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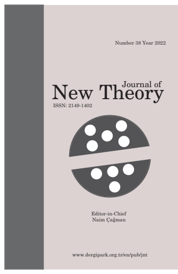


# $\mathcal{I}$-Statistical Rough Convergence of Order $\alpha$

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**Abstract** — The aim of this paper is to define the concept of $\mathcal{I}$-statistical ($\mathcal{I}$-st) rough convergence of order $\alpha$ ($0 < \alpha \leq 1$). It proposes the concept of $\mathcal{I}$-st bounded of order $\alpha$. Moreover, the necessary and sufficient condition for a sequence $(x_k)$ to be $\mathcal{I}$-st bounded of order $\alpha$ is studied. In addition, the necessary and sufficient condition for a sequence $(x_k)$ to be $\mathcal{I}$-st convergent of order $\alpha$ is examined. Finally, the need for further research studies are discussed.

***Keywords*** — *Ideal rough convergence, Ideal statistical rough convergence, Ideal statistical rough convergence of order $\alpha$.*

***Mathematics Subject Classification (2020)*** — 40A05, 40A35

## 1. Introduction

Statistical convergence is a generalization of the concept of convergence based on the concept of natural density of a subset of $\mathbb{N}$, the set of all natural numbers. This concept has been defined independently by Fast [1] and Steinhaus [2] in 1951. Further, Schoenberg [3] has defined statistical convergence as a summability method. Many mathematicians particularly Salat [4], Freedman and Sember [5], Fridy [6], Connor [7], Kolk [8], and Fridy and Orhan [9, 10], have contributed to the development of statistical convergence.

Let $K \subseteq \mathbb{N}$ and $K(n) = \{k \in K : k \leq n\}$, for all $n \in \mathbb{N}$. In this case, the natural density of the set $K$ is defined as follows:

$$\delta(K) = \lim_{n\to\infty} \frac{|K(n)|}{n}$$

such that $|K(n)|$ signifies the number of elements in $K(n)$ [5]. If $K$ is a finite set, its natural density is zero. Let $x = (x_k)$ be a sequence in $\mathbb{R}$ and $x_0 \in \mathbb{R}$. For every $\varepsilon > 0$, if the natural density of the set $\{k \in \mathbb{N} : ||x_k - x_0|| \geq \varepsilon\}$ is zero, that is

$$\lim_{n\to\infty} \frac{|\{k \leq n : ||x_k - x_0|| \geq \varepsilon\}|}{n}$$

is zero, then the sequence $x = (x_k)$ is said to be statistical convergent to $x_0$ and is denoted by $st - \lim x = x_0$. Since the natural density of finite sets is zero, every convergence sequence is statistical convergence. Kostryko et al. [11] has introduced the concept of ideal convergence (or briefly $\mathcal{I}$-convergence), a generalization of the concept of statistical convergence. They have defined the concept of $\mathcal{I}$-convergence by the concept of an ideal. They have investigated many properties of $\mathcal{I}$-convergence. Statistical convergence by degree has been first provided by Gadjiev and Orhan [12] as the relationship

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to the statistical convergence of a set of positive linear operators. In 2010, Çolak [13] has generalized the concept of statistical convergence by defining density of order $\alpha$ of a set and defined the concept of the statistical convergence of order $\alpha$, for sequence of real numbers. After, Savaş and Das [14] has put forward $\mathcal{I}$-statistical ($\mathcal{I}$-st) convergence of order $\alpha$ ($\alpha \in (0,1]$).

Phu [15] has suggested the concept of rough convergence in finite-dimensional normed spaces and examined between the relation rough convergence and other convergence types. He has also proved that the set of all rough limit points is bounded, closed, and convex. Later on, the concept of rough statistical convergence has been studied by Aytar [16]. Moreover, Aytar [17, 18] has also studied sets of rough statistical limit points and rough statistical cluster points. Pal et al [19] have studied rough ideal convergence and properties of the set of rough $\mathcal{I}$-limit points. Moreover, Dündar and Çakan [20] have also investigated rough ideal convergence. Savaş et al. [21] have defined the concept of $\mathcal{I}$-st rough convergence. Furthermore, the concept of rough statistical convergence of order $\alpha$ has been propounded and studied some properties of the set of all rough statistical limit points of order $\alpha$ by Maity [22].

