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Interval Valued Bipolar Fuzzy Prioritized Weighted Dombi Averaging Operator Based On Multi-Criteria Decision Making Problems

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Abstract

In this paper, we develop to a new operator which Interval Valued Bipolar Fuzzy Prioritized Weighted Dombi Averaging operator (IVBFPWDA) by using together IVBFS and Dombi operators. This construction is important because of presenting priotrized flexible approach. The soft concept provides to rank into its own structure. Thus, the obtained rankings act to find to ideal alternative and non-ideal alternative for many k values. Then, the some properties of this concept are introduced like addition, scalar multiplication, scalar power, multiplication. Moreover, we present to score function over (IVBFSs) [8] for ranking of aggregated values. Also, the offered operator is applied over an investment example under the Multi Criteria Decision Making (MCDM) method with five steps. Firstly, the components of Decision Making Matrix are turned into aggregated values thanks to IVBFPWDA and score values are calculated. The obtained values are ranked and determined the most desired alternative. The results show that our operator is reality, objective and agreement in its own. Also, a comparative analysis is proposed by using Hamacher operators over Interval Valued Bipolar fuzzy sets (IVBFSs) [8]. As can be seen from comparison analysis, the results are agreement. But our defined operator has several advantages than operators over IVBFS for example; our operator is flexible and variable. Thus; prefer, need and requirement of decision makers can be satisfied.

1. INTRODUCTION

Fuzzy set concept [1] was proposed in 1965 and developed by novel authors under different disciplines like medical diagnosis, MCDM problems, algebraic constructions, artificial intelligence etc. Especially, MCDM problems have been used many times with fuzzy sets, see [2,3,4] but this cluster has not satisfied enough requirement due to some reasons like increasing of noise information, non-objective interpretations of decision makers, density data flowing so on. Therefore, new concepts were started to be discussed that can be ordered as follow; Type-2 fuzzy sets [5], Bipolar Valued fuzzy sets [6, 7], Interval Valued Bipolar fuzzy sets [8] etc.. One of the most important structures is interval valued bipolar fuzzy sets. The concept was developed to eliminate the biggest obstacle of decision makers like giving only membership value. The decision makers have easily overcome indeterminate information by utilizing two membership instead a membership value together with interval valued bipolar fuzzy concept. An interval valued bipolar fuzzy set is proposed by two fuzzy membership functions as the membership degree of which is a fuzzy set pair in interval $[0,1]\times[0,1]$ and $[-1,0]\times[-1,0]$ instead using a value in [0,1] and [-1,0].

In daily life, human and environment want opposite opinion as white-black, front-back, positive-negative etc.. Therefore, mathematical environment needs to opposite ideas especially into decision making problems. In here, positive and negative values explain to positive or negative ideas of experts about an element. Therefore, bipolarity is very essential for mathematic. In this direction, Lee [9] and Zhang [6, 7] have put forward to idea of bipolar fuzzy set. Bipolarity construction is surveyed with three type. First of all, it is obtained by dividing membership value such as interval valued fuzzy sets (IVFS), Interval valued hesitant fuzzy sets (IVHFS), second type contains to have two different positive values that one of values shows degree of belonging of an element, if second of values, presents degree of non-belonging of an element for example; Intuitionistic fuzzy sets, Intuitionistic hesitant fuzzy sets, Dual hesitant fuzzy sets. If third type, bipolar-valued fuzzy set (BVFS) has been offered by Zhang and Lee [9, 6, 7]. Zhang and Lee

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defined two different memberships for an element as positive and negative. Increasing or decreasing of positive value and negative value indicates that increasing or decreasing of positive and negative comments of decision makers about an element.

T-norm and T-conorm constructions of Dombi operators were defined into [10]. But these operators propose more flexible approach according to other operators because of having generalization structure. In short time, a lot of concepts have been defined together with this construction. Picture fuzzy Dombi aggregation operators were presented by Jana et al. [11], then He [12] proposed Dombi hesitant fuzzy information aggregation operators and applied over medical diagnosis, Akram and Khan [13] proposed to Complex Pythagorean Dombi fuzzy graphs, Pythagorean Dombi fuzzy aggregation operators were surveyed by Akram and coauthors [14], Liu et al. [15] presented some intuitionistic fuzzy Dombi Bonferroni mean operators and tested an application over MCDM problems, Chen and Ye [16] defined to Dombi operations over single-valued neutrosophic set and consisted of series of aggregation operators, Complex Pythagorean Dombi fuzzy operators based on aggregation operators were tested by Akram et al. [17], Jana and others [18] offered to Bipolar fuzzy Dombi aggregation operators and Shi [20] defined to Dombi aggregation operators of neutrosophic cubic sets for multiple attribute decision- making.

The prioritized approach was developed over aggregation operators by Yager [21] in 2007. This approach provides prioritization relationship for criterions over aggregation problems. In this approach, while weights of criterions are determined, the weights are calculated based on criterions and the highest weight of criterion is prioritized from weights of other criterions. Thus, the revealed weights of criterions associated with decision makers can be protected from non-objective comments. This statement induces to error margin while the best alternative and the worst alternative are determined. In next time, this construction has been started to be worked by several authors as follow; Jana and others [19] offered Bipolar fuzzy Dombi prioritized aggregation operators in 2020, Gao et al. [22] defined to dual hesitant bipolar fuzzy Hamacher prioritized aggregation operators over MCDM, He and coauthors [23] defined to scaled prioritized geometric aggregation operators, Jin et al. [24] developed Interval valued hesitant fuzzy Einstein prioritized aggregation operators, Ye [25] defined prioritized aggregation operators over trapezoidal fuzzy sets, Akram et al. [26] proposed complex spherical fuzzy prioritized weighted aggregation operators, Liu and coauthors [27] gave extensions of prioritized aggregation operators over complex q-rung orthopair fuzzy information, Akram et al. [28] revealed prioritized weighted aggregation operators under complex pythagorean fuzzy sets. Although above proposed works; fuzzy, complexity, vagueness information has not be removed because of density data channels, non-objective decisions of experts, increasing relationship of disciplines so on. In this paper, we define to Interval valued Bipolar Fuzzy Prioritized Weighted Dombi Averaging operator (IVBFPWDA). This concept has novel advantages as follow;

1. The flexible structure has been revealed and thus comparative analysis has been presented in its own,

2. The positive and negative opinions have been given together with Dombi operators,

3. An application has been proposed over a reality example and the results are completely consistent in its own,

4. A comparative analysis has been developed with Hamacher operators and no change has been observed between the results. The results show that our operator is reality, objective and soft.

The remaining of paper is organized as follow; in section 2, some basic definitions are given as dombi operators so on, in section 3, 4, Interval valued bipolar fuzzy sets (IVBFS) and basic dombi properties are presented and also we offer Interval valued Bipolar Fuzzy Prioritized Weighted Dombi Averaging operator (IVBFPWDA) and Characteristic properties of this operator are presented, in section 5, we give decision making algorithm and an example in daily life and in section 6, we develop a comparative analysis.

2. PRELIMINARY

In this section, we recall some basic notions of fuzzy sets, dombi operators.

Definition 2.1 *[1] Let E be a universe. A fuzzy set X over E is a mapping defined as follows:*

$$X = \{(\mu_X(x)/x): x \in E\}$$

where $\mu_X: E \to [0.1]$.

