Pre-Service Classroom Teachers' Proof Schemes in Geometry: A Case Study of Three Pre-service Teachers

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Suggested Citation:

Oflaz, G., Bulut, N. & Akcakin, V. (2016). Pre-service classroom teachers' proof schemes in geometry: a case study of three pre-service teachers. *Eurasian Journal of Educational Research*, 63, 133-152, http://dx.doi.org/10.14689/ejer.2016.63.8

Abstract

Problem Statement: Recent research and evaluation reports show that students are not learning geometry efficiently. One identifier of student understanding related to geometry is teachers' knowledge structures. Understanding what a proof is and writing proofs are essential for success in mathematics. Thus, school mathematics should include proving activities. Proofs are at the heart of mathematics, and proving is complex; teachers should help their students develop these processes in the early grades. The success of this process depends on teachers' views about the essence and forms of proofs. Hence, it is necessary to investigate the classroom teachers' perceptions related to proofs.

Purpose of the Study: The purpose of this study is to determine the proof scheme of pre-service teachers when proving a geometry theorem. In this sense, the study is oriented by the research question: which proof schemes do pre-service teachers use when making proofs in geometry?

Method: The current case study is a detailed examination of a particular subject. Firstly, an open ended question was asked, and then semi-structured interviews were conducted. The three students investigated in this study were selected by considering their Basic Mathematics scores. Two girls having maximum and average scores and a boy having a

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minimum score voluntarily participated. The students were asked to proof "the sum of the interior angle measurements of a triangle is 180°". After proving this, each student was interviewed about what they think about proofs and proving.

Findings: The findings of the study reveal that pre-service classroom teachers have difficulties related to proving. Also, the participants' attitudes are not parallel to their achievements in the lesson. Another result of this study concerns using proofs in teaching and learning processes. When students are asked about their opinions regarding proofs, it is understood that they have the common idea that the lessons should be made with proofs.

Conclusion and Recommendations: The results of this study and other studies in the field reveal that pre-service teachers are not able to prove even a simple geometry theorem. What underlies this is that pre-service teachers are thought to have insufficient knowledge about the definitions of geometric concepts as well as the misconceptions concerning the topic. Another reason can be that participants do not experience any proving processes in previous education. Hence, students should realize how valuable proving and acquiring knowledge is through the counsel of a teacher.

Keywords: Teaching mathematics, proving, teacher candidate

Introduction

According to the needs of the 21st century, mathematics education aims to construct the real life connections of mathematical concepts. When viewed from this aspect, geometry is the study of space and shape, which helps students represent and make sense of the world (Clements, 1998). NCTM (2000) emphasized the importance of geometry by stating "geometric ideas are useful in representing and solving problems in other areas of mathematics and in real-world situations" (p.41). The study of geometry helps students develop the skills of critical thinking, deductive reasoning, and logical argument. Thus, it helps students efficiently solve the problems they face in daily life (Van de Walle, 2004; Driscoll, 2007). Herewith, geometry is one of the bridges between concrete objects and abstract thought. Regrettably, international and national reports execute that students are not learning geometry meaningfully and efficiently (Yildirim, Yildirim, Yetisir, & Ceylan, 2013; TEDMEM, 2014; Buyukozturk, Cakan, Tan, & Atar, 2014a; Buyukozturk, Cakan, Tan, & Atar, 2014b). One of the determinants influencing students' learning of geometry is teachers' knowledge structures, which play a fundamental role in student learning. When considered from this point of view, classroom teachers' geometric understanding and thinking becomes crucial.

Geometry becomes meaningful within geometric thought. According to van Hiele, a student who is learning geometry progresses through levels in geometric thinking: recognition, analysis, order, deduction, and rigor (Van Hiele, 1986; Van Hiele Geldof, 1984). The geometry curriculum students have up to the end of high school contains an almost ordered level (Crowley, 1987). As geometry is not taught through reasoning and is not proof based in primary and secondary schools, this situation is far from beneficial to student thought processes and is, instead, restrictive.

One of the substantial components of geometric thought is proof. Van Hiele levels focus on geometrical understanding and reasoning, and thus on proofs (Burger & Shaughnessy, 1986; Senk, 1989). The deduction level of geometric thinking is concerned with the axiomatic structure of mathematical understanding (Crowley, 1987). As to the van Hiele model, only if/when the student reaches the deduction level (the level at which the student understands the meaning of deduction and the roles of axioms, theorems, postulates, and proof) can s/he be able to write formal proofs (Senk, 1989). Development of geometric thought is directly associated with the proof process.

As an outstanding part of geometric thought, proof plays a significant role in terms of mathematics education. Numerous research shows that the concept of proofs has an important place in geometry teaching (Hanna, 2000; Martin & Harel, 1989; Moutsios-Rentzos & Spyrou, 2015; Usiskin, 1982). Bell (1976) describes proofs as an essentially public activity that follows the reaching of conviction, though it may be conducted internally against an imaginary potential doubter. According to Harel (2008), a proof is a product of our mental act; the particular argument one produces to ascertain for oneself or to convince others that an assertion is true. As for Bell (1976), a proof has three senses. The first is verification or justification, which is the validation of the truth of a proposition; the second is illumination, which is an explanation for why the proposition is true; the third and last is systematization, which is the organization of results into a deductive system of axioms, major concepts, and theorems. It is seen that a proof helps learners make sense of a result, giving some insight into why it must be true, exhibits the logical structure of ideas, and makes deductive chains of reasoning explicit (Coe & Ruthven, 1994). Through this process learners scrutinize geometric concepts.