In the second part of the present study, some basic definitions and properties to be required for the next section are provided. Section 3 proposes the concept of $\mathcal{I}$-st rough convergence of order $\alpha$ such that $0 < \alpha \leq 1$. Additionally, the necessary and sufficient conditions for a sequence $(x_k)$ to be $\mathcal{I}$-st convergent of order $\alpha$ and $\mathcal{I}$-st bounded of order $\alpha$ are proved. Finally, the need for further research studies is discussed.

## 2. Preliminaries

Throughout this study, a normed space $(X, ||.||)$ will be denoted by $X$.

**Definition 2.1.** [15] Let $x = (x_k)$ be a sequence in $X$ and $r \geq 0$. $x = (x_k)$ is said to be rough convergent ($r$-convergent) to $x_0 \in X$, if for every $\varepsilon > 0$ there exists a $k_\varepsilon \in \mathbb{N}$ such that $k \geq k_\varepsilon$ implies

$$||x_k - x_0|| < r + \varepsilon$$

or equivalently

$$\limsup ||x_k - x_0|| < r$$

and denoted by $x_k \xrightarrow{r} x_0$. Here, $r$ is called the roughness degree of the sequence $(x_k)$. If $r = 0$, then the concept of the $r$-convergence is equivalent to the concept of the classical convergence. Here, $x_0$ is called an $r$-limit point of $(x_k)$ and the set of all $r$-limit points is denoted by

$$\text{LIM}_r x = \left\{ x_0 \in X : x_k \xrightarrow{r} x_0 \right\}$$

If the set $\text{LIM}_r x$ is non-empty, then the sequence $x = (x_k)$ is $r$-convergent.

**Definition 2.2.** [16] Let $x = (x_k)$ be a sequence in $X$ and $r \geq 0$. $x = (x_k)$ is said to be statistical rough convergent to $x_0 \in X$, if, for every $\varepsilon > 0$,

$$\lim_{n\to\infty} \frac{|\{k \leq n : ||x_k - x_0|| \geq r + \varepsilon\}|}{n} = 0$$

or equivalently

$$st - \limsup ||x_k - x_0|| \leq r$$

and denoted by $x_k \xrightarrow{st-r} x_0$. Thus,

$$st - \text{LIM}_r x = \left\{ x_0 \in X : x_k \xrightarrow{st-r} x_0 \right\}$$

If $r = 0$, then the concept of statistical rough convergence is equivalent to the concept of the statistical convergence. If the set $st - \text{LIM}_r x$ is non-empty, then the sequence $x = (x_k)$ is statistical rough convergent.

**Definition 2.3.** [11] Let $X \neq \emptyset$ and $\mathcal{I} \subseteq P(X)$. If

*i.* $\emptyset \in \mathcal{I}$,

*ii.* $A, B \in \mathcal{I}$ imply $A \cup B \in \mathcal{I}$,

*iii.* $(A \in \mathcal{I} \wedge B \subseteq A) \Rightarrow B \in \mathcal{I}$,

then $\mathcal{I}$ is called an ideal of $X$.

**Definition 2.4.** [11] Let $\mathcal{I}$ be an ideal of $X$. Then,

*i.* $\mathcal{I}$ is called non-trivial ideal, if $\mathcal{I} \neq \emptyset$ and $X \notin I$.

*ii.* A non-trivial ideal $\mathcal{I} \subseteq P(X)$ is called admissible if and only if $\{\{x\} : x \in X\} \subseteq \mathcal{I}$.

