(REFEREED RESEARCH)

PREDICTION OF LARGE DEFORMATION BEHAVIOR OF FABRICS USING LAGRANGIAN FINITE ELEMENT FORMULATION

LAGRANGE SONLU ELEMANLAR YÖNTEMİYLE KUMAŞLARIN BÜYÜK DEFORMASYON DAVRANIŞLARININ HESAPLANMASI

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ABSTRACT

In this study, the Lagrangian Finite Element formulation, covering both the tensile and bending rigidities, is extended to include fabric weights and then exploited for the prediction of the large bending and buckling deformations of fabrics. The deformations results are compared with those published in the literature. It is shown that there is an excellent agreement between the calculated results and the previously published results.

Key Words: Lagrangian finite element, Large deformation, Fabric bending, Fabric buckling.

ÖZET

Bu çalışmada, eğilme ve eksenel deformasyon rijitliğini içeren Lagrange Sonlu Elemanlar formülasyonu, kumaş ağırlığını içerecek şekilde genişletilmiş ve kumaşların büyük eğilme ve burkulma deformasyonlarının hesaplanması için incelenmiştir. Deformasyon sonuçları, literatürdeki sonuçlarla karşılaştırılmıştır. Hesaplanan sonuçların, daha önce yayınlanan sonuçlarla oldukça iyi uyum içinde olduğu gösterilmiştir.

Anahtar Kelimeler: Lagrange Sonlu Elemanlar, Büyük deformasyon, Kumaşların eğilmesi, Kumaşların burkulması.

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1. INTRODUCTION

Bending and buckling play important role in determining the aesthetic appearance and the performance of textile fabrics. As known, textile fabrics show both geometric and material nonlinearities, due to their microstructure and very small bending stiffness. Consequently, fabric deformation problems pose more difficulties, when compared to solid mechanics. For the geometric nonlinearity, in the majority of studies in textile science, either the Bernoulli-Euler theorem or the strain-displacement relationship together with Bernoulli-Euler theorem is adopted.

In the former approach, the governing differential equation is derived using the Bernoulli-Euler theorem. In order to solve the highly nonlinear differential equation, the analytical and numerical solutions such as Runge Kutta, shooting, finite difference and finite element (FE) methods were developed (1-14). In these studies, the fabric deformation is determined in terms of the bending rigidity. The effect of the tensile rigidity is not taken into account.

For the case of using the strain-displacement relationship, FE method appears to be well-established powerful computational tool for handling the beams, plates and shells with general geometries, loadings and boundary conditions in the solid mechanics and in the textile applications it has been successfully applied to drape problems (15-17). It should be noted that all FE formulations require relatively more elements with the load incrementation procedure. FE formulations for the large deformation of beams in the solid mechanics are well covered in the literature (18-21) and they may need to be exploited further for fabric large deformation problems. One of the FE formulations in the solid mechanics is the Lagrangian FE formulation for the large displacements presented by Milner (21), in which axial and bending displacements are functions of a coordinate measured along the elastic curve and it was applied to one problem, a beam under a bending moment at the its free end without resorting to load incrementation procedure, using up to three elements. This formulation requires both the tensile and bending rigidities, thus promising a complete-cover the large deformation behavior of beams.

In this work, the Lagrangian FE is exploited for the fabric large bending and buckling deformation problems. The details of the resulting nonlinear solution matrixes are given for all possible loading conditions including fabric weights. The results are compared with those published previously, where possible.

2. LAGRANGIAN FINITE ELEMENT SOLUTION

Because of the nature of the nonlinear strain-displacement relationship, the displacements u and v are regarded as function of a coordinate s measured along the curved centroidal axis of the deformed member, requiring the use of one extra freedom $\frac{du}{ds}$ at each node (21). Hence, the strain ε and the curvature ρ are defined as

$$\{\varepsilon\} = \begin{Bmatrix} \varepsilon \\ \rho \end{Bmatrix} = \begin{Bmatrix} \dfrac{du}{ds} + \dfrac{1}{2}\left(\dfrac{du}{ds}\right)^2 + \dfrac{1}{2}\left(\dfrac{dv}{ds}\right)^2 \\ \dfrac{d^2v}{ds^2} \Big/ \left[1 - \left(\dfrac{dv}{ds}\right)^2\right]^{1/2} \end{Bmatrix} \tag{1}$$