Researchers have classified the proving process in terms of different dimensions (Bell, 1976; van Dormolen, 1977; Coe & Ruthven, 1994; Harel & Sowder, 1998). Of these, Harel and Sowder (1998) put forward what is called a proof scheme, which was taxonomized on the basis of students' work and historical development. A proof scheme is a collective cognitive characteristic of the proofs one produces. The taxonomy consists of three classes: external conviction, empirical, and deductive, each group comprising subclasses. In the externally based proof scheme, what convinces the student and what the student offers to convince others comes from an outside source (Flores, 2006). The outside source may be an authority such as a teacher or a book (the authoritative proof scheme); symbol manipulations, with the symbols or the manipulations having no coherent system of referents in the eyes of the student (the non-referential symbolic proof scheme). Additionally, students may remember the appearance of an argument that has been proved before (the ritual proof scheme) (Harel, 2008). Empirical proof schemes include arguments in which students appeal to specific examples or perceived patterns for validation (Martin, Soucy McCrone, Wallace Bower, & Dindyal, 2005). Students may rely on either evidence from examples of direct measurements of quantities, substitutions of specific numbers in algebraic expressions, and so forth (the inductive proof scheme), or they may use one or more examples to convince themselves or others of the truth of a conjecture (the perceptual proof scheme) (Harel, 2008; Flores, 2006). Psychologists have found that most natural human formation of concepts is based on examples and often on just one specific example. Students also appreciate the examples, either as a means of understanding the situation or as a way of checking their understanding (Flores, 2006). The deductive proof scheme class consists of two categories: the transformational proof scheme category and the modern axiomatic proof scheme category. If a student deals with the general aspects of a situation and the proof process is oriented toward the conjecture, then the student is at a transformational proof scheme. In the axiomatic proof scheme, proofs depend on new theorems, axioms, and undefined terms. According to mathematicians, deductive proof schemes are appropriate types of justifications in mathematics (Flores, 2006; Martin et al., 2005; Harel, 2008). When a student is at a deductive proof scheme, he or she will be able to make a generality, an operational thought, and a logical inference (Harel, 2008). Geometry teaching should not attach importance only to geometric proofs. Proof processes in terms of proof schemes have a key position in order to explain the meaning of learners' configuration of geometric knowledge.

In order to teach geometry efficiently, integration of proof into geometry curriculum comes into prominence. It is obvious that a proof-free ''geometry curriculum'' does not teach geometry (Harel, 2008). Understanding what proving means and being able to write proofs are essential for success in mathematics (Senk, 1989). According to Aksu and Koruklu (2015), generating mathematical formulas and creating generalizations will be promoted. Thus, school mathematics curricula should include proving activities.

Teaching geometry should focus on geometric understanding instead of rote learning. In Harel (2008), proofs are explained as ways of understanding associated with the mental act of proving. Likewise, proof schemes are ways of thinking that represent the collective mental characteristics of one's proofs. This means the methods of understanding one produces impact the quality of the ways of thinking being formed, and the ways of thinking one has formed impact the quality of the methods of understanding being produced.

Despite the importance of geometry in our school and daily lives, there have been numerous studies concerned with the failure of students in geometry (Burger & Shaughnessy, 1986; Mason, 1997; Gutierrez, Jaime & Fortuny, 1991; Yavuz Mumcu & Cansiz Aktas, 2015). Since proofs are the heart of mathematics, and proving is complex, teachers should help their students develop these processes in the early grades. The success of this process depends on the teachers' views about the essence and forms of proofs (Hanna et al., 2009). From this point of view, the purpose of this study is to determine the proof scheme of pre-service teachers when proving a geometry theorem. In this sense, the study is oriented by these research questions: which proof schemes do pre-service teachers' use, which difficulties do they encounter, and what are their views about using proofs in lessons when making proofs in geometry?

Method

Research Design

The purpose of this study is to investigate the proof schemes of pre-service classroom teachers. Thus, this case study is a detailed examination of a particular subject or event (Bogdan & Biklen, 1992). A case study was performed because it can enable researchers to gain a deeper understanding of a phenomenon. Three pre-service teachers were selected, so in this study multiple case design was used. In multiple case designs more than one participant is selected in order to prescribe resembling results throughout the cases (Yin, 2003). Firstly, an open ended question was asked, and then semi-structured interviews were conducted.

Research Sample

The participants of this study were three freshman students enrolled in a classroom teacher training program at a state university in Turkey. In Turkey, classroom teacher training program students take two mathematics courses: Basic Mathematics I and Basic Mathematics II. The rationale for selecting the participants from the first year students is that they had just taken Basic Mathematics I and Basic Mathematics II courses. Therefore, in this study the students were selected as a purposive sampling procedure, defined by Fraenkel and Wallen (2006) as based on previous knowledge of a population and the specific purpose of the research. Thus, the three students investigated in this study were selected considering their scores in the Basic Mathematics courses. Two girls having high (94 out of 100) and middle (76 out of 100) scores and a boy having a minimum (60 out of 100) score participated voluntarily. It was assured that their identity would be kept confidential.

Research Instrument and Procedure

An open ended question was used in order to obtain the data. Students were asked to prove that "the sum of the interior angles measurements of a triangle is 180^{o}". After proving the theorem, each student was interviewed in order to reveal what they thought about proofs and proving. Each interview lasted for approximately 30 minutes and was video recorded by the first author.

Data Analysis

The data were gathered through students' answer sheet responses and video records. The video records were transcribed and then both the video records and answer sheets were analyzed. Students' responses to the theorem were investigated to expose the proof schemes in the angles of a triangle concept. Predetermined categories derived from related literature were identified and participants' written and verbal responses were examined in terms of these categories. Three proof schemes were distinguished as three analysis categories. While categorizing students' answers, the proof schemes reported by Harel and Sowder (1998) were used. In this way, students' responses were categorized as externally based proof schemes (sub-categories: authoritarian, ritual, or symbolic), empirical proof schemes (sub-categories: perceptual or example based), or analytical proof schemes (sub-categories: transformational or axiomatic). The researchers analyzed both records and sheets separately and checked against the findings. As a result, their findings were demonstrated to be compatible.

Validity and Reliability

Related to the ethical issues, participants were informed about the scope of the study, the data collection process, and the privacy of their names; their consent was received. All participants joined the study voluntarily. Participants were given pseudonyms. Data were collected and transcribed by the first author of this study and two other researchers verified the data collection process and made the consistency checks between data and transcripts. Researchers did not confront any problems within the data collection process. In terms of the reliability of examining the participants' responses according to three categories, the categorizing process was repeated by all researchers.

Results

In this section, the students' proof processes, video records, and answer sheets were analyzed. These analyses were supported with direct quotations.