From now on, let $\mathcal{I}$ be a non-trivial admissible ideal of $X$.

**Definition 2.5.** [11] Let $X \neq \emptyset$ and $\emptyset \neq F \subseteq P(X)$. If

*i.* $\emptyset \notin F$,

*ii.* $A, B \in F \Rightarrow A \cap B \in F$,

*iii.* $(A \in F \wedge B \subseteq A) \Rightarrow B \in F$,

then $F$ is called a filter on $X$.

**Remark 2.6.** [11] Let $\mathcal{I} \subseteq P(X)$ be a non-trivial ideal. Then, the family

$$F(\mathcal{I}) = \{M \subset X : M = X \setminus A, \text{ for some } A \in \mathcal{I}\}$$

is a filter on $X$, is called the filter associated with the ideal $\mathcal{I}$.

**Definition 2.7.** [19] Let $x = (x_k)$ be a sequence in $X$ and $r \geq 0$. $x = (x_k)$ is said to be ideal rough convergent to $x_0 \in X$ if, for every $\varepsilon > 0$,

$$\{k \in \mathbb{N} : ||x_k - x_0|| \geq r + \varepsilon\} \in \mathcal{I}$$

or equivalently

$$\mathcal{I} - \limsup ||x_k - x_0|| \leq r$$

and denoted by $x_k \xrightarrow{\mathcal{I}-r} x_0$. Thus,

$$\mathcal{I} - \mathrm{LIM}_r x = \left\{x_0 \in X : x_k \xrightarrow{\mathcal{I}-r} x_0\right\}$$

If $r = 0$, then the concepts of the ideal rough convergence and the $\mathcal{I}$-convergence are equivalent. If the set $\mathcal{I} - \mathrm{LIM}_r x$ is non-empty, then the sequence $x = (x_k)$ is $\mathcal{I} - r$ convergent.

**Definition 2.8.** [23] Let $x = (x_k)$ be a sequence in $X$. $x = (x_k)$ is said to be $\mathcal{I}$-statistical convergent to $x_0 \in X$, if, for any $\varepsilon > 0$ and $\delta > 0$,

$$\left\{n \in \mathbb{N} : \frac{1}{n}|\{k \leq n : ||x_k - x_0|| \geq \varepsilon\}| \geq \delta\right\} \in \mathcal{I}$$

and denoted by $\mathcal{I} - st - \mathrm{LIM}x = x_0$ or $x_k \xrightarrow{\mathcal{I}-st} x_0$. If $\mathcal{I} = \mathcal{I}_f = \{A \subseteq \mathbb{N} : A \text{ is a finite set}\}$, then $\mathcal{I}$-statistical convergence is the same as the classical convergence. If $\mathcal{I} = \mathcal{I}_\delta = \{A \subseteq \mathbb{N} : \delta(A) = 0\}$, then $\mathcal{I}$-statistical convergence is the same as the statistical convergence.

**Definition 2.9.** [17] Let $x = (x_k)$ be a sequence in $X$. $x = (x_k)$ is said to be $\mathcal{I}$-statistical convergent of order $\alpha \in (0, 1]$ to $x_0 \in X$, if, for any $\varepsilon > 0$ and $\delta > 0$, $\{n \in \mathbb{N} : \frac{1}{n^\alpha}|\{k \leq n : ||x_k - x_0|| \geq \varepsilon\}| \geq \delta\} \in \mathcal{I}$ and denoted by $\mathcal{I} - st - \text{LIM}^\alpha x = x_0$ or $x_k \xrightarrow{\mathcal{I}-st\alpha} x_0$.

**Definition 2.10.** [19] Let $x = (x_k)$ be a sequence in $X$ and $r \geq 0$. $x = (x_k)$ is said to be $\mathcal{I}$ statistical rough convergent ($\mathcal{I}$-st rough convergent) to $x_0 \in X$ if, for every $\varepsilon > 0$,

$$\left\{n \in \mathbb{N} : \frac{1}{n}|\{k \leq n : \|x_k - x_0\| \geq r + \varepsilon\}| \geq \delta\right\} \in \mathcal{I}$$

and denoted by $\mathcal{I} - st - \text{LIM}_r x = x_0$ or $x_k \xrightarrow{\mathcal{I}-st-r} x_0$.

