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Authors: Eren TOSYALI, Fatma AYDOĞMUŞ

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Synchronization of Gursey System

Eren TOSYALI*[1], Fatma AYDOĞMUŞ[2]

Abstract

Gursey Model, the only possible four-dimensional pure spinor model, proposed as a possible basis for a unitary description of elementary particles. The model exhibits chaotic behaviors depending on the system parameter values. In this study, we investigate the synchronization of chaotic dynamic in the Gursey wave equation that has particle-like solutions derived classical field equations. Numerical results for synchronization of the Gursey system are performed to indicate the accuracy of the used method.

Keywords: Synchronization, Gursey, chaos

1. INTRODUCTION

The field equation proposed by Feza Gursey in 1956 is the first nonlinear spinor wave equation with conformal invariance [1]. Gursey Model is the first 4D conformal invariant fermionic model [1]. The exact solution of 4D Gursey Model via Heisenberg ansatz was found by Kortel. This exact solution had instantonic character [2]. Instantons are corresponding to classical topological solution with zero energy for the QCD (Quatum Chromo Dynamic) field equations [3]. In addition, soliton-type solutions are found by adding the mass term to the equation for certain values of the coupling constant [4, 5]. Also, soliton solutions of the expanded form of Gursey wave equation and Wu-Yang type monopole solutions were found [1, 4].

In nature, systems are well described by nonlinear equations, which have rich solutions such as regular or chaotic behavior. Therefore, chaos and nonlinear dynamics are widely used in many applied fields, from natural science (physics, biology …) to social science [6]. The Gursey system exhibits regular or chaotic behaviors depending on the system parameters. Recently, many studies have been done on the dynamics of the Gursey wave equation [7 - 9].

Synchronization in chaotic systems has been a topic of great interest in recent years. The first study on the coupling and synchronization of

* Corresponding author: eren.tosyali@bilgi.edu.tr
[1] İstanbul Bilgi University
ORCID: https://orcid.org/0000-0001-9118-851X
[2] İstanbul University
E-mail: fatmaa@istanbul.edu.tr
ORCID: https://orcid.org/0000-0003-1434-2143

identical chaotic systems was by Pecora-Carroll [10]. After the Pecora-Caroll studies, identical chaotic systems' synchronization problems get much popular in this field. Especially, active control method is used to synchronize chaotic flows and maps such as Lorenz, Duffing, Gross-Pitaevskii Equation (Bose-Einstein Condensate) and HIV-AIDS dynamical system [11-18]. In this paper, master-slave synchronization based on open-plus-closed-loop (OPCL) method is used to synchronize chaotic dynamic of Gursey wave equation. The OPCL method was proposed by Jackson and Grosu in 1995 [19]. They applied this method to synchronize chaotic identical Lorenz, Duffing and Chua systems. The effectiveness of the method was also investigated for complex network and hyper chaotic maps [20, 21]. In this paper, the synchronized and unsynchronized phase space diagrams and control-activated diagrams are given to show the effectiveness of the used method.

2. GURSEY MODEL

Gursey spinor wave equation is

$$i\partial\psi + g(\bar{\psi}\psi)\psi = 0. \tag{1}$$

here the fermion field ψ has scale dimension $\frac{3}{2}$ and g is the positive dimensionless coupling constant. The Heisenberg ansatz [22].

$$\psi = \left[\mathrm{i}x_\mu\,\gamma_\mu\chi(s) + \phi(s)\right]C, \tag{2}$$

here C is an arbitrary spinor constant; $\chi(s)$ and $\phi(s)$ are real functions of $s = x_\mu = r^2 + t^2 (x_1 = x, x_2 = y, x_3 = z, x_4 = t)$ in the Euclidean space-time, *i.e.* $r^2 = x_1^2 + x_2^2 + x_3^2$. Inserting Eq. (2) into Eq. (1) we obtain the following nonlinear differential equations system

$$4\chi(s) + 2s\frac{d\chi(s)}{ds} - \alpha[s\chi(s)^2 + \phi(s)^2]^{\frac{1}{3}}\phi(s) = 0 \tag{3a}$$

