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# Some Continuous Neutrosophic Distributions with Neutrosophic Parameters Based on Neutrosophic Random Variables

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### Abstract

In this paper, we apply continuous distributions such as the exponential distribution, gamma distribution, beta distribution and uniform distribution and discontinuous random distribution such as Poisson distribution by using neutrosophic random variables. This study opens a new way for dealing with issues that follow the classical distributions which appear in classical random variables and at the same time contain data not specified accurately.

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### 1. Introduction and Background

The notion of neutrosophic probability measure as a function  $\mathcal{NP} : Y \rightarrow [0, 1]^3$  was introduced by F. Smarandache where  $U$  is a neutrosophic sample space, and defined the probability mapping to take the form  $\mathcal{NP}(S) = (ch(S), ch(neutS), ch(antiS)) = (\alpha, \beta, \gamma)$  where  $0 \leq \alpha, \beta, \gamma \leq 1$  and  $0 \leq \alpha + \beta + \gamma \leq 3$  [36]. Besides, many researchers have investigated many neutrosophic probability distributions like Poisson,

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exponential, binomial, normal, uniform, Weibull,...etc. (See, [35], [2], [19], [27]). Additionally, researchers have investigated the notion of neutrosophic queueing theory in [38], [39] this is one branch of neutrosophic stochastic modelling. Furthermore, researchers have also studied neutrosophic time series prediction and modelling in many cases like neutrosophic moving averages, neutrosophic logarithmic models, neutrosophic linear models and so on. [3], [4], [12]. Recently, researchers have started to study the notion of neutrosophic random variable (see, Definition 1.5). Bisher and Hatip in 2020 [7] presented the first notion of neutrosophic random variables in which they presented some basics notions. later on, Granados in 2021 [13] showed new notions on neutrosophic random variables and then Granados and Sanabria [14] studied independence neutrosophic random variables. On the other hand, neutrosophic logic is an extension of intuitionistic fuzzy logic by adding indeterminacy component ( $I$ ) where  $I^2 = I, \dots, I^n = I, 0.I = 0; n \in \mathbb{N}$  and  $I^{-1}$  is undefined (see [21], [35]). Neutrosophic logic has a huge brand of applications in many fields including decision making [30], [20], [26], machine learning [6], [28], intelligent disease diagnosis [33], [11], communication services [8], pattern recognition [29], social network analysis and e-learning systems [22], physics [37], sequences spaces [15] and so on. Neutrosophic logic has solved many decision-making problems efficiently like evaluating green credit rating, personnel selection, . . . etc. [23], [24], [25], [1]. For more notions related to neutrosophic theory, we refer the reader to [15, 17, 9, 16, 18, 10].

In this paper, we highlight the use of neutrosophic random variables [7] with the classical probability distributions, particularly Poisson distribution, Exponential distribution and Uniform distribution, which opens the way for dealing with issues that follow the classical distributions and at the same time contain data not specified accurately and neutrosophic probability distributions. In this paper, we discuss continuous random distributions such as the Exponential distribution and Uniform distribution, and discontinuous random distribution such as Poisson distribution by using neutrosophic random variables.

Throughout this paper, the set of real number is denoted by  $\mathbb{R}$  or  $\mathbb{R}$ ,  $\Omega$  denotes the set of sample space and  $\omega$  denotes an event of the sample space,  $X_N$  and  $Y_N$  denote neutrosophic random variables.

Next, we show some well-known definitions and properties of neutrosophic logic and neutrosophic probability which are useful for the development of this paper.

**Definition 1.1.** (see [34]) Let  $X$  be a non-empty fixed set. A neutrosophic set  $A$  is an object having the form  $\{x, (\mu A(x), \delta A(x), \gamma A(x)) : x \in X\}$ , where  $\mu A(x)$ ,  $\delta A(x)$  and  $\gamma A(x)$  represent the degree of membership, the degree of indeterminacy, and the degree of non-membership respectively of each element  $x \in X$  to the set  $A$ .

**Definition 1.2.** (see [5]) Let  $K$  be a field, the neutrosophic field generated by  $K$  and  $I$  is denoted by  $\langle K \cup I \rangle$  under the operations of  $K$ , where  $I$  is the neutrosophic element with the property  $I^2 = I$ .

**Definition 1.3.** (see [35]) Classical neutrosophic number has the form  $a + bI$  where  $a, b$  are real or complex numbers and  $I$  is the indeterminacy such that  $0.I = 0$  and  $I^2 = I$  which results that  $I^n = I$  for all positive integers  $n$ .