**Definition 2.11.** [24] Let $x = (x_k)$ be a sequence in $X$. If there exist a real number $M$ such that $\{n \in \mathbb{N} : \frac{1}{n}|\{k \leq n : ||x_k|| > M\}| > \delta\} \in \mathcal{I}$, then $(x_k)$ is called $\mathcal{I}$-st bounded.

**Definition 2.12.** [21] Let $x = (x_k)$ be a sequence in $X$ and $c \in X$. If, for any $\varepsilon > 0$ and $\delta > 0$,

$$\left\{n \in \mathbb{N} : \frac{1}{n}|\{k \leq n : ||x_k - c|| \geq \varepsilon\}| < \delta\right\} \notin \mathcal{I}$$

then $c$ is called an $\mathcal{I}$-statistical cluster point of $x = (x_k)$.

The set of all $\mathcal{I}$-statistical ($\mathcal{I} - st$) cluster points denoted by $\mathcal{I} - S(\Gamma_x)$.

**Theorem 2.13.** [21] Let $x = (x_k)$ be an $\mathcal{I} - st$ bounded sequence. If the sequence $x = (x_k)$ has one cluster point, then it is $\mathcal{I} - st$ convergent.

## 3. Main Results

This section defines the concept of ideal statistical rough convergence of order $\alpha$ and studies some of its basic properties.

**Definition 3.1.** [22] Let $x = (x_k)$ be a sequence in $X$, $r \geq 0$, and $0 < \alpha \leq 1$. $x = (x_k)$. For all $\varepsilon > 0$, if

$$\lim_{n\to\infty} \frac{1}{n^\alpha}|\{k \leq n : ||x_k - x_0|| \geq r + \varepsilon\}| = 0$$

then the sequence $x = (x_k)$ is said to be statistical rough convergence of order $\alpha$ to $x_0$ and denoted by $x_k \xrightarrow{st-r\alpha} x_0$. Then, the set of all statistical rough of order $\alpha$ limit points of a sequence $(x_k)$ is denoted by $st - \text{LIM}_r^\alpha x$.

**Definition 3.2.** Let $x = (x_k)$ be a sequence in $X$. For any $\varepsilon > 0$ and $\delta > 0$, if

$$\left\{n \in \mathbb{N} : \frac{1}{n^\alpha}|\{k \leq n : ||x_k - x_0|| \geq r + \varepsilon\}| \geq \delta\right\} \in \mathcal{I}, \quad \alpha \in (0, 1]$$

then $x = (x_k)$ is called to be ideal statistical rough convergent of order $\alpha$ to $x_0$ and denoted by $x_k \xrightarrow{\mathcal{I}-st-r\alpha} x_0$. Then, the set of all ideal statistical rough of order $\alpha$ limit points is denoted by $\mathcal{I} - st - \text{LIM}_r^\alpha x$.

**Definition 3.3.** Let $x = (x_k)$ be a sequence in $X$. Then, $x = (x_k)$ is called $\mathcal{I}$-statistical bounded of order $\alpha$, if there exists a real number $H$ such that

$$\left\{n \in \mathbb{N} : \frac{1}{n^\alpha}|\{k \leq n : ||x_k|| > H\}| > \delta\right\} \in \mathcal{I}$$

Limit of a convergent sequence is unique, however limit of a rough convergent sequence is not need to be unique, for the degree of roughness $r > 0$.

**Theorem 3.4.** Let $x = (x_k)$ be a sequence in $X$. $x$ is $\mathcal{I}$-st bounded of order $\alpha$ if and only if there exists $r \geq 0$ such that $\mathcal{I} - st - \text{LIM}_r^\alpha x \neq \emptyset$.

