

Static analysis of beams on elastic foundation by the method of discrete singular convolution

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Abstract

A discrete singular convolution method is presented for computation of the deflection analysis of beams resting on elastic foundation. In the method of discrete singular convolution partial space derivatives of a function appearing in a differential equation are approximated by means of some kernels. Results are compared with existing solutions available from other analytical and numerical methods. The method presented gives accurate results and is computationally efficient.

Keywords: discrete singular convolution, beams on elastic foundation.

1. Introduction

Real physical systems or engineering problems are often described by partial differential equations, either linear or nonlinear and in most cases, their closed form solutions are extremely difficult to establish. As a result, approximate numerical methods have been widely used to solve partial differential equations which arise in almost all engineering disciplines. The most commonly used numerical methods for such applications are the finite element, finite difference and boundary element method and nowadays, most engineering problems can be solved by these methods to satisfactory accuracy if a proper and sufficient number of grid points are used.

The analysis of structures on elastic foundations is of considerable interest and widely used in several fields such as foundation engineering, pavement and railroad engineers, pipelines application, and some aero-space structures. Many problems in the engineering applications related to soil-structure interaction can be modeled by means of a beam or a beam-column on elastic foundation. Although few type foundation model are exist, the Winkler foundation model is extensively used by engineers and researchers because of its simplicity. Generally, the foundation is considered as an array of springs uniformly distributed along the length of the beam. A detailed explained of foundation models can be found in related references [1-15]. In this paper, discrete singular convolution method technique is presented for computation of the static analysis of beams on elastic foundation. The accuracy of the solutions is inferred by comparison with analytical and other numerical solutions.

2. Discrete singular convolution (DSC)

Discrete singular convolutions (DSC) algorithm introduced by Wei [16]. Wei and his co-workers [17-26] first applied the DSC algorithm to solve solid and fluid mechanics problem. These studies indicates that the DSC algorithm work very well for the vibration analysis of plates, especially for vibration and buckling analysis of micro and macro beams, plates and shells [27-66]. Furthermore, it is also concluded that the DSC algorithm has global methods' accuracy and local methods' flexibility for solving differential equations in applied mechanics. In a general definition, numerical solutions of differential equations are formulated by some singular kernels. The mathematical foundation of the DSC algorithm is the theory of distributions and wavelet analysis. Consider a distribution, T and $\eta(t)$ as an element of the space of the test function. A singular convolution can be defined by [16]

$$F(t) = (T * \eta)(t) = \int_{-\infty}^{\infty} T(t-x)\eta(x)dx, \quad (1)$$

where $T(t-x)$ is a singular kernel. For example, singular kernels of delta type [17]

$$T(x) = \delta^{(n)}(x); \quad (n=0,1,2,\dots). \quad (2)$$

Kernel $T(x) = \delta(x)$ is important for interpolation of surfaces and curves, and $T(x) = \delta^{(n)}(x)$ for $n>1$ are essential for numerically solving differential equations. With a sufficiently smooth approximation, it is more effective to consider a discrete singular convolution [18]

$$F_{\alpha}(t) = \sum_k T_{\alpha}(t-x_k)f(x_k), \quad (3)$$

where $F_{\alpha}(t)$ is an approximation to $F(t)$ and $\{x_k\}$ is an appropriate set of discrete points on which the DSC (3) is well defined. Note that, the original test function $\eta(x)$ has been replaced by $f(x)$. This new discrete expression is suitable for computer realization. The mathematical property or requirement of $f(x)$ is determined by the approximate kernel T_{α} . Recently, the use of some new kernels and regularizer such as delta regularized [19] was proposed to solve applied mechanics problem. The Shannon's kernel is regularized as [20]

$$\delta_{\Delta,\sigma}(x-x_k) = \frac{\sin[(\pi/\Delta)(x-x_k)]}{(\pi/\Delta)(x-x_k)} \exp\left[-\frac{(x-x_k)^2}{2\sigma^2}\right]; \quad \sigma>0 \quad (4)$$