$$2\,\frac{d\phi(s)}{ds} + \alpha[s\chi(s)^2 + \phi(s)^2\,]^{\frac{1}{3}}\chi(s) = 0, \tag{3b}$$

here $\alpha = g(\bar{C}C)^{\frac{1}{3}}$ for short. Substituting $\chi = As^{-\sigma}F(u)$ and $\phi = Bs^{-\tau}\,G(u)$, $u = \ln s$ and $\tau = \frac{3}{4}$ and $A^2 = B^2$ [2], the dimensionless form of the nonlinear coupled differential equations system is obtained as

$$2\frac{dF(u)}{du} + \frac{3}{2}F(u) - \alpha(AB)^{\frac{1}{3}}[F(u)^2 + G(u)^2]^{\frac{1}{3}}G(u) = 0, \tag{4a}$$

$$2\frac{dG(u)}{du} - \frac{3}{2}G(u) + \alpha(AB)^{\frac{1}{3}}[F(u)^2 + G(u)^2]^{\frac{1}{3}}F(u) = 0. \tag{4b}$$

Here F and G are dimensionless functions of u and A, B are constants [7].

3. MASTER SLAVE SYNCHRONIZATION METHOD

In this section, we describe our master-slave synchronization process based on OPCL method to synchronize identical systems. Let us consider two identical systems and relate them some coupling function. Systems are defined on $\mathbb{R}^3$, so they have three degrees of freedom. The generalized coordinates for master system are described by $\boldsymbol{x} \equiv (x, y, z)$ and slave system $\boldsymbol{x}_s \equiv (x_s, y_s, z_s)$. Their evolution is described by the same vector field $f\colon \mathbb{R}^3 \to \mathbb{R}^3$, then we have

$$\dot{\boldsymbol{x}}(t) = f\big(x(t)\big) \tag{7a}$$

$$\dot{\boldsymbol{x}}_s(t) = f\big(x_s(t)\big), \tag{7b}$$

The difference between master and slave systems named error function is given by $\boldsymbol{x} - \boldsymbol{x}_s$, with coefficients dependent on the master variables [23, 24], that is,

$$\boldsymbol{k}(x).(\boldsymbol{x} - \boldsymbol{x}_s) = \begin{pmatrix} k_{11}(x) & k_{12}(x) & k_{31}(x) \\ k_{21}(x) & k_{22}(x) & k_{32}(x) \\ k_{31}(x) & k_{23}(x) & k_{33}(x) \end{pmatrix}.\begin{pmatrix} x - x_s \\ y - y_s \\ z - z_s \end{pmatrix}. \tag{8}$$

This coupling function $\boldsymbol{k}(x).(\boldsymbol{x} - \boldsymbol{x}_s)$ is added to the slave subsystem. Therefore, this function generates a response which synchronize master and slave system

$$\dot{\boldsymbol{x}}(t) = f\big(\boldsymbol{x}(t)\big) \tag{9a}$$

$$\dot{\boldsymbol{x}}_s(t) = f(\boldsymbol{x}_s(t)) + k(x).(\boldsymbol{x} - \boldsymbol{x}_s), \quad (9b)$$

If $\boldsymbol{\xi}(t) = \boldsymbol{x}(t) - \boldsymbol{x}_s(t)$ go to zero, the system will reach asymptotically stable, implying that the states $\boldsymbol{x}(t)$ and $\boldsymbol{x}_s(t)$ will approach each other which means master and slave system is synchronized along the flow. If the regularity class of f is at least C^1, this is an easy consequence of the Mean Value Theorem in several variables and a judicious choice of the matrix k [21]: We consider it equal to

$$k = d_{\boldsymbol{x}(\boldsymbol{t})}\boldsymbol{f} - \boldsymbol{H}, \quad (10)$$