Here, μ_X called membership function of X, and the value $\mu_X(x)$ is called the grade of membership of $x \in E$. The value represents the degree of x belonging to the fuzzy set X.

Definition 2.2 *[5] Let X be a non-empty set. Then, a interval fuzzy set (shortly IFS) in X is in terms of a function that when applied to X return a subset of* $[0,1] \times [0,1]$. *We express the IFS by*

$$\hat{A} = \{(x, \hat{h}_A(x)): x \in X\},$$

where $\hat{h}_A(x)$ consist of from pairs in $[0,1] \times [0,1]$, denoting the interval membership degree of the element $x \in X$ to the set $\hat{A}$, $\hat{h} = \hat{h}_A(x)$ is called a interval fuzzy element (IFE).

Definition 2.3 *[10] Dombi t-norm and t- conorm are defined, respectively where $k \geq 1$ and $x, y \in [0,1]$ as follow;*

- $D(x,y) = \frac{1}{1+\{(\frac{1-x}{x})^k+(\frac{1-y}{y})^k\}^{\frac{1}{k}}}$
- $D^{\star}(x,y) = 1 - \frac{1}{1+\{(\frac{x}{1-x})^k+(\frac{y}{1-y})^k\}^{\frac{1}{k}}}$

3. INTERVAL VALUED BIPOLAR FUZZY SETS

The concept of IVBFSs was defined by Gao and coauthors [8] in 2019. In this section, the concept of IVBFSs is combined with Dombi operators.

Definition 3.1 [8] *Let X be a reference set. An Interval Valued Bipolar fuzzy set $\hat{A}$ is defined as follows:*

$$\hat{A} = \{(x, \hat{h}_{\hat{A}_p}(x), \hat{h}_{\hat{A}_n}(x)): x \in X\},$$

where $\hat{h}_{\hat{A}_p}(x): X \mapsto [0,1] \times [0,1]$ and $\hat{h}_{\hat{A}_n}(x): X \mapsto [-1,0] \times [-1,0]$ are interval fuzzy sets. Moreover, $\hat{A} = \{(x, [\mu_{\hat{A}_p}(x), \nu_{\hat{A}_p}(x)], [\mu_{\hat{A}_n}(x), \nu_{\hat{A}_n}(x)]): x \in X\}$ is indicated. Also, $\hat{h}_{\hat{A}_p}(x)$ and $\hat{h}_{\hat{A}_n}(x)$ are set of values $[0,1] \times [0,1]$ and $[-1,0] \times [-1,0]$, respectively and $\hat{h} = (\hat{h}_{\hat{A}_p}(x), \hat{h}_{\hat{A}_n}(x))$ is called an Interval Valued Bipolar fuzzy element. In special cases, if $\hat{h}_{\hat{A}_p}(x) \neq \emptyset$ and $\hat{h}_{\hat{A}_n}(x) = \emptyset$, $IVBFS$ is called as interval fuzzy sets and ideal element $\{[1,1],[0,0]\}$ and if non-ideal element, $\{[0,0],[-1,-1]\}$.

Definition 3.2 $A = \{(x, [\mu_{A_p}(x), \nu_{A_p}(x)], [\mu_{A_n}(x), \nu_{A_n}(x)]): x \in X\}$ *and*

$B = \{(x, [\mu_{B_p}(x), \nu_{B_p}(x)], [\mu_{B_n}(x), \nu_{A_n}(x)]): x \in X\}$ be two IVBFSs where $k > 0$. The basic Dombi operations of $IVBFEs$ are proposed with (1), (2), (3) and (4) equations as follows:

$$A \oplus B = \left\{ \begin{matrix} \left[1 - \frac{1}{1+\left\{\left(\frac{\breve{\mu}_{A_p}}{1-\breve{\mu}_{A_p}}\right)^k + \left(\frac{\breve{\mu}_{B_p}}{1-\breve{\mu}_{B_p}}\right)^k\right\}^{\frac{1}{k}}}, 1 - \frac{1}{1+\left\{\left(\frac{\breve{\nu}_{A_p}}{1-\breve{\nu}_{A_p}}\right)^k + \left(\frac{\breve{\nu}_{B_p}}{1-\breve{\nu}_{B_p}}\right)^k\right\}^{\frac{1}{k}}}\right], \\ \left[\frac{-1}{1+\left\{\left(\frac{1+\breve{\mu}_{A_n}}{|\breve{\mu}_{A_n}|}\right)^k + \left(\frac{1+\breve{\mu}_{B_n}}{|\breve{\mu}_{B_n}|}\right)^k\right\}^{\frac{1}{k}}}, \frac{-1}{1+\left\{\left(\frac{1+\breve{\nu}_{A_n}}{|\breve{\nu}_{A_n}|}\right)^k + \left(\frac{1+\breve{\nu}_{B_n}}{|\breve{\nu}_{B_n}|}\right)^k\right\}^{\frac{1}{k}}}\right] \end{matrix} \right\} \quad (1)$$

$A \otimes B =$

$$\left\{\begin{array}{l}\left[\dfrac{1}{1+\left\{\left(\frac{1-\mu_{A_p}}{\mu_{A_p}}\right)^k+\left(\frac{1-\mu_{B_p}}{\mu_{B_p}}\right)^k\right\}^{\frac{1}{k}}}, \dfrac{1}{1+\left\{\left(\frac{1-\nu_{A_p}}{\nu_{A_p}}\right)^k+\left(\frac{1-\nu_{B_p}}{\nu_{B_p}}\right)^k\right\}^{\frac{1}{k}}}\right], \\ \left[-1+\dfrac{1}{\left\{\left(\frac{\mu_{A_n}}{1+|\mu_{A_n}|}\right)^k+\left(\frac{\mu_{B_n}}{1+|\mu_{B_n}|}\right)^k\right\}^{\frac{1}{k}}}, -1+\dfrac{1}{1+\left\{\left(\frac{\nu_{A_n}}{1+|\nu_{A_n}|}\right)^k+\left(\frac{\nu_{B_n}}{1+|\nu_{B_n}|}\right)^k\right\}^{\frac{1}{k}}}\right]\end{array}\right\} \quad (2)$$

$A^{\lambda} =$

$$\left\{\begin{array}{l}\left[\dfrac{1}{1+\left\{\lambda\left(\frac{1-\mu_{A_p}}{\mu_{A_p}}\right)^k\right\}^{\frac{1}{k}}}, \dfrac{1}{1+\left\{\lambda\left(\frac{1-\nu_{A_p}}{\nu_{A_p}}\right)^k\right\}^{\frac{1}{k}}}\right], \\ \left[-1+\dfrac{1}{\left\{\lambda\left(\frac{\mu_{A_{n_1}}}{1+|\mu_{A_{n_1}}|}\right)^k\right\}^{\frac{1}{k}}}, -1+\dfrac{1}{1+\left\{\lambda\left(\frac{\nu_{A_{n_1}}}{1+|\nu_{A_{n_1}}|}\right)^k\right\}^{\frac{1}{k}}}\right]\end{array}\right\} \quad (3)$$

