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On Soft Uniform Spaces

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Abstract

The purpose of this paper is to introduce some properties of soft uniform spaces. The notions of closure, comparison of soft uniformities and soft uniform open function on soft uniform space are defined and their basic properties are investigated.

Keywords: Soft set, soft topological space, soft entourage, soft uniform structure, soft uniform space, soft uniform continuous function, soft uniform open function.

1.Introduction

Many practical problems in economics, engineering, environment, social science, medical science etc, cannot be dealt with classical methods because classical methods have inherent difficulties. The reason for these difficulties may be due to the inadequacy of the theories of parameterization tools. MOLODTSOV (1999) initiated the concept of soft set theory as a new mathematical tool for dealing with uncertainties. MAJI *et al*. (2002 AND 2003) research deal with operations over soft set. The algebraic structure of set theories dealing with uncertainties is an important problem. Many researchers have contributed towards the algebraic structure of soft set theory. AKTAŞ and ÇAĞMAN (2007) defined soft groups and derived their basic properties. ACAR *et al.* (2010) introduced initial concepts of soft rings. FENG *et al.* (2008) defined soft semi rings and several related notions to establish a connection between soft sets and semi rings. SHABIR and IRFAN (2009) studied soft ideals over a semigroup. SUN *et al.* (2008) defined soft modules and investigated their basic properties. GUNDUZ and BAYRAMOV (2011) introduced fuzzy soft modules and intuitionistic fuzzy soft modules and investigated some basic properties. OZTURK and BAYRAMOV (2012) defined chain complexes of soft modules and their soft homology modules. OZTURK *et al.* (2013) introduced the concept of inverse and direct systems in the category of soft modules

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Recently, SHABIR and NAZ (2011) have initiated the study of soft topological spaces. Theoretical studies of soft topological spaces have also be researched by some authors in (ÇAĞMAN *et al.* 2011*;* HUSSAIN and AHMAD 2011*;* MIN 2011;SHABIR and BASHIR 2011*;* ZORLUTUNA *et al.* 2012). In the study BAYRAMOV and GUNDUZ (2013), a soft point concept was given differently from the concepts that were given in the studies (AYGUNOGLU and AYGUN 2011*;* ÇAĞMAN *et al.* 2011*;* HUSSAIN and AHMAD 2011*;* MIN 2011*;* SHABIR and BASHIR 2011; ZORLUTUNA *et al.* 2012). In this study, we use the soft point concept that is defined in the study BAYRAMOV and GUNDUZ (2013).

Uniform spaces are alike to metric spaces however the application area of uniform spaces is more extensive than of the metric spaces. Since every uniform space can be transformed to a topological space, there exists a relation between uniform and topological spaces. Thus, it is important to carry uniform spaces to soft sets.

In this study, based on the study OZTURK (2015), we introduce new properties of soft uniform space. Furthermore, we define the notions of closure, comparison of soft uniformities and soft uniform open function on soft uniform space and give some interesting examples.

2.Preliminaries

In this section we will introduce necessary definitions and theorems for soft sets MOLODTSOV (1999) defined the soft set in the following way:

Let X be an initial universe set and E be a set of parameters. Let $P(X)$ denotes the power set of X and $A,B \subseteq E$.

Definition 1. (MOLODTSOV 1999) A pair (F,A) and (G,B) over X, where F is a mapping given by $F:A \to P(X)$.

In other words, the soft set is a parameterized family of subset of the set X. For $e \in A$, $F(e)$ may be considered as the set of e −elements of the soft set (F,A), or as the set of e −approximate elements of the soft set.

Definition 2. (MAJI *et al. 2003)* For two soft sets (F,A) and (G,B) over X, (F,A) is called soft subset of (G,B) if

(1) $A \subset B$ and
(2) $\forall e \in A$, $F(e)$ and $G(e)$ are identical approximations.

This relationship is denote by $(F,A) \tilde{\subset} (G,B)$. Similarly, (F,A) is called a soft superset of (G,B) if (G,B) is a soft subset of (F,A). This relationship is denoted by $(F,A) \tilde{\supset} (G,B)$. Two soft sets (F,A) and (G,B) over X are called soft equal if (F,A) is a soft subset of (G,B) is a soft subset of (F,A).

Definition 3. (MAJI *et al. 2003)* The intersection of two soft set (F,A) and (G,B) over X is the soft set (H,C), Where $C = A \cap B$ and $\forall e \in C$, $H(e) = F(e) \cap G(e)$. This is denoted by $(F,A)\tilde{\cap}(G,B) = (H,C)$.

Definition 4. (MAJI *et al. 2003)* The union of two soft sets (F,A) and (G,B) over X is the soft set, where $C = A \cup B$ and $\forall e \epsilon C$,

$$H(\varepsilon) = \begin{cases} F(e), & if\ e\epsilon A - B \\ G(e), & if\ e\epsilon B - A \\ F(e) \cup G(e) & if\ e \in A \cup B. \end{cases}$$

This relationship is denoted by $(F,A)\tilde{\cup}(G,B) = (H,C)$.

Definition 5. (MAJI *et al. 2003)* A soft set (F,A) over X is said to be a NULL soft set denoted by Φ if for all $e\epsilon A, F(e) = \emptyset$ (null set).