PROOF. Let $X$ be a normed space and $x = (x_k)$ be a sequence in $X$.
($\Rightarrow$) Assume that $x = (x_k)$ be $\mathcal{I}$-st bounded sequence. Then, there exists a real number $H$ such that

$$A = \left\{n \in \mathbb{N} : \frac{1}{n^\alpha}|\{k \leq n : ||x_k|| > H\}| > \delta\right\} \in \mathcal{I}$$

Let $\bar{r} = \sup_{\substack{t \leq m \\ m \in \mathbb{N} \setminus A}} \{||x_t||\}$. $\mathcal{I} - st - \text{LIM}_{\bar{r}}^\alpha x$ includes the origin of $X$ $\mathcal{I} - st - \text{LIM}_{\bar{r}}^\alpha x \neq \emptyset$. Here, since the normed space $X$ is a vector space, then the origin of $X$ is $\theta_X$.
($\Leftarrow$) Suppose that for an arbitrary $r \geq 0$, $\mathcal{I} - st - \text{LIM}_r^\alpha x \neq \emptyset$. Then, for any $\varepsilon > 0$ and $\delta > 0$, there exists $x_0 \in \mathcal{I} - st - \text{LIM}_r^\alpha x$ such that

$$\left\{n \in \mathbb{N} : \frac{1}{n^\alpha}|\{k \leq n : ||x_k - x_0|| \geq r + \varepsilon\}| \geq \delta\right\} \in \mathcal{I}$$

Choose $\varepsilon = \|x_0\|$. Then, for each $\delta > 0$ and $H = r + \|x_0\|$,

$$\left\{n \in \mathbb{N} : \frac{1}{n^\alpha}|\{k \leq n : \|x_k - x_0\| \geq H\}| \geq \delta\right\} \in \mathcal{I}$$

Therefore, $x = (x_k)$ is an $\mathcal{I}-$ st bounded sequence of order $\alpha$. □

**Theorem 3.5.** Let $x = (x_k)$ be a sequence in $X$. The set $\mathcal{I} - st - \text{LIM}_r^\alpha x$ is closed.

PROOF. If $\mathcal{I} - st - \text{LIM}_r^\alpha x = \emptyset$, then the proof is clear. Let $\mathcal{I} - st - \text{LIM}_r^\alpha x \neq \emptyset$ and there exists $(y_n) \subseteq \mathcal{I} - st - \text{LIM}_r^\alpha x$ such that $y_n \to x_0$ as $n \to \infty$. Then, for all $\varepsilon > 0$ there exists $n_\varepsilon \in \mathbb{N}$ such that $n > n_\varepsilon \Rightarrow ||y_n - x_0|| < \varepsilon$. Choose $n_0 \in \mathbb{N}$, $y_{n_0} \in (y_n) \subseteq \mathcal{I} - st - \text{LIM}_r^\alpha x$, then

$$A = \left\{n \in \mathbb{N} : \frac{1}{n^\alpha}|\{k \leq n : ||x_k - y_{n_0}|| \geq r + \frac{\varepsilon}{2}\}| < \delta\right\} \in F(\mathcal{I})$$

For $t \in A$, $\frac{1}{t^\alpha}|\{k \leq t : ||x_k - y_{n_0}|| \geq r + \frac{\varepsilon}{2}\}| < \delta$. For maximum $k \leq t$, $||x_k - y_{n_0}|| < r + \frac{\varepsilon}{2}$ and for an arbitrary $n_0 > n_\varepsilon$

$$||x_k - x_0|| \leq ||x_k - y_{n_0}|| + ||y_{n_0} - x_0|| < r + \varepsilon$$

and for maximum $k \leq t \in A$

$$\left\{n \in \mathbb{N} : \frac{1}{n^\alpha}|\{k \leq n : ||x_k - x_0|| \geq r + \varepsilon\}| < \delta\right\} \in \mathcal{I}$$