$\lambda A =$

$$\left\{\begin{array}{l}\left[\dfrac{1}{1+\left\{\left(\frac{1-\mu_{A_p}}{\mu_{A_p}}\right)^k\right\}^{\frac{1}{k}}}, \dfrac{1}{1+\left\{\lambda\left(\frac{1-\nu_{A_p}}{\nu_{A_p}}\right)^k\right\}^{\frac{1}{k}}}\right], \\ \left[\dfrac{-1}{1+\left\{\left(\frac{1+\mu_{A_n}}{|\mu_{A_n}|}\right)^k\right\}^{\frac{1}{k}}}, \dfrac{-1}{1+\left\{\lambda\left(\frac{1+\nu_{A_n}}{|\nu_{A_n}|}\right)^k\right\}^{\frac{1}{k}}}\right]\end{array}\right\} \quad (4)$$

Definition 3.3 [8] *Let determine a IVBFS that* $A = \{(x, [\mu_{A_p}, \nu_{A_p}], [\mu_{A_n}, \nu_{A_n}])\}$, *score function and accuracy function are proposed by using following (5) and (6) equations, respectively;*

$$s(h) = \frac{1}{4}(1 + \mu_{A_p} + \mu_{A_n}) + \frac{1}{4}(1 + \nu_{A_p} + \nu_{A_n}) \quad (5)$$

and

$$a(h) = \frac{1}{4}(\mu_{A_p} - \nu_{A_p}) + \frac{1}{4}(\mu_{A_n} - \nu_{A_n}) \quad (6)$$

Definition 3.4 *Let determine collection of IVBFe that* $A_\ell = \{(x, \{(\breve{\mu}_{A_{p_\ell}}(x), \breve{\nu}_{A_{p_\ell}}(x))\}, \{(\breve{\mu}_{A_{n_\ell}}(x), \breve{\nu}_{A_{n_\ell}}(x))\}): x \in X\}$*;*

$IVBFPWDA$: $\Phi^{\varpi} \to \Phi$ is a mapping called as Interval Valued Bipolar Fuzzy Prioritized Weighted Dombi Averaging operator for $k > 0$ and is defined with equation (7) as below;

$$IVBFPWDA(A_1, A_2, \ldots, A_\varpi) = \oplus_{\ell=1}^{\varpi}\left(\frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell}(A_\ell)\right) = \frac{T_1}{\sum_{\ell=1}^{\varpi} T_\ell}A_1 \oplus \frac{T_2}{\sum_{\ell=1}^{\varpi} T_\ell}A_2 \oplus \ldots \oplus$$

$$\frac{T_\varpi}{\sum_{\ell=1}^{\varpi} T_\ell}A_\varpi = \left\{\begin{matrix}\left[1 - \frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\mu}_{A_{p_\ell}}}{1-\breve{\mu}_{A_{p_\ell}}}\right)^k+\right\}^{\frac{1}{k}}}, 1 - \frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\nu}_{A_{p_\ell}}}{1-\breve{\nu}_{A_{p_\ell}}}\right)^k+\right\}^{\frac{1}{k}}}\right], \\ \left[\frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_\ell}}}{|\breve{\mu}_{A_{n_\ell}}|}\right)^k+\right\}^{\frac{1}{k}}}, \frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\nu}_{A_{n_\ell}}}{|\breve{\nu}_{A_{n_\ell}}|}\right)^k+\right\}^{\frac{1}{k}}}\right]\end{matrix}\right\} \quad (7)$$

where $T_\ell = \prod_{\ell=1}^{\varpi-1} s(A_\ell)$ and $(\ell = 2,3,\ldots,\varpi)$, $T_1 = 1$ and $s(A_\ell)$ indicates to score values of A_ℓ.

4. CHARACTERISTIC OF IVBFPWDA OPERATOR

Theorem 4.1 *Let determine collection of IVBFe that* $A_\ell = \{(x, \{(\breve{\mu}_{A_{p_\ell}}(x), \breve{\nu}_{A_{p_\ell}}(x))\}, \{(\breve{\mu}_{A_{n_\ell}}(x), \breve{\nu}_{A_{n_\ell}}(x))\}) : x \in X\}$ *with equiation 7;*

$$IVBFPWDA(A_1, A_2, \ldots, A_\varpi) = \oplus_{\ell=1}^{\varpi}\left(\frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell}(A_\ell)\right) = \frac{T_1}{\sum_{\ell=1}^{\varpi} T_\ell}A_1 \oplus \frac{T_2}{\sum_{\ell=1}^{\varpi} T_\ell}A_2 \oplus \ldots \oplus \frac{T_\varpi}{\sum_{\ell=1}^{\varpi} T_\ell}A_\varpi$$

$$= \left\{\begin{matrix}\left[1 - \frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\mu}_{A_{p_\ell}}}{1-\breve{\mu}_{A_{p_\ell}}}\right)^k+\right\}^{\frac{1}{k}}}, 1 - \frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\nu}_{A_{p_\ell}}}{1-\breve{\nu}_{A_{p_\ell}}}\right)^k+\right\}^{\frac{1}{k}}}\right], \\ \left[\frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_\ell}}}{|\breve{\mu}_{A_{n_\ell}}|}\right)^k+\right\}^{\frac{1}{k}}}, \frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\nu}_{A_{n_\ell}}}{|\breve{\nu}_{A_{n_\ell}}|}\right)^k+\right\}^{\frac{1}{k}}}\right]\end{matrix}\right\}$$

where $T_\ell = \prod_{\ell=1}^{\varpi-1} s(A_\ell)$ and $(\ell = 2,3,\ldots,\varpi)$, $T_1 = 1$ and $s(A_\ell)$ indicates to score values of A_ℓ.

Proof. Let use mathematical induction on ϖ and look for $\varpi = 2$;

$$\frac{T_1}{\sum_{\ell=1}^{\varpi} T_\ell}A_1 = \left\{\begin{matrix}\left[1 - \frac{1}{1+\left\{\frac{T_1}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\mu}_{A_{p_1}}}{1-\breve{\mu}_{A_{p_1}}}\right)^k\right\}^{\frac{1}{k}}}, 1 - \frac{1}{1+\left\{\frac{T_1}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\nu}_{A_{p_1}}}{1-\breve{\nu}_{A_{p_1}}}\right)^k\right\}^{\frac{1}{k}}}\right], \\ \left[\frac{-1}{1+\left\{\frac{T_1}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_1}}}{|\breve{\mu}_{A_{n_1}}|}\right)^k\right\}^{\frac{1}{k}}}, \frac{-1}{1+\left\{\frac{T_1}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\nu}_{A_{n_1}}}{|\breve{\nu}_{A_{n_1}}|}\right)^k\right\}^{\frac{1}{k}}}\right]\end{matrix}\right\}$$

and

$$\frac{T_2}{\sum_{\ell=1}^{\varpi} T_\ell}A_2 = \left\{\begin{matrix}\left[1 - \frac{1}{1+\left\{\frac{T_2}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\mu}_{A_{p_2}}}{1-\breve{\mu}_{A_{p_2}}}\right)^k\right\}^{\frac{1}{k}}}, 1 - \frac{1}{1+\left\{\frac{T_2}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\nu}_{A_{p_2}}}{1-\breve{\nu}_{A_{p_2}}}\right)^k\right\}^{\frac{1}{k}}}\right], \\ \left[\frac{-1}{1+\left\{\frac{T_2}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_2}}}{|\breve{\mu}_{A_{n_2}}|}\right)^k\right\}^{\frac{1}{k}}}, \frac{-1}{1+\left\{\frac{T_2}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\nu}_{A_{n_2}}}{|\breve{\nu}_{A_{n_2}}|}\right)^k\right\}^{\frac{1}{k}}}\right]\end{matrix}\right\}$$

