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Application of Mathematical Modeling in Multi Criteria Decision Making Process: Intuitionistic Fuzzy PROMETHEE

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Abstract

In this paper, the intuitionistic fuzzy PROMETHEE method is explained in detail and an original application has been made. The aim of the paper is to bring innovation to the evaluation system in the field of education by using the intuitionistic fuzzy PROMETHEE method. The advantages of using the PROMETHEE method, which is one of the many methods used in multi-criteria decision-making problems, in the intuitionistic fuzzy sense are explained in detail. Intuitionistic fuzzy PROMETHEE is a method that attracts our attention thanks to its benefits such as allowing the researcher to observe the positive and negative rankings simultaneously, expressing the degree of hesitation, changing the significance weights of the criteria, and using different methods when identifying the significance levels for each criterion, putting the degree of hesitation of significance weights into action, and enabling decision-makers, who are given the opportunity to use different criteria types and different criteria types for alternatives and criteria, to establish a unique system; moreover, the method also provides us numerous advantages while using it in our application area. It is of great importance for decision-makers to determine the specific importance level for each criterion. In this paper, controlled sets are used to express the importance of the criteria in the form of intuitionistic fuzzy values. The intuitionistic fuzzy-based PROMETHEE algorithm, aiming to contribute to the education system by examining the factors affecting the students' achievement, is a unique algorithm, and it is the first to shed light on various researchers.

1. Introduction

The main purpose of this paper is to explain the intuitionistic fuzzy PROMETHEE method and present an application that demonstrates the usefulness of the method. In addition, there is an example to improve the evaluation system, which is a first in the field of education, in this application where the intuitionistic fuzzy PROMETHEE method is used. This paper, it is aimed to create an intuitionistic fuzzy logic-based intelligent assessment system in which factors affecting students' success at the same time with their exam results are also activated. In order to create a training module suitable for individuals, a multi-criteria decision-making (MCDM) algorithm in intuitionistic fuzzy logic has been created using the PROMETHEE (Preference Ranking Organization Method for Enrichment of Evaluations) method. Creating a new algorithm with PROMETHEE method, it is aimed to solve this MCDM problem in the healthiest way with the help of the intuitionistic fuzzy theory-based PROMETHEE method. With the algorithm we did present, it is aimed to evaluate students' success not only according to the exam results, but also by considering their psychological, economic, social, and talented characteristics. Fuzzy logic was firstly defined by Zadeh in 1965 [1]. Then, the Intuitionistic fuzzy set (shortly IFS) was defined by K. Atanassov [2, 3]. If we give a brief information about these issues; we cannot say that every situation is completely true-false, good-bad as a fact and the problem. Therefore, the classical (Aristotle) logic used is insufficient in solving our problems or in situations we encounter. "There are shades of gray between black and white." According to the fuzzy logic that can be explained with the sentence, we can solve more of the problems we encounter in daily life. Fuzzy logic is built on the foundations of classical logic to include classical logic. While the studies on fuzzy logic were continuing, researchers encountered events in which they realized that fuzzy logic was also insufficient and that hesitation should have value in the