Hence, $x_0 \in \mathcal{I} - st - \text{LIM}_r^\alpha x$. Consequently, $\mathcal{I} - st - \text{LIM}_r^\alpha x$ is a closed set. □

**Theorem 3.6.** Let $x = (x_k)$ be a sequence in $X$. The set $\mathcal{I} - st - \text{LIM}_r^\alpha x$ is convex.

PROOF. Let $y_0, y_1 \in \mathcal{I} - st - \text{LIM}_r^\alpha x$. For any $\varepsilon > 0$ and $\delta > 0$,

$$A_1 = \left\{n \in \mathbb{N} : \frac{1}{n^\alpha}|\{k \leq n : ||x_k - y_0|| \geq r + \varepsilon\}| \geq \delta\right\} \in \mathcal{I}$$

$$A_2 = \left\{n \in \mathbb{N} : \frac{1}{n^\alpha}|\{k \leq n : ||x_k - y_1|| \geq r + \varepsilon\}| \geq \delta\right\} \in \mathcal{I}$$

As $S = \mathbb{N} \setminus (A_1 \cup A_2) \in F(\mathcal{I})$, then $S$ is a finite set. For $s \in S$, let $B_1 = \{k \leq s : ||x_k - y_0|| \geq r + \varepsilon\}$ and $B_2 = \{k \leq s : ||x_k - y_1|| \geq r + \varepsilon\}$. Thus, $\lim_{n\to\infty} \frac{1}{n^\alpha}|B_1| = \lim_{n\to\infty} \frac{1}{n^\alpha}|B_2| = 0$. For all $k \in B_1^c \cap B_2^c$ and $\lambda \in [0, 1]$,

$$||x_k - ((1 - \lambda)y_0 + \lambda y_1)|| = ||(1 - \lambda)(x_k - y_0) + \lambda(x_k - y_1)|| < r + \varepsilon$$

and so $\lim\limits_{k\to\infty} \frac{1}{k^\alpha}|B_1^c \cap B_2^c| = 1$. Thereby,

$$\left\{n \in \mathbb{N} : \frac{1}{n^\alpha}|\{k \le n : ||x_k - ((1-\lambda)y_0 + \lambda y_1)|| \ge r + \varepsilon\}| < \delta\right\} \supseteq S \in F(\mathcal{I})$$

Consequently, $\mathcal{I} - st - \mathrm{LIM}_r^\alpha x$ is convex. $\square$

**Theorem 3.7.** For an arbitrary $b \in \mathcal{I} - S(\Gamma_x)$. Then, for all $x_* \in \mathcal{I} - st - \mathrm{LIM}_r^\alpha x$, $||x_* - b|| \le r$.

Proof. Assume that $b \in \mathcal{I}-S(\Gamma_x)$ and $x_* \in \mathcal{I}-st-\mathrm{LIM}_r^\alpha x$ such that $||x_*-b|| > r$. Let $\varepsilon = \frac{||x_*-b||-r}{2}$. Therefore,

$$A = \left\{n \in \mathbb{N} : \frac{1}{n^\alpha}|\{k \le n : ||x_k - b|| \ge \varepsilon\}| < \delta\right\} \notin \mathcal{I}$$

Let

$$B = \left\{n \in \mathbb{N} : \frac{1}{n^\alpha}|\{k \le n : ||x_k - x_*|| \ge r + \varepsilon\}| \ge \delta\right\}$$

For $t \in A$, $\frac{1}{m^\alpha}|\{k \le t : ||x_k - b|| \ge \varepsilon\}| < \delta$. So, for maximum $k \le t, ||x_k - b|| < \varepsilon$. Hence, for all $k \le t \in A$,

$$||x_k - x_*|| \ge ||x_* - b|| - ||x_k - b|| > r + \varepsilon$$

Therefore, $B \supseteq A$ implies that $B$ is not an element of $\mathcal{I}$ and this is a contradiction. Thus, $||x_*-b|| \le r$, for all $x_* \in \mathcal{I} - st - \mathrm{LIM}_r^\alpha x$ and $b \in \mathcal{I} - S(\Gamma_x)$. $\square$