Ayşe earned the highest average score in the Basic Mathematics course the previous semester. She was an active participant in the class and interested in the lessons. When she was asked to solve the problem, Ayşe first read the question aloud.

Ayşe: "First of all, we have learned that two straight lines are one ray; and a circle crosses this line segment; and we have proven that one circle is 360^{o}; and we have learned that one straight line divides it into two. Like this…" (She draws a circle and places a coordinate system in the circle, (0,0) point being at the center.) "We consider it as a rope. We know that the center is 360^{o} and these (upper part of the x-axis) are 180^{o}. And here (center angles of 1st, 2nd, 3rd, and 4th regions of the coordinate system) are 90-90. Then we form these parts of the triangle equally." (She makes a triangle by drawing a chord from points where the coordinate system crosses the circle.) "Then here, we form a right-angled triangle, I just think a right-angled one. Here is 45-45, and here is 90-90. For example, let's take this triangle, let's name it ABC triangle, and shade it." (She draws the triangle and shades near the circle again.) "We take the right-angled triangle like this, and it is divided into equal parts, too. From there $90^o+45^o+45^o=180^o$."

Researcher: "How do you know that this angle (the surrounding angle facing the diameter) is 90^o?"

Ayşe: "Hmm… First of all, I thought of a circle and we know that a circle's circumference is 360^o. Let's take it from that point. For example, you consider it not as a straight line but as a rope. We are dividing the ropes into equal parts and we see that a right angle is formed; and we know that a right angle is 90^o..."

Researcher: "You said rope, where did you learn that?"

Ayşe: "In primary school, our teacher first taught us that. Like, when we consider it as a rope we can make it. We stretch the ropes; for example, let's put a pen here," (she holds the pen perpendicular on the paper), "when we stretch the ropes, we also stretch one at the bottom, like this." (She shows it with her hand.) "We already understand that these angles are 90^o; and we also show that the parts here are also 90^o. I thought like that."

Researcher: "Right, but how do you know that those angles (the base angles of the isosceles triangle formed) are 45^o?"

Ayşe: "We took from here, in the same way, a straight line," (showing the base of the triangle), "and that line perpendicularly…"

Researcher: "Your thinking is correct but this is valid for an isosceles right triangle."

Ayşe: "Yes. Then, let's draw another triangle. Hmm... Let's name these angles x, y, and z angles. I am again thinking of a circle… Let's use the parallels. For example, let's name the straight line d1 and d2. Let's draw a right angle to them…" (She gives values to the angles formed.) "Here we have a new problem. The sum of these two internal angles' measurements equals here (shows this), but of course we have to show it." (She wants to use the theorem that the sum of two interior angles' measurements equals one exterior angle's measure; however she shows an incorrect exterior angle.)

Researcher: "How do you know that the sum of those angles' measurements are equal to there?"

Ayşe: "Yes, we have to consider this, too, in fact…"

Ayşe gives values to the new angles she formed. After a few steps she completes the proof by depending on the fact that a supplementary angle is 180^o and that the sum of the interior angles of the rectangle she formed is 360^o.

When Ayşe read the question she remembered the example that her primary school teacher gave, and she structured the proof according to that. Beginning to prove with the example of her teacher instead of using her own mental processes, Ayşe completed the proof on her second try. Ayşe's paper shows that it is very disorganized and unsystematic (Figure 1).

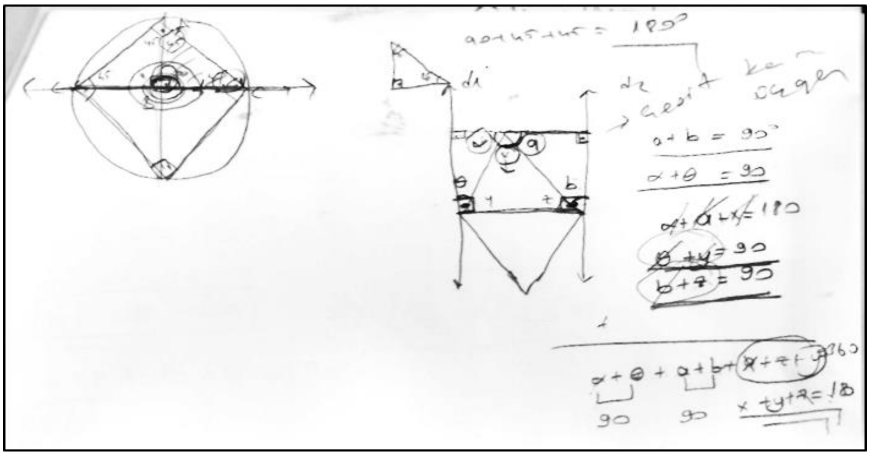

Figure 1. Ayşe's Answer Sheet

Ayşe's thoughts about making lessons with proofs are as follows. "I will not do it again." (She smiles.) "Well, it should definitely be done; first of all, I should say that children must be taught like that. Without proofs, it doesn't work. For example, I haven't thought of this problem before. I wish I had thought of it a little before I came. For example, a lesson from last semester included all of these proofs. If we hadn't studied them, I couldn't have shown these."

Elif is a student who passed the Basic Mathematics course with an average score. She was not active in the class and not interested in the lessons. When Elif was asked to make the proof, she first repeated some of the operations, her voice low. Then she was stimulated by the researcher to think out loud.

Elif: "Hmm… how can we prove this is an isosceles triangle? I don't know in fact. How can we prove it? …" (By the way she shows the interior angles with algebraic expressions)

Researcher: "Why do you think it is an isosceles triangle?"

Elif: "Oh, I don't know, because its sides are equal, it's equal to 180…, but it isn't working. Or it's an equilateral triangle, but how are going to prove it?"

Researcher: "Let's say it is equal for an equilateral triangle; there is the possibility that it's not equal in other triangles."

Elif: "That's true, too. So, we cannot make it an isosceles triangle."

Researcher: "What do you need to show?"

Elif: "For example, we know that the sum of the exterior angles' measurements is 360^o."

Researcher: "How do you know?"