**Definition 1.4.** (see [36]) The neutrosophic probability of event  $A$  occurrence is  $NP(A) = (ch(A), ch(neutA), ch(antiA), (T, I, F))$  where  $T, I, F$  are standard or non-standard subsets of the non-standard unitary interval  $]^{-0, 1^+}$ .

Recently, Bisher and Hatip [7] introduced and studied the notions of neutrosophic random variables by using the concepts presented by [36], these notions were defined as follows:

**Definition 1.5.** Consider the real valued crisp random variable  $X$  which is defined as follows:

$$X : \Omega \rightarrow \mathbb{R}$$

where  $\Omega$  is the events space. Now, they defined a neutrosophic random variable  $X_N$  as follows:

$$X_N : \Omega \rightarrow \mathbb{R}(I)$$

and

$$X_N = X + I$$

where  $I$  is indeterminacy.

**Theorem 1.6.** Consider the neutrosophic random variable  $X_N = X + I$  where cumulative distribution function of  $X$  is  $F_X(x) = P(X \leq x)$ . Then, the following statements hold:

1.  $F_{X_N}(x) = F_X(x - I)$ ,
2.  $f_{X_N}(x) = f_X(x - I)$ .

Where  $F_{X_N}$  and  $f_{X_N}$  are cumulative distribution function and probability density function of  $X_N$ , respectively.

**Theorem 1.7.** Consider the neutrosophic random variable  $X_N = X + I$ , expected value can be found as follows:

$$E(X_N) = E(X) + I.$$

**Proposition 1.8** (Properties of expected value of a neutrosophic random variable). Let  $X_N$  and  $Y_N$  be neutrosophic random variables, then the following properties holds:

1.  $E(aX_N + b + cI) = aE(X_N) + b + cI; a, b, c \in \mathbb{R}$ ,
2. If  $X_N$  and  $Y_N$  are neutrosophic random variables, then  $E(X_N \pm E(Y_N)) = E(X_N) \pm E(Y_N)$ ,
3.  $E[(a + bI)X_N] = aE(X_N) + bIE(X_N); a, b \in \mathbb{R}$ ,
4.  $|E(X_N)| \leq E|X_N|$ .

**Theorem 1.9.** Consider the neutrosophic random variable  $X_N = X + I$ , variance of  $X_N$  is equal to variance of  $X$ , i.e.  $V(X_N) = V(X)$ .

Granados [13, 14] studied the notions of neutrosophic random vector and joint neutrosophic random variable, these notions were defined as follows:

**Definition 1.10.** A neutrosophic random vector of two dimension is a vector  $(X_N, Y_N)$  in which each coordinate is a neutrosophic random variable. Analogously, we can define a neutrosophic random vector multidimensional as follows  $(X_{N_1}, X_{N_2}, \dots, X_{N_n})$  in which  $X_{N_1}, X_{N_2}, \dots, X_{N_n}$  are neutrosophic random variables for each  $n = 1, 2, \dots$

**Definition 1.11.** Let  $(X_N, Y_N)$  be a neutrosophic random vector, we define probability function of a neutrosophic continuous random vector  $(X_N, Y_N)$ . Then, joint probability neutrosophic function of a discrete random vector  $(X_N, Y_N)$   $f_N(x, y) : \mathbb{R}^2 \rightarrow [0, \infty)$  in which is non-negative and integrable, and for any  $(x, y) \in \mathbb{R}^2$ , it is defined as follows

$$P(X_N \leq x, Y_N \leq y) = P(X \leq x - I, Y \leq y - I) = \int_{-\infty}^{y-I} \int_{-\infty}^{x-I} f_{(X_N, Y_N)}(u, v) dv du$$

Similarly, probability function of a neutrosophic discrete random vector  $(X_N, Y_N)$  is defined similar by using sum.

**Definition 1.12.** Let  $(X_N, Y_N)$  be a neutrosophic random vector, we define neutrosophic joint distribution function which will be denoted by  $F_{(X_N, Y_N)}(x, y) = P(X_N \leq x, Y_N \leq y) = P(X \leq x - I, Y \leq y - I)$ .