**Theorem 3.8.** Let $r \ge 0$. $x = (x_k)$ is $\mathcal{I}$-statistical rough convergent of order $\alpha$ to $x_*$ if and only if there exists a sequence $y = (y_k)$ such that $\mathcal{I} - st - \mathrm{LIM}^\alpha y = x_*$ and for all $k \in \mathbb{N}$, $||x_k - y_k|| \le r$.

Proof. ($\Leftarrow$) Assume that $\mathcal{I} - st - \mathrm{LIM}^\alpha y = x_*$ and for all $k \in \mathbb{N}$, $||x_k - y_k|| \le r$. For any $\varepsilon > 0$ and $\delta > 0$

$$A = \left\{n \in \mathbb{N} : \frac{1}{n^\alpha}|\{k \le n : ||y_k - x_*|| \ge \varepsilon\}| < \delta\right\} \in \mathcal{I}$$

Moreover,

$$||x_k - x_*|| \le ||x_k - y_k|| + ||y_k - x_*|| < r + \varepsilon, \quad k \le s \in A^c$$

Therefore,

$$\left\{n \in \mathbb{N} : \frac{1}{n^\alpha}|\{k \le n : ||x_k - x_*|| \ge r + \varepsilon\}| < \delta\right\} \supset A^c \in F(\mathcal{I})$$

Then, $x = (x_k)$ is $\mathcal{I}$-statistical rough convergent of order $\alpha$ to $x_*$
($\Rightarrow$) Suppose that $\mathcal{I} - st - \mathrm{LIM}^\alpha x_k = x_*$. For any $\varepsilon > 0$,

$$\left\{n \in \mathbb{N} : \frac{1}{n^\alpha}|\{k \le n : ||x_k - x_*|| \ge \varepsilon\}| > \delta\right\} \in \mathcal{I}$$

For the sequence $y = (y_k)$ defined by

$$y_k = \begin{cases} x_*, & \text{if } ||x_k - x_*|| \le r \\ x_k + r\frac{x_* - x_k}{||x_k - x_*||}, & \text{otherwise} \end{cases}$$

the following inequalities, for $k \in \mathbb{N}$,

$$||y_k - x_*|| \le \begin{cases} 0, & \text{if } ||x_k - x_*|| \le r \\ ||x_k - x_*|| + r, & \text{if otherwise} \end{cases}$$

and

$$||x_k - y_k|| \le r$$

are hold. Thereby,

$$\left\{n \in \mathbb{N} : \frac{1}{n^\alpha}|\{k \le n : ||y_k - x_*|| \ge r + \varepsilon\}| \ge \delta\right\} \in \mathcal{I}$$

Consequently, $\mathcal{I} - st - \mathrm{LIM}^\alpha y = x_*$ and for all $k \in \mathbb{N}$, $||x_k - y_k|| \le r$. $\square$

## 4. Conclusion

In this study, ideal statistical rough convergence of order $\alpha$ was defined. Moreover, some important properties of the set of all ideal statistical rough of order $\alpha$ limit points were studied. In addition, this study proved two theorems that "a sequence $(x_k)$ in a normed space is $\mathcal{I}$-st bounded of order $\alpha$ if and only if there exists $r \geq 0$ such that $\mathcal{I} - st - \text{LIM}_r^\alpha x \neq \emptyset$" and "a sequence $(x_k)$ in a normed space is $\mathcal{I}$-statistical rough convergent of order $\alpha$ to $x_*$ if and only if there exists a sequence $y = (y_k)$ such that $\mathcal{I} - st - \text{LIM}^\alpha y = x_*$ and for all $k \in \mathbb{N}$, $||x_k - y_k|| \leq r$".

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