where Δ is the grid spacing. It is also known that the truncation error is very small due to the use of the Gaussian regularizer, the above formulation given by Eq. (1) is practically and has an essentially compact support for numerical interpolation. Equation (1) can also be used to provide discrete approximations to the singular convolution kernels of the delta type [18]

$$f^{(n)}(x) \approx \sum_{k=-M}^M \delta_{\Delta}(x-x_k) f(x_k), \quad (5)$$

In the DSC method, the function $f(x)$ and its derivatives with respect to the x coordinate at a grid point x_i are approximated by a linear sum of discrete values $f(x_k)$ in a narrow bandwidth $[x-x_M, x+x_M]$. This can be expressed as

$$\left. \frac{d^n f(x)}{d x^n} \right|_{x=x_i} = f^{(n)}(x) \approx \sum_{k=-M}^M \delta_{\Delta,\sigma}^{(n)}(x_i-x_k) f(x_k); \quad (n=0,1,2,\dots). \quad (6)$$

where superscript n denotes the n th-order derivative with respect to x . For example the second order derivative at $x=x_i$ of the DSC kernels for directly given

$$\delta_{\Delta,\sigma}^{(2)}(x-x_j) = \left. \frac{d^2}{d x^2} \left[\delta_{\Delta,\sigma}(x-x_j) \right] \right|_{x=x_i}, \quad (7)$$

The discretized forms of Eq. (7) can then be expressed as

$$f^{(2)}(x) = \left. \frac{d^2 f}{d x^2} \right|_{x=x_i} \approx \sum_{k=-M}^M \delta_{\Delta,\sigma}^{(2)}(k\Delta x_N) f_{i+k,j}. \quad (8)$$

3. Beams on elastic foundation

Numerous researchers have treated the linear and nonlinear analysis of beams on elastic foundation having various support conditions. Closed form solutions of the governing differential equations have been proposed in the literature [1-8]. In recent years, the problem of stability and vibration analysis of beam or beam-columns has been studied [9-15]. Consider a linear elastic beam of stiffness EI on a winkler elastic foundation of modulus of k . The governing differential equation for the deflection of the beam resting on elastic foundation in Fig.1 is

$$EI \frac{d^4 v}{d x^4} + kv = -q(x) \quad (9)$$

where v defines the deflection of the beam, E is the modulus of the elasticity of the beam material, I is the moment of inertia of the cross-section, $q(x)$ is the distributed lateral load, and k is the foundation modulus. By using DSC discretization the Eq. (9) takes the form

$$EI \sum_{j=1}^N \delta_{\pi/\Delta,\sigma}^{(4)}(\Delta x) v(x_i) + kv_i = -q(x) \quad (10)$$

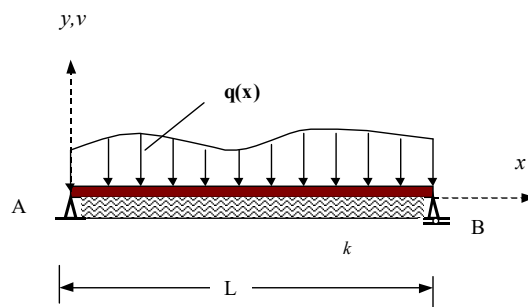


Fig.1. a. Beam on elastic foundation

4. Results and conclusions

A partially loaded beam on elastic foundation with 5 m is considered [67]. The uniformly distributed load ($q=1$ kN/m) is partially effected to the beam (from the 1 m inside of B support during 1 m). The results are listed in Table 1. The results are listed for two points (at $x=3$ m and at $x=4$ m). The beam have $EI=45*10^3$ kN /m². The foundation stiffness is $k=10^6$ kN/m². It is shown that the reasonable accurate results have been obtained for DSC method using 7 points.

Table 1. Deflections for beams for two points

x (m)	Ref. 67	DSC (N=7)	DSC (N=9)	(N=11)
3	$0.497*10^{-6}$	$0.495*10^{-6}$	$0.497*10^{-6}$	$0.497*10^{-6}$
4	$0.518*10^{-6}$	$0.517*10^{-6}$	$0.518*10^{-6}$	$0.518*10^{-6}$

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