Research Article GU J Sci 36(4): 1834-1852 (2023) DOI:10.35378/gujs.1056453

Gazi University

Journal of Science

http://dergipark.gov.tr/gujs

# Classes of Population Mean Estimators using Transformed Variables in Double Sampling

Natthapat THONGSAK, Nuanpan LAWSON*

*Department of Applied Statistics, Faculty of Applied Science, King Mongkut's University of Technology North Bangkok, Bangkok, 10800, Thailand*

**Highlights**

- Classes of population mean estimators using the transformation method have been proposed.
- An application on the production of rubber in Thailand was used.
- The proposed estimators perform much better than other existing estimators.

**Article Info**

*Received: 11 Jan 2022*
*Accepted: 26 Sep 2022*

**Keywords**

*Double sampling*
*Ratio estimators*
*Bias*
*Mean square error*

**Abstract**

Transformation techniques have been used to increase the efficiency of estimators in sample surveys. In this paper, some classes of population mean estimators using transformation on an auxiliary variable and on both the auxiliary and study variables have been proposed under double sampling. The proposed estimators' biases and mean square errors are approximated up to the first order. A simulation study and application to a rubber production dataset have been used to illustrate the proposed estimators' performance. The results show that they perform much better than other existing estimators under given conditions.

## 1. INTRODUCTION

Auxiliary information has become interesting data in research for surveys used to estimate possible outcomes for several decades, for example, the amount of agricultural products can be estimated from the area of cultivation in which they are expected. Cochran [1] proposed a usual ratio estimator by utilizing auxiliary information and found that the ratio method is very effective when the study variable and the auxiliary variable are positively correlated with each other. If the two variables are negatively related, the product estimator can be utilized successfully to estimate the population mean of the study variable [2]. To improve the precision of estimation of the study variable's population mean, the auxiliary variable's population parameters such as the correlation coefficient $(\rho)$, coefficient of variation $(C_x)$, coefficient of skewness $(\beta_1(x))$, coefficient of kurtosis $(\beta_2(x))$ were used by several researchers (e.g. [3-16]).

If the population parameters of the auxiliary variable are not obtained, Neyman [17] suggested a double sampling technique which is a beneficial method for estimating population parameters. Let $(x_i, y_i)(i=1,2,3,\ldots,N)$ be the pair of observations of the auxiliary variable $x_i$ and the study variable $y_i$. In the first step of sampling, from a population of size $N$, $n'$ random samples are drawn, to measure the auxiliary variable through simple random sampling without replacement (SRSWOR). In the second phase, a random sample of size $n(n<n')$ is drawn from the first sample of size $n'$ to measure both auxiliary and study variables by using SRSWOR. After that, many researchers also suggested population mean estimators

*Corresponding author, email: nuanpan.n@sci.kmutnb.ac.th

under double sampling. For example, Malik and Tailor [18] modified the ratio estimator under double sampling by modification of the Singh and Tailor [6] estimator which used the correlation coefficient between the auxiliary and study variables. Amin et al. [19] modified estimators under simple random sampling that were suggested by Kadilar and Cingi [8] according to the double sampling scheme. Following the work of Kadilar and Cingi [8,10] and Raja et al. [16], Akingbade and Okafor [20] proposed population mean estimators that covered some existing ratio estimators under double sampling such as the usual ratio estimator, Malik and Tailor's [18] estimator or Amin et al.'s [19] estimators. Other studies utilizing auxiliary information under double sampling can be seen in Mohanty [21], Chand [22], Kiregyera [23,24], Bedi [25], Srivastava et al. [26], Singh [27], Singh et al. [28], Samiuddin and Hanif [29], Singh et al. [30], Singh and Agnihotri [31], Zaman and Kadilar [32,33].

The transformation of variables has been used to increase the performance of population mean estimators by many researchers. Under simple random sampling, Srivenkataramana [34] proposed to increase its efficiency by utilizing the transformation of an auxiliary variable and many authors have also used this technique (e.g., [35-37]). Later, Adewara et al. [38] transformed both study and auxiliary variables to increase the efficiency of the estimator and this strategy was further used in several studies (e.g., [39,40]). Recently, Thongsak and Lawson [41] proposed two classes of estimators using auxiliary variable transformation and the suggested estimators were found to be more efficient than the non-transformed estimators under theoretical study, simulation study, and application to a rubber production dataset. Thongsak and Lawson [41]'s estimators are given as:

$$\hat{\bar{Y}}_{\text{TL1.SRS}} = \bar{y}\left(\frac{A\bar{x}^* + D}{A\bar{X} + D}\right), \tag{1}$$

$$\hat{\bar{Y}}_{\text{TL2.SRS}} = \left[\bar{y} + b\left(\bar{X} - \bar{x}^*\right)\right]\left(\frac{G\bar{x}^* + H}{G\bar{X} + H}\right) \tag{2}$$

where $\bar{x}^* = (1+\pi')\bar{X} - \pi'\bar{x}$ is the sample mean of an auxiliary variable that is transformed under simple random sampling, $\pi' = n/(N-n)$, $b$ is the sample regression coefficient, $(A \neq 0, D, G \neq 0, H)$ are constants or functions of the auxiliary variable. The biases and mean square errors (MSEs) of this estimator are given as:

$$Bias\left(\hat{\bar{Y}}_{\text{TL1.SRS}}\right) \cong -\gamma\pi'\theta_1\bar{Y}\rho C_x C_y, \tag{3}$$

$$Bias\left(\hat{\bar{Y}}_{\text{TL2.SRS}}\right) \cong \gamma\pi'\frac{1}{\bar{X}}\left[\beta\bar{X}S_x\left(\frac{\lambda_{12}}{\rho} - \lambda_{03}\right) - \theta_2 S_{xy}\left(1+\pi\right)\right], \tag{4}$$

$$MSE\left(\hat{\bar{Y}}_{\text{TL1.SRS}}\right) \cong \gamma\bar{Y}^2\left(C_y^2 + \theta_1^2\pi'^2C_x^2 - 2\theta_1\pi'\rho C_x C_y\right), \tag{5}$$

$$MSE\left(\hat{\bar{Y}}_{\text{TL2.SRS}}\right) \cong \gamma\bar{Y}^2\left[C_y^2 + \left(\theta_2 - \beta K\right)^2\pi'^2C_x^2 - 2\left(\theta_2 - \beta K\right)\pi'\rho C_x C_y\right] \tag{6}$$

where $\theta_1 = \frac{A\bar{X}}{A\bar{X}+D}, \theta_2 = \frac{G\bar{X}}{G\bar{X}+H}, K = \frac{\bar{X}}{\bar{Y}}, \beta = \frac{\rho S_y}{S_x}, \lambda_{rs} = \frac{\mu_{rs}}{\mu_{20}^{r/2}\mu_{02}^{s/2}}, \mu_{rs} = \frac{1}{N}\sum_{i=1}^{N}\left(y_i - \bar{Y}\right)^r\left(x_i - \bar{X}\right)^s$,

$\gamma = \frac{1}{n} - \frac{1}{N}$, $\rho$ is the correlation coefficient between the auxiliary and study variables, $C_x, C_y$ are the coefficients of variation of the auxiliary variable and study variable, respectively.

Kumar and Bahl [42] suggested a dual to ratio estimator under the double sampling scheme by using the transformation of an auxiliary variable. Consider the variable transformation,

$$x_i^{*d} = \frac{n'\bar{x}' - nx_i}{n' - n}; i = 1, 2, 3, \ldots, N. \tag{7}$$

The sample mean of the auxiliary and study variables are:

$$\bar{x}^{*d} = \frac{n'\bar{x}' - n\bar{x}}{n' - n} = (1+\pi)\bar{x}' - \pi\bar{x}, \tag{8}$$

$$\bar{y}^{*d} = \frac{n'\bar{y}' - n\bar{y}}{n' - n} = (1+\pi)\bar{y}' - \pi\bar{y}, \tag{9}$$

where $\pi = \frac{n}{n' - n}$, $\bar{y}', \bar{y}$ are the sample means of the study variable based on the first and second phases, respectively, $\bar{x}', \bar{x}$ are the sample means of the auxiliary variable based on the first and second phases, respectively.

Later, many other researchers have also used this transformation under the double sampling scheme. Singh and Choudhury [43] proposed a class of product-cum-dual to ratio estimators by combining the usual product estimator with the dual to ratio estimator and the results based on theoretical and empirical studies concluded that the combined estimator is more exceptional than the conventional ones. Choudhury and Singh [44] suggested a dual to ratio-cum-product estimator by using transformation of two auxiliary variables and the results from comparisons study indicate that the proposed estimator produced the most optimal performance under given conditions. The transformation technique under double sampling can be seen in Boonrodrak [45], Jaroengeratikun and Lawson [46], Kamba et al. [47].

