The Numerical Approximation for the Single-walled Carbon Nanotubes Conveying Fluid

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Abstract
This study introduces the free vibration of single-walled carbon nanotubes conveying fluid. The variational iteration method is applied to the problem of single-walled carbon nanotubes. The governing equation is based on the model of the beam with the local, coriolis and centripetal acceleration components. The velocity of fluid flow is assumed as a constant mean velocity. The numerical results are illustrated for this method with high accuracy and simplicity.

Keywords — Carbon nanotube, linear vibration, variational iteration method

1 Introduction

Many models of carbon nanotubes (CNTs) due to some excellent properties appear extensively in the literature. There are many new applications of CNTs in nanobiological devices and nanomechanical systems [1]. Recently, the CNTs conveying fluid have become quite popular. In the literature, many investigations exist about the CNTs filled with fluid as well as the studies involving the elastic properties of CNTs. The vibration behaviors of CNTs conveying fluid are widely studied by using the theories based continuum [2-4].

Yoon [2] et al. introduced the free vibration of cantilever carbon nanotubes with a continuum elastic model. The stability of the single-walled carbon nanotubes (SWCNTs) was structurally studied by Reddy [5]. He presented how the flow velocity influences natural frequency and the fundamental mode shape. The effects of flow velocity on the structure were investigated by Lee and Chang [1]. Also, they [6] used the Timoshenko beam theory for obtaining the velocity of flow fluid and geometric parameters on the frequency and mode shape of the SWCNT.

Recently, Lin and Qiao [7] applied the differential quadrature method to discretize the governing equation of CNTs conveying fluid. Thus, they analysed the vibration and instability of CNTs. Wang [8] introduced the model of fluid-conveying SWCNTs based on elastic Bernoulli-Euler beam.

Natsuki et al. [9] used wave propagation approach method for the vibrations of CNTs conveying fluid. Yan et al. [10] presented the analysis of the fluid-filled MWCNTs. Aminikhah and Hemmatnezhad [11] obtained the nonlinear vibrations of MWCNTs by applying the variational iteration method. Wang et al. [12] considered free vibration of multi-walled carbon nanotubes (MWCNTs) based on the Timoshenko beam model. Zhang et al. [13] studied transverse vibrations of double-walled carbon nanotubes subject to compressive axial load via the beam modelled with the Bernoulli–Euler beam theory.

In this study, the single-walled carbon nanotubes are presented. The model of SWCNT in [5] is considered as dimensionless. The variational iteration method (VIM) is applied to the governing equation. VIM is a highly effective method for approximate solutions of many vibrating systems [14-18]. Besides, this method gives the approximations converging rapidly to the exact solutions for the free vibrations of a SWCNTs conveying fluid.

2 Variational Iteration Method

For an approximate solution, we present the basic ideas of VIM [11]. The general nonlinear differential equation is considered in the form

$$Lu(t)+Nu(t)=f(t)$$

where L is a linear operator, N is nonlinear operator, and $f(t)$ is a known function. Using the variational iteration method, the iteration formulation is constructed as

$$u_{n+1}(t)=u_n(t)+\int_0^t \lambda\left[Lu_n(\xi)+N\tilde{u}_n(\xi)-g(\xi)\right]d\xi$$

so called a correction functional with initial approximation $u_0(t)$. Here, λ is a general Lagrange multiplier, which can be optimally determined via variational theory; the subscript n means the n th approximation; $\tilde{u}_n$ is a restricted variation, i.e. $\delta\tilde{u}_n=0$ [19]. Now, let us apply VIM to the problem of the free vibrations of SWCNT.

3 Applying VIM for free vibration analysis of a SWCNT

Consider the SWCNT with length L, the mass per unit length m_c, the mass of fluid per unit length m_f, the uniform mean flow velocity of the fluid $\hat{v}$ and the bending rigidity YI. The governing equation gives the flexural vibration motion of SWCNT. The vibration equation for the carbon nanotube conveying fluid [5] is

$$\left(m_f+m_c\right)\frac{\partial^2\hat{w}}{\partial\hat{t}^2}+2m_f\hat{v}\frac{\partial^2\hat{w}}{\partial\hat{x}\,\partial\hat{t}}+m_f\hat{v}^2\frac{\partial^2\hat{w}}{\partial\hat{x}^2}+YI\frac{\partial^4\hat{w}}{\partial\hat{x}^4}=0 \qquad (1)$$