Elif: "We don't know it, too..."

(Elif makes drawings randomly and speaks in a low voice.)

Researcher: "You can use auxiliary drawings." (The student, after thinking for a long time, draws an internal bisectrix.)

Researcher: "Usually, we draw a straight line that is parallel to a side…"

Elif: "Where can we draw a parallel?" (She thinks, drawing an internal bisectrix from all corners) "Hmm..." (She draws a straight line parallel to the base that crosses other sides of the triangle. She then writes values of the corresponding angles and after incorrect operations, she finds that it is an isosceles triangle.)

Researcher: "Why are those two angles' measurements equal?"

(She explains the operations she made.)

Researcher: "Well, how can you show that angle (the supplement of the corresponding angle parallel to the base) in another way?"

Elif: "I can tell here a+b. Because, the sum of two interior angles' measurements equals one exterior angle. Here it is also 180^o. We can equalize these two." (The student, without realizing that she completed the demonstration, writes other equations).

Researcher: "What are you trying to show?"

Elif: "That a+b+c is 180^o. Hmm, err…. oh, with this equation, if we take c to the other side, we will probably find it."

(The student is not sure of what she makes.)

Researcher: "Did you really find it?"

Elif: "Did I? Here is a, here is b, and here is c. Their sum is 180. I don't know, I think I showed it." (She pauses and thinks.) "If these angles are not corresponding angles, it means I couldn't show it. But these angles are corresponding and their measurements are equal to each other." (She tells the operation steps she made, she hesitates at one interior angle to which she gave the unknown c and tries to find why she gave it c). "We also need to show why we gave c here. In the small angle, the sum of a, b, and c is 180^o, and then it must be 180^o in the big angle, too. Yes, equal, I showed it."

As can be seen, Elif is not sure of her operations; she makes them a little bit randomly. Investigating Elif's answer sheet shows that she tried to solve the question with scribbles (Figure 2).

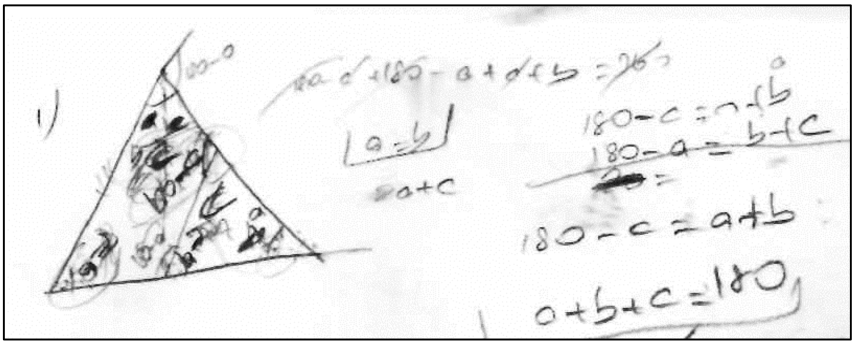

Figure 2. Elif's Answer Sheet

Elif's thoughts about making lessons with proofs are as follows: "In high school they told us, in the final class our teacher told us, but I didn't listen because I wasn't interested. I didn't listen to anything; I just looked at the result."

Researcher: "Well, do you think a lesson using proofs would be useful?"

Elif: "Of course it would be useful. You know, you force your mind, it isn't something idle. You know it can improve one's intelligence. You know, it is more useful to know where it comes from rather than doing it without any grounds."

The third and last student is Ali. He got the lowest score in the Basic Mathematics course. He was interested in the lessons a bit. Ali started the proof as shown below:

Ali: "We will show that the sum of the interior angles' measurements of a triangle is 180°. When we draw with rays, it makes a triangle." (He draws three straight lines, intersecting in doubles). "If we call this x and this y, since x+y is half an angle," (the angle he shows is a supplementary angle), "it is 180°. And here is y from the opposite angles. Later, what can we call this? If we draw a line from here, this will be y." (He draws a line parallel to the base, which passes from the other corner.)

Researcher: "Why will it be y?"

Ali: "Because I drew parallel to this side, it is an opposite angle."

Ali goes on giving unknowns to angles. Since he gave z to the supplement of the angle that he also gave z, after a series of operations, he gets an incorrect result. The researcher emphasizes this and asks why the two angles are equal to z.

Ali: "Pardon, not here, from the internal opposite angles, here is z."

Since Ali goes on with the incorrect unknowns he gave before, after a few operations, he again gets a wrong result. Then he makes a new drawing. He draws the triangle, with an intersection of three lines at three points.

Ali: "Now, if we call this x and this y, from half an angle," (he means the supplementary angle), "their sum will be 180°. Then if we draw a parallel to this ray, from corresponding angles," (in fact, an internal opposite angle) "this would be y. If we add all of the angles, it will equal x+y. And we have already shown it is 180°."

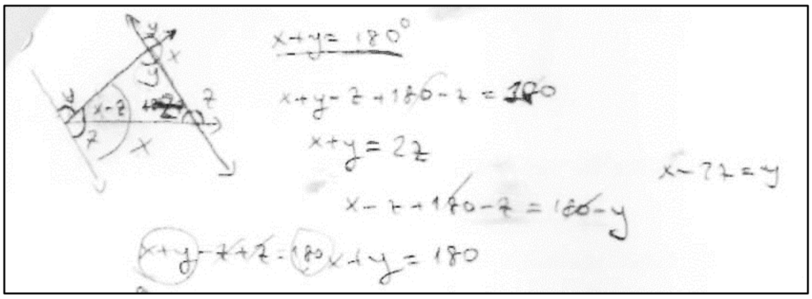

Figure 3. Ali's Answer Sheet

As can be seen, in his second try Ali achieved the correct result. Furthermore, his answer sheet shows that operation steps have been followed in a good order and the demonstration is clear (Figure 3). However, Ali drew the triangle as an intersection of three lines at three points.

Researcher: "Why did you draw the triangle like that?"

Ali: "I made it in order to show the corresponding angles or opposite angles. Normally triangles are like this, is what I mean; we only give names to points."

Researcher: "Normally how can we show a triangle?"