where $d_{\boldsymbol{x}(\mathrm{t})}\boldsymbol{f}$ is the differential of the vector field $\boldsymbol{f}$ evaluated along the trajectory $\boldsymbol{x}(t)$ of the master system, and $\boldsymbol{H}$ is a constant matrix all of whose eigenvalues have strictly negative real parts (this is known as a Hurwitz matrix). For any fixed t, subtracting Eq. 9b from the Eq. 9a we obtain

$$\dot{\boldsymbol{\xi}}(t) = \frac{d}{dt}(\boldsymbol{x}(t) - \boldsymbol{x}_s(t)) = \boldsymbol{f}(\boldsymbol{x}) - \boldsymbol{f}(\boldsymbol{x}_s) - \boldsymbol{k}.\boldsymbol{\xi} \cong (d_{\boldsymbol{x}(\boldsymbol{t})}\boldsymbol{f} - \boldsymbol{k}).\boldsymbol{\xi} = \boldsymbol{H}.\boldsymbol{\xi}, \quad (11)$$

here, $\boldsymbol{H}$ is Hurwitz implies that the exponential is a decaying one, leading to the asymptotic $\lim_{t\to\infty}\|\boldsymbol{\xi}(\mathbf{t})\| = 0$.

3.1. GURSEY SYNCHRONIZATION

Let us consider a driven and damped master Gursey system as given below,

$$\frac{dx_1}{du} = \left(-\frac{3}{4}\right)x_1 + 0.5y_1(x_1^2 + y_1^2)^{\frac{1}{3}}, \quad (12a)$$

$$\frac{dy_1}{du} = \frac{3}{4}y_1 - 0.5x_1(x_1^2 + y_1^2)^{\frac{1}{3}} + 0.5A\,Cos(wu) - 0.5\gamma y_1. \quad (12b)$$

For simplification we use (x_1, y_1) and (x_2, y_2) instead of (F_1, G_1) and (F_2, G_2) for master and slave systems, respectively. Jacobian matrix of master system is

$$detf = \begin{pmatrix} -\frac{3}{4} + \frac{0.333x_1y_1}{(x_1^2+y_1^2)^{(2/3)}} & \frac{0.333y_1^2}{(x_1^2+y_1^2)^{(2/3)}} + 0.5(x_1^2 + y_1^2)^{(1/3)} \\ \frac{0.419974x_1^2}{(x_1^2)^{(2/3)}} - 0.629961(x_1^2)^{(1/3)} & -\frac{3}{4} - 0.5\gamma \end{pmatrix}, \quad (13)$$

We take parameters and constants of Jacobi matrix (detf) as

$$\begin{pmatrix} -\frac{3}{4} + \mathrm{p} & 0 \\ 0 & -\frac{3}{4} - 0.5\gamma + p \end{pmatrix}. \quad (14)$$

Eigenvalues of Eq. 14 are $\lambda_1 = \frac{1}{4}(-3 + 4p)$ and $\lambda_2 = 0.75 + \mathrm{p} - 0.5\gamma$. The largest p value for the synchronization is 0.592 depending on $\lambda_1 < 0$ and $\lambda_2 < 0$. We took $p = -1$ which is smaller then 0.592 and $\gamma = 0.316$.

If we substitute the p and γ value into Eq. 14 we reach the H matrix which is given below,

$$\begin{pmatrix} -\frac{7}{4} & 0 \\ 0 & -0.408 \end{pmatrix}, \quad (15)$$

$\boldsymbol{k} = detf - H$ and $error = \begin{pmatrix} x_1 & x_2 \\ y_1 & y_2 \end{pmatrix}$. $\boldsymbol{k.error}$ is

$$\begin{pmatrix} (x_1 - x_2)\left(1 + \frac{0.333x_1y_1}{(x_1^2+y_1^2)^{(2/3)}}\right) + \left(\frac{0.333y_1^2}{(x_1^2+y_1^2)^{(2/3)}} + 0.5(x_1^2 + y_1^2)^{(1/3)}\right)(y_1 - y_2) \\ -1.04993(x_1^2)^{\frac{1}{3}}(x_1 - x_2) + 1(y_1 - y_2) \end{pmatrix}. \quad (16)$$