Motivated by Thongsak and Lawson [41], we proposed four classes of estimators of population mean in double sampling by using the transformation technique of an auxiliary variable on both auxiliary and study variables through double sampling. The formulas of biases and MSEs of the proposed estimators are discovered. The proposed estimators are compared with the usual ratio estimators using the MSEs as a criterion via a simulation study and an application to the production of rubber in Thailand, which plays an important role for export from Thailand.

## 2. SOME EXISTING ESTIMATORS

Under double sampling, the usual ratio estimator which was proposed by Neyman [17] is defined as

$$\hat{\hat{Y}}_{\text{Neyman}} = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right). \tag{10}$$

Malik and Tailor [18] made use of the correlation coefficient between the auxiliary and study variables and proposed a modified ratio estimator. Malik and Tailor's [18] estimator is given by

$$\hat{\hat{Y}}_{\text{MT}} = \bar{y}\left(\frac{\bar{x}' + \rho}{\bar{x} + \rho}\right). \tag{11}$$

Motivated by Kadilar and Cingi [8], Amin et al. [19] obtained the population mean estimators through the regression type estimator. Some estimators are represented as

$$\hat{\hat{\bar{Y}}}_{\text{Amin1}} = \left[\bar{y} + b\left(\bar{x}' - \bar{x}\right)\right]\left(\frac{\bar{x}'}{\bar{x}}\right), \tag{12}$$

$$\hat{\hat{\bar{Y}}}_{\text{Amin2}} = \left[\bar{y} + b\left(\bar{x}' - \bar{x}\right)\right]\left(\frac{\bar{x}' + C_x}{\bar{x} + C_x}\right). \tag{13}$$

Akingbade and Okafor [20] suggested a general class of regression type ratio estimators that cover some existing estimators such as the usual ratio estimator, Malik and Tailor's [18] estimator or Amin et al.'s [19] estimators under the double sampling scheme. The estimator is given by

$$\hat{\hat{\bar{Y}}}_{\text{AO}} = \left[\bar{y} + t\left(\bar{x}'^{\delta} - \bar{x}^{\delta}\right)\right]\left(\frac{A\bar{x}' + D}{A\bar{x} + D}\right)^{\alpha} \tag{14}$$

where $t, \delta$ are scalars, $\alpha = -1$ or 0 or 1 for product-type estimator or sample mean or ratio-type estimator, respectively. Note that Akingbade and Okafor [20] considered only the case that $\alpha = 1$.

If $t = 0, \alpha = 1$ and $t = b, \delta = 1, \alpha = 1$, $\hat{\hat{\bar{Y}}}_{AO}$ in Equation (14) is reduced to $\hat{\hat{\bar{Y}}}_{\text{R}}$ in Equation (15) and $\hat{\hat{\bar{Y}}}_{\text{Reg}}$ in Equation (16), respectively

$$\hat{\hat{\bar{Y}}}_{\text{R}} = \bar{y}\left(\frac{A\bar{x}' + D}{A\bar{x} + D}\right), \tag{15}$$

$$\hat{\hat{\bar{Y}}}_{\text{Reg}} = \left[\bar{y} + b\left(\bar{x}' - \bar{x}\right)\right]\left(\frac{G\bar{x}' + H}{G\bar{x} + H}\right). \tag{16}$$

The biases and MSEs of $\hat{\hat{\bar{Y}}}_{\text{R}}$ and $\hat{\hat{\bar{Y}}}_{\text{Reg}}$ are given as follows

$$Bias\left(\hat{\hat{\bar{Y}}}_{\text{R}}\right) \cong \bar{Y}\left(\gamma - \gamma^*\right)\left(\theta_1^2 C_x^2 - \theta_1 \rho C_x C_y\right), \tag{17}$$

$$Bias\left(\hat{\hat{\bar{Y}}}_{\text{Reg}}\right) \cong \bar{Y}\left(\gamma - \gamma^*\right)\left(\left(\theta_2^2 + \beta K \theta_2\right)C_x^2 - \theta_2 \rho C_x C_y\right), \tag{18}$$

$$MSE\left(\hat{\hat{\bar{Y}}}_{\text{R}}\right) \cong \bar{Y}^2\left[\gamma C_y^2 + \left(\gamma - \gamma^*\right)\left(\theta_1^2 C_x^2 - 2\theta_1 \rho C_x C_y\right)\right], \tag{19}$$

$$MSE\left(\hat{\hat{\bar{Y}}}_{\text{Reg}}\right) \cong \bar{Y}^2\left[\gamma C_y^2 + \left(\gamma - \gamma^*\right)\left[\left(\theta_2 + \beta K\right)^2 C_x^2 - 2\left(\theta_2 + \beta K\right)\rho C_x C_y\right]\right] \tag{20}$$

where $\gamma^* = \frac{1}{n'} - \frac{1}{N}$.

Some members of $\hat{\hat{\bar{Y}}}_{\text{R}}$ and $\hat{\hat{\bar{Y}}}_{\text{Reg}}$ utilizing some auxiliary parameters are shown in Table 1.

***Table 1.*** *Some existing estimators*

| Estimator | A or G | D or H |
|---|---|---|
| $\hat{\bar{Y}}_{\text{R1}} = \hat{\bar{Y}}_{\text{Neyman}} = \bar{y}\left(\frac{\bar{x}'}{\bar{x}}\right)$ | 1 | 0 |
| $\hat{\bar{Y}}_{\text{R2}} = \bar{y}\left(\frac{\bar{x}'+C_x}{\bar{x}+C_x}\right)$ | 1 | $C_x$ |
| $\hat{\bar{Y}}_{\text{R3}} = \hat{\bar{Y}}_{\text{MT}} = \bar{y}\left(\frac{\bar{x}'+\rho}{\bar{x}+\rho}\right)$ | 1 | $\rho$ |
| $\hat{\bar{Y}}_{\text{Reg1}} = \hat{\bar{Y}}_{\text{Amin1}} = \left[\bar{y}+b\left(\bar{x}'-\bar{x}\right)\right]\left(\frac{\bar{x}'}{\bar{x}}\right)$ | 1 | 0 |
| $\hat{\bar{Y}}_{\text{Reg2}} = \hat{\bar{Y}}_{\text{Amin2}} = \left[\bar{y}+b\left(\bar{x}'-\bar{x}\right)\right]\left(\frac{\bar{x}'+C_x}{\bar{x}+C_x}\right)$ | 1 | $C_x$ |
| $\hat{\bar{Y}}_{\text{Reg3}} = \left[\bar{y}+b\left(\bar{x}'-\bar{x}\right)\right]\left(\frac{\bar{x}'+\rho}{\bar{x}+\rho}\right)$ | 1 | $\rho$ |

## 3. THE ESTIMATION OF POPULATION MEAN

### 3.1. Proposed General Classes of Estimators

We propose to modify the estimators proposed by Thongsak and Lawson [41], which used the transformed auxiliary variable under simple random sampling, by using the transformation technique on an auxiliary variable and on both auxiliary and study variables to accelerate the efficacy of the estimators under double sampling, which is a useful technique when some auxiliary information is not known. The proposed population mean estimators are:

$$\hat{\bar{Y}}_{\text{N1}} = \bar{y}\left(\frac{A\bar{x}^{*d}+D}{A\bar{x}'+D}\right)^{\alpha}, \tag{21}$$

$$\hat{\bar{Y}}_{\text{N2}} = \left[\bar{y}+b\left(\bar{x}'-\bar{x}^{*d}\right)\right]\left(\frac{G\bar{x}^{*d}+H}{G\bar{x}'+H}\right)^{\alpha}, \tag{22}$$

$$\hat{\bar{Y}}_{\text{N3}} = \bar{y}^{*d}\left(\frac{A\bar{x}'+D}{A\bar{x}^{*d}+D}\right)^{\alpha}, \tag{23}$$

$$\hat{\bar{Y}}_{\text{N4}} = \left[\bar{y}^{*d}+b\left(\bar{x}'-\bar{x}^{*d}\right)\right]\left(\frac{G\bar{x}'+H}{G\bar{x}^{*d}+H}\right)^{\alpha} \tag{24}$$

where $\bar{x}^{*d} = (1+\pi)\bar{x}'-\pi\bar{x}$ and $\bar{y}^{*d} = (1+\pi)\bar{y}'-\pi\bar{y}$ are the transformed sample means of an auxiliary variable and study variable under double sampling, respectively. $\alpha$ is a constant to be determined in order to acquire the minimum mean square error.