where the first term corresponds to the inertial term regarding both structure and fluid, the second and third terms are coriolis and centrifugal terms related to the fluid flow, the last term is exactly related to SWCNT. Here, $\hat{w}$ represents the transverse displacement depending on time $\hat{t}$ and the spatial coordinate $\hat{x}$. Here, the bending rigidity YI instead of the assumed values of Young's modulus and wall thickness in literature is used as in [1]. The boundary conditions given for Eq. (1) correspond to the conditions which the displacement and slope equal to zero at $x=0$ and $x=1$ such that

$$\hat{w}(0,t)=\hat{w}(1,t)=\left.\frac{\partial\hat{w}}{\partial\hat{x}}\right|_{x=0}=\left.\frac{\partial\hat{w}}{\partial\hat{x}}\right|_{x=1}=0$$

(2)

The dimensionless parameters are defined as

$$w=\frac{\hat{w}}{L},\qquad x=\frac{\hat{x}}{L},\qquad t=\frac{\hat{t}}{L^2}\sqrt{\frac{YI}{m_c+m_f}},$$

$$m_r=\frac{m_f}{m_c+m_f},\; v=\hat{v}L\sqrt{\frac{m_f}{YI}} \qquad (3)$$

where m_r is the mass ratio. Then, Eq. (1) transforms to the following dimensionless equation

$$\frac{\partial^2 w}{\partial t^2}+2v\sqrt{m_r}\frac{\partial^2 w}{\partial x\,\partial t}+v^2\frac{\partial^2 w}{\partial x^2}+\frac{\partial^4 w}{\partial x^4}=0$$

(4)

$$w(0,t)=w(1,t)=\left.\frac{\partial w}{\partial x}\right|_{x=0}=\left.\frac{\partial w}{\partial x}\right|_{x=1}=0$$

(5)

Applying VIM, the correction functional on Eq. (4) is obtained as

$$w_{n+1}(x,t)=w_n(x,t)+\int_0^t\lambda(s)\left[\frac{\partial^2 w_n(x,s)}{\partial s^2}+2v\sqrt{m_r}\frac{\partial^2\tilde{w}(x,s)}{\partial x\partial s}+v^2\frac{\partial^2\tilde{w}(x,s)}{\partial x^2}+\frac{\partial^4\tilde{w}(x,s)}{\partial x^4}\right]ds \qquad (6)$$

where w_n is n th approximate solution and $\tilde{w}_n$ is a restricted variation, i.e. $\delta\tilde{w}_n=0$. Applying $\delta w_n(x,0)=0$ for making the correction functional stationary, we get

$$\delta w_{n+1}(x,t) = \delta w_n(x,t) + \lambda(s)\,\delta w_n'(x,s)\Big|_0^t$$

$$-\lambda'(s)\,\delta w_n(x,s)\Big|_0^t + \int_0^t \{\lambda''(s)\,\delta w_n(x,s)\}\,ds = 0$$

(7)

Then, the stationary conditions are obtained as

$$\lambda''(s) = 0$$

$$\lambda(s)\Big|_{s=t} = 0$$

$$1 - \lambda'(s)\Big|_{s=t} = 0$$

(8)

If the problem is solved, the Lagrange multiplier is readily found

$$\lambda(s) = s - t$$

(9)

Substituting the Lagrange multiplier into Eq. (6), the iteration formula becomes

$$w_{n+1}(x,t) = w_n(x,t) + \int_0^t (s-t)\left[\frac{\partial^2 w_n(x,s)}{\partial s^2} + 2v\sqrt{m_r}\,\frac{\partial^2 w(x,s)}{\partial x \partial s} + v^2\frac{\partial^2 w(x,s)}{\partial x^2} + \frac{\partial^4 w(x,s)}{\partial x^4}\right] ds$$

(10)

For approximate solution of Eq. (4), the initial approximation is assumed as

$$w_0(x,t) = X(x)\exp(i\omega t)$$

(11)

where ω is the natural frequency. Substituting the initial approximation into Eq. (4) leads to

$$X^{iv} + v^2 X'' + 2iv\omega\sqrt{m_r}\,X' - \omega^2 X = 0$$

(12) $X(0) = X(1) = X'(0) = X'(1) = 0$

(13)

for which the solution is the following form

$$X(x) = \sum_{n=1}^{4} c_n e^{i r_n x}$$

(14)

where $r_n\ (n = 1,2,3,4)$ satisfies the characteristic equation

$$r^4 + v^2 r^2 + 2iv\omega\sqrt{m_r}\,r - \omega^2 = 0$$

(15)