Definition 6. (MAJI *et al. 2003)* A soft set (F,A) over X is said to be an absolute soft set denoted by $\tilde{X}$ if for all $e\epsilon A, F(e) = X$.

Definition 7. (SHABIR and NAZ 2011) The difference (H,E) of two soft sets (F,E) and (G,E) over X, denoted by $(F,E) \setminus (G,E)$, is defined as $H(e) = (F,E) \setminus (G,E)$ for all $e \in E$.

Definition 8. (SHABIR and NAZ 2011) The complement of a soft set (F,E), denoted by $(F,E)^c$, is defined $(F,E)^c = (F^c,E)$ where $F^c: E \to$ P(X) is a mapping given by $F^c = X \setminus F(e), \forall e\epsilon E$ and F^c is called the soft complement function of F.

Definition 9. (SHABIR and NAZ 2011) Let Y be a non-empty subset of X, then $\tilde{Y}$ denotes the soft set (Y,E) over X for which $Y(e) = Y$, for all $e \in E$.

In particular, (X,E) will be denoted by $\tilde{X}$.

Definition 10. (SHABIR and NAZ 2011) Let (F,E) be a soft set over X and Y be a non-empty subset of X. Then the soft subset of (F,E) over Y denoted by $({}^Y F, E)$, is defined as follows ${}^Y F(e) = Y \cap F(e)$, for all $e \in E$. In other words $\left({}^Y F, E\right) = \tilde{Y} \cap (F,E)$.

Definition 11. (BABITHA and SUNIL 2010) Let (F,A) and (G,B) be two soft sets over X_1 and X_2, respectively. The cartesian product $(F,A) \times (G,B)$ is defined by $(F \times A)_{(A\times B)}$ where

$$(F \times A)_{(A\times B)}(e,k) = F(e) \times G(k), \forall (e,k) \in A \times B$$

According to this definition the soft set $(F,A) \times (G,B)$ is soft set over $X_1 \times X_2$ and its parameter universe is $E_1 \times E_2$.

Definition 12. (SHABIR and NAZ 2011) Let τ be the collection of set over X, then τ is said to be a soft topology on X if

1) Φ , $\tilde{X}$ belongs to τ;
2) the union of any number of soft sets in τ belongs to τ ;
3) the intersection of any two soft sets in τ belongs to τ.

The triplet (X, τ, E) is called a soft topological space over X.

Definition 13. (SHABIR and NAZ 2011) Let (X, τ, E) be a soft topological space over , then members of τ are said to be soft open sets in X.

Definition 14. (SHABIR and NAZ 2011) Let (X, τ, E) be a soft topological space over X. A soft set (F,E) over X is said to be a soft closed in X , if its relative complement $(F,E)'$ belongs to τ.

Proposition 1. (SHABIR and NAZ 2011) Let (X, τ, E) be a soft topological space over X. Then the collection $\tau_{e=\{F(e):(F,E\epsilon\tau\}}$ for each $e \in E$, defines a topology on X.

Definition 15. (SHABIR and NAZ 2011) Let (X, τ, E) be a soft topological space over X and (F,E) be a soft set over X. Then the soft closure of (F,E), denoted by $(F,E)^c$ is the intersection of all soft closed super sets of (F,E). Clearly $(F,E)^c$ is the smallest soft closed set over X which contains (F,E).

Definition 16. (BAYRAMOV and GUNDUZ 2013) Let (F, E) be a soft set over X. The soft set (F, E) is called a soft point, denoted by (x_e, E), if for the element $e \in E$, $F(e) = \{x\}$ and $F(e') = \emptyset$ for all $e' \in E - \{e\}$(briefly denoted by x_e).

Definition 17. (BAYRAMOV and GUNDUZ 2013) Let two soft points (x_e, E) and $(y_{e'}, E)$ over a common universe X, we say that the points are different points if $x \neq y$ or $e \neq e'$.

Definition 18. (BAYRAMOV and GUNDUZ 2013) Let (X, τ, E) be a soft topological space over X. A soft set (F, E) in (X, τ, E) is called a soft neighbourhood of the soft point $(x_e, E) \in (F,E)$ if there exits a soft open set (G,E) such that $(x_e, E) \in (G,E) \tilde{\subset} (F, E)$.

Definition 19. (HAZRA *et al.* 2012) Let (f,g): $(X, E) \to (Y, E')$ be a soft mapping from (X,E) to (Y,E'). A soft mapping (f,g) is said to be injective if f,g are both injective. A soft mapping (f,g) is said to be surjective if f,g are both surjective. A soft mapping (f,g) is said to be bijective if f,g are both bijective.

Definition 20. (HAZRA *et al.* 2012) Let (X,τ,E) and (Y,τ',E) be two soft topological spaces, $f:(X,\tau,E)\to(Y,\tau',E)$ be a mapping. For each soft neighborhood (H,E) of $(f(x)_e,E)$, if there exists a soft neighborhood $f\big((F,E)\big)\subset(H,E)$, then f is said to be soft continuous mapping (x_e,E).

If f is soft continuous mapping for all (x_e,E), then f is called soft continuous mapping.

Definition 21. (HAZRA *et al.* 2012) Let (X,τ,E) and (Y,τ',E) be two soft topological spaces, $f:X\to Y$ be a mapping. If f is a bijection, soft continuous and f^{-1} is soft a homomorphism f exists between X and Y, we say that X is soft homomorphic to Y.