To acquire the biases and MSEs of the proposed estimators, these notations are used:
$\varepsilon_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$ then $\bar{y} = (1+\varepsilon_0)\bar{Y}$, $\varepsilon_0 = \frac{\bar{y}-\bar{Y}}{\bar{Y}}$ then $\bar{y} = (1+\varepsilon_0)\bar{Y}$, $\varepsilon_1 = \frac{\bar{x}-\bar{X}}{\bar{X}}$ then $\bar{x} = (1+\varepsilon_1)\bar{X}$, $\varepsilon_2 = \frac{\bar{x}'-\bar{X}}{\bar{X}}$ then $\bar{x}' = (1+\varepsilon_2)\bar{X}$ and $\bar{x}^{*d} = (1+\varepsilon_2+\pi\varepsilon_2-\pi\varepsilon_1)\bar{X}$, $\varepsilon_3 = \frac{\bar{y}'-\bar{Y}}{\bar{Y}}$ then $\bar{y}' = (1+\varepsilon_3)\bar{Y}$ and

$\bar{y}^{*d} = (1+\varepsilon_3+\pi\varepsilon_3-\pi\varepsilon_0)\bar{Y}$ such that $E(\varepsilon_0)=E(\varepsilon_1)=E(\varepsilon_2)=E(\varepsilon_3)=0$, $E(\varepsilon_0^2)=\gamma C_y^2$, $E(\varepsilon_1^2)=\gamma C_x^2$, $E(\varepsilon_2^2)=\gamma^* C_x^2$, $E(\varepsilon_3^2)=\gamma^* C_y^2$, $E(\varepsilon_0\varepsilon_1)=\gamma\rho C_x C_y$, $E(\varepsilon_0\varepsilon_2)=\gamma^*\rho C_x C_y$, $E(\varepsilon_0\varepsilon_3)=\gamma^* C_y^2$, $E(\varepsilon_1\varepsilon_2)=\gamma^* C_x^2$, $E(\varepsilon_1\varepsilon_3)=E(\varepsilon_2\varepsilon_3)=\gamma^*\rho C_x C_y$.

Rewriting Equation (21) to (24) in terms of $\varepsilon_0,\ \varepsilon_1, \varepsilon_2,\ \varepsilon_3$, we have:

$$\hat{\hat{Y}}_{\mathrm{N1}} = (1+\varepsilon_0)\bar{Y}\left(\frac{(A\bar{X}+D)+(\varepsilon_2+\pi\varepsilon_2-\pi\varepsilon_1)A\bar{X}}{(A\bar{X}+D)+\varepsilon_2 A\bar{X}}\right)^{\alpha}, \tag{25}$$

$$\hat{\hat{Y}}_{\mathrm{N2}} = \left[(1+\varepsilon_0)\bar{Y}+b(\varepsilon_1-\varepsilon_2)\pi\bar{X}\right]\left(\frac{(G\bar{X}+H)+(\varepsilon_2+\pi\varepsilon_2-\pi\varepsilon_1)G\bar{X}}{(G\bar{X}+H)+\varepsilon_2 G\bar{X}}\right)^{\alpha}, \tag{26}$$

$$\hat{\hat{Y}}_{\mathrm{N3}} = (1+\varepsilon_3+\pi\varepsilon_3-\pi\varepsilon_0)\bar{Y}\left(\frac{(A\bar{X}+D)+\varepsilon_2 A\bar{X}}{(A\bar{X}+D)+(\varepsilon_2+\pi\varepsilon_2-\pi\varepsilon_1)A\bar{X}}\right)^{\alpha}, \tag{27}$$

$$\hat{\hat{Y}}_{\mathrm{N4}} = \left[(1+\varepsilon_3+\pi\varepsilon_3-\pi\varepsilon_0)\bar{Y}+b(\pi\varepsilon_1-\pi\varepsilon_2)\bar{X}\right]\left(\frac{(G\bar{X}+H)+\varepsilon_2 G\bar{X}}{(G\bar{X}+H)+(\varepsilon_2+\pi\varepsilon_2-\pi\varepsilon_1)G\bar{X}}\right)^{\alpha}. \tag{28}$$

The biases and MSEs of the proposed estimators approximated up to the first degree are:

$$Bias\left(\hat{\hat{Y}}_{\mathrm{N1}}\right) \cong \bar{Y}(\gamma-\gamma^*)\left[\frac{\alpha(\alpha-1)}{2}\pi^2\theta_1^2 C_x^2-\alpha\pi\theta_1\rho C_x C_y\right], \tag{29}$$

$$Bias\left(\hat{\hat{Y}}_{\mathrm{N2}}\right) \cong \bar{Y}(\gamma-\gamma^*)\left[\left(\frac{\alpha(\alpha-1)}{2}\theta_2^2-\alpha\beta K\theta_2\right)\pi^2 C_x^2-\alpha\pi\theta_2\rho C_x C_y\right], \tag{30}$$

$$Bias\left(\hat{\hat{Y}}_{\mathrm{N3}}\right) \cong \pi^2\bar{Y}(\gamma-\gamma^*)\left(\frac{\alpha(\alpha+1)}{2}\theta_1^2 C_x^2-\alpha\theta_1\rho C_x C_y\right), \tag{31}$$

$$Bias\left(\hat{\hat{Y}}_{\mathrm{N4}}\right) \cong \pi^2\bar{Y}(\gamma-\gamma^*)\left[\left(\frac{\alpha(\alpha+1)}{2}\theta_2^2+\alpha\beta K\theta_2\right)C_x^2-\alpha\theta_2\rho C_x C_y\right], \tag{32}$$

$$MSE\left(\hat{\hat{Y}}_{\mathrm{N1}}\right) \cong \bar{Y}^2\left[\gamma C_y^2+(\gamma-\gamma^*)\left(\alpha^2\theta_1^2\pi^2 C_x^2-2\alpha\theta_1\pi\rho C_x C_y\right)\right], \tag{33}$$

$$MSE\left(\hat{\hat{Y}}_{\mathrm{N2}}\right) \cong \bar{Y}^2\left[\gamma C_y^2+(\gamma-\gamma^*)\left\{(\alpha\theta_2-\beta K)^2\pi^2 C_x^2-2(\alpha\theta_2-\beta K)\pi\rho C_x C_y\right\}\right], \tag{34}$$

$$MSE\left(\hat{\hat{Y}}_{\mathrm{N3}}\right) \cong \bar{Y}^2\left[\gamma^* C_y^2+\pi^2(\gamma-\gamma^*)\left(C_y^2+\alpha^2\theta_1^2 C_x^2-2\alpha\theta_1\rho C_x C_y\right)\right], \tag{35}$$

$$MSE\left(\hat{\bar{Y}}_{N4}\right) \cong \bar{Y}^2\left[\gamma^* C_y^2 + \pi^2\left(\gamma-\gamma^*\right)\left(C_y^2 + \left(\alpha\theta_2+\beta K\right)^2 C_x^2 - 2\left(\alpha\theta_2+\beta K\right)\rho C_x C_y\right)\right]. \tag{36}$$

Note that the difference $E(b)-\beta$ was omitted [49].

**3.2. Optimum Choice of Scalar** $\alpha$

To find the minimum value of MSE in Equation (21) to (24), we find the optimum value of $\alpha$ by taking a partial derivative of MSE in Equation (33) to (36) with respect to $\alpha$ and equating it to zero.