and from here

$$\frac{T_1}{\sum_{\ell=1}^{\varpi} T_\ell} A_1 \oplus \frac{T_2}{\sum_{\ell=1}^{\varpi} T_\ell} A_2 =$$

$$\left\{\begin{array}{l}\left[\begin{array}{l}1-\dfrac{1}{1+\left\{\frac{T_1}{\sum_{\ell=1}^{\varpi} T_\ell}\left(\frac{\breve{\mu}_{A_{p_1}}}{1-\breve{\mu}_{A_{p_1}}}\right)^k+\frac{T_2}{\sum_{\ell=1}^{\varpi} T_\ell}\left(\frac{\breve{\mu}_{A_{p_2}}}{1-\breve{\mu}_{A_{p_2}}}\right)^k\right\}^{\frac{1}{k}}}, \\ 1-\dfrac{1}{1+\left\{\frac{T_1}{\sum_{\ell=1}^{\varpi} T_\ell}\left(\frac{\breve{\nu}_{A_{p_1}}}{1-\breve{\nu}_{A_{p_1}}}\right)^k+\frac{T_2}{\sum_{\ell=1}^{\varpi} T_\ell}\left(\frac{\breve{\nu}_{A_{p_2}}}{1-\breve{\nu}_{A_{p_2}}}\right)^k\right\}^{\frac{1}{k}}}\end{array}\right], \\ \left[\begin{array}{l}\dfrac{-1}{1+\left\{\frac{T_1}{\sum_{\ell=1}^{\varpi} T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_1}}}{|\breve{\mu}_{A_{n_1}}|}\right)^k+\frac{T_2}{\sum_{\ell=1}^{\varpi} T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_2}}}{|\breve{\mu}_{A_{n_2}}|}\right)^k\right\}^{\frac{1}{k}}}, \\ \dfrac{-1}{1+\left\{\frac{T_1}{\sum_{\ell=1}^{\varpi} T_\ell}\left(\frac{1+\breve{\nu}_{A_{n_1}}}{|\breve{\nu}_{A_{n_1}}|}\right)^k+\frac{T_2}{\sum_{\ell=1}^{\varpi} T_\ell}\left(\frac{1+\breve{\nu}_{A_{n_2}}}{|\breve{\nu}_{A_{n_2}}|}\right)^k\right\}^{\frac{1}{k}}}\end{array}\right]\end{array}\right\}$$

$$\frac{T_1}{\sum_{\ell=1}^{\varpi} T_\ell} A_1 \oplus \frac{T_2}{\sum_{\ell=1}^{\varpi} T_\ell} A_2 = \left\{\begin{array}{l}\left[1-\dfrac{1}{1+\left\{\sum_{\ell=1}^{2}\frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell}\left(\frac{\breve{\mu}_{A_{p_\ell}}}{1-\breve{\mu}_{A_{p_\ell}}}\right)^k+\right\}^{\frac{1}{k}}}, 1-\dfrac{1}{1+\left\{\sum_{\ell=1}^{2}\frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell}\left(\frac{\breve{\nu}_{A_{p_\ell}}}{1-\breve{\nu}_{A_{p_\ell}}}\right)^k+\right\}^{\frac{1}{k}}}\right], \\ \left[\dfrac{-1}{1+\left\{\sum_{\ell=1}^{2}\frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_\ell}}}{|\breve{\mu}_{A_{n_\ell}}|}\right)^k\right\}^{\frac{1}{k}}}, \dfrac{-1}{1+\left\{\sum_{\ell=1}^{2}\frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell}\left(\frac{1+\breve{\nu}_{A_{n_\ell}}}{|\breve{\nu}_{A_{n_\ell}}|}\right)^k\right\}^{\frac{1}{k}}}\right]\end{array}\right\}$$

from here for $\varpi = \zeta$, IVBFPWDA holds as follow;

$$IVBFPWDA(A_1, A_2, \ldots, A_\zeta) = \oplus_{\ell=1}^{\zeta}\left(\frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell}(A_\ell)\right)$$

$$= \left\{\begin{array}{l}\left[1-\dfrac{1}{1+\left\{\sum_{\ell=1}^{\zeta}\frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell}\left(\frac{\breve{\mu}_{A_{p_\ell}}}{1-\breve{\mu}_{A_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}}, 1-\dfrac{1}{1+\left\{\sum_{\ell=1}^{\zeta}\frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell}\left(\frac{\breve{\nu}_{A_{p_\ell}}}{1-\breve{\nu}_{A_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}}\right], \\ \left[\dfrac{-1}{1+\left\{\sum_{\ell=1}^{\zeta}\frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_\ell}}}{|\breve{\mu}_{A_{n_\ell}}|}\right)^k\right\}^{\frac{1}{k}}}, \dfrac{-1}{1+\left\{\sum_{\ell=1}^{\zeta}\frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell}\left(\frac{1+\breve{\nu}_{A_{n_\ell}}}{|\breve{\nu}_{A_{n_\ell}}|}\right)^k\right\}^{\frac{1}{k}}}\right]\end{array}\right\}$$

and for $\varpi = \zeta + 1$;

$$\oplus_{\ell=1}^{\zeta+1}\left(\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}(A_\ell)\right)=\oplus_{\ell=1}^{\zeta}\left(\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}(A_\ell)\right)\oplus\frac{T_{\zeta+1}}{\sum_{\ell=1}^{\varpi}T_\ell}(A_{\zeta+1})=$$

$$\left\{\begin{matrix}\left[\begin{matrix}1-\dfrac{1}{1+\left\{\sum_{\ell=1}^{\zeta}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\mu}_{A_{p_\ell}}}{1-\breve{\mu}_{A_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}},\\ 1-\dfrac{1}{1+\left\{\sum_{\ell=1}^{\zeta}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\nu}_{A_{p_\ell}}}{1-\breve{\nu}_{A_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}}\end{matrix}\right],\\ \left[\begin{matrix}\dfrac{-1}{1+\left\{\sum_{\ell=1}^{\zeta}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_\ell}}}{|\breve{\mu}_{A_{n_\ell}}|}\right)^k\right\}^{\frac{1}{k}}},\\ \dfrac{-1}{1+\left\{\sum_{\ell=1}^{\zeta}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\nu}_{A_{n_\ell}}}{|\breve{\nu}_{A_{n_\ell}}|}\right)^k\right\}^{\frac{1}{k}}}\end{matrix}\right]\end{matrix}\right\}\oplus$$

$$\left\{\begin{matrix}\left[1-\dfrac{1}{1+\left\{\frac{T_{\zeta+1}}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\mu}_{A_{p_{\zeta+1}}}}{1-\breve{\mu}_{A_{p_{\zeta+1}}}}\right)^k\right\}^{\frac{1}{k}}},1-\dfrac{1}{1+\left\{\frac{T_{\zeta+1}}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\nu}_{A_{p_{\zeta+1}}}}{1-\breve{\nu}_{A_{p_{\zeta+1}}}}\right)^k\right\}^{\frac{1}{k}}}\right],\\ \left[\dfrac{-1}{1+\left\{\frac{T_{\zeta+1}}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_{\zeta+1}}}}{|\breve{\mu}_{A_{n_{\zeta+1}}}|}\right)^k\right\}^{\frac{1}{k}}},\dfrac{-1}{1+\left\{\frac{T_{\zeta+1}}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\nu}_{A_{n_{\zeta+1}}}}{|\breve{\nu}_{A_{n_{\zeta+1}}}|}\right)^k\right\}^{\frac{1}{k}}}\right]\end{matrix}\right\}$$