Definition 22. (AYGUNOGLU and AYGUN 2011) Let $\{(\varphi_s,\psi_s):(X,\tau,E)\to(Y_s,\tau_s,E_s)\}_{s\epsilon S}$ be a family of soft mappings and $\{(Y_s,\tau_s,E_s)\}_{s\epsilon S}$ is a family of soft topological spaces. Then, the topology τ genarated from the subbase $\delta=\{(\varphi_s,\psi_s)^{-1}_{s\epsilon S}(F,E):(F,E)\epsilon\tau_s, s\epsilon S\}$is called the soft topology (or initial soft topology) induced by the family of soft mappings $\{(\varphi_s,\psi_s)\}_{s\epsilon S}$.

Definition 23. (AYGUNOGLU and AYGUN 2011) Let $\{(X_s,\tau_s,E_s)\}_{s\epsilon S}$ be a family of soft topological spaces. Then, the initial soft topology on $X=\prod_{s\epsilon S}X_s$ generated by the family $\{(p_s,q_s)\}_{s\epsilon S}$ is called product soft topology on X. (Here, (p_s,q_s) is the soft projection mapping from X to $X_s s\in S$).

The product soft topology is denoted by $\prod_{s\epsilon S}\tau_s$.

Definition 24. (ÇAĞMAN *et l. 2011)* Let (X,τ,E) be a soft topological space and $x,y\in X$ such that $x\neq y$. If there exist soft open sets (F,A) and (G,B) such that $x\in(F,A), y\in(G,B)$ and $(F,A)\cap(G,B)=\Phi$, then (X,τ,E) is called a soft Hausdorff space.

Definition 25. (DAS and SAMANTA 2012) Let $\mathbb{R}$ be the set of real numbers and $B(\mathbb{R})$ the collection of all non-empty bounded subsets of $\mathbb{R}$ and E take as a set of parameters. Then a mapping $F:E\to B(\mathbb{R})$ is called a soft real set. If a soft real set is a singleton soft set, it will be called a soft real number and denoted by $\tilde{r},\tilde{s},\tilde{t}$ *etc*.

$\bar{0},\bar{1}$ are the soft real numbers where $\bar{0}(e)=0,\bar{1}(e)=1$ for all $e\epsilon E$, respectively.

Definition 26. (DAS and SAMANTA 2013) Let $\mathbb{R}(E)^*$ denote the set of all non-negative soft real numbes and $SS(X,E)$ denote the set of all soft points on the set X. A mapping $\tilde{d}:SS(X,E)\times SS(X,E)\to\mathbb{R}(E)^*$ is said to be a soft metric on the soft set (X,E) if $\tilde{d}$ satisfies the following conditions:

$(M1)$: $\tilde{d}(\tilde{x}_{e_1},\tilde{y}_{e_2})\cong\tilde{0}$ for all $\tilde{x}_{e_1},\tilde{y}_{e_2}\epsilon SS(X,E)$

$(M2)$: $\tilde{d}(\tilde{x}_{e_1},\tilde{y}_{e_2})=\tilde{0}$ if and only if $\tilde{x}_{e_1}=\tilde{y}_{e_2}\epsilon SS(X,E)$

$(M3)$: $\tilde{d}(\tilde{x}_{e_1},\tilde{y}_{e_2})=\tilde{d}(\tilde{y}_{e_2},\tilde{x}_{e_1})$ for all $\tilde{x}_{e_1},\tilde{y}_{e_2}\epsilon SS(X,E)$

$(M4)$: For all $\tilde{x}_{e_1},\tilde{y}_{e_2},\tilde{z}_{e_3}\epsilon SS(X,E)$, $\tilde{d}(\tilde{x}_{e_1},\tilde{z}_{e_3})\cong\tilde{d}(\tilde{x}_{e_1},\tilde{y}_{e_2})+\tilde{d}(\tilde{y}_{e_2},\tilde{z}_{e_3})$.

The soft set (X,E) with a soft metric $\tilde{d}$ is a called a soft metric space and denoted by $(X,\tilde{d},E)$.

Definition 27. (OZTURK 2015) a) If (X,E) is a soft set, then the soft set

$$\Delta_{(X,E)}=\{(x_e,x_e):x_e\epsilon(X,E)\}$$

is called as the diagonal of $(X,E)\times(X,E)$. Here $\Delta_{(X,E)}=(\Delta_X,\Delta_E)$ is defined by $\big((x,x),(e,e)\big)=(x,x)_{(e,e)}=(x_e,x_e)$.

b) If $(A,E\times E)\tilde{\subseteq}(X,E)\times(X,E)$ is a soft subset and the soft subset $(A,E\times E)$ is denoted by $\tilde{A}$, then

$\tilde{A}^{-1}=\{(y_{e'},x_e):(x_e,y_{e'})\epsilon\tilde{A}\}$.

If $\tilde{A}=\tilde{A}^{-1}$, then the soft set $\tilde{A}$ is called soft symmetric.

c) If $\tilde{A},\tilde{B}\tilde{\subseteq}(X,E)\times(X,E)$ are soft subsets, then

$\tilde{A}\circ\widetilde{B=}$

$\Big\{(x_e,y_{e'}):\exists_{z_{e''}}\epsilon(X,E),(x_e,z_{e''})\epsilon\tilde{A},(z_{e''},y_{e'})\epsilon\tilde{B}\Big\}$.

d) $\tilde{A}^1=\tilde{A}$ and $\tilde{A}^n=\tilde{A}^{n-1}\circ\tilde{A}$ for $n=1,2,3,\ldots$.