1) Optimum value of $\alpha$ for $\hat{\bar{Y}}_{N1}$ is given as:

$$\alpha = \frac{\rho C_y}{\pi\theta_1 C_x} = \alpha_1^{opt}, \ (\text{say}). \tag{37}$$

Substituting Equation (37) into Equation (21), the optimum of $\hat{\bar{Y}}_{N1}$ is given as:

$$\hat{\bar{Y}}_{N1}^{opt} = \bar{y}\left(\frac{A\bar{x}^{*d}+D}{A\bar{x}'+D}\right)^{\frac{\rho C_y}{\pi\theta_1 C_x}}. \tag{38}$$

Substituting Equation (37) into Equation (33), the MSE of the optimum estimator $\hat{\bar{Y}}_{N1}^{opt}$ is given as:

$$MSE_{\min}\left(\hat{\bar{Y}}_{N1}^{opt}\right) \cong \bar{Y}^2 C_y^2\left[\gamma-\left(\gamma-\gamma^*\right)\rho^2\right]. \tag{39}$$

2) Optimum value of $\alpha$ for $\hat{\bar{Y}}_{N2}$ is given as:

$$\alpha = \frac{\rho C_y + \pi\beta K C_x}{\pi\theta_2 C_x} = \alpha_2^{opt}, \ (\text{say}). \tag{40}$$

Substituting Equation (40) into Equation (22), the optimum of $\hat{\bar{Y}}_{N2}$ is given as:

$$\hat{\bar{Y}}_{N2}^{opt} = \left[\bar{y} + b\left(\bar{x}' - \bar{x}^{*d}\right)\right]\left(\frac{G\bar{x}^{*d}+H}{G\bar{x}'+H}\right)^{\frac{\rho C_y+\pi\beta K C_x}{\pi\theta_2 C_x}}. \tag{41}$$

Substituting Equation (40) into Equation (34), the MSE of the optimum estimator $\hat{\bar{Y}}_{N2}^{opt}$ is given as:

$$MSE_{\min}\left(\hat{\bar{Y}}_{N2}^{opt}\right) \cong \bar{Y}^2 C_y^2\left[\gamma-\left(\gamma-\gamma^*\right)\rho^2\right]. \tag{42}$$

3) Optimum value of $\alpha$ for $\hat{\bar{Y}}_{N3}$ is given as:

$$\alpha = \frac{\rho C_y}{\theta_1 C_x} = \alpha_3^{opt}, \ (\text{say}). \tag{43}$$

Substituting Equation (43) into Equation (23), the optimum of $\hat{\hat{\bar{Y}}}_{\mathrm{N3}}$ is given as:

$$\hat{\hat{\bar{Y}}}_{\mathrm{N3}}^{\mathrm{opt}} = \bar{y}^{*d}\left(\frac{A\bar{x}' + D}{A\bar{x}^{*d} + D}\right)^{\frac{\rho C_y}{\theta_1 C_x}}. \tag{44}$$

Substituting Equation (43) into Equation (35), the MSE of the optimum estimator $\hat{\hat{\bar{Y}}}_{\mathrm{N3}}^{\mathrm{opt}}$ is:

$$MSE_{\min}\left(\hat{\hat{\bar{Y}}}_{\mathrm{N3}}^{\mathrm{opt}}\right) \cong \bar{Y}^2 C_y^2\left[\gamma^* + \left(1-\rho^2\right)\left(\gamma - \gamma^*\right)\pi^2\right]. \tag{45}$$

4) Optimum value of $\alpha$ for $\hat{\hat{\bar{Y}}}_{\mathrm{N4}}$ is given as:

$$\alpha = \frac{\rho C_y - \beta K C_x}{\theta_2 C_x} = \alpha_4^{opt}, \ (\text{say}). \tag{46}$$

Substituting Equation (46) into Equation (24), the optimum of $\hat{\hat{\bar{Y}}}_{\mathrm{N4}}$ is:

$$\hat{\hat{\bar{Y}}}_{\mathrm{N4}}^{\mathrm{opt}} = \left[\bar{y}^{*d} + b\left(\bar{x}' - \bar{x}^{*d}\right)\right]\left(\frac{G\bar{x}' + H}{G\bar{x}^{*d} + H}\right)^{\frac{\rho C_y - \beta K C_x}{\theta_2 C_x}}. \tag{47}$$

Substituting Equation (46) into Equation (36), the MSE of the optimum estimator $\hat{\hat{\bar{Y}}}_{\mathrm{N4}}^{\mathrm{opt}}$ is:

$$MSE_{\min}\left(\hat{\hat{\bar{Y}}}_{\mathrm{N4}}^{\mathrm{opt}}\right) \cong \bar{Y}^2 C_y^2\left[\gamma^* + \left(1-\rho^2\right)\left(\gamma - \gamma^*\right)\pi^2\right]. \tag{48}$$

### 3.3. Some Members of Proposed Estimators

Some members of the proposed estimators are shown in Table 2.

***Table 2.*** *Some members of the proposed estimators*

| Estimator | $\alpha$ | A or G | D or H |
|---|---|---|---|
| $\hat{\hat{\bar{Y}}}_{\mathrm{N11}} = \bar{y}\left(\frac{\bar{x}^{*d}}{\bar{x}'}\right)$ | 1 | 1 | 0 |
| $\hat{\hat{\bar{Y}}}_{\mathrm{N12}} = \bar{y}\left(\frac{\bar{x}^{*d} + C_x}{\bar{x}' + C_x}\right)$ | 1 | 1 | $C_x$ |
| $\hat{\hat{\bar{Y}}}_{\mathrm{N13}} = \bar{y}\left(\frac{\bar{x}^{*d} + \rho}{\bar{x}' + \rho}\right)$ | 1 | 1 | $\rho$ |
| $\hat{\hat{\bar{Y}}}_{\mathrm{N21}} = \left[\bar{y} + b\left(\bar{x}' - \bar{x}^{*d}\right)\right]\left(\frac{\bar{x}^{*d}}{\bar{x}'}\right)$ | 1 | 1 | 0 |
| $\hat{\hat{\bar{Y}}}_{\mathrm{N22}} = \left[\bar{y} + b\left(\bar{x}' - \bar{x}^{*d}\right)\right]\left(\frac{\bar{x}^{*d} + C_x}{\bar{x}' + C_x}\right)$ | 1 | 1 | $C_x$ |
| $\hat{\hat{\bar{Y}}}_{\mathrm{N23}} = \left[\bar{y} + b\left(\bar{x}' - \bar{x}^{*d}\right)\right]\left(\frac{\bar{x}^{*d} + \rho}{\bar{x}' + \rho}\right)$ | 1 | 1 | $\rho$ |

| | | | |
|---|---|---|---|
| $\hat{\hat{Y}}_{\mathrm{N31}} = \bar{y}^{*d}\left(\dfrac{\bar{x}'}{\bar{x}^{*d}}\right)$ | 1 | 1 | 0 |
| $\hat{\hat{Y}}_{\mathrm{N32}} = \bar{y}^{*d}\left(\dfrac{\bar{x}'+C_x}{\bar{x}^{*d}+C_x}\right)$ | 1 | 1 | $C_x$ |
| $\hat{\hat{Y}}_{\mathrm{N33}} = \bar{y}^{*d}\left(\dfrac{\bar{x}'+\rho}{\bar{x}^{*d}+\rho}\right)$ | 1 | 1 | $\rho$ |
| $\hat{\hat{Y}}_{\mathrm{N41}} = \left[\bar{y}^{*d} + b\left(\bar{x}' - \bar{x}^{*d}\right)\right]\left(\dfrac{\bar{x}'}{\bar{x}^{*d}}\right)$ | 1 | 1 | 0 |
| $\hat{\hat{Y}}_{\mathrm{N42}} = \left[\bar{y}^{*d} + b\left(\bar{x}' - \bar{x}^{*d}\right)\right]\left(\dfrac{\bar{x}'+C_x}{\bar{x}^{*d}+C_x}\right)$ | 1 | 1 | $C_x$ |
| $\hat{\hat{Y}}_{\mathrm{N43}} = \left[\bar{y}^{*d} + b\left(\bar{x}' - \bar{x}^{*d}\right)\right]\left(\dfrac{\bar{x}'+\rho}{\bar{x}^{*d}+\rho}\right)$ | 1 | 1 | $\rho$ |

## 4. EFFICIENCY COMPARISONS

We compared the efficacies of the proposed estimators with the usual ratio estimator ($\hat{\hat{Y}}_{\mathrm{Neyman}}$) and Akingbade and Okafor's [20] estimators ($\hat{\hat{Y}}_{\mathrm{R}}$ and $\hat{\hat{Y}}_{\mathrm{Reg}}$) through the double sampling scheme, and also with the usual ratio estimator under the simple random sampling scheme ($\hat{\hat{Y}}_{\mathrm{Ratio}} = \bar{y}\left(\dfrac{\bar{X}}{\bar{x}}\right)$) [1] in case of sample size equal to $n'$, the MSEs are used as a criterion.

1) The proposed estimator ($\hat{\hat{Y}}_{\mathrm{N1}}^{\mathrm{opt}}$) is superior to the existing estimators under the certain conditions as below:

$$\left(\omega C_x - \rho C_y\right)^2 > 0. \tag{49}$$

We can see that from Equation (49), the inequality is always satisfied. We can conclude that the proposed estimator $\hat{\hat{Y}}_{\mathrm{N1}}^{\mathrm{opt}}$ is more efficient than $\hat{\hat{Y}}_{\mathrm{Neyman}}$, $\hat{\hat{Y}}_{\mathrm{R}}$, and $\hat{\hat{Y}}_{\mathrm{Reg}}$.