Applying the boundary conditions (13) yields

$$c_1 + c_2 + c_3 + c_4 = 0$$

(16)

$$r_1 c_1 + r_2 c_2 + r_3 c_3 + r_4 c_4 = 0$$

(17)

$$c_1 e^{i r_1} + c_2 e^{i r_2} + c_3 e^{i r_3} + c_4 e^{i r_4} = 0$$

(18)

$$c_1 r_1 e^{i r_1} + c_2 r_2 e^{i r_2} + c_3 r_3 e^{i r_3} + c_4 r_4 e^{i r_4} = 0$$

(19)

ω and r_n can be numerically calculated by using the characteristic equation and the boundary condition. The coefficients c_n are obtained by elimination from Eqs. (16)-(19).

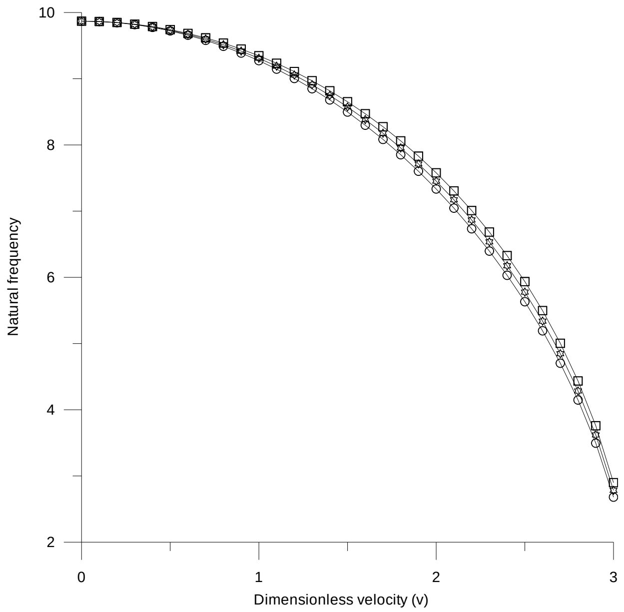

Figure 1: Variation of natural frequency with the velocity for the different value m_r . Circles, stars and squares represent the values 0.1, 0.5 and 0.9 of m_r , respectively (for first mode).

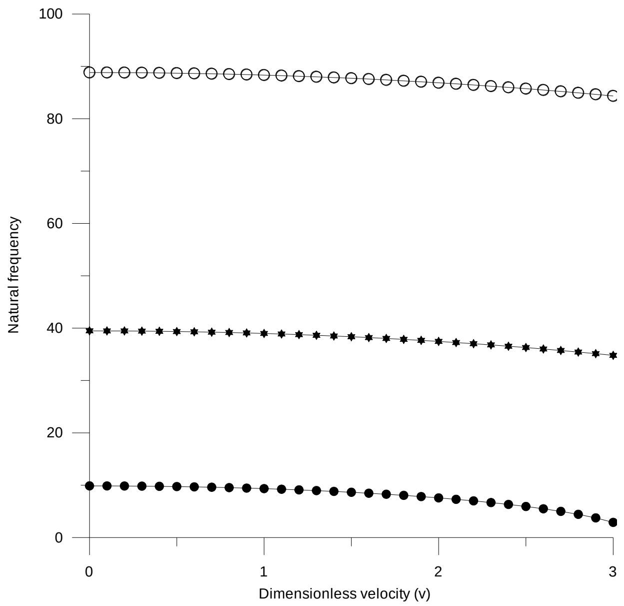

Figure 2: Variation of natural frequency with the velocity for the various modes. Dots, stars and circles denote first, second and third modes, respectively.

In Fig. 1, the natural frequency-velocity graphics are shown for the different values of mass ratio. For first mode, the natural frequency decreases as the flow velocity increases. Thus, the fluid flow in the nanotube does not influence damping of the SWCNT. As the flow velocity increases to approximately π , natural frequency is equal to zero.

In Fig. 2, the natural frequency becomes zero. Then, the velocity is defined as the critical velocity which is equal to approximately π . Also, SWCNT is unstable at this velocity.

4 Conclusion

The free vibrations of a SWCNTs conveying fluid are analysed. The model presented for SWCNT is reduced to dimensionless form. The equation of motion is modelled as the beam with local, Coriolis and centripetal acceleration components. This partial differential equation is solved by variational iteration method. The stability of the

SWCNT is structurally studied. Also, the critical velocity of fluid flow is determined. The numerical results are graphically given.

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