Example 1. Let $X=\{x^1,x^2,x^3\}$ be any set and $E=\{e_1,e_2\}$ be a set of parameters. In this case, the soft set $SS(X,E)$ is formed by the following soft points

$$\{x^1_{e_1},x^1_{e_2},x^2_{e_1},x^2_{e_2},x^3_{e_1},x^3_{e_2}\}.$$

Thus, the soft diagonal $\Delta_{(X,E)}$ of this soft set is

$\Delta_{(X,E)}=$

$\{(x^1_{e_1},x^1_{e_1}),(x^1_{e_2},x^1_{e_2}),(x^2_{e_1},x^2_{e_1}),(x^2_{e_2},x^2_{e_2}),(x^3_{e_1},x^3_{e_1}),(x^3_{e_2},x^3_{e_2})\}$

Proposition 2. (OZTURK 2015) For every $e\epsilon E$, we have

$$\big(\Delta_{(X,E)}\big)_e=\{(x^i_e,x^i_e):x^i\epsilon X\}=\Delta_X.$$

Therefore, the diogonal of the soft set (X,E) is the family of diagonals that is parametrised on the set E of X. That is $\Delta_{(X,E)}=\cup_{e\in E}(\Delta_X)_e$.

Definition 28. (OZTURK 2015) Let (X,E) be a soft set $(V,E\times E)\tilde{\subseteq}(X,E)\times(X,E)$ be a soft set that is defined as $V:E\times E\to P(X\times X)$ and the soft set $(V,E\times E)$ is denoted by $\tilde{V}$. If $\Delta_{(X,E)}\tilde{\subseteq}\tilde{V}$ and $\tilde{V}=\tilde{V}^{-1}$ are satisfied, then the soft set $\tilde{V}$ is called the soft entourage of the soft diagonal. All soft entourage of the soft diagonal $\Delta_{(X,E)}$ is denoted by $D_{(X,E)}$.

Example 2. According to the Example 1, we can give a few examples of the soft entourage $\tilde{V}$.

$\tilde{V}_1=\{\Delta_{(X,E)},(x^1_{e_1},x^2_{e_2}),(x^2_{e_2},x^1_{e_1})\}$

$\tilde{V}_2 = \{\Delta_{(X,E)}, (x^1_{e_1}, x^2_{e_1}), (x^2_{e_1}, x^1_{e_1})\}$

$\tilde{V}_1 = \{\Delta_{(X,E)}, (x^1_{e_1}, x^2_{e_1}), (x^2_{e_1}, x^1_{e_1}), (x^2_{e_2}, x^3_{e_2}), (x^3_{e_2}, x^2_{e_2})\}$

. . .

Definition 29. (OZTURK 2015) Let (X,E) be a soft set, $(F,E) \tilde{\subset} (X,E)$ be a soft subset, $\tilde{V} \epsilon D_{(X,E)}$ be a soft entourage and $x_e, y_{e'} \epsilon (X,E)$ be soft points. Then,

a) if $(x_e, y_{e'}) \epsilon \tilde{V}$, then it is said that the distance between the soft point x_e and the soft point $y_{e'}$ is smaler than $\tilde{V}$ and deoted by $|x_e - y_{e'}| \tilde{<} \tilde{V}$. Otherwise, $|x_e - y_{e'}| \tilde{>} \tilde{V}$;
b) if $(F,E) \times (F,E) \tilde{\subset} \tilde{V}$, then it is said that the diameter of the soft set (F,E) is smaller than $\tilde{V}$ and denoted by $\delta((F,E)) \tilde{<} \tilde{V}$.

Definition 30. (OZTURK 2015) Let (X,E) be a soft set, $x^0_e \in (X,E)$ be a soft point and $\tilde{V} \epsilon D_{(X,E)}$. The set

$$B(x^0_e, \tilde{V}) = \{y_{e'} \in (X,E) : |x^0_e - y_{e'}| \tilde{<} \tilde{V}\}$$

is called the soft sphere whose center is the soft point x^0_e with the diameter $\tilde{V}$. $B((F,E),\tilde{V}) = \cup^{B(x_e,\tilde{V})}_{x_e \in (F,E)}$ is defined for every soft set $(F,E) \tilde{\subset} (X,E)$.

Definition 31. (OZTURK 2015) Let $(X,\tilde{\vartheta},E)$ be a soft uniform space and $\tilde{\Gamma} \tilde{\subset} \tilde{\vartheta}$ be a soft subfamily. If for every $\tilde{V} \in \tilde{\vartheta}$ there exists $\tilde{W} \tilde{\subset} \tilde{\Gamma}$ such that $\tilde{W} \tilde{\subset} \tilde{V}$ is satisfied, then the family of $\tilde{\Gamma}$ is called a soft base of the soft uniform structure $\tilde{\vartheta}$.