2) The proposed estimator ($\hat{\hat{Y}}_{\mathrm{N2}}^{\mathrm{opt}}$) proved more excellent to the existing estimators under the certain condition as follows:

$$\left(\omega C_x - \rho C_y\right)^2 > 0. \tag{50}$$

We can see that from Equation (50), the inequality is always true. We can conclude that the proposed estimator $\hat{\hat{Y}}_{\mathrm{N2}}^{\mathrm{opt}}$ is better than $\hat{\hat{Y}}_{\mathrm{Neyman}}$, $\hat{\hat{Y}}_{\mathrm{R}}$, and $\hat{\hat{Y}}_{\mathrm{Reg}}$.

3) The proposed estimator ($\hat{\hat{Y}}_{\mathrm{N3}}^{\mathrm{opt}}$) performed more highly than the existing estimators under the certain condition as follows:

$$\pi^2 < \frac{C_y^2 + \omega^2 C_x^2 - 2\omega\rho C_x C_y}{C_y^2\left(1-\rho^2\right)}. \quad (51)$$

If $\omega = 1$, then the proposed estimator ($\hat{\hat{Y}}_{\mathrm{N3}}^{\mathrm{opt}}$) is more efficient than $\hat{\hat{Y}}_{\mathrm{Neyman}}$.

If $\omega = \theta_1$, then the proposed estimator ($\hat{\hat{Y}}_{\mathrm{N3}}^{\mathrm{opt}}$) is more efficient than $\hat{\hat{Y}}_{\mathrm{R}}$.

If $\omega = \theta_2 + \beta K$, then the proposed estimator ($\hat{\hat{Y}}_{\mathrm{N3}}^{\mathrm{opt}}$) is more efficient than $\hat{\hat{Y}}_{\mathrm{Reg}}$.

4) The proposed estimator ($\hat{\hat{Y}}_{\mathrm{N4}}^{\mathrm{opt}}$) was superior to the existing estimators under the certain condition as follows:

$$\pi^2 < \frac{C_y^2 + \omega^2 C_x^2 - 2\omega\rho C_x C_y}{C_y^2\left(1-\rho^2\right)}. \quad (52)$$

If $\omega = 1$, then the proposed estimator ($\hat{\hat{Y}}_{\mathrm{N4}}^{\mathrm{opt}}$) is superior to $\hat{\hat{Y}}_{\mathrm{Neyman}}$.

If $\omega = \theta_1$, then the proposed estimator ($\hat{\hat{Y}}_{\mathrm{N4}}^{\mathrm{opt}}$) is superior to $\hat{\hat{Y}}_{\mathrm{R}}$.

If $\omega = \theta_2 + \beta K$, then the proposed estimator ($\hat{\hat{Y}}_{\mathrm{N4}}^{\mathrm{opt}}$) is superior to $\hat{\hat{Y}}_{\mathrm{Reg}}$.

5) The proposed estimators ($\hat{\hat{Y}}_{\mathrm{N1}}^{\mathrm{opt}}$ and $\hat{\hat{Y}}_{\mathrm{N2}}^{\mathrm{opt}}$) are better than the usual ratio estimator in simple random sampling ($\hat{\hat{Y}}_{\mathrm{Ratio}}$) under the certain condition:

$$\frac{N\left(n'-n\right)}{n\left(N-n'\right)} < \frac{C_x^2 - 2\rho C_x C_y}{C_y^2\left(1-\rho^2\right)}. \quad (53)$$

6) The proposed estimators ($\hat{\hat{Y}}_{\mathrm{N3}}^{\mathrm{opt}}$ and $\hat{\hat{Y}}_{\mathrm{N4}}^{\mathrm{opt}}$) proved more excellent to the usual ratio estimator in simple random sampling ($\hat{\hat{Y}}_{\mathrm{Ratio}}$) under the certain condition as follows:

$$\frac{N\left(n'-n\right)}{n\left(N-n'\right)} < \frac{C_x^2 - 2\rho C_x C_y}{\left(1-\left(1-\rho^2\right)\pi^2\right)C_y^2}. \quad (54)$$

Next to compare among the proposed estimators, according to Equations (39), (42), (45), and (48), the MSEs of $\hat{\hat{Y}}_{\mathrm{N1}}^{\mathrm{opt}}$ and $\hat{\hat{Y}}_{\mathrm{N2}}^{\mathrm{opt}}$ are equal and the MSEs of $\hat{\hat{Y}}_{\mathrm{N3}}^{\mathrm{opt}}$ and $\hat{\hat{Y}}_{\mathrm{N4}}^{\mathrm{opt}}$ are equal. Then, we compare the proposed estimators ($\hat{\hat{Y}}_{\mathrm{N1}}^{\mathrm{opt}}$ and $\hat{\hat{Y}}_{\mathrm{N2}}^{\mathrm{opt}}$) with the proposed estimators ($\hat{\hat{Y}}_{\mathrm{N3}}^{\mathrm{opt}}$ and $\hat{\hat{Y}}_{\mathrm{N4}}^{\mathrm{opt}}$).

7) The proposed estimators ($\hat{\hat{Y}}_{\mathrm{N3}}^{\mathrm{opt}}$ and $\hat{\hat{Y}}_{\mathrm{N4}}^{\mathrm{opt}}$) proved more excellent to the proposed estimators ($\hat{\hat{Y}}_{\mathrm{N1}}^{\mathrm{opt}}$ and $\hat{\hat{Y}}_{\mathrm{N2}}^{\mathrm{opt}}$) under the certain condition as follows:

$$n < \frac{n'}{2}. \quad (55)$$

## 5. SIMULATION STUDIES

We compared the performance of the proposed estimators with the existing estimators via simulation studies. A bivariate normal distribution is generated by:

$N = 2000,\ \bar{Y} = 50,\ \bar{X} = 40,\ C_y = 0.8,\ C_x = 2.3,\quad \rho = 0.7,\ 0.9.$

Samples of sizes $n' = 200,\ 400,$ and $800$ units are selected from $N$ population units in the first phase of sampling using the SRSWOR scheme and sample of sizes $n$ ($n = 60$ and $80$ for $n' = 200$, $n = 120$ and $160$ for $n' = 400$, and $n = 240$ and $320$ for $n' = 800$) are selected from $n'$ units using the SRSWOR scheme in the second phase of sampling, repeated 10,000 times. The biases and MSEs of the proposed and existing estimators are calculated by the following formulas:

$$Bias\left(\hat{\bar{Y}}\right) = \frac{1}{10{,}000}\sum_{i=1}^{10{,}000}\left|\hat{\bar{Y}}_i - \bar{Y}\right|, \tag{56}$$

$$MSE\left(\hat{\bar{Y}}\right) = \frac{1}{10{,}000}\sum_{i=1}^{10{,}000}\left(\hat{\bar{Y}}_i - \bar{Y}\right)^2. \tag{57}$$

The biases and MSEs of the proposed and existing estimators are presented in Tables 3-4.

According to Table 3, we found that the proposed estimators proved superior to all existing estimators in terms of smaller biases and MSEs for all correlation levels. When the correlation between the auxiliary and study variables is increased, the biases and MSEs of all estimators are decreased. Similar results have been found in Table 4, we found a big improvement in the proposed estimators using a transformation technique compared to the existing estimators especially the proposed estimators using both transformed auxiliary and study variables. The optimum proposed estimators performed much better than other estimators for all situations, especially a class of $\hat{\bar{Y}}_{\mathrm{N3}}^{\mathrm{opt}}$ and $\hat{\bar{Y}}_{\mathrm{N4}}^{\mathrm{opt}}$ which transformed both auxiliary and study variables.

Moreover, when we compared the proposed estimators to the usual ratio estimator ($\hat{\bar{Y}}_{\mathrm{Ratio}}$) under the simple random sampling scheme when the sample size is equal to $n'$, all of the proposed estimators were also more efficient than $\hat{\bar{Y}}_{\mathrm{Ratio}}$ for all situations.