Finally, slave systems are

$$\frac{dx_2}{du} = -\frac{3}{4}x_2 + 0.5y_2(x_2^2 + y_2^2)^{\frac{1}{3}} + (x_1 - x_2)\left(1 + \frac{0.333x_1y_1}{(x_1^2+y_1^2)^{(2/3)}}\right) + \left(\frac{0.333y_1^2}{(x_1^2+y_1^2)^{(2/3)}} + 0.5(x_1^2 + y_1^2)^{(1/3)}\right)(y_1 - y_2), \quad (17a)$$

$$\frac{dy_2}{du} = \frac{3}{4}y_2 - 0.5x_2(x_2^2 + y_2^2)^{\frac{1}{3}} + 0.5A\,Cos(wu) - 0.5\gamma y_2 - 1.04993(x_1^2)^{\frac{1}{3}}(x_1 - x_2) + 1(y_1 - y_2). \quad (17b)$$

4. NUMERICAL RESULTS

In this section, we investigate the simulation results for synchronization of the master and slave Gursey systems using the fourth-order Runge Kutta algorithm. The sets of differential equations related to the master and slave systems are solved with step size 0.1 , length 1000, $A = 0.71$, $\omega = 1.04898$ and $\gamma = 0.316$. The initial values of the master and slave systems are taken as$(x_1(0); y_1(0)) = (0.2; 0.1)$, $(x_2(0); y_2(0)) = (1.6; 2.2)$, respectively. The bifurcation diagram is given in Figure 1. Gursey system shows regular and chaotic dynamics depending on the amplitude of driven force. The system exhibits regular dynamics until $A = 0.6$. After this value of A, system exhibits chaotic dynamics. There is only one stable fix points for less than $A = 0.4$. After that point there is periodic dumpling until $A = 0.6$. $A = 0.6$ is threshold point for transition regular to chaotic behavior. In order to prove chaotic dynamics of Gursey system we calculate Lyapunov Characteristic Exponents (LCEs) [25,26]. LCEs are $\lambda_1 = 0.0846347$, $\lambda_2 = -0.242635$, $\lambda_3 = 0$. In Figure 2, we show evolution of LCEs depending on u. In addition, for regular case, one LCE is 0 and all the other LCEs are less than zero (negative). There is one LCE, which is bigger than zero shows us chaotic dynamics of Gursey system. We start controlling at $u = 100$. After that point system exhibits synchronization.

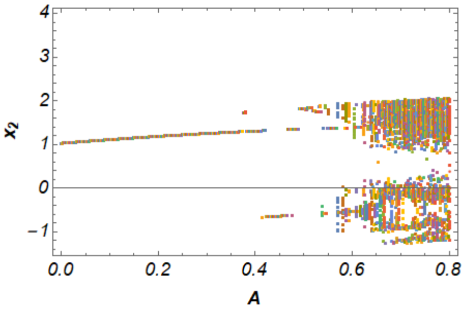


Figure 1 Bifurcation diagram for Gursey system.

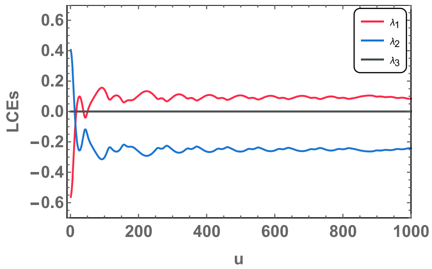
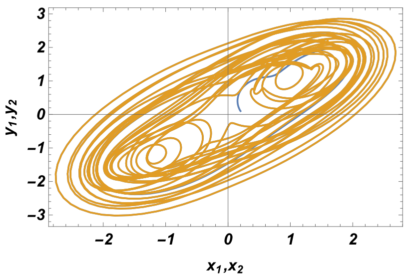
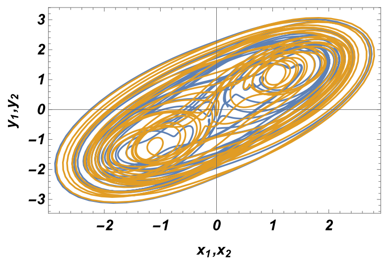


Figure 2 LCEs for chaotic Gursey system.