Theorem 1. (OZTURK 2015) Let (X,E) be a soft set and $\tilde{\Gamma}$ be the family of soft subsets of soft subsets of $(X,E) \times (X,E)$. If the conditions

a): If $\tilde{B} \tilde{\subset} \tilde{\Gamma} \Rightarrow \Delta_{(X,E)} \tilde{\subset} \tilde{B}$,

b): $\tilde{B}_1, \tilde{B}_2 \in \tilde{\Gamma} \Rightarrow \exists \tilde{B}_3 \in \tilde{\Gamma} : \tilde{B}_3 \tilde{\subset} \tilde{B}_1 \cap \tilde{B}_2$,

c): $\tilde{B} \tilde{\subset} \tilde{\Gamma} \Rightarrow \exists \tilde{C} \in \tilde{\Gamma} : \tilde{C} \circ \tilde{C} \tilde{\subset} \tilde{B}$,

d): $\tilde{B} \tilde{\subset} \tilde{\Gamma} \Rightarrow \exists \tilde{C} \in \tilde{\Gamma} : \tilde{C}^{-1} \tilde{\subset} \tilde{B}$,

e): $\tilde{\cap} \tilde{\Gamma} = \Delta_{(X,E)}$,

are satisfied for soft family $\tilde{\Gamma}$, then the soft family $\tilde{\Gamma}$ is a soft base of a soft uniform structure on the soft set (X,E).

3.On Soft Uniform Spaces

Definition 32. (OZTURK 2015) Let (X,E) be a soft set and $\tilde{\vartheta} \tilde{\subset} D_{(X,E)}$ be a soft subfamily. If the conditions

a): If $\tilde{V} \in \tilde{\vartheta}$ and $\tilde{V} \tilde{\subset} \tilde{W} \in D_{(X,E)}$, then $\tilde{W} \in \tilde{\vartheta}$

b): If $\tilde{V}_1, \tilde{V}_2 \in \tilde{\vartheta}$ then $\tilde{V}_1 \tilde{\cap} \tilde{V}_2 \in \tilde{\vartheta}$

c): For every $\tilde{V} \in \tilde{\vartheta}$, there exists $\tilde{W} \in \tilde{\vartheta}$ such that $\widetilde{W^2} \tilde{\subset} \tilde{V}$

d): $\tilde{\bigcap}_{\tilde{V} \in \tilde{\vartheta}} \tilde{V} = \Delta_{(X,E)}$

are satisfied for the soft family $\tilde{\vartheta}$, then the soft family $\tilde{\vartheta}$ is called a soft uniform structure and the triple $(X,\tilde{\vartheta},E)$ is called a soft uniform space.

Remark 1. Let $(X,\tilde{\vartheta},E)$ be a soft uniform space. Then the axioms (c) of Definition 31. is a equivalent to the following axioms:

c′) For each $\tilde{V} \in \tilde{\vartheta}$, there exists $\tilde{W} \in \tilde{\vartheta}$ such that $\tilde{W} \circ \tilde{W}^{-1} \subset \tilde{V}$.

Proof. Actually, if $\tilde{W} \in \tilde{\vartheta}$, then $\tilde{W}$ is a soft entourages and $\tilde{W} = \tilde{W}^{-1}$. Therefore,

$\tilde{W} \circ \tilde{W}^{-1} \subset \tilde{V}$.

Conversely, if (c') is satisfied, then $\tilde{W}^{-1} = \Delta_{(X,E)} \circ \tilde{W}^{-1} \subset \tilde{W} \circ \tilde{W}^{-1} \subset \tilde{V}$. Hence

$\tilde{W} \tilde{\subseteq} \in \tilde{\vartheta}$. Let $\tilde{W}' = \tilde{W} \cap \tilde{W}^{-1}$; then $\tilde{W}' \in \tilde{\vartheta}$ and we have $\tilde{W}' \circ \tilde{W}' \subset \tilde{W} \circ \tilde{W}^{-1} \subset \tilde{V}$.

Example 3. Let $(X,\tilde{\vartheta},E)$ be a soft uniform space. Then,

(1) $\tilde{\vartheta}$ is called soft discrete uniform structure if every superset of the soft diagonal is a soft entourage of $\tilde{\vartheta}$. In this case, $(X,\tilde{\vartheta},E)$ is called soft discrete uniform space.
(2) $\tilde{\vartheta}$ is called soft discrete uniform structure if the soft full set $(X,E) \times (X,E)$ is the single soft entourage of $\tilde{\vartheta}$. In this case, $(X,\tilde{\vartheta},E)$ is called soft indiscrete uniform spaces.

Definition 33. Let $\tilde{\vartheta}_1$ and $\tilde{\vartheta}_2$ be two soft uniform structures on the some soft set (X,E). $\tilde{\vartheta}_1$ is said to be finer than $\tilde{\vartheta}_2$ (and $\tilde{\vartheta}_2$ coarser than $\tilde{\vartheta}_1$) if each soft entourage of $\tilde{\vartheta}_2$ is also a soft entourage of $\tilde{\vartheta}_1$. If $\tilde{\vartheta}_1$is finer than $\tilde{\vartheta}_2$and distinct from $\tilde{\vartheta}_2$, we say that $\tilde{\vartheta}_1$ is strictly finer than $\tilde{\vartheta}_2$(and that $\tilde{\vartheta}_2$ *is strictly coarser than* $\tilde{\vartheta}_1$).

Therefore, two soft uniformities are said to be comparable if one is finer than the other.

Example 4. It is clear that,

(1) The soft discrete uniform structure is the finnest every other soft uniform structure on (X,E);
(2) The soft indiscrete uniform structure is the coarsest every other soft uniform structure on (X,E).