***Table 3.*** *Biases and MSEs of the proposed and existing estimators when* $\rho = 0.7$

| Estimator | $n' = 200$ | | | | $n' = 400$ | | | | $n' = 800$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | n=60 | | n=80 | | n=120 | | n=160 | | n=240 | | n=320 | |
| | Bias | MSE | Bias | MSE | Bias | MSE | Bias | MSE | Bias | MSE | Bias | MSE |
| $\hat{\bar{Y}}_{\text{Ratio}}$ (SRS) | 5.54 | 54.90 | 5.54 | 54.90 | 3.54 | 20.87 | 3.54 | 20.87 | 2.14 | 7.28 | 2.14 | 7.28 |
| $\hat{\bar{Y}}_{\text{R1}} = \hat{\bar{Y}}_{\text{Neyman}}$ | 17.48 | 344667.82 | 8.25 | 189.10 | 6.67 | 85.00 | 5.19 | 47.84 | 4.47 | 34.24 | 3.55 | 20.99 |
| $\hat{\bar{Y}}_{\text{R2}}$ | 9.99 | 548.70 | 7.57 | 339.54 | 6.14 | 69.47 | 4.81 | 40.14 | 4.15 | 29.03 | 3.30 | 18.03 |
| $\hat{\bar{Y}}_{\text{R3}}$ | 11.26 | 1482.11 | 8.04 | 186.28 | 6.52 | 80.24 | 5.08 | 45.52 | 4.37 | 32.64 | 3.48 | 20.09 |
| $\hat{\bar{Y}}_{\text{Reg1}}$ | 21.37 | 472802.42 | 10.24 | 288.14 | 8.39 | 135.29 | 6.48 | 75.12 | 5.63 | 54.47 | 4.46 | 33.18 |
| $\hat{\bar{Y}}_{\text{Reg2}}$ | 12.67 | 863.48 | 9.52 | 512.71 | 7.83 | 113.75 | 6.09 | 64.68 | 5.30 | 47.45 | 4.21 | 29.25 |
| $\hat{\bar{Y}}_{\text{Reg3}}$ | 14.09 | 2197.39 | 10.01 | 284.31 | 8.22 | 128.70 | 6.37 | 71.98 | 5.53 | 52.33 | 4.38 | 31.99 |
| $\hat{\bar{Y}}_{\text{N11}}^{\text{opt}}$ | 3.29 | 16.95 | 2.92 | 13.27 | 2.23 | 7.88 | 1.98 | 6.20 | 1.50 | 3.53 | 1.30 | 2.68 |
| $\hat{\bar{Y}}_{\text{N12}}^{\text{opt}}$ | 3.29 | 16.90 | 2.91 | 13.23 | 2.23 | 7.87 | 1.98 | 6.19 | 1.50 | 3.53 | 1.30 | 2.68 |
| $\hat{\bar{Y}}_{\text{N13}}^{\text{opt}}$ | 3.29 | 16.94 | 2.91 | 13.26 | 2.23 | 7.87 | 1.98 | 6.20 | 1.50 | 3.53 | 1.30 | 2.68 |
| $\hat{\bar{Y}}_{\text{N21}}^{\text{opt}}$ | 3.43 | 18.47 | 3.07 | 14.91 | 2.27 | 8.19 | 2.04 | 6.57 | 1.52 | 3.61 | 1.32 | 2.77 |
| $\hat{\bar{Y}}_{\text{N22}}^{\text{opt}}$ | 3.42 | 18.33 | 3.05 | 14.73 | 2.27 | 8.17 | 2.03 | 6.54 | 1.51 | 3.60 | 1.32 | 2.76 |
| $\hat{\bar{Y}}_{\text{N23}}^{\text{opt}}$ | 3.42 | 18.43 | 3.07 | 14.87 | 2.27 | 8.19 | 2.04 | 6.57 | 1.52 | 3.61 | 1.32 | 2.76 |
| $\hat{\bar{Y}}_{\text{N31}}^{\text{opt}}$ | 2.35 | 8.76 | 2.49 | 9.76 | 1.60 | 4.00 | 1.70 | 4.52 | 1.02 | 1.63 | 1.10 | 1.87 |
| $\hat{\bar{Y}}_{\text{N32}}^{\text{opt}}$ | 2.36 | 8.77 | 2.49 | 9.77 | 1.60 | 4.01 | 1.70 | 4.52 | 1.02 | 1.63 | 1.10 | 1.87 |
| $\hat{\bar{Y}}_{\text{N33}}^{\text{opt}}$ | 2.35 | 8.76 | 2.49 | 9.76 | 1.60 | 4.00 | 1.70 | 4.52 | 1.02 | 1.63 | 1.10 | 1.87 |
| $\hat{\bar{Y}}_{\text{N41}}^{\text{opt}}$ | 2.40 | 9.14 | 2.53 | 10.13 | 1.61 | 4.07 | 1.71 | 4.59 | 1.03 | 1.65 | 1.10 | 1.89 |
| $\hat{\bar{Y}}_{\text{N42}}^{\text{opt}}$ | 2.40 | 9.11 | 2.53 | 10.11 | 1.61 | 4.06 | 1.71 | 4.58 | 1.03 | 1.64 | 1.10 | 1.89 |
| $\hat{\bar{Y}}_{\text{N43}}^{\text{opt}}$ | 2.40 | 9.13 | 2.53 | 10.12 | 1.61 | 4.07 | 1.71 | 4.58 | 1.03 | 1.65 | 1.10 | 1.89 |

***Table 4.*** *Biases and MSEs of the proposed and existing estimators when* $\rho = 0.9$

| Estimator | $n' = 200$ | | | | $n' = 400$ | | | | $n' = 800$ | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | n=60 | | n=80 | | n=120 | | n=30 | | n=40 | | n=120 | |
| | Bias | MSE | Bias | MSE | Bias | MSE | Bias | MSE | Bias | MSE | Bias | MSE |
| $\hat{\hat{Y}}_{\text{Ratio}}$ (SRS) | 4.87 | 42.79 | 4.87 | 42.79 | 3.12 | 16.20 | 3.12 | 16.20 | 1.89 | 5.65 | 1.89 | 5.65 |
| $\hat{\hat{Y}}_{\text{R1}} = \hat{\hat{Y}}_{\text{Neyman}}$ | 11.16 | 8275.57 | 7.36 | 156.48 | 5.93 | 67.01 | 4.64 | 38.03 | 3.95 | 26.87 | 3.16 | 16.52 |
| $\hat{\hat{Y}}_{\text{R2}}$ | 8.86 | 623.50 | 6.64 | 153.73 | 5.41 | 53.71 | 4.26 | 31.35 | 3.64 | 22.32 | 2.91 | 13.93 |
| $\hat{\hat{Y}}_{\text{R3}}$ | 9.73 | 653.20 | 7.11 | 179.23 | 5.73 | 61.72 | 4.49 | 35.41 | 3.83 | 25.05 | 3.06 | 15.50 |
| $\hat{\hat{Y}}_{\text{Reg1}}$ | 15.23 | 13850.58 | 10.00 | 283.68 | 8.22 | 130.23 | 6.37 | 72.44 | 5.51 | 52.28 | 4.37 | 31.83 |
| $\hat{\hat{Y}}_{\text{Reg2}}$ | 12.45 | 1185.83 | 9.21 | 287.29 | 7.66 | 109.27 | 5.97 | 62.21 | 5.18 | 45.40 | 4.12 | 27.97 |
| $\hat{\hat{Y}}_{\text{Reg3}}$ | 13.45 | 1181.85 | 9.74 | 322.43 | 8.01 | 121.93 | 6.22 | 68.46 | 5.38 | 49.56 | 4.28 | 30.31 |
| $\hat{\hat{Y}}_{\text{N11}}^{\text{opt}}$ | 2.71 | 11.49 | 2.52 | 9.97 | 1.79 | 5.06 | 1.67 | 4.42 | 1.15 | 2.09 | 1.06 | 1.77 |
| $\hat{\hat{Y}}_{\text{N12}}^{\text{opt}}$ | 2.70 | 11.40 | 2.51 | 9.88 | 1.78 | 5.04 | 1.67 | 4.40 | 1.15 | 2.09 | 1.06 | 1.76 |
| $\hat{\hat{Y}}_{\text{N13}}^{\text{opt}}$ | 2.70 | 11.46 | 2.51 | 9.94 | 1.79 | 5.05 | 1.67 | 4.41 | 1.15 | 2.09 | 1.06 | 1.77 |
| $\hat{\hat{Y}}_{\text{N21}}^{\text{opt}}$ | 2.96 | 14.19 | 2.81 | 12.95 | 1.88 | 5.66 | 1.78 | 5.08 | 1.19 | 2.24 | 1.10 | 1.93 |
| $\hat{\hat{Y}}_{\text{N22}}^{\text{opt}}$ | 2.94 | 13.94 | 2.78 | 12.63 | 1.88 | 5.61 | 1.77 | 5.03 | 1.18 | 2.23 | 1.09 | 1.92 |
| $\hat{\hat{Y}}_{\text{N23}}^{\text{opt}}$ | 2.95 | 14.11 | 2.80 | 12.85 | 1.88 | 5.64 | 1.78 | 5.06 | 1.19 | 2.23 | 1.10 | 1.93 |
| $\hat{\hat{Y}}_{\text{N31}}^{\text{opt}}$ | 2.22 | 7.72 | 2.27 | 8.09 | 1.49 | 3.50 | 1.53 | 3.69 | 0.94 | 1.36 | 0.97 | 1.45 |
| $\hat{\hat{Y}}_{\text{N32}}^{\text{opt}}$ | 2.22 | 7.74 | 2.27 | 8.11 | 1.49 | 3.50 | 1.53 | 3.69 | 0.94 | 1.36 | 0.97 | 1.45 |
| $\hat{\hat{Y}}_{\text{N33}}^{\text{opt}}$ | 2.22 | 7.72 | 2.27 | 8.09 | 1.49 | 3.50 | 1.53 | 3.69 | 0.94 | 1.36 | 0.97 | 1.45 |
| $\hat{\hat{Y}}_{\text{N41}}^{\text{opt}}$ | 2.31 | 8.41 | 2.36 | 8.81 | 1.52 | 3.63 | 1.56 | 3.82 | 0.94 | 1.39 | 0.98 | 1.48 |
| $\hat{\hat{Y}}_{\text{N42}}^{\text{opt}}$ | 2.30 | 8.38 | 2.36 | 8.77 | 1.52 | 3.62 | 1.56 | 3.82 | 0.94 | 1.39 | 0.98 | 1.48 |

| | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\hat{\bar{Y}}_{N43}^{opt}$ | 2.31 | 8.40 | 2.36 | 8.80 | 1.52 | 3.63 | 1.56 | 3.82 | 0.94 | 1.39 | 0.98 | 1.48 |

## 6. APPLICATION ON RUBBER PRODUCTION DATA IN THAILAND

To compare the efficiencies of the proposed estimators with the existing estimators under double sampling, we applied all estimators to rubber production data in Thailand following Thongsak and Lawson [41]. The data was collected from 746 Thai districts [49]. We assigned the yield of rubber (kilogram/hectare) in each district as a study variable and the cultivated area (hectare) in each district as an auxiliary variable.

The description of population parameters is summarized as follows:

$$N = 746,\ \bar{Y} = 1129.98,\ \bar{X} = 4{,}900.92,\ C_y = 0.29,\ C_x = 1.70,\ \rho = 0.59$$

In the first phase of sampling, a sample of size $n' = 150$ is selected from the population size $N = 746$ using the SRSWOR scheme. After that, in the second phase of sampling a sample of size $n = 30$ is selected from $n' = 150$ using the SRSWOR scheme. The biases and MSEs of the proposed and existing estimators are presented in Table 5.

***Table 5.*** *The biases and MSEs of the proposed and existing estimators when applied to rubber production data in Thailand*

| Estimator | Bias | MSE |
|---|---|---|
| $\hat{\bar{Y}}_{\text{Cochran}}$ (SRS) | 100.40 | 10080.00 |
| $\hat{\bar{Y}}_{\text{R1}} = \hat{\bar{Y}}_{\text{Neyman}}$ | 83.69 | 7003.54 |
| $\hat{\bar{Y}}_{\text{R2}}$ | 83.68 | 7002.65 |
| $\hat{\bar{Y}}_{\text{R3}}$ | 83.68 | 7003.17 |
| $\hat{\bar{Y}}_{\text{Reg1}}$ | 85.67 | 7340.15 |
| $\hat{\bar{Y}}_{\text{Reg2}}$ | 85.67 | 7339.23 |
| $\hat{\bar{Y}}_{\text{Reg3}}$ | 85.67 | 7339.77 |
| $\hat{\bar{Y}}_{\text{N11}}^{\text{opt}}$ | 68.16 | 4645.41 |
| $\hat{\bar{Y}}_{\text{N12}}^{\text{opt}}$ | 68.16 | 4645.42 |
| $\hat{\bar{Y}}_{\text{N13}}^{\text{opt}}$ | 68.16 | 4645.42 |
| $\hat{\bar{Y}}_{\text{N21}}^{\text{opt}}$ | 68.15 | 4644.31 |
| $\hat{\bar{Y}}_{\text{N22}}^{\text{opt}}$ | 68.15 | 4644.32 |
| $\hat{\bar{Y}}_{\text{N23}}^{\text{opt}}$ | 68.15 | 4644.31 |
| $\hat{\bar{Y}}_{\text{N31}}^{\text{opt}}$ | 31.13 | 969.32 |
| $\hat{\bar{Y}}_{\text{N32}}^{\text{opt}}$ | 31.13 | 969.32 |
| $\hat{\bar{Y}}_{\text{N33}}^{\text{opt}}$ | 31.13 | 969.32 |
| $\hat{\bar{Y}}_{\text{N41}}^{\text{opt}}$ | 31.18 | 972.25 |
| $\hat{\bar{Y}}_{\text{N42}}^{\text{opt}}$ | 31.18 | 972.25 |
| $\hat{\bar{Y}}_{\text{N43}}^{\text{opt}}$ | 31.18 | 972.25 |

The results in Table 5 showed that the proposed estimators have smaller biases and MSEs than the existing estimators. We can see that the proposed estimators performed much better than the existing estimators for estimating population mean under the double sampling scheme, especially the classes of $\hat{\hat{Y}}_{\mathrm{N3}}^{\mathrm{opt}}$ and $\hat{\hat{Y}}_{\mathrm{N4}}^{\mathrm{opt}}$ performed the best for this situation.

## 7. CONCLUSION

The general classes of population mean estimators using transformation on an auxiliary variable and on both auxiliary and study variables have been proposed under double sampling to improve the efficiency of ratio estimators. The biases and mean square errors for all proposed estimators were obtained along with the optimum value of $\alpha$ to make the minimum mean square error for the proposed estimators have been investigated.

The results from the simulation studies showed that all proposed estimators performed very well which gave smaller biases and mean square errors as opposed to all existing estimators including the usual ratio estimator. The biases and mean square errors decreased when the correlation increased. Among the optimum proposed estimators $\hat{\hat{Y}}_{\mathrm{N3}}^{\mathrm{opt}}$ and $\hat{\hat{Y}}_{\mathrm{N4}}^{\mathrm{opt}}$ which comprised of both the transformed auxiliary and study variables performed a lot better than $\hat{\hat{Y}}_{\mathrm{N1}}^{\mathrm{opt}}$ and $\hat{\hat{Y}}_{\mathrm{N2}}^{\mathrm{opt}}$ which had only the auxiliary variable transformed. Similar results had shown the application to rubber data, the proposed estimators were more excellent than all existing estimators especially for the optimum proposed estimators $\hat{\hat{Y}}_{\mathrm{N3}}^{\mathrm{opt}}$ and $\hat{\hat{Y}}_{\mathrm{N4}}^{\mathrm{opt}}$ which transformed both auxiliary and study variables. We can see that using the transformation on both auxiliary and study variables can result in a substantial improvement by reducing the mean square errors at least four times more than the proposed estimators using transformation on the auxiliary variable only. The optimum proposed estimators $\hat{\hat{Y}}_{\mathrm{N3}}^{\mathrm{opt}}$ and $\hat{\hat{Y}}_{\mathrm{N4}}^{\mathrm{opt}}$ perfomed better than $\hat{\hat{Y}}_{\mathrm{N1}}^{\mathrm{opt}}$ and $\hat{\hat{Y}}_{\mathrm{N2}}^{\mathrm{opt}}$ when the second phase sample size is less than half of the first phase sample size. The benefits of the proposed estimators using the transformation technique under the double sampling scheme could be an alternative approach to apply in some applications. For example, in business, economics, social studies, agricultural to estimate the population mean or population total of the variable of interest when the population mean of the auxiliary variable is unknown which usually occurs in the real world. Using the transformation technique could ameliorate the efficacy of the estimators which is ideal to gain more accuracy in the estimation stage under sample survey data. For future studies, the proposed estimators can also be applied to more complex survey designs such as stratified sampling and cluster sampling to achieve the utmost efficacy for the population mean estimators.

## ACKNOWLEDGEMENTS

This research was funded by National Science, Research and Innovation Fund (NSRF), and King Mongkut's University of Technology North Bangkok with Contract no. KMUTNB-FF-65-28.

## CONFLICTS OF INTEREST

No conflict of interest was declared by the authors.

## REFERENCES

[1] Cochran, W. G., "Sampling Techniques", 3rd edition, India: Wiley Eastern Limited, (1940).

[2] Murthy, M. N., "Sampling Theory and Methods", India: Statistical Publishing Society, (1967).