**Definition 1.13.** Let  $f_{(X_N, Y_N)}(x, y)$  be a joint probability neutrosophic function of a continuous random variable  $(X_N, Y_N)$ . We define neutrosophic marginal function of  $X_N$  as follows:

$$f_{X_N}(x) = \int_{-\infty}^{+\infty} f_{(X_N, Y_N)}(x, y) dy$$

and we define neutrosophic marginal function of  $Y_N$  as follows:

$$f_{Y_N}(y) = \int_{-\infty}^{+\infty} f_{(X_N, Y_N)}(x, y) dx$$

## 2. Main Results

In this section, we use the notion of neutrosophic continuous distribution which were introduced by [2], and we apply them in neutrosophic random variable and present some examples.

### 2.1. Neutrosophic Poisson Distribution in Neutrosophic Random Variables

Neutrosophic Poisson distribution of a neutrosophic random variable  $X_N$  is a neutrosophic Poisson distribution of  $X + I$ , but its parameter is imprecise and it has a level of indeterminacy. For example,  $\lambda$  can be set with two or more elements and  $I$  is a function which is defined as a classical neutrosophic theory which takes value in  $[0, 1]$ . This distribution is defined as follows:

$$NP_N(x) = e^{-\lambda_N} \frac{(\lambda_N)^{(x-I)}}{(x - I)!},$$

for  $x = 1, 2, 3, \dots$

Where,  $\lambda_N$  is the neutrosophic distribution parameter and it is the expected value i.e.,  $\lambda_M + I$ ,  $\lambda_M$  is the distribution parameter.

**Example 2.1.** *In a company, Phone employee receives phone calls, the calls arrive with rate of  $[2, 3]$  calls per minute, we will calculate the probability that the employee will not receive any call within a minute with indeterminacy  $I \in [0, 1]$ . **Solution:** Let us consider  $x$  the number of calls in a minute, then*

$$NP(x = I) = e^{-\lambda_N} \frac{(\lambda_N)^0}{0!} = e^{-\lambda_N} = e^{-[2,3]+[0,1]} = e^{[-2,-2]} = e^{-2} = 0.1353.$$

*Thus, the probability that employee won't receive any call, within a minute is 0.1353.*

*Now, consider that we want to find the probability that employee won't receive any call within 5 minutes. So, we have  $\lambda_N = 5[2, 3] + 5[0, 1] = [10, 15] + [0, 5] = [10, 20]$ . Hence,*

$$NP(x = I) = e^{-[10,20]} \frac{([10, 20])^0}{0!} = e^{-[10,20]}.$$

*For  $\lambda = 10$ ,  $NP(I) = e^{-10} = 0.00004$  and for  $\lambda = 20$ ,  $NP(I) = e^{-20} = 0.000000002$ . Thus, the probability that employee won't receive any call, within a five minutes is  $[0.000000002, 0.00004]$ .*

*If we make this exercise in classical neutrosophic probability, we obtain  $NP(0) = [0.00000003054, 0.00004] \in [0.000000002, 0.00004] = NP(I)$ .*

### 2.2. Neutrosophic Exponential Distribution in Neutrosophic Random Variables

Neutrosophic exponential distribution is defined as a generalization of classical exponential distribution, Neutrosophic exponential distribution in random variables can deals with all the data even non-specific, we express the density function as:

$$X_N \sim exp(\lambda_M) = f_X(x - I) = \lambda_M e^{-(x-I)\lambda_M}; I < x < \infty.$$

Where  $exp(\lambda_M)$  is the neutrosophic exponential distribution,  $X_N$  is a neutrosophic random variable and  $\lambda_M$  is the distribution parameter.

By mentioned above, we have the following properties:

1.  $E(X_N) = \frac{1}{\lambda} + I,$
2.  $Var(X_N) = \frac{1}{(\lambda)^2}.$

**Example 2.2.** *The time required to terminate client's service in the bank follow an exponential distribution, with an average of  $[0.67, 2]$  minute. let us write a density function that represents the time required for terminating client's service, and then calculate the probability of terminating client's service in less than a minute with an indeterminacy probability of 0.1.*

**Solution** We have,  $\frac{1}{\lambda_M} = [0, 67, 2]$ , then  $\lambda_M = [0.5, 1.5]$ . Thus, the probability density function is defined as follows

$$f_X(x - I) = [0.5, 1.5]e^{-(x-0.1)[0.5,1.5]}.$$

Probability to terminate the client's service in less than a minute is

$$NP(X \leq 0.9) = (1 - e^{-(0.9)[0.5,1.5]}) = (1 - e^{-[0.45,1.35]}).$$

We can see that for  $\lambda = 0.45$ ,

$$NP(X \leq 0.9) = 1 - e^{-0.45} = 1 - 0.6376 = 0.3624,$$

and for  $\lambda = 1.35$ ,

$$NP(X \leq 0.9) = 1 - e^{-1.35} = 1 - 0.2231 = 0.7769.$$

That is, the probability of terminating client's service in less than a minute ranges between  $[0.3624, 0.7407]$ .