Thus

$$\oplus_{\ell=1}^{\zeta+1}\left(\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}(A_\ell)\right)=\left\{\begin{matrix}\left[\begin{matrix}1-\dfrac{1}{1+\left\{\sum_{\ell=1}^{\zeta+1}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\mu}_{A_{p_\ell}}}{1-\breve{\mu}_{A_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}},\\ 1-\dfrac{1}{1+\left\{\sum_{\ell=1}^{\zeta+1}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\nu}_{A_{p_\ell}}}{1-\breve{\nu}_{A_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}}\end{matrix}\right],\\ \left[\begin{matrix}\dfrac{-1}{1+\left\{\sum_{\ell=1}^{\zeta+1}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_\ell}}}{|\breve{\mu}_{A_{n_\ell}}|}\right)^k\right\}^{\frac{1}{k}}},\\ \dfrac{-1}{1+\left\{\sum_{\ell=1}^{\zeta+1}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\nu}_{A_{n_\ell}}}{|\breve{\nu}_{A_{n_\ell}}|}\right)^k\right\}^{\frac{1}{k}}}\end{matrix}\right]\end{matrix}\right\}$$

it holds for $\ell=\zeta+1$ so it provides for all ℓ.

Theorem 4.2 *(idempotency) Let propose collection of IVBFe that*

$A_\ell=\{(x,\{[\breve{\mu}_{A_{p_\ell}}(x),\breve{\nu}_{A_{p_\ell}}(x)]\},\{[\breve{\mu}_{A_{n_\ell}}(x),\breve{\nu}_{A_{n_\ell}}(x)]\}):x\in X\}$. Let be $A_\ell=A$ for $(\ell=1,2,\ldots,\varpi)$. Thus, $IVBFPWDA(A_1,A_2,\ldots A_\ell)=A$ where $T_\ell=\prod_{\ell=1}^{\varpi-1}s(A_\ell)$ and $(\ell=2,3,\ldots,\varpi)$, $T_1=1$ and $s(A_\ell)$ indicates to score values of A_ℓ.

Proof. Firstly let write $IVBFPWDA$ operator;

$$IVBFPWDA(A_1, A_2, \dots, A_\varpi) = \bigoplus_{\ell=1}^{\varpi} \left(\frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell}(A_\ell)\right) = \left\{ \begin{array}{l} \left[\begin{array}{l} 1 - \dfrac{1}{1 + \left\{ \sum_{\ell=1}^{\varpi} \frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell} \left(\frac{\breve{\mu}_{A_{p_\ell}}}{1 - \breve{\mu}_{A_{p_\ell}}} \right)^k \right\}^{\frac{1}{k}}}, \\ 1 - \dfrac{1}{1 + \left\{ \sum_{\ell=1}^{\varpi} \frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell} \left(\frac{\breve{\nu}_{A_{p_\ell}}}{1 - \breve{\nu}_{A_{p_\ell}}} \right)^k \right\}^{\frac{1}{k}}} \end{array} \right], \\ \left[\begin{array}{l} \dfrac{-1}{1 + \left\{ \sum_{\ell=1}^{\varpi} \frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell} \left(\frac{1 + \breve{\mu}_{A_{n_\ell}}}{|\breve{\mu}_{A_{n_\ell}}|} \right)^k \right\}^{\frac{1}{k}}}, \\ \dfrac{-1}{1 + \left\{ \sum_{\ell=1}^{\varpi} \frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell} \left(\frac{1 + \breve{\nu}_{A_{n_\ell}}}{|\breve{\nu}_{A_{n_\ell}}|} \right)^k \right\}^{\frac{1}{k}}} \end{array} \right] \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \left[1 - \dfrac{1}{1 + \left\{ \left(\frac{\breve{\mu}_{A_{p_\ell}}}{1 - \breve{\mu}_{A_{p_\ell}}} \right)^k \right\}^{\frac{1}{k}}}, 1 - \dfrac{1}{1 + \left\{ \left(\frac{\breve{\nu}_{A_{p_\ell}}}{1 - \breve{\nu}_{A_{p_\ell}}} \right)^k \right\}^{\frac{1}{k}}} \right], \\ \left[\dfrac{-1}{1 + \left\{ \left(\frac{1 + \breve{\mu}_{A_{n_\ell}}}{|\breve{\mu}_{A_{n_\ell}}|} \right)^k \right\}^{\frac{1}{k}}}, \dfrac{-1}{1 + \left\{ \left(\frac{1 + \breve{\nu}_{A_{n_\ell}}}{|\breve{\nu}_{A_{n_\ell}}|} \right)^k \right\}^{\frac{1}{k}}} \right] \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \left[1 - \dfrac{1}{1 + \left\{ \left(\frac{\breve{\mu}_{A_{p_\ell}}}{1 - \breve{\mu}_{A_{p_\ell}}} \right) \right\}}, 1 - \dfrac{1}{1 + \left\{ \left(\frac{\breve{\nu}_{A_{p_\ell}}}{1 - \breve{\nu}_{A_{p_\ell}}} \right) \right\}} \right], \\ \left[\dfrac{-1}{1 + \left\{ \left(\frac{1 + \breve{\mu}_{A_{n_\ell}}}{|\breve{\mu}_{A_{n_\ell}}|} \right) \right\}}, \dfrac{-1}{1 + \left\{ \left(\frac{1 + \breve{\nu}_{A_{n_\ell}}}{|\breve{\nu}_{A_{n_\ell}}|} \right) \right\}} \right] \end{array} \right\}$$

Thus, $IVBFPWDA(A_1, A_2, \dots A_\varpi) = A$.

Theorem 4.3 *(monotonicity) Let determine two IVBFSs that*

$A_\ell = \{(x, \{[\breve{\mu}_{A_{p_\ell}}(x), \breve{\nu}_{A_{p_\ell}}(x)]\}, \{(\breve{\mu}_{A_{n_\ell}}(x), \breve{\nu}_{A_{n_\ell}}(x))\}): x \in X\}$ and

$A_\ell^* = \{(x, \{[\breve{\mu}_{A_{p_\ell}^*}(x), \breve{\nu}_{A_{p_\ell}^*}(x)]\}, \{[\breve{\mu}_{A_{n_\ell}^*}(x), \breve{\nu}_{A_{n_\ell}^*}(x)]\}): x \in X\}$ $(\ell = 1,2, \dots, \varpi)$,

$IVBFPWDA(A_1, A_2, \dots, A_\varpi) \leq IVBFPWDA(A_1^*, A_2^*, \dots, A_\varpi^*)$.