The stress resultants P and M are obtained as

$$\begin{Bmatrix} P \\ M \end{Bmatrix} = \begin{bmatrix} EA & 0 \\ 0 & EI \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \rho \end{Bmatrix} \tag{2}$$

or

$$\{\sigma\} = [D]\{\varepsilon\} \tag{3}$$

To arrive solution equations, the virtual work principle can be applied and it is stated as (20)

$$\int_{V^e} \delta\varepsilon_{ij}\sigma_{ij}\, dV - \int w\delta v ds - \int f\delta u ds - \sum_{i=1}^{8} Q_i^e\, \delta\Delta_i^e = 0 \tag{4}$$

where V^e denotes the element volume, $w(s)$ is the distributed transverse load or weight, $f(s)$ is the distributed axial load, Q_i^e are the generalized nodal forces, and $\delta\Delta_i^e$ are the virtual generalized nodal displacements of the element defined by

$$\Delta_1^e = u(s_a), \quad \Delta_2^e = v(s_a), \quad \Delta_3^e = \left[\frac{du}{ds}\right]_{s_a}, \quad \Delta_4^e = \left[\frac{dv}{ds}\right]_{s_a} \equiv \theta(s_a)$$

$$\Delta_5^e = u(s_b), \quad \Delta_6^e = v(s_b), \quad \Delta_7^e = \left[\frac{du}{ds}\right]_{s_b}, \quad \Delta_8^e = \left[\frac{dv}{ds}\right]_{s_b} \equiv \theta(s_b) \tag{5}$$

The axial displacement u and the transverse deflection v are interpolated as:

$$u(s) = \sum_{j=1}^{4} u_j \psi_j(s) \tag{6a}$$

$$v(s) = \sum_{j=1}^{4} v_j \phi_j(s) \tag{6b}$$

where ψ_i and ϕ_i are the cubic interpolation functions.

To solve the resulting in non-linear solution equations, the Newton-Raphson procedure is employed and it can be expressed in the general form as:

$$\left[K_T\left(\{\Delta\}^{n-1}\right)\right]\{\Delta\}^n=\{F\}-\left(\left[K^e\right]\left\{\Delta^e\right\}\right)^{n-1} \tag{7}$$

where K_T is the tangential stiffness matrix and K^e is the direct stiffness matrix. Taking appropriate variations of Equation 4 and substituting the stress-strain relation of Equations 1 and 2 into this expression and then the solution matrices in the Newton-Raphson iteration form can be obtained as follows:

$$\begin{bmatrix}
\begin{bmatrix} K_{011}+K_{L11}+K_{\sigma11} & K_{012}+K_{L12}+K_{\sigma12} \\ K_{021}+K_{L21}+K_{\sigma21} & K_{022}+K_{L22}+K_{\sigma22} \end{bmatrix}^{11} & \begin{bmatrix} K_{L11} & K_{L12} \\ K_{L21} & K_{L22} \end{bmatrix}^{12} & \begin{bmatrix} K_{013}+K_{L13}+K_{\sigma13} & K_{014}+K_{L14}+K_{\sigma14} \\ K_{023}+K_{L23}+K_{\sigma23} & K_{024}+K_{L24}+K_{\sigma24} \end{bmatrix}^{11} & \begin{bmatrix} K_{L13} & K_{L14} \\ K_{L23} & K_{L24} \end{bmatrix}^{12} \\
\begin{bmatrix} K_{L11} & K_{L12} \\ K_{L21} & K_{L22} \end{bmatrix}^{21} & \begin{bmatrix} K_{L11}+K_{\sigma11} & K_{L12}+K_{\sigma12} \\ K_{L21}+K_{\sigma21} & K_{L22}+K_{\sigma22} \end{bmatrix}^{22} & \begin{bmatrix} K_{L13} & K_{L14} \\ K_{L23} & K_{L24} \end{bmatrix}^{21} & \begin{bmatrix} K_{L13}+K_{\sigma13} & K_{L14}+K_{\sigma14} \\ K_{L23}+K_{\sigma23} & K_{L24}+K_{\sigma24} \end{bmatrix}^{22} \\
\begin{bmatrix} K_{031}+K_{L31}+K_{\sigma31} & K_{032}+K_{L32}+K_{\sigma32} \\ K_{041}+K_{L41}+K_{\sigma41} & K_{042}+K_{L42}+K_{\sigma42} \end{bmatrix}^{11} & \begin{bmatrix} K_{L31} & K_{L32} \\ K_{L41} & K_{L42} \end{bmatrix}^{12} & \begin{bmatrix} K_{033}+K_{L33}+K_{\sigma33} & K_{034}+K_{L34}+K_{\sigma34} \\ K_{043}+K_{L43}+K_{\sigma43} & K_{044}+K_{L44}+K_{\sigma44} \end{bmatrix}^{11} & \begin{bmatrix} K_{L33} & K_{L34} \\ K_{L43} & K_{L44} \end{bmatrix}^{12} \\
\begin{bmatrix} K_{L31} & K_{L32} \\ K_{L41} & K_{L42} \end{bmatrix}^{21} & \begin{bmatrix} K_{L31}+K_{\sigma31} & K_{L32}+K_{\sigma32} \\ K_{L41}+K_{\sigma41} & K_{L42}+K_{\sigma42} \end{bmatrix}^{22} & \begin{bmatrix} K_{L33} & K_{L34} \\ K_{L43} & K_{L44} \end{bmatrix}^{21} & \begin{bmatrix} K_{L33}+K_{\sigma33} & K_{L34}+K_{\sigma34} \\ K_{L43}+K_{\sigma43} & K_{L44}+K_{\sigma44} \end{bmatrix}^{22}
\end{bmatrix}$$

$$\begin{Bmatrix} u_1 \\ \frac{du_1}{ds} \\ v_1 \\ \frac{dv_1}{ds} \\ u_2 \\ \frac{du_2}{ds} \\ v_2 \\ \frac{dv_2}{ds} \end{Bmatrix}^n = \begin{Bmatrix} F_{X1} \\ 0 \\ \int w\phi_1 ds + F_{Y1} \\ \int w\phi_2 ds + M_1/\cos\theta \\ F_{X2} \\ 0 \\ \int w\phi_3 ds + F_{Y2} \\ \int w\phi_4 ds + M_2/\cos\theta \end{Bmatrix} - \begin{bmatrix}
\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}^{11} & \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}^{12} & \begin{bmatrix} K_{13} & K_{14} \\ K_{23} & K_{24} \end{bmatrix}^{11} & \begin{bmatrix} K_{13} & K_{14} \\ K_{23} & K_{24} \end{bmatrix}^{12} \\
\begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}^{21} & \begin{bmatrix} K_{11} & K_{12} \\ K_{21} & K_{22} \end{bmatrix}^{22} & \begin{bmatrix} K_{13} & K_{14} \\ K_{23} & K_{24} \end{bmatrix}^{21} & \begin{bmatrix} K_{13} & K_{14} \\ K_{23} & K_{24} \end{bmatrix}^{22} \\
\begin{bmatrix} K_{31} & K_{32} \\ K_{41} & K_{42} \end{bmatrix}^{11} & \begin{bmatrix} K_{31} & K_{32} \\ K_{41} & K_{42} \end{bmatrix}^{12} & \begin{bmatrix} K_{33} & K_{34} \\ K_{43} & K_{44} \end{bmatrix}^{11} & \begin{bmatrix} K_{33} & K_{34} \\ K_{43} & K_{44} \end{bmatrix}^{12} \\
\begin{bmatrix} K_{31} & K_{32} \\ K_{41} & K_{42} \end{bmatrix}^{21} & \begin{bmatrix} K_{31} & K_{32} \\ K_{41} & K_{42} \end{bmatrix}^{22} & \begin{bmatrix} K_{33} & K_{34} \\ K_{43} & K_{44} \end{bmatrix}^{21} & \begin{bmatrix} K_{33} & K_{34} \\ K_{43} & K_{44} \end{bmatrix}^{22}
\end{bmatrix} \begin{Bmatrix} u_1 \\ \frac{du_1}{ds} \\ v_1 \\ \frac{dv_1}{ds} \\ u_2 \\ \frac{du_2}{ds} \\ v_2 \\ \frac{dv_2}{ds} \end{Bmatrix}^{n-1} \tag{8}$$