Ali: "Normally it is like this, too, but there are no extensions of the ray."

When Ali was asked whether or not he was pleased that the lesson was made with proofs, he answered, "It happens because we haven't seen proofs before. We are not used to this. That's why we are not pleased with proofs."

Researcher: "Didn't you get used to it?"

Ali: "So so, but because the class is very crowded, it wasn't gone into more detail. It's only taught once and that's all. But if the class had a smaller number of students or these proofs were seen with mutual conversations, they would be understood more easily. That's why I don't understand. Also in the classroom, I hesitate to ask questions."

Researcher: "How should the lesson be made better?"

Ali: "In fact, when there is a proof, we understand better what comes from where, but as long as it isn't very difficult. Nothing is very difficult though. Err, how would it be with the old style? If only rules were given, without showing what comes from where, then it would be more enjoyable. If we solved questions, it would be better. But that is because of high school, because we saw it all the time like that; we didn't see any proofs. But here, when there are details, it becomes difficult to understand."

Discussion and Conclusion

The results of the study show that participants' attitudes are not parallel to their achievements in the lesson. Ayşe, contrary to her success in the class, failed to show what was expected from her when answering the question. She started the proof with an example to help her, and drew a circle and then an isosceles right triangle inside the circle. When telling the operations steps she made, she used the example that her primary school teacher had given. However, she failed to remember and explain this example properly. Ayşe's beginning the proof with an example and giving explanations through a remembered example from her teacher shows that she is in Harel and Sowder's (1998) inductive proof scheme. Her beginning with a circle when answering the question supports this opinion. But Ayşe made the proof with a special example (an isosceles right angle); she could not generalize the proof. Then, she made a second drawing and, using the auxiliary drawings, she completed the proof. Housman and Porter (2003), in the study they conducted with students who had a high average grade, found that students have analytic, empiric, and external proof schemes. As a difference in categorization, Hoyles (1997), in the study he conducted with the aim of finding the proof schemes that primary and secondary school students use in algebra and geometry, asked students to show that the sum of the interior angles' measurements of a triangle is 180^o. As a result of the study, he found that students' proof schemes can be categorized as empirical, enactive, narrative, and visual.

Although an intermediate level student, when Elif started to answer the question, she was mute for a long time and tried to remember something. Her drawing figures randomly shows that she did not know what she had to do. Then, with the help of the researcher, she used some known proving processes and in her second try she completed the proof. That Elif was not sure of what she was doing and that she completed the proof a little bit randomly is reminiscent of Harel and Sowder's (1998) non-referential symbolic proof scheme approach. Coe and Ruthven (1994), in their studies, found that most students act in accordance with the empirical proof scheme approach.

Although Ali was an unsuccessful student, he showed a better performance than what was expected from him. Firstly, he drew the auxiliary element to the triangle, and then he went on to give values to the angles formed. Furthermore, that Ali formed the triangle with an intersection of three lines at three points is another interesting issue. When the researcher asked why he drew the triangle like that, Ali said he had been shown a triangle like that before. For these reasons, Ali acts in a ritual proof scheme approach (Harel and Sowder, 1998).

Although Ali was an unsuccessful student, looking at his grade point average, he nearly showed a similar performance to that of the other attendants. However, when investigated, Ali had a more basic problem. Ali did not know the most basic concepts in geometry, such as internal opposite angles and supplementary angles. Similarly, there are studies that show that students have insufficient information and misconceptions relating to basic concepts in geometry.

Another result of this study concerns using proofs in teaching and learning processes. When the students were asked for their opinions about proofs, they had the common idea that lessons should be made with proofs. They thought that when students, without memorizing the rules, understand its logic, mental processes start to run and they gain different points of view. However, the students stated that they had difficulty understanding the proof since during their primary and secondary education they did not see proofs, which indicated that even if lessons were made with proofs they would not be interested. Although the students stated that lessons should be made with proof demonstrations, they preferred to have lessons with traditional methods, where rules are taught and students solve the questions. There are studies that have similar findings (Coe & Ruthven, 1994; Gokkurt & Soylu, 2012). In these studies only a small number of students are interested in how rules and patterns are established. Accordingly, Morali, Ugurel, Turnuklu, and Yesildere (2006) state that pre-service mathematics teachers do not know the necessity of the proving process in mathematics. The foundation of mathematical proof that students meet at university is built on earlier experiences. If a student has never met proofs in the previous learning process, he or she will have difficulty in understanding proofs. So previous lessons (Tall, 2004) are essential in learning effectively. Curriculum builders should consider that new mathematical structures are built on previous ones. In fact, learning is a student building on his or her own personal previous lessons in ways that may not fit with the intended new developments (Tall, 2005).

Proofs are the heart of mathematics; they help us see what is true as well as why it is true (Rav, 1999; Hanna, 2000). Proofs help in the development of mathematical concepts and lead to further mathematics (Thurstone, 1994). As with Rav's analogy (1999), proofs are a network of roads in a public transportation system, and statements of theorems are the bus stops.

Proving is the establishment of truth by deductive means and reasoning. These means distinguish mathematics from other subjects. The development of the ability to construct a proof is an important goal of mathematics education. By this means other mathematical competencies are developed. Therefore, it is strongly believed that mathematics curricula should contain proofs (Porteous, 1990; Mariotti, 2006).

Although proofs and reasoning are fundamental aspects of mathematics, there are researches concerning students' failure in proving. There are many reasons for this failure. Because students are not used to proving, they do not know the accurate meaning of a proof. Thus, they are insufficient in understanding a theorem or a proof and misapply it. Proving requires the coordination of a great deal of competencies. For a student, learning to prove means making a transition from a computational view of mathematics to a view of intricately related structures (Weber, 2001).

Another reason for students' failure in proving may be the attitude of teachers about proofs. When investigated, it is shown that school curricula is not sufficient in terms of developing proving ability (Tall, 1995). What a teacher thinks about the role of a proof in mathematical thinking is important in in-class training (Martin & Harel, 1989). As Hersh (1993) stated, due to restricted time, the subject being difficult for a student's level, and the insipidity of the proving process, the expected importance of a proof is not extended in the classroom.