Theorem 2. (OZTURK 2015) If (X,ϑ,E) is a soft uniform space, then the soft family

$$\tilde{\tau} = \{(G,E) \tilde{\subset} (X,E) : for\ every\ x_e \in (G,E), \exists \tilde{V} \in \tilde{\vartheta}\ such\ that\ B(X_e,\tilde{V}) \tilde{\subset} (G,E)\}$$

is a soft topology on (X,E). The soft set (X,E) is a T_1 −space together with this soft topology $\tilde{\tau}$ is called the generated soft uniform topology form the soft uniform structure $\tilde{\vartheta}$.

Example 5. Let $\tilde{\vartheta}_1$ and $\tilde{\vartheta}_2$ be two soft uniform structure on a soft set (X, E) and suppose that $\tilde{\vartheta}_1$ is finer than $\tilde{\vartheta}_2$; then the soft topology induced by $\tilde{\vartheta}_1$ is finer than the soft topology induced by $\tilde{\vartheta}_2$.

Theorem 3. (OZTURK 2015) If (X, ϑ, E) is a soft uniform space $(F, E) \times (F, E)$ is a soft subset and $\tilde{\tau}$ is a soft uniform topology generated from $\widetilde{\vartheta}$ on the soft set (X, E), then the soft set

$$\tilde{B} = \{x_e \in (X, E): \exists \tilde{V} \in \tilde{\vartheta} \text{ such that } B(X_e, \tilde{V}) \tilde{\subset} (G, E)\}$$

is equal to $\text{Int}(F, E)$.

Theorem 4. Let $(X, \tilde{\vartheta}, E)$ be a soft uniform spaces for every soft entourage $\tilde{V}$ of $\widetilde{\vartheta}$ and every soft subset $\widetilde{M}$ of $(X, E) \times (X, E)$. If $\tilde{V}\text{o}\widetilde{M}o\tilde{V}$ is a soft neighbourhood of $\widetilde{M}$ in the soft product space $(X, E) \times (X, E)$, then the soft closure of $\widetilde{M}$ in this soft space is given by the formula

$$\widetilde{M}^c = \widetilde{\bigcap}_{\tilde{V} \in \widetilde{\vartheta}} \tilde{V}\text{o}\widetilde{M}o\tilde{V}$$

Proof. Let $\tilde{V}$ be a soft entourage of $\widetilde{\vartheta}$. The relation $(x_e, y_{e'}) \in \tilde{V}\text{o}\widetilde{M}o\tilde{V}$ means that there is an element of $\widetilde{M}$ such that $(x_e, p_{e''}) \in \tilde{V}$ and $(q_e{'''}, y_{e'}) \in \tilde{V}$. In other words $x_e \tilde{\in} B(p_{e''}, \tilde{V})$ and $y_{e'} \in B(q_e{'''}, \tilde{V})$ that is, $(x_e, y_{e'}) \tilde{\in} B(p_{e''}, \tilde{V}) \times B(q_e{'''}, \tilde{V})$. Since $B(p_{e''}, \tilde{V}) \times B(q_e{'''}, (X, E) \times (X, E))$ is a soft neighbourhood of $(p_{e'''}, p_{e'''})$ in $(X, E) \times (X, E)$, the first part of proposition is proved.

The relations $(x_e, p_{e''}) \in \tilde{V}$, $(y_{e'}, q_e{'''}) \in \tilde{V}$ can also be written $p_{e''} \in B(x_e, \tilde{V})$, $q_e{'''} \in B(y_{e'}, V)$ or $(p_{e''}, q_{e'''}) \in B(x_e, \tilde{V}) \times B(y_{e'}, V)$ .The sets $B(x_e, \tilde{V}) \times B(y_{e'}, V)$ form a soft base of neighbourhoods of $(x_e, y_{e'})$ in $(X, E) \times (X, E)$; for if V_1, V_2 are any two soft entourages there is always a soft antourages there is always a soft entourage $\tilde{V} \tilde{\subset} \tilde{V}_1 \cap \tilde{V}_2$, so that $B(x_e, \tilde{V}) \times B(y_{e'}, V) \tilde{\subset} B(x_e, \tilde{V}_1) \times B(y_{e'}, \tilde{V}_2)$. Hence $B(x_e, \tilde{V}) \times B(y_{e'}, V)$ meets $\widetilde{M}$ for each $\tilde{V} \in \vartheta$ if and only if $(x_{e'}, y_{e'}) \tilde{\in} \widetilde{M}^c$

Proposition 3. Let $(X, \tilde{\vartheta}, E)$ be a soft uniform spaces. The soft interiors (resp. the soft closures) of the soft entourages of $\widetilde{\vartheta}$ form a soft entourages.

Proof. Let $\tilde{V}$ be a soft entourage of soft diagonal $\widetilde{\vartheta}$. In this case, there exits a soft entourages $\widetilde{W}$ such that $\widetilde{W}^3 \subset \tilde{V}$. Since $\widetilde{W}^3$ is a soft neighbourhood of $\widetilde{W}$, the interior of $\tilde{V}$ in $(X, E) \times (X, E)$ contains $\widetilde{W}$ and is therefore a soft entourage. Furthermore, we have $\widetilde{W} \tilde{\subset} \widetilde{W}^3 \tilde{\subset} \widetilde{W}^3 \subset \tilde{V}$ by Theorem 4. and hence $\tilde{V}$ contains the clouser of a soft entourage.