[3] Sisodia, B. V., Dwivedi, V. K., "A modified ratio estimator using coefficient of variation of auxiliary variable", Journal Indian Society of Agricultural Statistics, 8(1): 20-25, (1981).

[4] Upadhyaya, L. N., Singh, H. P., "Use of transformed auxiliary variable in estimating the finite population mean", Biometrical Journal, 41(5): 627-636, (1999).

[5] Singh, G. N., “On the improvement of product method of estimation in sample surveys”, Journal of the Indian Society of Agricultural Statistics, 56(3): 267-275, (2003).

[6] Singh, H. P., Tailor, R., “Use of known correlation co-efficient in estimating the finite population means”, Statistics in Transition, 6(4): 555-560, (2003).

[7] Singh, H. P., Tailor, R., Tailor, R., and Kakran, M. S., “An improved estimator of population mean using power transformation”, Journal of the Indian Society of Agricultural Statistics, 58(2): 223-230, (2004).

[8] Kadilar, C., Cingi, H., “Ratio estimators in simple random sampling”, Applied Mathematics and Computation, 151(3): 893-902, (2004).

[9] Kadilar, C., Cingi, H., “New ratio estimators using correlation coefficient”, Interstat, 4: 1-11, (2006).

[10] Kadilar, C., Cingi, H., “An improvement in estimating the population mean by using the correlation coefficient”, Hacettepe Journal of Mathematics and Statistics, 35(1): 103-109, (2006).

[11] Yan, Z., Tian, B., “Ratio method to the mean estimation using coefficient of skewness of auxiliary variable”, Proceeding of the ICICA 2010, Part II, CCIS 106: 103-111, (2010).

[12] Subramani, J., Kumarapandiyan, G., “A class of almost unbiased modified ratio estimators for population mean with known population parameters”, Elixir Statistics, 44: 7411-7415, (2012).

[13] Subramani, J., Kumarapandiyan, G., “Estimation of population mean using coefficient of variation and median of an auxiliary variable”, International Journal of Probability and Statistics, 1: 111-118, (2012).

[14] Subramani, J., Kumarapandiyan, G., “Estimation of population mean using known median and coefficient of skewness”, American Journal of Mathematics and Statistics, 2: 101-107, (2012).

[15] Subramani, J., Kumarapandiyan, G., “Modified ratio estimators using known median and co-efficient of kurtosis”, American Journal of Mathematics and Statistics, 2: 95-100, (2012).

[16] Raja, T. A., Subair, M., Maqbool, S., and Hakak, A., “Enhancing the mean ratio estimator for estimating population mean using conventional parameters”, International Journal of Mathematics and Statistics Invention, 5(1): 58-61, (2017).

[17] Neyman, J., “Contribution to the theory of sampling human populations”, Journal of the American Statistical Association, 33: 101-116, (1938).

[18] Malik, K. A., Tailor, R., “Ratio type estimator of population mean in double sampling”, International Journal of Advanced Mathematics and Statistics, 1(1): 34-39, (2013).

[19] Amin, M. N. U., Shahbaz, M. Q., and Kadilar, C., “Ratio estimators for population mean using robust regression in double sampling”, Gazi University Journal of Science, 29(4): 793-798, (2016).

[20] Akingbade, T. J., Okafor, F. C., “A class of ratio type estimators in double sampling using an auxiliary variable with some known population parameters”, Journal of the Nigerian Statistical Association, 30: 14-29, (2018).

[21] Mohanty, S., "Combination of regression and ratio estimate", Journal of the Indian Statistical Association, 5: 16-19, (1967).

[22] Chand, L., "Some Ratio-type Estimators based on two or more Auxiliary Variables", Phd.Thesis, Iowa State University, Iowa, (1975).

[23] Kiregyera, B., "A chain ratio-type estimator in finite population double sampling using two auxiliary variables", Metrika, 27: 217-223, (1980).

[24] Kiregyera, B., "Regression-type estimator using two auxiliary variables and model of double sampling from finite populations", Metrika, 31: 215-226, (1984).

[25] Bedi, P. K., "On two-phase multivariate sampling estimator", Journal of the Indian Society of Agricultural Statistics, 37(2): 158-162, (1985).

[26] Srivastava, S. Rani, Khare, B. B., and Srivastava, S. R., "A generalized chain ratio estimator for mean of finite population", Journal of the Indian Society of Agricultural Statistics, 42(1): 108-117, (1990).

[27] Singh, G. N., "On the use of transformed auxiliary variable in the estimation of population mean in two phase sampling", Statistics in Transition, 5(3): 405-416, (2001).

[28] Singh, H. P., Upadhyaya, L. N., and Chandra, P., "A general family of estimators for estimating population mean using two auxiliary variables in two-phase sampling", Statistics in Transition, 6(7): 1055-1077, (2004).

[29] Samiuddin, M., Hanif, M., "Estimation of population mean in single and two phase sampling with or without additional information", Pakistan Journal of Statistics, 23(2): 99-118, (2007).

[30] Singh, R., Chuhan, P., and Sawan, N., "A family of estimators for estimating population mean using known correlation coefficient in two phase sampling", Statistics in Transition, 8(1): 89-96, (2007).

[31] Singh, H. P., Agnihotri, N., "A general procedure of estimating population mean using auxiliary information in sample survey", Statistics in Transition, 9(1): 71-87, (2008).

[32] Zaman, T., Kadilar, C., "New class of exponential estimators for finite population mean in two-phase sampling", Communications in Statistics-Theory and Methods, 50(4): 874-889, (2021).

[33] Zaman, T., Kadilar, C., "Exponential ratio and product type estimators of the mean in stratified two-phase sampling", AIMS Mathematics, 6(5): 4265-4279, (2021).

[34] Srivenkataramana, T., "A dual to ratio estimator in sample surveys", Biometrika, 67: 199-204, (1980).

[35] Bandyopadhyaya, S., "Improved ratio and product estimators", Sankhya, Series C, 42: 45-49, (1980).

[36] Singh, H. P., Upadhyaya, L. N., "A dual to modified ratio estimator using coefficient of variation of auxiliary variable", Proceedings of the National Academy of Sciences, India-Section A, 56(A): 336-340, (1986).

[37] Onyeka, A. C., Nlebedim, V. U., and Izunobi, C. H., "Estimation of population ratio in simple random sampling using variable transformation", Global Journal of Science Frontier Research Mathematics and Decision Sciences, 13(4): 57-65, (2013).

[38] Adewara, A. A., “Effect of improving both the auxiliary and variable of interest in ratio and product estimators”, Proceedings of the Pakistan Academy of Sciences, 43(4): 275-278, (2006).

[39] Adewara, A. A., Singh, R., and Kumar, M., “Efficiency of some modified ratio and product estimators using known value of some population parameters”, International Journal of Applied Science and Technology, 2(2): 76-79, (2012).

[40] Singh, R., Malik, S., and Smarandache, F., Uses of Sampling Techniques & Inventory Control with Capacity Constraints, Malik, S., Kumar, N., and Smarandache, F. editors, Pons Editions, Belgium, 19-29, (2016).

[41] Thongsak, N., Lawson, N., “Classes of dual to modified ratio estimators for estimating population mean in simple random sampling”, Proceeding of the 2021 Research, Invention and Innovation Congress, 211-215, (2021).

[42] Kumar, M., Bahl, S., “Class of dual to ratio estimators for double sampling”, Statistical Papers, 47: 319-326, (2006).

[43] Singh, B. K., Choudhury, S., “Dual to product estimator for estimating population mean in double sampling”, International Journal of Statistics and Systems, 7: 31-39, (2012).

[44] Choudhury, S., Singh, B. K., “Study of dual to ratio-cum-product estimator of finite population mean under double sampling in sample surveys”, Journal of Statistical Theory and Applications, 14(2): 214-221, (2015).

[45] Boonrodrak, N., “Ratio estimators of population mean using transformed auxiliary variable under two-phase sampling”, MSc. Thesis, King Mongkut’s University of Technology North Bangkok, Bangkok, (2015).

[45] Jaroengeratikun, U., Lawson, N., “Improved ratio estimators of population mean using transformed variable in double sampling”, The Journal of Applied Science, 17(2): 9-18, (2018).

[47] Kamba, A. I., Adewara, A. A., and Ahmed, A., “Modification of ratio estimator under two phase sampling”, FUW Trends in Science & Technology Journal, 4: 495-500, (2019).

[48] Cochran, W. G., “Sampling Techniques”, New York: John Wiley and Sons, (1977).

[49] http://www.oae.go.th/assets/portals/1/fileups/prcaidata/files/1_rubber_dit%2060.pdf. Access date: 16.09.2020