in which n represents the number of the iterations.

The mathematical expresions for the element stiffness matrix K can be obtained as follows:

$$K_{ij}^{11}=\int EA\frac{d\psi_i}{ds}\frac{d\psi_j}{ds}ds+\frac{3}{2}\int\left(EA\frac{du}{ds}\right)\frac{d\psi_i}{ds}\frac{d\psi_j}{ds}ds+\frac{1}{2}\int\left[EA\left(\frac{du}{ds}\right)^2\right]\frac{d\psi_i}{ds}\frac{d\psi_j}{ds}ds \tag{9a}$$

$$K_{ij}^{12}=\frac{1}{2}\int\left(EA\frac{dv}{ds}\right)\frac{d\psi_i}{ds}\frac{d\phi_j}{ds}ds+\frac{1}{2}\int\left(EA\frac{du}{ds}\frac{dv}{ds}\right)\frac{d\psi_i}{ds}\frac{d\phi_j}{ds}ds \tag{9b}$$

$$K_{ij}^{21}=\int\left(EA\frac{dv}{ds}\right)\frac{d\phi_i}{ds}\frac{d\psi_j}{ds}ds+\frac{1}{2}\int\left(EA\frac{du}{ds}\frac{dv}{ds}\right)\frac{d\phi_i}{ds}\frac{d\psi_j}{ds}ds \tag{9c}$$

$$K_{ij}^{22}=\frac{1}{2}\int\left[EA\left(\frac{dv}{ds}\right)^2\right]\frac{d\phi_i}{ds}\frac{d\phi_j}{ds}ds+\int\frac{EI\dfrac{d^2\phi_i}{ds^2}\dfrac{d^2\phi_j}{ds^2}}{\left[1-\left(\dfrac{dv}{ds}\right)^2\right]}+\int\frac{\left(EI\dfrac{dv}{ds}\dfrac{d^2v}{ds^2}\right)\dfrac{d\phi_i}{ds}\dfrac{d^2\phi_j}{ds^2}}{\left[1-\left(\dfrac{dv}{ds}\right)^2\right]^2} \tag{9d}$$

The right side matrix of the Equation 8 is the tangent matrix K_T and the mathematical expresion for K_T can be given by

$$K_T=K_0+K_L+K_\sigma \tag{10}$$

In this expresion, K_0 represents the small displacement stiffness matrix, K_L is the large displacement stiffness matrix, covering both axial and bending deformations, and K_σ is the geometric stiffness matrix. K_0, K_L and K_σ terms can be obtained by

$$K_{0ij}^{11} = \int EA \frac{d\psi_i}{ds}\frac{d\psi_j}{ds} ds \tag{11}$$

$$K_{Lij}^{11} = K_{0ij}^{11} + 2\int\left(EA\frac{du}{ds}\right)\frac{d\psi_i}{ds}\frac{d\psi_j}{ds} ds + \int\left[EA\left(\frac{du}{ds}\right)^2\right]\frac{d\psi_i}{ds}\frac{d\psi_j}{ds} ds \tag{12a}$$

