$g = w^2[du\otimes du + dv\otimes dv + \frac{1}{w^2}dw\otimes dw].$$

Let η *be the* 1*-form have the significance*

$$\eta(\vec{U}) = g(\vec{U},\vec{e}_3)$$

for any $\vec{U} \in \Gamma(T\vec{M})$ *and* $\breve{\phi}$ *be the* $(1,1)$*-tensor field defined by*

$$\breve{\phi}\vec{e}_1 = -\vec{e}_2, \ \ \breve{\phi}\vec{e}_2 = \vec{e}_1, \ \ \breve{\phi}\vec{e}_3 = 0.$$

Making use of the linearity of $\breve{\phi}$ *and g we have*

$$\eta(\vec{e}_3) = 1, \quad \breve{\phi}^2(\vec{U}) = -\vec{U} + \eta(\vec{U})\vec{e}_3, \quad g(\breve{\phi}\vec{U},\breve{\phi}\vec{V}) = g(\vec{U},\vec{V}) - \eta(\vec{U})\eta(\vec{V}),$$

for any $\vec{U},\vec{W} \in \Gamma(T\vec{M})$*. Now we can easily calculate*

$$[\vec{e}_1,\vec{e}_2] = 0, \quad [\vec{e}_1,\vec{e}_3] = -\frac{1}{w}\vec{e}_2, \quad [\vec{e}_2,\vec{e}_3] = -\frac{1}{w}\vec{e}_1.$$

The Riemannian connection $\vec{\nabla}$ *of the metric tensor g is given by the Koszul's formula, i. e.,*

$$2g(\vec{\nabla}_{\vec{U}}\vec{V},\vec{W}) = \vec{U}(g(\vec{V},\vec{W})) + \vec{V}(g(\vec{W},\vec{X})) - \vec{W}(g(\vec{U},\vec{V})) - g(\vec{U},[\vec{V},\vec{W}]) - g(\vec{V},[\vec{U},\vec{W}]) + g(\vec{W},[\vec{U},\vec{V}]).$$

Making use of Koszul's formula we get the following:

$$\begin{array}{lll} \vec{\nabla}_{\vec{e}_2}\vec{e}_3 = -\frac{1}{w}\vec{e}_2, & \vec{\nabla}_{\vec{e}_2}\vec{e}_2 = \frac{1}{w}\vec{e}_3, & \vec{\nabla}_{\vec{e}_2}\vec{e}_1 = 0, \\ \vec{\nabla}_{\vec{e}_3}\vec{e}_3 = 0, & \vec{\nabla}_{\vec{e}_3}\vec{e}_2 = 0, & \vec{\nabla}_{\vec{e}_3}\vec{e}_1 = 0, \\ \vec{\nabla}_{\vec{e}_1}\vec{e}_3 = -\frac{1}{w}\vec{e}_1, & \vec{\nabla}_{\vec{e}_1}\vec{e}_2 = 0, & \vec{\nabla}_{\vec{e}_1}\vec{e}_1 = \frac{1}{w}\vec{e}_3. \end{array}$$

Consequently it is clear that $\vec{M}$ *satisfies the condition* $\vec{\nabla}_{\vec{U}}\xi = f\{\vec{U} - \eta(\vec{U})\xi\}$ *for* $\vec{e}_3 = \xi$*, where* $f = -\frac{1}{w}$*. Thus we conclude that* $\vec{M}$ *leads to* f*-Kenmotsu manifold. Also* $f^2 + \dot{f} = \frac{2}{w^2} \neq 0$*. That implies* $\vec{M}$ *is a regular* f*-Kenmotsu* 3*-manifold. Also the Schouten-van Kampen connection* $\widetilde{\nabla}$ *on* $\vec{M}$ *as follows*

$$\begin{array}{lll} \widetilde{\nabla}_{\vec{e}_2}\vec{e}_3 = -(\frac{1}{w}+f)\vec{e}_2, & \widetilde{\nabla}_{\vec{e}_2}\vec{e}_2 = (\frac{1}{w}+f)\vec{e}_3, & \widetilde{\nabla}_{\vec{e}_2}\vec{e}_1 = 0, \\ \widetilde{\nabla}_{\vec{e}_3}\vec{e}_3 = 0, & \widetilde{\nabla}_{\vec{e}_3}\vec{e}_2 = 0, & \widetilde{\nabla}_{\vec{e}_3}\vec{e}_1 = 0, \\ \widetilde{\nabla}_{\vec{e}_1}\vec{e}_3 = -(\frac{1}{w}+f)\vec{e}_1, & \widetilde{\nabla}_{\vec{e}_1}\vec{e}_2 = 0, & \widetilde{\nabla}_{\vec{e}_1}\vec{e}_1 = (\frac{1}{w}+f)\vec{e}_3. \end{array}$$