The results of this study are limited to the proof schemes of three pre-service teachers. Consequently, we cannot make a generalization with the proof schemes of pre-service teachers. However, according to the results of this study and other studies in the field, it is understood that pre-service teachers are not able to prove even a simple geometry theorem. What underlies this is thought to be insufficient knowledge in the students about definitions of geometric concepts, as well as the misconceptions they have. Furthermore, another reason for this can be that they did not experience any proving processes in their former education. Hence, students should realize how valuable proving is and acquire knowledge through the counseling of a teacher.

References

Aksu, G. & Koruklu, N. (2015). Determination the effects of vocational high school students' logical and critical thinking skills on mathematic success. *Eurasian Journal of Educational Research*, *59*, 181-206.

Bell, A. W. (1976). A Study of pupils' proof-explanations in mathematical situations. *Educational Studies in Mathematics*, *7*(½), 23-40.

Bogdan, R. C., & Biklen, S. K. (1992). *Qualitative research for education: An introduction to theory and methods.* London: Ally and Bacon.

Burger, W. F., & Shaugnessy, J. M. (1986). *Assesing children's intellectual growth in geometry, final report.* Oregon State University.

Buyukozturk, S.; Cakan, M.; Tan, S. & Atar, H.Y. (2014a). *TIMSS 2011 ulusal matematik ve fen raporu – 8. Siniflar[TIMSS 2011 national mathematics and science report-8th grades].* Ankara: Iskur.

Buyukozturk, S.; Cakan, M.; Tan, S. & Atar, H.Y. (2014b). *TIMSS 2011 ulusal matematik ve fen raporu – 4. Siniflar [TIMSS 2011 national mathematics and science report-4th grades.* Ankara: Iskur.

Clements, D. H. (1998). *Geometric and spatial thinking in young children*. National Science Foundation, Arlington, VA.

Coe, R., & Ruthven, K. (1994). Proof practices and constructs of advanced mathematics students. *British Educational Research Journal*, *20*(1), 41-53.

Crowley, M. (1987). The Van Hiele model of development of geometric thought. In M. Lindquist & A. P. Shulte (Eds), *Learning and teaching geometry, K-12* (pp. 1-16). Reston, VA: NCTM.

Driscoll, M. (2007). *Fostering geometric thinking a guide for teachers, grades 5-10.* Porsmouth: Heinemann.

Flores, A. (2006). How do students know what they learn in middle school mathematics is true? *School Science and Mathematics*, *106*(3), 124-132.

Fraenkel, J.R., & Wallen, N.E. (2006). *How to design and evaluate research in education.* New York: McGraw-Hill.

Gokkurt, B. & Soylu, Y. (2012). Universite ögrencilerinin matematiksel ispat yapmaya yonelik gorusleri [University students' ideas related to mathematical proof]. *Egitim ve Ogretim Arastirmalari Dergisi*, *1*(4), 56-64.

Gutierrez, A., Jaime, A. & Fortuny, J. (1991). An alternative paradigm to evaluate the acquisition of the Van Hiele levels. *Journal for Research in Mathematics Education*, *22*(3), 237-251.

Hanna, G. (2000). Proof, explanation and exploration: An overview. *Educational Studies in Mathematics*, *44*, 5–23.

Hanna, G., de Villiers, M. Arzarello, F., Dreyfus, T., Durand-Guerrier, V., Jahnke, H.N., Lin, F.L., Selden, A., Tall, D., & Yevdokimov, O. (2009). Icmi study 19: Proof and proving in mathematics education: Discussion document. In Fou-Lai L., Feng-Jui H., Gila H., Michael de V. (Eds.). *Proceedings of the ICMI Study 19 conference: Proof and Proving in Mathematics Education: Vol. 1.* (pp. xix - xxviii). Taipei, Taiwan.

Harel, G. (2008). DNR perspective on mathematics curriculum and instruction, Part I: focus on proving. *ZDM Mathematics Education*, *40*, 487–500.

Harel, G., & Sowder, L. (1998). Students' proof schemes: results from exploratory studies. In A. H. Schoenfeld, J. Kaput, & E. Dubinsky (Eds.), *Research in collegiate mathematics III* (pp.234–282). Providence, Rhode Island: American Mathematical Society.

Hersh, R. (1993). Proving is convincing and explaining. *Educational Studies in Mathematics, 24*(4), 389-399.

Housman, D., & Porter, M. (2003). Proof schemes and learning strategies of above-average mathematics students. *Educational Studies in Mathematics*, *53*, 139–158.

Hoyles, C. (1997). The curricular shaping of students' approaches to proof. *For the Learning of Mathematics*, *17*(7), 7-16.

Mariotti, M. A. (2006). Proof and proving in mathematics education. In A. Gutiérrez & P. Boero (Eds.), *Handbook of research on the psychology of mathematics education: Past, present and future* (pp.173–204). Rotterdam: Sense.

Martin, T.S., Soucy McCrone, S. M., Wallace Bower, M. L. & Dindyal, D. (2005). The interplay of teacher and student actions in the teaching and learning of geometric proof. *Educational Studies in Mathematics*, *60*, 95- 124.

Martin, W. G., Harel, G. (1989). Proof frames of pre-service elementary teachers. *Journal for Research in Mathematics Education*, *20*(1), 41-51.

Mason, M. M. (1997). The Van Hiele model of geometric understanding and mathematically talented students. *Journal for the Education of the Gifted*, *21*(1), 39-53.

Morali, S., Ugurel, I., Turnuklu, E. & Yesildere, S. (2006). Matematik ogretmen adaylarinin ispat yapmaya yonelik gorusleri [The views of the mathematics teachers on proving]. *Kastamonu Egitim Dergisi*, *14*(1), 147-160

Moutsios-Rentzos, A., & Spyrou, P. (2015, February). The genesis of proof in ancient Greece: The pedagogical implications of a Husserlian reading. In *CERME 9-Ninth Congress of the European Society for Research in Mathematics Education* (pp. 164-170).

National Council for Teachers of Mathematics (2000). *Curriculum and evaluation standards for school mathematics.* Reston, VA: Author.