$$K_{Lij}^{12} = 2K_{ij}^{12} \tag{12b}$$

$$K_{Lij}^{21} = 2K_{ji}^{12} \tag{12c}$$

$$K_{Lij}^{22} = \int\left[EA\left(\frac{dv}{ds}\right)^2\right]\frac{d\phi_i}{ds}\frac{d\phi_j}{ds} ds + \int\frac{EI\frac{d^2\phi_i}{ds^2}\frac{d^2\phi_j}{ds^2}}{\left[1-\left(\frac{dv}{ds}\right)^2\right]} ds + \int\frac{\left(EI\frac{dv}{ds}\frac{d^2v}{ds^2}\right)\frac{d\phi_i}{ds}\frac{d^2\phi_j}{ds^2}}{\left[1-\left(\frac{dv}{ds}\right)^2\right]^2} ds +$$

$$\int\frac{\left(EI\frac{dv}{ds}\frac{d^2v}{ds^2}\right)\frac{d^2\phi_i}{ds^2}\frac{d^2\phi_j}{ds^2}}{\left[1-\left(\frac{dv}{ds}\right)^2\right]^2} ds + \int\frac{\left[EI\left(\frac{dv}{ds}\frac{d^2v}{ds^2}\right)^2\right]\frac{d\phi_i}{ds}\frac{d\phi_j}{ds}}{\left[1-\left(\frac{dv}{ds}\right)^2\right]^3} ds \tag{12d}$$

$$K_{\sigma ij}^{11} = \frac{1}{2}\int\left(EA\frac{du}{ds}\right)\frac{d\psi_i}{ds}\frac{d\psi_j}{ds} ds + \frac{1}{2}\int\left[EA\left(\frac{du}{ds}\right)^2\right]\frac{d\psi_i}{ds}\frac{d\psi_j}{ds} ds + \frac{1}{2}\int\left[EA\left(\frac{dv}{ds}\right)^2\right]\frac{d\psi_i}{ds}\frac{d\psi_j}{ds} ds \tag{13a}$$

$$K_{\sigma ij}^{22} = \int\left(EA\frac{du}{ds}\right)\frac{d\phi_i}{ds}\frac{d\phi_j}{ds} ds + \frac{1}{2}\int\left[EA\left(\frac{du}{ds}\right)^2\right]\frac{d\phi_i}{ds}\frac{d\phi_j}{ds} ds + \frac{1}{2}\int\left[EA\left(\frac{dv}{ds}\right)^2\right]\frac{d\phi_i}{ds}\frac{d\phi_j}{ds} ds +$$

$$\int\frac{\left(EI\frac{dv}{ds}\frac{d^2v}{ds^2}\right)\frac{d^2\phi_i}{ds^2}\frac{d\phi_j}{ds} + \left(EI\frac{dv}{ds}\frac{d^2v}{ds^2}\right)\frac{d\phi_i}{ds}\frac{d^2\phi_j}{ds^2} + \left[EI\left(\frac{d^2v}{ds^2}\right)^2\right]\frac{d\phi_i}{ds}\frac{d\phi_j}{ds}}{\left[1-\left(\frac{dv}{ds}\right)^2\right]^2} ds +$$

$$\int\frac{3\left[EI\left(\frac{dv}{ds}\frac{d^2v}{ds^2}\right)^2\right]\frac{d\phi_i}{ds}\frac{d\phi_j}{ds}}{\left[1-\left(\frac{dv}{ds}\right)^2\right]^3} ds \tag{13b}$$

3. NUMERICAL RESULTS AND DISCUSSION

The predicted bending deformation results with the Lagrangian FE formulation were compared with the experimental and theoretical results given in the study of Kang and Yu (16). The mechanical properties of the samples from their study are given in Table 1. The Figures 1 and 2 show the bending deformation results of the wool and cotton woven fabrics, respectively. It is clear from the figures that calculated results are in an excellent agreement with the numerical results of Kang and Yu (16).

The Lagrangian FE can also handle fabric buckling problems under the loading in terms of prescribed displacements, which may be well-suit to textile applications. The buckling deformation results are given in the dimensionless form. In order to bring the Lagrangian FE formulation to the dimensionless form, simple cantilever beam was assumed to be inextensible. Therefore, the tensile rigidity is assigned a very large value in comparison with the bending rigidity.