It is clear that for $\vec{e}_3 = \xi$ *and* $f = -\frac{1}{w}$*, we get* $\widetilde{\nabla}_{\vec{e}_i}\vec{e}_j = 0$ $(1 \leq i,j \leq 3)$*. So the manifold* $\vec{M}$ *is a* f*-Kenmotsu* 3*-manifold with the Schouten-van Kampen connection* $\widetilde{\nabla}$*. Also one can seen that* $\widetilde{R} = 0$*. Thus the manifold* $\vec{M}$ *is a flat manifold with respect to the Schouten-van Kampen connection* $\widetilde{\nabla}$*. Since a flat manifold is a Ricci-flat manifold with respect to the Schouten-van Kampen connection* $\widetilde{\nabla}$*. So from* (3.4)*, we get* $\lambda = 0$*, that is Ricci solition is always steady on regular* f*-Kenmotsu* 3*-manifold with the Schouten-van Kampen connection* $\widetilde{\nabla}$*. In case of Ricci soliton, from* (3.4) *it is sufficient to verify that*

$$\widetilde{Ric}(\vec{e}_i,\vec{e}_i) = -(\lambda + f)g(\vec{e}_i,\vec{e}_i) + f\eta(\vec{e}_i)\eta(\vec{e}_i), \ \ i = 1,2,3. \tag{10.1}$$

It is clear that $\lambda = 0$*, that is Ricci solition is always steady on regular* f*-Kenmotsu* 3*-manifold with the Schouten-van Kampen connection* $\widetilde{\nabla}$*. Hence Proposition* 3.2*, Corollary* 3.6 *and Corollary* 3.7 *are hold.*

Example 10.2. *We consider the* 3*-dimensional manifold* $\vec{M} = \{(u,v,w) \in \Re^3, w \neq 0\}$*, where* (u,v,w) *are the standard coordinate in* $\Re^3$*. Let* $(\vec{e}_1,\vec{e}_2,\vec{e}_3)$ *be linearly independent vector fields at each point of* $\vec{M}$*, given by*

$$\vec{e}_1 = \sin^2 w\frac{\partial}{\partial u}, \quad \vec{e}_2 = \sin^2 w\frac{\partial}{\partial v}, \quad \vec{e}_3 = \sin w\frac{\partial}{\partial w}.$$

are linearly independent at each point of $\vec{M}$*. Let g be the Riemannian metric defined*

$$g(\vec{e}_1,\vec{e}_2) = g(\vec{e}_2,\vec{e}_3) = g(\vec{e}_1,\vec{e}_3) = 0, \quad g(\vec{e}_1,\vec{e}_1) = g(\vec{e}_2,\vec{e}_2) = g(\vec{e}_3,\vec{e}_3) = 1.$$

and given by

$$g=\sin^4 w[du\otimes du+dv\otimes dv+\frac{1}{\sin^2 w}dw\otimes dw].$$

Let η be the 1-form have the significance

$$\eta(\vec{U})=g(\vec{U},\vec{e}_3)$$

for any $\vec{U}\in\Gamma(TM)$ and $\check{\phi}$ be the $(1,1)$-tensor field defined by

$$\check{\phi}\vec{e}_1=-\vec{e}_2,\ \ \check{\phi}\vec{e}_2=\vec{e}_1,\ \ \check{\phi}\vec{e}_3=0.$$

Making use of the linearity of $\check{\phi}$ and g we have

$$\eta(\vec{e}_3)=1,\quad \check{\phi}^2(\vec{U})=-\vec{U}+\eta(\vec{U})\vec{e}_3,\quad g(\check{\phi}\vec{U},\check{\phi}\vec{V})=g(\vec{U},\vec{V})-\eta(\vec{U})\eta(\vec{V}),$$

for any $\vec{U},\vec{W}\in\Gamma(T\vec{M})$. Now we can easily calculate

$$[\vec{e}_1,\vec{e}_2]=0,\quad [\vec{e}_1,\vec{e}_3]=-2\cos w\,\vec{e}_2,\quad [\vec{e}_2,\vec{e}_3]=-2\cos w\,\vec{e}_1.$$

The Riemannian connection $\vec{\nabla}$ of the metric tensor g is given by the Koszul's formula, that is.,

$$2g(\nabla_{\vec{U}}\vec{V},\vec{W})=\vec{U}(g(\vec{V},\vec{W}))+\vec{V}(g(\vec{W},\vec{X}))-\vec{W}(g(\vec{U},\vec{V}))-g(\vec{U},[\vec{V},\vec{W}])-g(\vec{V},[\vec{U},\vec{W}])+g(\vec{W},[\vec{U},\vec{V}]).$$

Making use Koszul's formula we get the following:

$$\begin{array}{lll}\vec{\nabla}_{\vec{e}_2}\vec{e}_3=-2\cos w\vec{e}_2, & \vec{\nabla}_{\vec{e}_2}\vec{e}_2=2\cos w\vec{e}_3, & \vec{\nabla}_{\vec{e}_2}\vec{e}_1=0,\\ \vec{\nabla}_{\vec{e}_3}\vec{e}_3=0, & \vec{\nabla}_{\vec{e}_3}\vec{e}_2=0, & \vec{\nabla}_{\vec{e}_3}\vec{e}_1=0,\\ \vec{\nabla}_{\vec{e}_1}\vec{e}_3=-2\cos w\vec{e}_1, & \vec{\nabla}_{\vec{e}_1}\vec{e}_2=0, & \vec{\nabla}_{\vec{e}_1}\vec{e}_1=2\cos w\vec{e}_3.\end{array}$$