Porteous, K. (1990). What do children really believe? *Educational Studies in Mathematics, 21*(6), 589-598.

Rav, Y. (1999). Why do we prove theorems? *Philosophia Mathematica*, *7*(3), 5-41.

Senk, S. L. (1989). Van Hiele levels and achievement in writing geometry proofs. *Journal for Research in Mathematics Education*, *20*(3), 309-321.

Tall, D. (1995). Cognitive growth in elementary and advanced mathematical thinking. *Plenary Lecture, Conference of the International Group for the Psychology of Learning Mathematics, Vol. 1.* (pp. 161-175). Recife, Brazil,

Tall, D. (2004), Thinking through three worlds of mathematics, *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education, Vol. 4.* (pp. 281-288). Bergen, Norway.

Tall, D. (2005). The transition from embodied thought experiment and symbolic manipulation to formal proof. *Plenary Lecture for the Delta Conference*, Frazer Island, Australia.

TEDMEM, Turk Egitim Dernegi (2014). *2014 Egitim degerlendirme raporu [2014 Education evaluation report]*. Ankara: Iskur.

Thurstone, W. P. (1994). On proof and progress in mathematics. *Bulletin of the American Mathematical Society*, *30*(2), 161-177.

Usiskin, Z. (1982). Van Hiele Levels and Achievement in Secondary School Geometry. Chicago: University Of Chicago. Retrieved April 2 2016 from http://ucsmp.uchicago. edu/van_Hiele.html

Van De Walle, J. A. (2004). *Elementary and middle school mathematics (5th ed.).* Boston: Pearson Education Inc.

Van Dormolen, J. (1977). Learning to understand what giving a proof really means. *Educational Studies in Mathematics*, *8*(1), 27-34.

Van Hiele, P. M. (1986). *Structure and insight*. New York: Academic Press.

van Hiele-Geldof, D. (1984). Last article written by Dina van Hiele-Geldof entitled: Didactics of geometry as learning process for adults. *English translation of selected writings of Dina van Hiele-Geldof and PM van Hiele*, 215-233. New York: Brooklyn College

Yavuz Mumcu, H., & Cansiz Aktas, M. (2015). Multi-program high school students' attitudes and self-efficacy perceptions toward mathematics. *Eurasian Journal of Educational Research*, 59, 207-226. http://dx.doi.org/10.14689/ejer.2015.59.12

Yildirim, H. H., Yildirim, S., Yetisir M. I., & Ceylan, E. (2013). *PISA uluslararasi ogrenci degerlendirme programi: PISA 2012 ulusal on raporu [PISA 2012 national snap report].* Ankara: Sebit Egitim ve Bilgi Teknolojileri A.S.

Weber, K. (2001). Student difficulty in constructing proofs: The need for strategic knowledge. *Educational Studies in Mathematics*, *48*, 101–119.

Yin, R. K. (2003). *Case study research: Design and methods* (3rd ed.). Thousand Oaks, CA: Sage.

Sınıf Öğretmeni Adaylarının Geometrideki İspat Şemaları: Üç Öğretmen Adayının Durum Çalışması

Atıf:

Oflaz, G., Bulut, N. & Akcakin, V. (2016). Pre-service classroom teachers' proof schemes in geometry: a case study of three pre-service teachers. *Eurasian Journal of Educational Research*, 63, 133-152, http://dx.doi.org/ 10.14689/ejer.2016.63.8

Özet

Problem Durumu: Matematiğin önemli bir parçası olan geometri, okullarda ve günlük yaşamda önemli bir yer edinmiştir. Geometri eğitimine verilen öneme rağmen, birçok kişi geometri karşısında çaresiz kalmakta, buna bağlı olarak da başarısızlık göstermektedir. Matematiği ezberlenmesi gereken kurallar bütünü olarak gören öğrenciler, matematikten kaçmaktadırlar. Buda onların matematik derslerinde başarısı gösterememesinin nedenlerinden birini oluşturmaktadır. Hâlbuki matematik belli bir mantık içinde gelişmiş, kavramların bir biri ile uyum içinde olduğu bir disiplindir. Matematiğin bu uyumlu mantıksal ilişkisini anlamanın yolu, kuralları ezberlemeden ziyade, kavramların nasıl geliştiğini görmekten geçmektedir. Bunun anlamanın yolu ise onları elde ederken yapılan ispatları incelemekten geçmektedir. Nasıl kalp insana hayat verirse, ispatlarda matematiğe hayat vermektedirler. Fakat ispatlar karmaşık gözüktüğünden, birçok öğretmen tarafından derslerde pek kullanılmamaktadır. Oysaki eğitimin ilk yıllarından itibaren, öğrencilerde bu becerinin gelişmesi için öğretmenlerin derslerde ispata yer vermeleri gerekmektedir. Bu açıdan bakıldığında öğretmenlerin derslerde ispat kullanımına yönelik görüşleri ve ispat yaparken ki süreçlerinin incelenmesi önem arz etmektedir.

Araştırmanın Amacı: Bu çalışmanın amacı öğretmen adaylarının geometrik bir teoremi ispatlarken, kullandıkları ispat şemalarını, ispat yaparken karşılaştıkları zorlukları ve derslerde ispat kullanımına yönelik görüşlerini belirlemektir.

Araştırmanın Yöntemi: Araştırmada nitel araştırma desenlerinden durum çalışması kullanılmıştır. Çünkü bir konuda derinlemesine bilgi edinmek istenildiğinde durum çalışmasının kullanılması uygundur. Çalışmanın örneklemini, seçkisiz desenlerden amaçlı örnekleme yöntemine göre belirlenen ikisi kız, diğeri erkek üç öğretmen adayı oluşturmaktadır. Çalışmada ilk önce öğretmen adaylardan bir üçgenin iç açıları ölçüleri toplamının 1800 olduğunu göstermeleri istenmiştir. Daha sonra yaptıkları ispata ve derslerde ispat kullanımına yönelik yaklaşık otuz dakika süren yarı yapılandırılmış görüşmeler yapılmıştır. Yapılan görüşmeler analiz edilmek üzere kayıt altına alınmıştır. Öğretmen adaylarının ispat şemalarına belirlemek için, elde edilen veriler her bir araştırmacı tarafından ayrı ayrı incelenmiştir. Sonuç olarak araştırmacıların uyum birliğine vardıkları görülmüştür.