Definition 34. (OZTURK 2015) Let $(X, \tilde{\vartheta}, E)$, $(Y, \tilde{\vartheta}', E')$ be two soft uniform space and $(f, g): (X, E) \to (Y, E')$ be a pair of function.

a) If for every $\tilde{V}' \in \tilde{\vartheta}'$ $\exists \tilde{V} \in \tilde{\vartheta}$: for every $|x^1{}_{e_1} - x^2{}_{e_2}| \tilde{<} \tilde{V}$, $|f(x^1{}_{g(e_1)}) - f(x^2{}_{g(e_2)})| \tilde{<} \tilde{V}'$

is satisfied, then soft function (f, g) is called as a soft uniform continuous.

b) If (f, g) is a soft bijection mapping and each of the soft functions (f, g) and $(f, g)^{-1}$ are soft uniform continuous, then the soft function (f, g) is called a soft uniform isomorphism and these two soft uniform spaces are called soft isomorphic spaces.

Theorem 5. Let $(X, \tilde{\vartheta}, E)$, $(Y, \tilde{\vartheta}', E')$ be two soft uniform space and $(f, g): (X, E) \to (Y, E')$ be a pair of function. Then (f, g) is a soft uniform continuous if $((f, g) \times (f, g))^{-1}(\tilde{V}') \in \tilde{\vartheta}$ is soft entourage for each soft entourage $\tilde{V}' \in \tilde{\vartheta}$.

Proof. Let (f, g) be a soft uniform continuous function and $\tilde{V}' \in \tilde{\vartheta}$. Let us show that $((f, g) \times (f, g))^{-1}(\tilde{V}') \in \tilde{\vartheta}$. Suppose that $(x_e, y_{e'}) \in ((f, g) \times (f, g))^{-1}(\tilde{V}')$. That is, $|x_e - y_{e'}| \tilde{\leq} ((f, g) \times (f, g))^{-1}(\tilde{V}')$. Since (f, g) is a soft uniform continuous and $|f(x)_{g(e)} - f(y)_{g(e')}| \tilde{<} \tilde{V}'$, then there exits a soft entourage $\tilde{V}$ of $\tilde{\vartheta}$such that $|x_e - y_{e'}| \tilde{<} \tilde{V}'$

Conversely, let $((f, g) \times (f, g))^{-1}(\tilde{V}') \in \tilde{\vartheta}$ be a soft entourage, for each soft entourage $\tilde{V}' \in \tilde{\vartheta}$. Suppose that $|f(x)_{g(e)} - f(y)_{g(e')}| \tilde{<} \tilde{V}'$ is a soft entourage of $\tilde{\vartheta}'$ for every soft point $x_e, y_{e'}$ over X. Then $|x_e - y_{e'}| \tilde{<} ((f, g) \times (f, g))^{-1}(\tilde{V}')$ is a soft entourage of $\tilde{\vartheta}$. That is (f, g) is a uniform continuous.

Proposition 4. Let $(X, \tilde{\vartheta}, E)$, $(Y, \tilde{\vartheta}', E')$ and , $(Z, \tilde{\vartheta}'', E'')$ be soft uniform spaces. If $(f, g): (X, E) \to (Y, E')$ and $: (Y, E') \to (Z, E'')$ are two soft uniform continuous then $(f, g)o(\varphi, k) = (f\ o\ \varphi, g\ o\ k): (X, E) \to (Z, E'')$ is soft uniform continuous.

Proof. Straightforward.

4. Soft Uniform Open Mappings

Definition 35. Let $(X, \tilde{\vartheta}, E)$, $(Y, \tilde{\vartheta}', E')$ be two soft uniform and $(f, g): (X, E) \to (Y, E')$ be a pair of function. Then (f, g) is called a soft uniform open function if for soft entourage $\tilde{V}$ of $\tilde{\vartheta}$ there exits a soft entourage of $\tilde{\vartheta}'$ such that

$$B\left(f(x)_{g(e)}, \tilde{V}'\right) \tilde{\subset} (f,g)\left(B\left(x_e, \tilde{V}\right)\right)$$

for all $x_e \in SS(X)_E$.

Example 6. Let $(X, \tilde{\vartheta}, E)$ be a soft uniform space and $(Y, \tilde{\vartheta}', E')$ be a soft discrete uniform space then $(f,g):(X,E) \to (Y,E')$ is always soft uniform open function.

Note that a soft bijection function is soft uniform open if and only if its inverse soft uniform continuous.

Proposition 5. Let $(f,g):(X,E) \to (Y,E')$ be a soft uniform continuous surjection and let $(\varphi,k):(Y,E') \to (Z,E'')$ be a soft function, where $(X, \tilde{\vartheta}, E)$, $(Y, \tilde{\vartheta}', E')$ and $(Z, \tilde{\vartheta}'', E'')$ are soft uniform spaces. If $(f\ o\ \varphi, g\ o\ k):(Y,E') \to (Z,E'')$ is soft uniform open so is (φ,k).