The buckling deformation results are given by using the same load values and boundary conditions proposed by Clapp and Peng (5). Therefore, the analysis is carried out by considering two boundary conditions, which are called as free-free ends and fixed-fixed ends. In the case of the free-free ends, the boundary conditions for one half of a buckled beam are $\theta = 0$ at $s = 0$ and $d\theta/ds = 0$ at $s = 1$. In the case of the fixed-fixed ends, the boundary conditions are $\theta = 0$ at $s = 0$ and $s = 1$. It should be noted that boundary conditions in the study of Clapp and Peng (5) are converted in Cartesian coordinates for the Lagrangian FE formulation.

For two different boundary conditions, free-free ends and fixed-fixed ends, the right half of deformation shapes of buckled fabrics are displayed in terms of the four different weight values $wL^3/EI = 0$, 2, 8 and 20 in Figures 3a-d and Figures 4a-d, respectively. For each weight values, the fabric buckling deformation shapes are given under the minimum and maximum buckling loads, PL^2/EI. Between these loads, several buckling loads are also applied for the prediction of the fabric buckling deformation behavior, not covered in the literature. Therefore, for comparison purposes, the Lagrangian FE formulation results are compared with those, obtained by author's FE formulation (12), the Galerkin FE formulation. Figures indicate that the Lagrangian FE results are very good agreement with the Galerkin FE results (12).

Table 1. Mechanical properties of the samples from Kang and Yu (16).

Material	Average length (cm)	Unit weight (gf/cm^2)	Tensile rigidity (gf/cm)		Bending rigidity (gfcm^2/cm)	
			warp	weft	warp	weft
Wool	5	0.019	1118.2	759.5	0.083	0.063
Cotton	5	0.0095	2531.6	1413.5	0.068	0.030

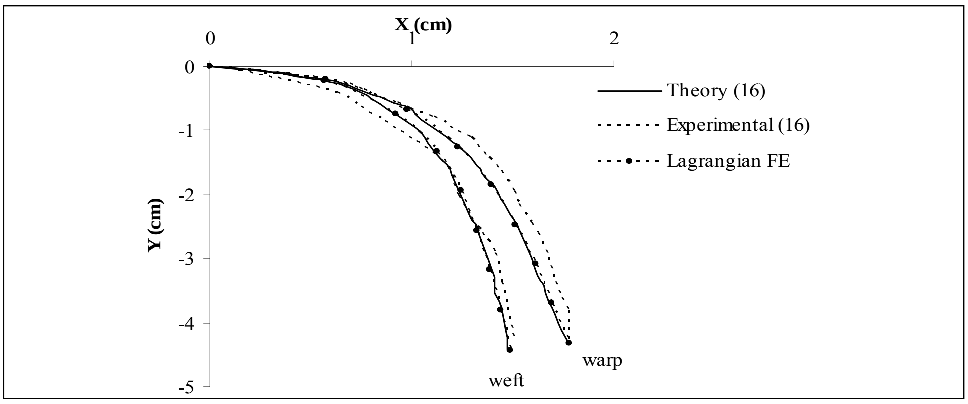

Figure 1. Deformation shapes of the wool fabric.