Consequently it is clear that $\vec{M}$ satisfies the condition $\vec{\nabla}_U\xi=f\{\vec{U}-\eta(\vec{U})\xi\}$ for $\vec{e}_3=\xi$, where f=-2$\cos w$. Thus we conclude that $\vec{M}$ leads to f-Kenmotsu manifold. Also $f^2+\dot{f}$=2$\cos w(2\cos w+\tan w)\neq 0$, which implies that $\vec{M}$ is a regular f-Kenmotsu 3-manifold
It is known that

$$\vec{R}(\vec{X},\vec{Y})\vec{Z}=\vec{\nabla}_{\vec{X}}\vec{\nabla}_{\vec{Y}}\vec{Z}-\vec{\nabla}_{\vec{Y}}\vec{\nabla}_{\vec{X}}\vec{Z}-\vec{\nabla}_{[\vec{X},\vec{Y}]}\vec{Z}.$$

Therefore, we find the component of curvature tensor as follows

$$\begin{array}{l}\vec{R}(\vec{e}_2,\vec{e}_3)\vec{e}_3=-2(\sin w+2\cos^2 w)\vec{e}_2,\quad \vec{R}(\vec{e}_3,\vec{e}_2)\vec{e}_2=-2(\sin w+2\cos^2 w)\vec{e}_3,\\ \vec{R}(\vec{e}_1,\vec{e}_3)\vec{e}_3=-2(\sin w+2\cos^2 w)\vec{e}_1,\quad \vec{R}(\vec{e}_3,\vec{e}_1)\vec{e}_1=-2(\sin w+2\cos^2 w)\vec{e}_2,\\ \vec{R}(\vec{e}_3,\vec{e}_1)\vec{e}_2=0,\quad \vec{R}(\vec{e}_1,\vec{e}_2)\vec{e}_2=-4\cos^2 w\vec{e}_1,\quad \vec{R}(\vec{e}_1,\vec{e}_2)\vec{e}_3=0,\\ \vec{R}(\vec{e}_2,\vec{e}_3)\vec{e}_1=0,\quad \vec{R}(\vec{e}_2,\vec{e}_1)\vec{e}_1=4\cos^2 w\vec{e}_3.\end{array}$$

The Schouten-van Kampen connection $\widetilde{\nabla}$ on $\vec{M}$ is given by

$$\begin{array}{lll}\widetilde{\nabla}_{\vec{e}_2}\vec{e}_3=(-2\cos w-f)\vec{e}_2, & \widetilde{\nabla}_{\vec{e}_2}\vec{e}_2=(-2\cos w-f)\vec{e}_3, & \widetilde{\nabla}_{\vec{e}_2}\vec{e}_1=0,\\ \widetilde{\nabla}_{\vec{e}_3}\vec{e}_3=0, & \widetilde{\nabla}_{\vec{e}_3}\vec{e}_2=0, & \widetilde{\nabla}_{\vec{e}_3}\vec{e}_1=0,\\ \widetilde{\nabla}_{\vec{e}_1}\vec{e}_3=(-2\cos w-f)\vec{e}_1, & \widetilde{\nabla}_{\vec{e}_1}\vec{e}_2=0, & \widetilde{\nabla}_{\vec{e}_1}\vec{e}_1=(-2\cos w-f)\vec{e}_3.\end{array}$$

It is clear that for $\vec{e}_3$=ξ and $f=-2\cos w$, we get $\widetilde{\nabla}_{\vec{e}_i}\vec{e}_j$=0 $(1\leq i,j\leq 3)$. So the manifold $\vec{M}$ is a f-Kenmotsu 3-manifold with the Schouten-van Kampen connection $\widetilde{\nabla}$. Also from above curvature component one can be seen that $\widetilde{R}$=0. Thus the manifold $\vec{M}$ is a flat manifold with respect to the Schouten-van Kampen connection $\widetilde{\nabla}$. Since a flat manifold is a Ricci-flat manifold with respect to the Schouten-van Kampen connection $\widetilde{\nabla}$.
In case of Ricci soliton, from (3.4) it is sufficient to verify that

$$\widetilde{Ric}(\vec{e}_i,\vec{e}_i)=-(\lambda+f)g(\vec{e}_i,\vec{e}_i)+f\eta(\vec{e}_i)\eta(\vec{e}_i),\ \ i=1,2,3. \tag{10.2}$$

It is clear that λ=0, that is Ricci solition is always steady on regular f-Kenmotsu 3-manifold with the Schouten-van Kampen connection $\widetilde{\nabla}$. Hence Proposition 3.2, Corollary 3.6 and Corollary 3.7 are hold.

11. Conclusion

In this study, we examine certain new curvature conditions of Q-curvature tensor on f-Kenmotsu 3-manifold admitting the Schouten-van Kampen connection $\widetilde{\nabla}$ and deduce some geometrical results. Also we explore the nature of Ricci soliton.

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