Proof. Let $\tilde{V}'$ be a any soft entourage of $\tilde{\vartheta}'$. Then $\tilde{V} = \left((f,g)\times(f,g)\right)^{-1}(\tilde{V}')$ is a soft entourage of $\tilde{\vartheta}$. If $(f\ o\ \varphi, g\ o\ k)$ is soft uniform open then there exists a soft entourage $\tilde{V}''$ of $\tilde{\vartheta}''$ such that

$$B\left(\varphi(f(x))_{K(g(e))}, \tilde{V}''\right) \tilde{\subset} (f\ o\ \varphi, g\ o\ k)\left(B\left(x_e, \tilde{V}\right)\right)$$

for all $x_e \in SS(X)_E$. Then

$$B\left(\varphi(y)_{K(e')}, \tilde{V}''\right) \tilde{\subset} (\varphi, g)\left(B\left(y_{e'}, \tilde{V}'\right)\right)$$

for all $y_{e'} \in SS(X)_{E'}$ and so (φ,k) is soft uniform open.

Proposition 6. Let $(f,g):(X,E) \to (Y,E')$ be a soft uniform continuous surjection and let $(\varphi,k):(Y,E') \to (Z,E'')$ be a soft uniform continuous injection, where $(X, \tilde{\vartheta}, E), (Y, \tilde{\vartheta}', E')$ and $(Z, \tilde{\vartheta}'', E'')$ are soft uniform spaces. If $(f\ o\ \varphi, g\ o\ k)$ is soft uniform open then so is (f,g).

Proof. Let $\tilde{V}'$ be a any soft entourage of $\tilde{\vartheta}'$. If $(f\ o\ \varphi, g\ o\ k)$ is soft uniform open there exists a soft entourage $\tilde{V}''$ of $\tilde{\vartheta}''$ such that

$$B\left(\varphi(f(x))_{K(g(e))}, \tilde{V}''\right) \tilde{\subset} (f\ o\ \varphi, g\ o\ k)\left(B\left(x_e, \tilde{V}\right)\right)$$

For all soft points $x_e \in SS(X)_E$. Since (ϑ,k) is soft uniform continuous, $\tilde{V}' = \left((\vartheta,k)\times(\vartheta,k)\right)^{-1}(\tilde{V}'')$ is a soft entourage of $\tilde{\vartheta}'$. Suppose that ${y^1}_{{e_1}'} \in B\left((f(x))_{g(e)}, \tilde{V}'\right)$. Then

$$\varphi(y^1)_{K({e_1}')} \in B\left(\varphi(f(x))_{K(g(e))}, \tilde{V}''\right) \tilde{\subset} (f\ o\ \varphi, g\ o\ k)\left(B\left(x_e, \tilde{V}\right)\right)$$

and so $\varphi(y^1)_{k({e_1}')} = \varphi(f(x^1))_{k(g(e))}$ for some ${x^1}_{e_1} \tilde{\in} B\left(x_e, \tilde{V}\right)$. Since (φ,k) is injective we have ${y^1}_{{e_1}'} = f(x^1)_{g(e_1)}$ and so ${y^1}_{{e_1}'} \tilde{\in} (f,g)\left(B\left(x, \tilde{V}\right)\right)$ thus $B\left(f(x)_{g(e)}, \tilde{V}'\right) \tilde{\subset} (f,g)\left(B\left(x_e, \tilde{V}\right)\right)$ and so (f,g) is soft uniform open.

Proposition 7. Let $(f,g):(X,E) \to (Y,E')$ and $(\varphi,k):(Y,E') \to (Z,E'')$ be two soft functions where $(X, \tilde{\vartheta}, E)$, $(Y, \tilde{\vartheta}', E')$ and $(Z, \tilde{\vartheta}'', E'')$ are soft uniform spaces. Suppose that (f,g) is a soft uniform open surjection function. If $(f\ o\ \varphi, g\ o\ k)$ is soft uniform continuous so is (φ,k).

Proof. Suppose that $(f\ o\ \varphi, g\ o\ k)$ is soft uniform continuous. If $\tilde{V}''$ is a soft entourage of $\tilde{\vartheta}''$, then $\tilde{V} = \left((f\ o\ \varphi, g\ o\ k)\times(f\ o\ \varphi, g\ o\ k)\right)^{-1}(\tilde{V}'')$ is a soft entourage of $\tilde{\vartheta}$. Since (f,g) is soft uniform open there exists a soft entourage $\tilde{V}'$ of $\tilde{\vartheta}$ such that

$$B\left(f(x)_{g(e)}, \tilde{V}'\right) \tilde{\subset} (f,g)\left(B\left(x_e, \tilde{V}\right)\right)$$

for all $x_e \in SS(X)_E$. So if $f(x^1)_{g(e_1)} \tilde{\in} B\left(f(x)_{g(e)}, \tilde{V}'\right)$ for some ${x_e}^1 \in SS(X)_E$, then $f(x^1)_{g(e_1)} = f(x^2)_{g(e_2)}$ for some ${x^2}_{e_2} \tilde{\in} B\left(x_e, \tilde{V}\right)$ and so $\left(\varphi(f(x))_{K(g(e))}, \varphi f(x^2)_{K(g(e_2))}\right) \tilde{\in} \tilde{V}''$. Since $(f(x^1))_{K(g(e_1))} = \varphi(f(x^2))_{K(g(e_2))}$, this shows that (φ,k) is soft uniform continuous.

5.Conclusion

The category of soft uniform spaces is a generalization of the category of uniform spaces. Therefore, the basic properties of soft uniform structure have been investigated.

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