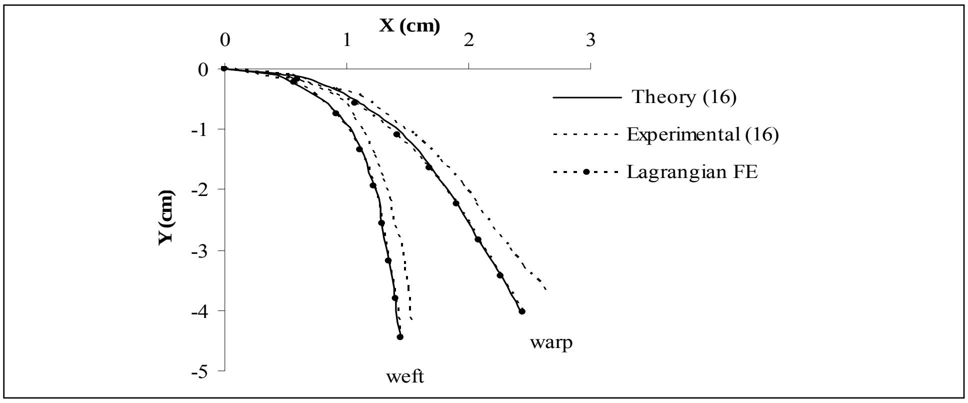

Figure 2. Deformation shapes of the cotton fabric

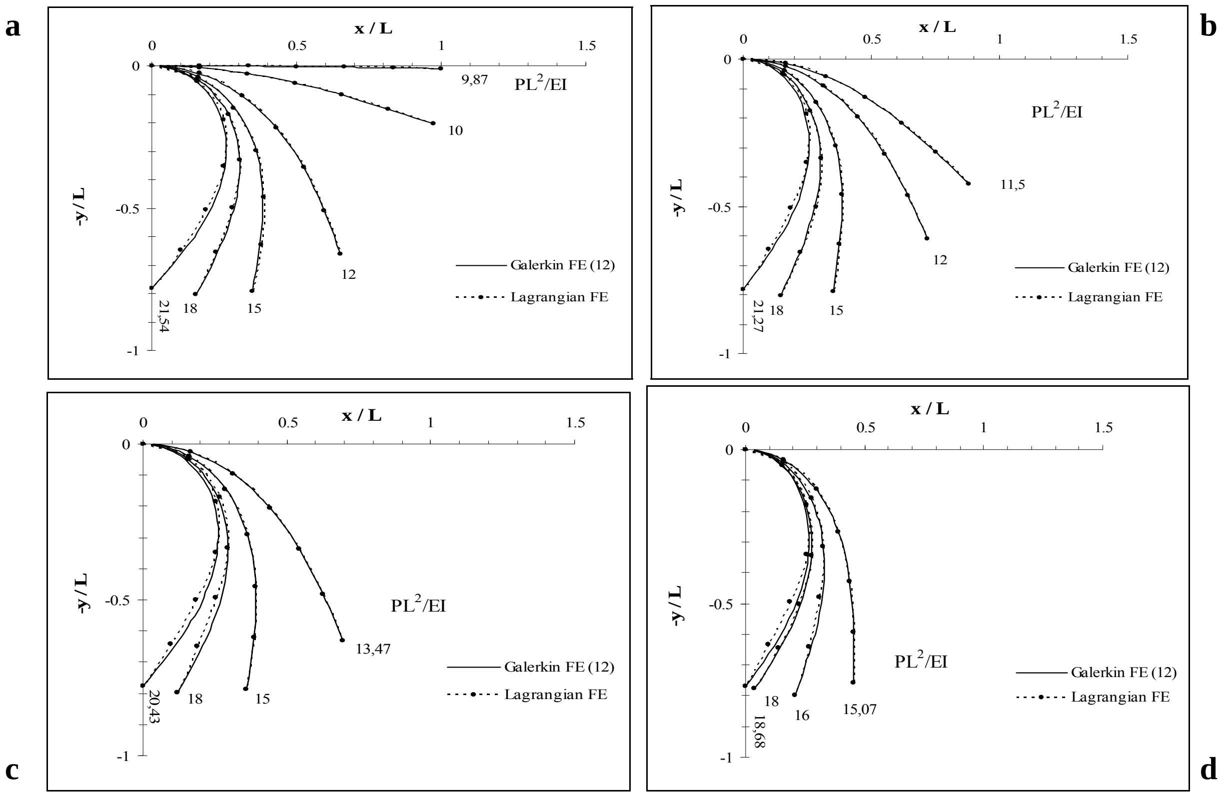

Figure 3. Normalized deformation shapes for a buckled fabric model with free-free ends.