Araştırmanın Bulguları: Araştırmadan elde edilen verilere göre, öğretmen adayları derslerdeki başarılarını ispat yaparken gösterememişlerdir ve temel geometrik kavramlara yönelik bilgilerinde eksiklikler olduğu görülmüştür. Örneğin Ayşe derste başarılı olmasına rağmen kendisinden istenen ispatı uygun şekilde göstermekte zorlanmıştır. İspatı yaparken ilköğretim yıllarındaki öğretmeninin verdiği örneği temel alarak ispatı yapmaya çalışmıştır. İlk denemesinde başarısız olan Ayşe, ikinci denemesinde ise ispatı özel bir durum üzerinde (ikizkenar dik üçgen için) göstermeye çalışmıştır. Yani genellemeye gidememiştir. Elif orta düzey başarı gösteren bir öğrencidir. İspat yaparken rastgele çizimler yapmaya çalışmıştır. Bu durum Elif'in tam olarak ne yapacağını bilmediğini göstermektedir. Ali derslerde başarılı olmamasına rağmen ispat yaparken iyi bir performans göstermiştir. İşlemleri düzenli bir şekilde yapmıştır. Fakat temel geometrik kavramlara yönelik bilgilerinde eksiklik olduğu görülmüştür. Bu ise Ali'nin tanımları tam olarak öğrenemediğinin göstergesidir. Ayrıca Ali'nin üçgeni oluştururken ışınları kullanması farklı bir bulgudur. Ali bunu daha önceki öğrenmelerine bağlamıştır.

Araştırmanın Sonuçları ve Önerileri: Sonuç olarak Ayşe'nin ispatı yaparken bir örnek ile başlaması ve öğretmenin verdiği örneği temel alarak ispatı yapmaya çalışması, onun tümevarımsal ispat şeması yaklaşımında olduğunu göstermektedir. Elif'in rastgele şekiller çizmesi, ne yapacağını tam olarak bilememesi onun sembolik ispat şeması yaklaşımında olduğunu göstermektedir. Ali'nin ispatı daha önceki öğrenmelerine göre yapması onun ritüel ispat şeması yaklaşımda olduğunu göstermektedir. Genel olarak bakıldığında Ayşe deneysel, Elif ve Ali dışsal bir yaklaşım göstermişlerdir. Bununla birlikte Coe ve Ruthven (1994) çalışmalarında birçok öğrencinin deneysel ispat şeması yaklaşımına sahip olduklarını görmüştür. Ayrıca öğretmen adayları derslerde ispat kullanımının yararlı olduğunu düşünmektedirler. Böylece kuralların ezberlenmeyip, mantıksal olarak öğrenileceğini belirtmektedirler. Fakat öğrencilik yıllarında derslerde ispat kullanılmışsa da ilgili olmadıkları görülmektedir. Benzer şekilde Ceo ve Ruthen (1994) çalışmalarında da çok az sayıda öğrencinin kuralların nasıl oluştuğunu öğrenmeye meraklı olduklarını belirtmişlerdir. Matematiğin kalbi olan ispatlar, bir teoremin doğru olup olmadığını anlamaya yarar ve matematiğin gelişmesine öncülük eder. Rav (1999) yaptığı bir analojide ispatları yollar ağına, teoremleri de otobüs durağına benzetmiştir. İspatlar matematiğin temelleri olmasına rağmen, birçok araştırmada öğrencilerin başarısız oldukları görülmüştür. Bunun birçok nedeni vardır. Öğrencilerin ispatı hiç kullanmaması, ne anlama geldiğini bilmemesi bunlardan bir kaçını oluşturmaktadır. Bundan dolayı öğrenciler teoremleri anlamada ve uygulamada yetersiz kalmaktadır. Tall (2004) ise üniversite öğrenimine kadar ispatlar ile ilgilenmemiş öğrencilerin, ispat yapmada zorlandıklarını belirtmiştir. Öğrencilerin başarısızlığın bir diğer nedeni öğretmenlerin ispata yönelik tutumundan olabilir. Çünkü programlar incelendiğinde ispat yapma becerisini geliştirecek şekilde düzenlenmediği görülmektedir (Tall, 1995). Programda ispatlara yeterince yer verilmediğinden dolayı öğrencilerin ispat becerilerinin geliştirilmesinde temel kaynak sınıf öğretmenleridir. Benzer şekilde Martin ve Harel (1989) öğretmen adaylarının ispatlara bakış açısının önemli olduğunu belirtmektedirler. Hersh (1993) ise derslerde sınırlı zamandan dolayı ispat süreçlerinin göz ardı edilmesine neden olduğunu belirtmektedir. Bu çalışmanın sonuçları üç öğretmen adayının ispat şemaları ile sınırlıdır. Bundan dolayı genelleme yapmak mümkün değildir. Bununla birlikte bu çalışmanın sonuçlarına göre, sınıf öğretmeni adaylarının basit bir geometrik teoremi ispatlama da bile zorlandıkları görülmüştür. Öğrencilerin geometrik kavramlardaki eksik bilgileri ve kavram yanılgıları ise göze çarpmaktadır. Öğrencilerin önceki eğitimlerinde ispatlar ile ilgili yeterince tecrübe kazanmamaları ve kavramsal bilgilerin göz ardı edilmesi buna neden olmuş olabilir. Oysaki İspatlar öğrencilerin işlemsel bakış açısından kavramsal bakış açısına geçmelerine yardım eder (Weber, 2001; Jones, 2002). Öğretmen adayları da ispatlamanın bilgiyi elde etmede önemli olduğunu belirtmişlerdir. Sonuç olarak ispat yapabilme matematik eğitiminde geliştirilmesi gereken bir beceridir. Bundan dolayı matematik eğitimi ispatları içermelidir.

Anahtar Sözcükler: Geometri, matematik eğitimi, öğretmen adayları, ispat şemaları