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decision-making process. The IF logic concept, which has proven useful for the solution of these and similar problems, is defined. Smart systems become more meaningful and applicable since the IFS includes the degree of membership, non-membership and hesitation. Since IFS also includes uncertainty, they are used in decision-making for incompletely defined and incomplete data, allowing us to obtain more consistent results. Intuitionistic fuzzy logic is based on fuzzy logic. As can be seen here, classical, fuzzy and intuitionistic fuzzy logic are intertwined. Intuitionistic fuzzy logic is a generalization of fuzzy logic. Intuitionistic fuzzy logic comes into play in situations where fuzzy logic cannot respond or is insufficient.The intuitionistic fuzzy set theory is useful in various application areas such as; medicine, medical diagnosis, medical application, career determination, real-life situations, education, decision making, multi-criteria decision making, artificial intelligence, networking, computer, smart systems, economy, and various fields. The decision-making process involves choosing between two or more choices. Multi-criteria decision-making (MCDM) is a well-known notion that aims to choose the best solution among various alternatives in decision-making. The working style of all MCDM methods is as follows: the Selection of Criteria, the Selection of Alternatives, the Selection of Aggregation Methods, and ultimately the Determination of Ranking which is based on weights or outranking [4]. Some of the MCDM methods are Analytical Hierarchy Process (AHP), Fuzzy Multi-Criteria Decision Making Process, ELECTRE, PROMETHEE, and the TOPSIS Method. Bellman and Zadeh were the first researchers who introduced decision-making in fuzzy logic. Multi-criteria fuzzy decision-making has been one of the rapidly growing areas in recent years thanks to its practicality. In MCDM problems, usually, the best alternative is chosen from those depending on the criteria. One of the multi-criteria decision-making methods PROMETHEE was developed by Jean Pierre Brans (1982) [5]. Using the PROMETHEE method could be taken a partial or complete ranking of alternatives based on the positive outranking flow, the negative outranking flow, and the net outranking flow. This method is different from other decision methods because it evaluates each alternative within itself. The main idea of PROMETHEE is to derive a partial or complete ranking of alternatives based on the positive outranking flow, the negative outranking flow, and the net outranking flow. Then it has been expanded in different forms. After being defined in the fuzzy field, it has been expanded in the sense of the intuitionistic fuzzy field. Many researchers have developed applications using PROMETHEE methods and intuitionistic fuzzy sets [6]- [16].

2. Preliminaries

Definition 2.1. *[2, 3] Let $X \neq \emptyset$. An intuitionistic fuzzy set A in X;*

$$A = \{\langle x, \mu_A(x), \nu_A(x)\rangle \mid x \in X\},$$

$$\mu_A(x), \nu_A(x), \pi_A(x) : X \to [0,1]$$

defined membership, nonmembership and hesitation degree of the element $x \in X$ respectively.

$$\mu_A(x) + \nu_A(x) + \pi_A(x) = 1.$$

Intuitionistic fuzzy value (IFV) defined by Xu [17]. Intuitionistic fuzzy value (IFV) is shown as follows: $\tilde{a} = (\mu_{\tilde{a}}, \nu_{\tilde{a}}, \pi_{\tilde{a}})$, where $\mu_{\tilde{a}}, \nu_{\tilde{a}}, \pi_{\tilde{a}} \in [0,1]$.
For each IFS $\tilde{A}$;

$$\pi_{\tilde{A}} = 1 - \mu_{\tilde{A}} - \nu_{\tilde{A}}. \tag{2.1}$$

In this paper; we will not write the third part so we'll show $\tilde{a} = (\mu_{\tilde{a}}, \nu_{\tilde{a}})$ shape instead of $\tilde{a} = (\mu_{\tilde{a}}, \nu_{\tilde{a}}, \pi_{\tilde{a}})$. The degree of hesitation can be obtained by equation (2.1).
For IFVs $\tilde{a} = (\mu_{\tilde{a}}, \nu_{\tilde{a}})$ and $\tilde{b} = (\mu_{\tilde{b}}, \nu_{\tilde{b}})$ the following operations have been carried out [17, 18]:

$$(1) \quad \tilde{a} \oplus \tilde{b} = \left(\mu_{\tilde{a}} + \mu_{\tilde{b}} - \mu_{\tilde{a}}\mu_{\tilde{b}}, \nu_{\tilde{a}}\nu_{\tilde{b}}\right) \tag{2.2}$$

$$(2) \quad \tilde{a} \otimes \tilde{b} = \left(\mu_{\tilde{a}}\mu_{\tilde{b}}, \nu_{\tilde{a}} + \nu_{\tilde{b}} - \nu_{\tilde{a}}\nu_{\tilde{b}}\right) \tag{2.3}$$

$$(3) \quad \oplus_{j=1}^{m} \tilde{a}_j = \left(1 - \prod_{j=1}^{m} (1 - \mu_j), \prod_{j=1}^{m} \nu_j\right) \tag{2.4}$$

$$(4) \quad \otimes_{j=1}^{m} \tilde{a}_j = \left(\prod_{j=1}^{m} \mu_j, \prod_{j=1}^{m} (1 - \nu_j)\right) \tag{2.5}$$