Proof. We know that $A_\ell \leq A_\ell^*$, in this statement for first part of equation 7;

$$\Leftrightarrow \sum_{\ell=1}^{\varpi} \frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell} \left(\frac{\breve{\mu}_{A_{p_\ell}}}{1 - \breve{\mu}_{A_{p_\ell}}} \right)^k \leq \sum_{\ell=1}^{\varpi} \frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell} \left(\frac{\breve{\mu}_{A_{p_\ell}^*}}{1 - \breve{\mu}_{A_{p_\ell}^*}} \right)^k$$

$$\Leftrightarrow 1 + \left\{ \sum_{\ell=1}^{\varpi} \frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell} \left(\frac{\breve{\mu}_{A_{p_\ell}}}{1 - \breve{\mu}_{A_{p_\ell}}} \right)^k \right\}^{\frac{1}{k}} \leq 1 + \left\{ \sum_{\ell=1}^{\varpi} \frac{T_\ell}{\sum_{\ell=1}^{\varpi} T_\ell} \left(\frac{\breve{\mu}_{A_{p_\ell}^*}}{1 - \breve{\mu}_{A_{p_\ell}^*}} \right)^k \right\}^{\frac{1}{k}}$$

$$\Leftrightarrow \frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\mu}_{A_{p_\ell}}}{1-\breve{\mu}_{A_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}} \geq \frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\mu}_{A^*_{p_\ell}}}{1-\breve{\mu}_{A^*_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}}$$

$$\Leftrightarrow \left\{1-\frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\mu}_{A_{p_\ell}}}{1-\breve{\mu}_{A_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}}\right\} \leq \left\{1-\frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\mu}_{A^*_{p_\ell}}}{1-\breve{\mu}_{A^*_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}}\right\}$$

similarly

$$\left\{1-\frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\nu}_{A_{p_\ell}}}{1-\breve{\nu}_{A_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}}\right\} \leq \left\{1-\frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\nu}_{A^*_{p_\ell}}}{1-\breve{\nu}_{A^*_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}}\right\}$$

It can be made like above proof and thus;

$$\left\{\left[1-\frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\mu}_{A_{p_\ell}}}{1-\breve{\mu}_{A_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}}, 1-\frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\nu}_{A_{p_\ell}}}{1-\breve{\nu}_{A_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}}\right]\right\}$$

$$\leq \left\{\left[1-\frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\mu}_{A^*_{p_\ell}}}{1-\breve{\mu}_{A^*_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}}, 1-\frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\nu}_{A^*_{p_\ell}}}{1-\breve{\nu}_{A^*_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}}\right]\right\}$$

Also,

$$\Leftrightarrow \frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_\ell}}}{|\breve{\mu}_{A_{n_\ell}}|}\right)^k \leq \frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A^*_{n_\ell}}}{|\breve{\mu}_{A^*_{n_\ell}}|}\right)^k$$

$$\Leftrightarrow 1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_\ell}}}{|\breve{\mu}_{A_{n_\ell}}|}\right)^k\right\}^{\frac{1}{k}} \leq 1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A^*_{n_\ell}}}{|\breve{\mu}_{A^*_{n_\ell}}|}\right)^k\right\}^{\frac{1}{k}}$$

$$\Leftrightarrow \left\{\frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_\ell}}}{|\breve{\mu}_{A_{n_\ell}}|}\right)^k\right\}^{\frac{1}{k}}}\right\} \geq \left\{\frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A^*_{n_\ell}}}{|\breve{\mu}_{A^*_{n_\ell}}|}\right)^k\right\}^{\frac{1}{k}}}\right\}.$$

similarly,

$$\left\{\frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\nu}_{A_{n_\ell}}}{|\breve{\nu}_{A_{n_\ell}}|}\right)^k\right\}^{\frac{1}{k}}}\right\} \geq \left\{\frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\nu}_{A^*_{n_\ell}}}{|\breve{\nu}_{A^*_{n_\ell}}|}\right)^k\right\}^{\frac{1}{k}}}\right\}$$

and from here

$$\left\{\left[\frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_\ell}}}{|\breve{\mu}_{A_{n_\ell}}|}\right)^k\right\}^{\frac{1}{k}}},\frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\nu}_{A_{n_\ell}}}{|\breve{\nu}_{n_\ell}|}\right)^k\right\}^{\frac{1}{k}}}\right]\right\}$$
$$\geq\left\{\left[\frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A^*_{n_\ell}}}{|\breve{\mu}_{A^*_{n_\ell}}|}\right)^k\right\}^{\frac{1}{k}}},\frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\nu}_{A^*_{n_\ell}}}{|\breve{\nu}_{A^*_{n_\ell}}|}\right)^k\right\}^{\frac{1}{k}}}\right]\right\}$$

the proof is completed.

Theorem 4.4 *(Boundedness) Let determine collection of IVBFSs that*

$A_\ell=\{(x,\{[\breve{\mu}_{A_{p_\ell}}(x),\breve{\nu}_{A_{p_\ell}}(x)]\},\{[\breve{\mu}_{A_{n_\ell}}(x),\breve{\nu}_{A_{n_\ell}}(x)]\}):x\in X\}$ and A_ℓ^+ and A_ℓ^- are maximum and minimum elements for $\ell=1,2,\ldots,\varpi$. Thus, $A^-\leq IVBFPWDA(A_1,A_2,\ldots,A_\varpi)\leq A^+$.

Proof. In here, it is open by using equation 7 that

$A^+=max\{A_1,A_2,\ldots,A_\ell\}=(\{[\breve{\mu}^+_{A_{p_\ell}}(x),\breve{\nu}^+_{A_{p_\ell}}(x)]\},\{[\breve{\mu}^+_{A_{n_\ell}}(x),\breve{\nu}^+_{A_{n_\ell}}(x)]\})$ and

$A^-=min\{A_1,A_2,\ldots,A_\ell\}=(\{[\breve{\mu}^-_{A_{p_\ell}}(x),\breve{\nu}^-_{A_{p_\ell}}(x)]\},\{[\breve{\mu}^-_{A_{n_\ell}}(x),\breve{\nu}^-_{A_{n_\ell}}(x)]\})$ where

$\breve{\mu}^+_{A_{p_\ell}}=max_\ell\{\breve{\mu}_{A_{p_\ell}}\}$, $\breve{\mu}^-_{A_{p_\ell}}=min_\ell\{\breve{\mu}_{A_{p_\ell}}\}$, $\breve{\nu}^+_{A_{p_\ell}}=max_\ell\{\breve{\nu}_{A_{p_\ell}}\}$, $\breve{\nu}^-_{A_{p_\ell}}=min_\ell\{\breve{\nu}_{A_{p_\ell}}\}$, $\breve{\mu}^+_{A_{n_\ell}}=min_\ell\{\breve{\mu}_{A_{n_\ell}}\}$, $\breve{\mu}^-_{A_{n_\ell}}=max_\ell\{\breve{\mu}_{A_{n_\ell}}\}$, $\breve{\nu}^+_{A_{n_\ell}}=min_\ell\{\breve{\nu}_{A_{n_\ell}}\}$, $\breve{\nu}^-_{A_{n_\ell}}=max_\ell\{\breve{\nu}_{A_{n_\ell}}\}$

from here

$$\left\{\left[1-\frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\mu}^-_{A_{p_\ell}}}{1-\breve{\mu}^-_{A_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}},1-\frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\nu}^-_{A_{p_\ell}}}{1-\breve{\nu}^-_{A_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}}\right]\right\}$$
$$\leq\left\{\left[1-\frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\mu}_{A_{p_\ell}}}{1-\breve{\mu}_{A_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}},1-\frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\nu}_{A_{p_\ell}}}{1-\breve{\nu}_{A_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}}\right]\right\}$$
$$\leq\left\{\left[1-\frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\mu}^+_{A_{p_\ell}}}{1-\breve{\mu}^+_{A_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}},1-\frac{1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{\breve{\nu}^+_{A_{p_\ell}}}{1-\breve{\nu}^+_{A_{p_\ell}}}\right)^k\right\}^{\frac{1}{k}}}\right]\right\}$$