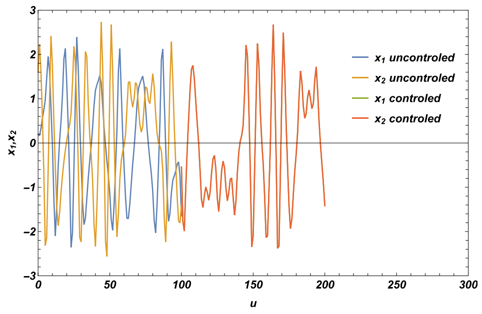
(a)



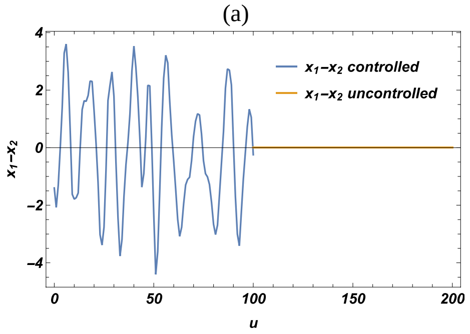
(b)

Figure 3 Synchronized and Unsynchronized Phase Space

The synchronized and unsynchronized phase space diagrams are given in Figure 3. Also Figure 4 and 5 show the dynamics of synchronized master and slave systems in the range 100 to 200. Control activated at $u = 100$. Before this value of u, systems are unsynchronized.

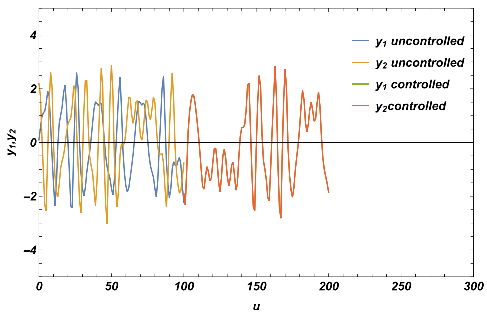


(a)

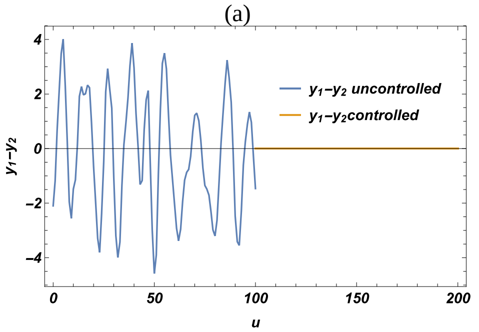


(b)

Figure 4 (a) Evolution graph (b) error graph, control function activated at $u = 100$, for x_1 and x_2



(a)



(b)

Figure 5 (a) Evolution graph (b) error graph, control function activated at $u = 100$, for y_1 and y_2

5. CONCLUSION AND DISCUSSION

In this paper, the validity of OPCL synchronization method is investigated for 4D fermionic Gursey model. The model exhibits chaotic dynamics depending on system parameters given in numerical results. In master-slave synchronization process, the selected slave and master systems are identical. Once the control function added to the slave system is activated, master and slave systems' orbits converge each other. The signals produced by control function stabilize the error between master and slave systems. The error signals go rapidly to the zero when control input function is activated at u=100 (Fig. 4 and Fig. 5). Two identical master and slave Gursey systems achieve the synchronization for different initial conditions. In Fig. 3 (a) Phase space and Fig. 4-5 (a) evolution graphs show synchronized dynamics after activated control signals.

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The authors of the paper declare that they comply with the scientific, ethical and quotation rules of SAUJS in all processes of the paper and that they do not make any falsification on the data collected. In addition, they declare that Sakarya University Journal of Science and its editorial board have no responsibility for any ethical violations that may be encountered, and that this study has not been evaluated in any academic publication environment other than Sakarya University Journal of Science.

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