If we make this exercise as a classical way, we will obtain that  $P(X \leq 1) = 0.63 \in [0.3624, 0.7407] = NP(X \leq 0.9) = NP(X_N \leq 1)$

### 2.3. Relationship Between Neutrosophic Poission Distribution and Neutrosophic Exponential Distribution in Neutrosophic Random Variables

If the occurrence of events follows the Poisson distribution, the duration between the occurrence of two events follow exponential distribution. For example, arrival of customers to a service centre follows the Poisson distribution, the time between the arrival of a customer and the next customer follow the exponential distribution. Thus, when the parameter  $\lambda_M$  and indeterminacy  $I$  are inaccurately defined, we are dealing with the neutrosophic exponential distribution and the neutrosophic Poisson distribution in neutrosophic random variables and we write,

If an event is repeated in time according to the neutrosophic Poisson distribution in neutrosophic random variables,

$$NP_N(x) = e^{-\lambda_N} \frac{(\lambda_N)^{(x-I)}}{(x-I)!}.$$

Then, the time between two events follows the neutrosophic exponential distribution in neutrosophic random variables,

$$f_X(t - I) = \lambda_M e^{-(t-I)\lambda_M}; t \geq I.$$

**Example 2.3.** *Consider that we have a machine in a factory. The rate of machine breakdowns is  $[1, 2]$  per week, let us calculate the possibility of no breakdowns per week, and calculate the possibility that at least two weeks pass before the appearance of the following breakdowns with with indeterminacy  $I \in [0, 1]$ .*

**Solution**

Consider that  $x$  is a variable that is subject to the neutrosophic Poisson distribution in neutrosophic random variables, the distribution parameter is defined as follows:

$$NP(x = I) = e^{-[1,3]}.$$

Then, the possibility of no breakdowns in the week ranges between  $[0.0497, 0.3678]$ . Now, assuming  $y$  represents the time before the appearance of the following breakdowns, we note that  $y$  is a variable following the neutrosophic exponential distribution in neutrosophic random variables. Then,  $NF = NP(X \leq x - I) = (1 - e^{-(x-I)\lambda_M})$ , thus

$$\begin{aligned} NP(y > 2 + I) &= 1 - NP(y \leq 2 + I) = 1 - NF(2 + I) = 1 - (1 - e^{-[2,3][1,3]}) \\ &= e^{[-3,-2][1,3]}. \end{aligned}$$

Then, we have to option,  $e^{[-3,3]}$  or  $e^{[-2,1]}$  i.e.,  $[0, 0.4978, 20.08]$  or  $[0.1353, 2.718]$ . Thus, the possibility that at least two weeks pass before the appearance of the following breakdowns, ranges between  $[0, 0.4978, 0.1353]$ .

#### 2.4. Neutrosophic Uniform Distribution in Neutrosophic Random Variables

Neutrosophic Uniform distribution in neutrosophic random variables of a neutrosophic continuous variable  $X_N$  is a classical Uniform distribution, but distribution parameters  $a$  or  $b$  or both are imprecise and determinate by indeterminacy  $I \in [0, 1]$ . For example,  $a$  or  $b$  or both are sets with two or more elements (may  $a$  or  $b$  or both are intervals) with  $a < b$ .

**Example 2.4.** Consider  $x$  is a variable represents a person’s waiting time to passengers’ bus (in minutes), bus’s arrival time is not specified, the station official said the bus arrival time is either from now to 5 minutes  $[0, 5]$  or will arrive after 15 to 20 minutes  $[15, 20]$  with indeterminacy  $[0.1, 0.5]$ , then we have two option

$$f_{X_N}(x) = \frac{1}{[15, 20] - [0, 5] - [0.1, 0.5]} = \frac{1}{[14.5, 14.9]} = [0.06711, 0.06896].$$

and

$$f_{X_N}(x) = \frac{1}{[15, 20] - [0, 5] + [0.1, 0.5]} = \frac{1}{[15.1, 15.5]} = [0.0526, 0.06451].$$

Thus, The solution is  $[0.0526, 0.06896]$  with the probability to moving  $[0, 5]$  and  $[15, 20]$  minutes.