similarly;

$$\left\{\left[\frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_\ell}}^{-}}{|\breve{\mu}_{A_{n_\ell}}^{-}|}\right)^{k}\right\}^{\frac{1}{k}}},\frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\nu}_{A_{n_\ell}}^{-}}{|\breve{\nu}_{A_{n_\ell}}^{-}|}\right)^{k}\right\}^{\frac{1}{k}}}\right]\right\}$$
$$\leq\left\{\left[\frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_\ell}}}{|\breve{\mu}_{A_{n_\ell}}|}\right)^{k}\right\}^{\frac{1}{k}}},\frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\nu}_{A_{n_\ell}}}{|\breve{\nu}_{A_{n_\ell}}|}\right)^{k}\right\}^{\frac{1}{k}}}\right]\right\}$$
$$\leq\left\{\left[\frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\mu}_{A_{n_\ell}}^{+}}{|\breve{\mu}_{A_{n_\ell}}^{+}|}\right)^{k}\right\}^{\frac{1}{k}}},\frac{-1}{1+\left\{\sum_{\ell=1}^{\varpi}\frac{T_\ell}{\sum_{\ell=1}^{\varpi}T_\ell}\left(\frac{1+\breve{\nu}_{A_{n_\ell}}^{+}}{|\breve{\nu}_{A_{n_\ell}}^{+}|}\right)^{k}\right\}^{\frac{1}{k}}}\right]\right\}$$

5. AN APPLICATION OF MULTI-ATTRIBUTE DECISION-MAKING METHOD UNDER IVBFS

In this section, we define the proposed IVBFPWDA into an algorithm and apply over a MCDM problem with m alternatives and t criterions to indicate effective of Dombi Aggregation operators over IVBFS. Let $A=\{A_1,A_2,\dots,A_m\}$ be a set of alternatives, in here there is a prioritization relationship between alternatives and is defined as a linear ordering that $A_1 \succ A_2 \succ \dots \succ A_m$ show that A_j has a higher priority than A_i, if $j<i$, $C=\{C_1,C_2,\dots,C_n\}$ be a set of criterions and interval valued bipolar fuzzy decision matrix is $B=(\hat{b}_{ij})_{m\times t}=([\mu_{\hat{b}_{p_{ij}}},\nu_{\hat{b}_{p_{ij}}}],[\mu_{\hat{b}_{n_{ij}}},\nu_{\hat{b}_{n_{ij}}}])_{m\times t}$ for $i=1,2,\dots,m$ and $j=1,2,\dots,t$. Moreover, $[\mu_{\hat{b}_{p_{ij}}},\nu_{\hat{b}_{p_{ij}}}]$ and $[\mu_{\hat{b}_{n_{ij}}},\nu_{\hat{b}_{n_{ij}}}]$ indicate to positive interval membership value and negative interval membership value to be appointed by decision makers, respectively.

Then, the following steps have been proposed for algorithm.

1. Determine values of T_{ij} where

$$T_{ij}=\prod_{\psi=1}^{j-1} s(\hat{b}_{i\psi}) \quad (8)$$

$$T_{i1}=1 \quad (9)$$

for $i=1,2,\dots,m$ and $j=1,2,\dots,t$.

2. Apply to Aggregation operators IVBFPWDA

$$\hat{b}_i = \text{IV}BFPWDA(\hat{b}_{i1}, \hat{b}_{i2}, \ldots, \hat{b}_{it}) = \oplus_{j=1}^{t} \left(\frac{T_{ij}}{\sum_{j=1}^{n} T_{ij}} (\hat{b}_{ij})\right)$$

$$= \frac{T_{i1}}{\sum_{j=1}^{t} T_{ij}} \hat{b}_{i1} \oplus \frac{T_{i2}}{\sum_{j=1}^{t} T_{ij}} \hat{b}_{i2} \oplus \ldots \oplus \frac{T_{it}}{\sum_{j=1}^{t} T_{ij}} \hat{b}_{it}$$

$$= \left\{ \begin{array}{l} \left[\begin{array}{l} 1 - \dfrac{1}{1 + \left\{ \sum_{j=1}^{t} \frac{T_{ij}}{\sum_{j=1}^{t} T_{ij}} \left(\frac{\mu_{\hat{b}p_{ij}}}{1 - \mu_{\hat{b}p_{ij}}} \right)^{k} \right\}^{\frac{1}{k}}}, \\ 1 - \dfrac{1}{1 + \left\{ \sum_{j=1}^{t} \frac{T_{ij}}{\sum_{j=1}^{t} T_{ij}} \left(\frac{\nu_{\hat{b}p_{ij}}}{1 - \nu_{\hat{b}p_{ij}}} \right)^{k} \right\}^{\frac{1}{k}}} \end{array} \right], \\ \left[\begin{array}{l} \dfrac{-1}{1 + \left\{ \sum_{j=1}^{t} \frac{T_{ij}}{\sum_{j=1}^{t} T_{ij}} \left(\frac{1 + \mu_{\hat{b}n_{ij}}}{|\mu_{\hat{b}n_{ij}}|} \right)^{k} \right\}^{\frac{1}{k}}}, \\ \dfrac{-1}{1 + \left\{ \sum_{j=1}^{t} \frac{T_{ij}}{\sum_{j=1}^{t} T_{ij}} \left(\frac{1 + \nu_{\hat{b}p_{ij}}}{|\nu_{\hat{b}p_{ij}}|} \right)^{k} \right\}^{\frac{1}{k}}} \end{array} \right] \end{array} \right\}$$

3. Calculate score values according to score function $s(\hat{b}_i)$ to order rank of alternatives for $i = 1,2,\ldots,m$;

4. Obtain rank of alternatives and select the most desirable value;

5. End.

Numerical example

A company thinks to invest over different business sectors so has made some studies together with the experts of subject and thus decision makers have developed five different alternatives based on five criterions as follow; A_1; A computer production company, A_2; A communication company, A_3; A food company, A_4; A car company, A_5; An airport company; if criterions,

C_1; environment impact: This factor is very important for a business because of environmental damage caused by the business. The CEO of business has to minimize the given harm over environmental;

C_2; the proximity to raw material: This criterion will reduce to cost by decreasing the need for gasoline and the number of workers so on;

C_3; cost: the employer should own the factory with the least cost, so she\he has to choose the most logical alternative;

C_4; economic fluctuations: Economic fluctuations within the country are a factor that directly affects the business;

C_5; experience: The owner of the business should have a certain backround about the business she\he will establish.

Decision makers construct to decision making matrix as follow in Table 1.