(a) wL^3 / EI =0 (b) wL^3 / EI =2 (c) wL^3 / EI =8 (d) wL^3 / EI =20

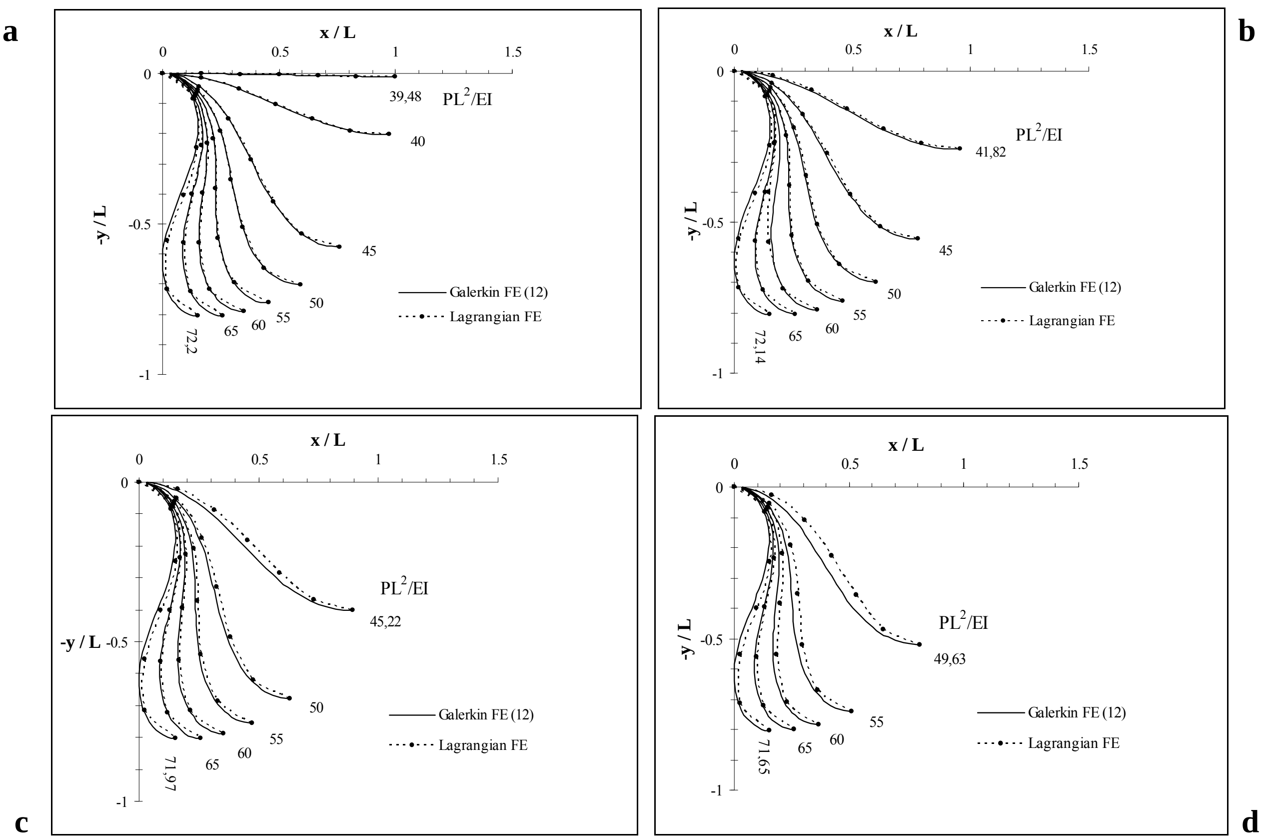

Figure 4. Normalized deformation shapes for a buckled fabric model with fixed-fixed ends.

(a) wL^3 / EI =0 (b) wL^3 / EI =2 (c) wL^3 / EI =8 (d) wL^3 / EI =20

4. CONCLUSION

In this work, the Lagrangian FE formulation (21) is extended to cover fabric weights and then exploited for the prediction of the large bending and buckling deformations of fabrics. The details of the resulting nonlinear solution matrixes are given for all possible loading conditions.

The formulation is applied to the fabric bending and buckling large deformation problems, covered in the literature. The formulation is verified by comparing the deformations results with those published in the literature and the calculated results. It is shown that the formulation, covering both the tensile and bending rigidities, gives a complete-cover the large deformation behavior of fabrics.

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