**Example 2.5.** Assume  $x$  is a variable represents a person’s waiting time to passengers’ bus (in minutes), bus’s arrival time is not specified, the station official said the bus arrival time is 5 minutes or will arrive after 15 to 20 minutes  $[15, 20]$  with indeterminacy  $[0.1, 0.5]$ , then we have two option

$$f_{X_N}(x) = \frac{1}{[15, 20] - 5 - [0.1, 0.5]} = \frac{1}{[9.9, 14.5]} = [0.06896, 0.1010].$$

and

$$f_{X_N}(x) = \frac{1}{[15, 20] - 5 + [0.1, 0.5]} = \frac{1}{[10.1, 15.5]} = [0.0990, 0.06451].$$

Thus, The solution is  $[0.06896, 0.1010]$  with the probability to moving 5 and  $[15, 20]$  minutes.

If we make this exercise in classical neutrosophic probability way, we obtain  $[0.067, 0, 1] \in [0.06896, 0.1010]$ .

2.5. Neutrosophic Gamma Distribution in Neutrosophic Random Variables

Let  $X_N$  be a neutrosophic random variable which has neutrosophic gamma distribution with parameters  $\alpha > 0$  and  $\lambda > 0$  where  $\alpha$  or  $\lambda$  or both are sets with two or more elements (may  $\alpha$  or  $\lambda$  or both are intervals), and we write  $X_N \sim \text{gamma}(\alpha, \lambda)$  and its neutrosophic density function is defined as follows:

$$f_X(x - I) = \lambda e^{-\lambda(x-I)} \frac{(\lambda(x - I))^{\alpha-1}}{\Gamma(\alpha)},$$

if  $x \geq I$ .

We shall recall that  $\Gamma(\alpha + 1) = \alpha\Gamma(\alpha)$  and  $\Gamma(\alpha + 1) = \alpha!$  if  $\alpha \in \mathbb{Z}^+$  i.e. positive integers. It is easy to check that  $E(X_N) = \frac{\alpha}{\lambda} + I$ .

By using the software project for statistical computing *R* we can obtain the values of the function  $\gamma(x - I)$  using the *gamma(x)* command. The values of the density function  $f_X(x - I)$  are obtained as follows

```
# dgamma(x, shape=\alpha, rate=\lambda) Evaluate f(x) in the distribution
# gamma(\alpha, \lambda)
> dgamma(2.5, shape=7, rate=3)
[1] 0.4101547
> dgamma(1.5, shape=7, rate=3)
[2] 0.00032107
```

where  $x = 2.5$  and  $I = [0, 1]$ , we had the following probability  $[0.00032107, 0.4101547]$ .

**Example 2.6.** Suppose that experience shows that the time (in minutes) required to perform periodic maintenance a dictaphone is followed by a gamma distribution with  $\alpha = [1, 2.1]$  and  $\lambda = 2$ . It takes a new maintenance technician 22.5 minutes check the machine with indeterminacy time period 0.1. Does this time used in the keeping the dictaphone with the previous period?

**Solution**

We can see that  $E(X_N) = \frac{\alpha}{\lambda} + I = \frac{[1, 2.1]}{2} + 0.1 = [0.6, 1.15]$  and  $Var(X_N) = Var(X) = [0.25, 0.525]$ . So, we have that  $\sigma = [0.5, 0.7245]$  ( $\sigma$  means typical deviation). Since  $x = 22.5 > E(X_N)$  by  $[21.35, 21.9]$  minutes. Therefore, applying neutrosophic gamma distribution, we have probabilities are  $[0.0466, 0.05624]$  and  $[0.9430, 0.9634]$ . Therefore, if we take  $P(X_N \geq 22.5)$  is  $[0.0466, 0.05624]$  and it is small, hence we should conclude that our new maintenance technician randomly generated a period long-term maintenance, which has a low probability to occur, or that is slower than the previous ones.

If we make this exercise in classical way, we will obtain  $P(X \geq 22.5) = 0.04998 \in [0.0466, 0.05624] = P(X_N \geq 22.5)$ .

**Proposition 2.7.** Let  $X_{N_n}$  with  $n = 1, 2, 3, \dots$  be independence neutrosophic random variables, for each neutrosophic random variable with has neutrosophic exponential distribution. Then,  $X_{N_n} \sim \text{gamma}(n, \lambda)$ .