***Table 1**. Evaluations of alternatives made by decision makers*

	C_1	C_2	C_3
A_1	$\{[0.10,0.30],[-0.40,-0.10]\}$	$\{[0.50,0.90],[-0.20,-0.10]\}$	$\{[0.40,0.58],[-0.30,-0.20]\}$
A_2	$\{[0.50,0.80],[-0.80,-0.60]\}$	$\{[0.30,0.50],[-0.60,-0.30]\}$	$\{[0.10,0.20],[-0.80,-0.70]\}$
A_3	$\{[0.40,0.54],[-0.80,-0.10]\}$	$\{[0.10,0.58],[-0.10,-0.01]\}$	$\{[0.20,0.30],[-0.50,-0.30]\}$
A_4	$\{[0.20,0.21],[-0.50,-0.10]\}$	$\{[0.30,0.40],[-0.30,-0.20]\}$	$\{[0.30,0.40],[-0.60,-0.50]\}$
A_5	$\{[0.30,0.40],[-0.30,-0.20]\}$	$\{[0.40,0.50],[-0.10,-0.01]\}$	$\{[0.20,0.50],[-0.90,-0.40]\}$
	C_4	C_5	
	$\{[0.30,0.80],[-0.80,-0.70]\}$	$\{[0.40,0.90],[-0.60,-0.50]\}$	
	$\{[0.20,0.90],[-0.54,-0.30]\}$	$\{[0.50,0.70],[-0.80,-0.50]\}$	
	$\{[0.50,0.70],[-0.91,-0.90]\}$	$\{[0.20,0.50],[-0.70,-0.10]\}$	
	$\{[0.20,0.30],[-0.90,-0.80]\}$	$\{[0.40,0.90],[-0.90,-0.50]\}$	
	$\{[0.10,0.20],[-0.60,-0.50]\}$	$\{[0.70,0.80],[-0.98,-0.50]\}$	

Consist of T_{ij} by using to Eqs 8 and 9 as follow;

$$T_{ij}=\begin{pmatrix}1 & 0.4750 & 0.3681 & 0.2282 & 0.0912\\ 1 & 0.4750 & 0.2256 & 0.0451 & 0.0254\\ 1 & 0.3350 & 0.3276 & 0.1392 & 0.0483\\ 1 & 0.4525 & 0.2488 & 0.0995 & 0.0199\\ 1 & 0.5750 & 0.3836 & 0.1342 & 0.0402\end{pmatrix}$$

Obtain interval valued Bipolar fuzzy elements by utilizing $IVBFPWDA$ operator $\hat{b}_i$ for $i=1,2,3,4,5$ and $k=1$ as follow;

$$\hat{b}_1=\mathrm{IV}BFPWDA(\hat{b}_{11},\hat{b}_{12},\hat{b}_{13})=\{(0.3064,0.7622),(-0.3209,-0.1171)\}$$

$$\hat{b}_2=\mathrm{IV}BFPWDA(\hat{b}_{21},\hat{b}_{22},\hat{b}_{23})=\{(0.4283,0.7473),(-0.7227,-0.4611)\}$$

$$\hat{b}_3=\mathrm{IV}BFPWDA(\hat{b}_{31},\hat{b}_{32},\hat{b}_{33})=\{(0.3409,0.5453),(-0.3125,-0.0349)\}$$

$$\hat{b}_4=\mathrm{IV}BFPWDA(\hat{b}_{41},\hat{b}_{42},\hat{b}_{43})=\{(0.2430,0.3498),(-0.4415,-0.1355)\}$$

$$\hat{b}_5=\mathrm{IV}BFPWDA(\hat{b}_{51},\hat{b}_{52},\hat{b}_{53})=\{(0.3322,0.4643),(-0.2873,-0.1781)\}$$

Score values are given for the all cases over IVBFPWDA as follow;

***Table 2** . Score Values according to IVBFPWDA*

	s_(b̂_1)	s_(b̂_2)	s_(b̂_3)	s_(b̂_4)	s_(b̂_5)	Ranking Alternatives
k=1	0.6576	0.4979	0.6346	0.5039	0.5827	A_1>A_3>A_5>A_4>A_2

k=2	0.6990	0.5280	0.6767	0.5627	0.6732	A_1>A_3>A_5>A_4>A_2
k=3	0.7196	0.5491	0.6924	0.6094	0.7076	A_1>A_3>A_5>A_4>A_2
k=4	0.7318	0.5641	0.7015	0.6345	0.7382	A_1>A_3>A_5>A_4>A_2
k=5	0.7397	0.5751	0.7083	0.6496	0.7605	A_1>A_3>A_5>A_4>A_2
k=6	0.7453	0.5836	0.7141	0.6598	0.7760	A_1>A_3>A_5>A_4>A_2
k=7	0.7494	0.5901	0.7193	0.6673	0.7870	A_1>A_3>A_5>A_4>A_2
k=8	0.7525	0.5953	0.7239	0.6732	0.7952	A_1>A_3>A_5>A_4>A_2
k=9	0.7550	0.5994	0.7281	0.6781	0.8015	A_1>A_3>A_5>A_4>A_2
k=10	0.6260	0.6028	0.7318	0.6821	0.8064	A_1>A_3>A_5>A_4>A_2

6 . COMPARATIVE ANALYSIS

In this section, we give a comparative analysis. To do this, firstly we utilize some aggregation operators over IVBFSs. Moreover, it should be noted that our proposed operator is to work as agreement with these aggregation operators but has more advantages bacause of having generalized concept like dombi operators. In here, we calculate for $k = 1,2,3,4,5$ to dombi operators. The results are as following;

Table 3*. Ranking alternatives of Score Function Values under IVBFPWDA*

	$s_{\hat{b}_1}$	$s_{\hat{b}_2}$	$s_{\hat{b}_3}$	$s_{\hat{b}_4}$	$s_{\hat{b}_5}$	Ranking Alternatives
$k = 1$	0.6079	0.5381	0.5789	0.3709	0.6285	$A_5 > A_1 > A_3 > A_2 > A_4$
$k = 2$	0.6569	0.5532	0.5891	0.4326	0.6422	$A_1 > A_5 > A_3 > A_2 > A_4$
$k = 3$	0.6796	0.5632	0.5946	0.4821	0.6531	$A_1 > A_5 > A_3 > A_4 > A_2$
$k = 4$	0.6942	0.5700	0.5987	0.5155	0.6613	$A_1 > A_5 > A_3 > A_4 > A_2$

$k = 5$	0.7057	0.5748	0.6022	0.5387	0.6675	$A_1 > A_5 > A_3 > A_4 > A_2$
IVBFHWA [8]	0.679	0.571	0.578	0.573	0.615	$A_1 > A_5 > A_3 > A_4 > A_2$
IVBFHWG [8]	0.575	0.480	0.516	0.497	0.557	$A_1 > A_5 > A_3 > A_4 > A_2$

7. CONCLUSION

In this paper, we present to IVBFPWDA by using IVBFS together with priotrized approach. The basic goal that while weights of the criteria are determined, the margin of error is minimized by eliminating the non-objective comments of the decision makers and also to make a comparison within itself by obtaining more flexible structure thanks to dombi operators. Then, we apply to defined opertaor over a realistic example. These results are almost agreement in their own. Moreover, we propose to Table 2 and Table 4 that these tables indicate that our operators are flexible, realistic and useful.

The contributions of paper can be ordered in literature as follow;

1. Interval Valued Bipolar Fuzzy Prioritized Weighted Dombi Averaging operator (IVBFPWDA) has been offered;

2. The decision making algorithm and a problem have been discussed for a operator and the results of presented operator have been compared in their own;

3. A comparative analysis has been made with operators over IVBFS;

Besides, in next time; we plan to combine to these operators with Hamacher aggregation operators, Power aggregation operator.

Compliance with Ethical Standards

Ethical Approval

This article does not contain any studies with human participants or animals performed by any of the authors.

Conflicts Of Interest

No conflict of interest was declared by the authors.

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