**Theorem 2.8.** Let  $X_N$  be a neutrosophic random variable with neutrosophic gamma distributuon and let  $c > 0 \in \mathbb{R}$ , then  $cX_N \sim \text{gamma}(\alpha, \lambda/c)$ .

*Proof.* For  $x > I$ ,

$$\begin{aligned} P(cX \leq x - I) &= P(X \leq \frac{x - I}{c}) \\ &= \int_0^{\frac{x-I}{c}} \frac{(\lambda u)^{\alpha-1}}{\Gamma(\alpha)} \lambda e^{-\lambda u} du \\ &= \int_0^{x-I} \frac{((\lambda/c)v)^{\alpha-1}}{\Gamma(\alpha)} (\lambda/c) e^{-(\lambda/c)v} dv. \end{aligned}$$

□

2.6. Neutrosophic Beta Distribution in Neutrosophic Random Variables

Let  $X_N$  be a neutrosophic random variable which has neutrosophic beta distribution with parameters  $a > 0$  and  $b > 0$  where  $a$  or  $b$  or both are sets with two or more elements (may  $a$  or  $b$  or both are intervals), and we write  $X_N \sim \text{beta}(a, b)$  and its neutrosophic density function is defined as follows:

$$f_X(x - I) = \frac{1}{B(a, b)}(x - I)^{a-1}(1 + I - x)^{b-1},$$

if  $I \leq x \leq 1 + I$ .

We shall recall that  $B(a, b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$ . It is easy to check that  $E(X_N) = \frac{a}{a+b} + I$ .

By using the software project for statistical computing  $R$  we can obtain the values of the function  $\beta(x - I)$  using the  $\text{beta}(x)$  command. The values of the density function  $f_X(x - I)$  are obtained as follows

```
# dbeta(x, a, b) Evaluate f(x) in the distribution beta(a, b)
> dbeta(0.3, 1, 2)
[1] 1.4
> dbeta(1.3, 1, 2)
[2] 2.6
```

where  $x = 0.7$  and  $I = [0, 1]$ , we had the following probability [1.4, 2.6].

**Example 2.9.** A wholesale gasoline distributor has large-capacity storage tanks with a fixed supply, which are filled every Monday with indeterminacy 20%. He wants to know the percentage of gasoline sold during the week. After several weeks of observation, the wholesaler discovers that this percentage could be described by a neutrosophic beta distribution with  $a = 4$  and  $b = 2$ . He wants to know the probability that sell less than 50 % of your stock in a week.

**Solution** To solve this exercise, we need to find  $P(X_N < 0.5)$ . Applying beta distribution,

$$P(X_N < 0.5) = P(X < 0.5 - I) = P(X < 0.5 - 0.2) = 0.378.$$

Therefore, probability of wholesale sells less than 50 of your stock in a week is 0.378.

**Proposition 2.10.** Let  $X_N$  and  $Y_N$  be two independence neutrosophic random variables with neutrosophic distribution  $\text{gamma}(a, \lambda)$  and  $\text{gamma}(b, \lambda)$ , respectively. Then,

$$\frac{X_N}{X_N + Y_N} \sim \text{beta}(a, b).$$

**Theorem 2.11.** If  $X_N \sim \text{beta}(a, b)$ , then  $1 - X_N \sim \text{beta}(b, a)$ .

*Proof.* For any  $x \in (I, 1 + I)$  and making a change of variable  $v = 1 - u$ ,

$$\begin{aligned} P(1 - X \leq x + I) &= P(X \geq 1 - x - I) \\ &= \int_{1-x-I}^1 u^{a-1}(1 - u)^{b-1} du \\ &= \int_0^{x-I} v^{b-1}(1 - v)^{a-1} dv. \end{aligned}$$

□



### 3. Conclusion

The neutrosophic probability distributions only deal with the specified undetermined values. In this paper, we contributed to the study of classical distributions and classical neutrosophic probability distribution and applied them in neutrosophic random variable and we define its continuous distribution. We called these distributions neutrosophic continuous distributions in neutrosophic random variables. On the other hand, We conclude from this paper that the neutrosophic continuous distributions in neutrosophic random variables gives us a more general and clarity study of the studied issue. In this paper, we presented several solved for the problems that classic logic and classical neutrosophic probability. We look forward in the future to study other types of neutrosophic distributions in random variables that have not yet been studied.

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### Conflicts of Interest

The authors declare no conflict of interest.

### Data availability statement

This manuscript has no associated data.

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