Many researchers have suggested approaches for comparing the IFVs [18, 19]. The following method will be used in this paper. This method was proposed by Szmidt and Kacprzyk, which leads to more consistent results than other methods [19]. This function is used to rank IFVs:

$$\rho(\alpha) = 0.5(1 + \pi_\alpha)(1 - \mu_\alpha)) \tag{2.6}$$

As the $\rho(\alpha)$ value decreases, the preferred value α increases.

2.1. The intuitionistic fuzzy PROMETHEE

When a MCDM problem is encountered, the decision maker is expected to choose the best alternative among the alternatives according to certain criteria. However, the significance of all the criteria can vary. In such cases, the weighting of the criteria shall be taken into consideration. The benefit of the PROMETHEE method is to assess considering the weight of the criterion. The criteria's weights indicate how important they are. Considering both intuitionistic fuzzy sets and weights of criteria at the same time, more consistent and rational results will be obtained. Therefore; using intuitionistic fuzzy PROMETHEE method will provide advantageous results. The criteria's weights could be depicted as IFVs: $\tilde{w}_j$ where $\mu_{\tilde{w}_j} \in [0,1], \nu_{\tilde{w}_j} \in [0,1], \mu_{\tilde{w}_j} + \nu_{\tilde{w}_j} \leq 1, j = 1,2,...,m$. According to the weights, $\mu_{\tilde{\omega}_j}$ and $\nu_{\tilde{\omega}_j}$ demonstrate the membership and non-membership degrees of the alternative x_i respectively. Indeed; the concept of weight represents the importance of that criteria. The weights are expressed as IFV in the intuitionistic fuzzy PROMETHEE. It is of great importance for decision-makers to determine the specific importance level for each criterion. In this paper, controlled sets are used to express the importance of the criteria in the intuitionistic fuzzy PROMETHEE method in the form of intuitionistic fuzzy values. Some methods can help decision makers in determining intuitionistic fuzzy weights [20–25]. The basic definitions for controlled sets are as follows:

Definition 2.2. *[22] Let E be an universe, α is a function from E to I then E is called $\alpha-$ set.*

Definition 2.3. *[22] Let E be an $\alpha-$ set. The set E is called $\alpha-$ controlled set if*

$$\forall x \in E, \exists y \in E \ni 1-\alpha(x) = \alpha(y).$$

The family of $\alpha-$ controlled set on an universe E is represented by $E \in CS(\alpha)$.

Definition 2.4. *[22] Let $E \in CS(\alpha)$ and $a \in E$. The following set is called control set of a,*

$$\bar{a} = \{b \in E | 1-\mu(a) = \mu(b)\}$$

Definition 2.5. *[21] Let E be an $\alpha-$ set. We define the following mapping on E so that*

$$\alpha^*(x) = \begin{cases} 1-\alpha(x), & x \in E_\alpha \\ sup_y \alpha(y), & y \in E \ni \alpha(x) < 1-\alpha(y) \\ 0, & otherwise. \end{cases} \tag{2.7}$$

where $E_\alpha = \cup_{a\in E} \bar{a}$.

Definition 2.6. *[21] Let E be $\alpha-$ set. Then the set $A = \{\langle x, \alpha(x), \alpha^*(x)\rangle | x \in E\}$ is called $(\alpha, \alpha^*)-$ controlled set.*

In this research, V shape criterion type has been used [26]:

$$P(d) = \begin{cases} 0, & d \le q \\ \frac{d-q}{p-q}, & q < d \le p \\ 1, & d > p \end{cases} \tag{2.8}$$

Parameter thresholds q and p are indicated as indifference and strict preference, respectively. Decision makers are free to change these thresholds according to the desired situation. Evaluate the alternatives $x_i (i = 1,2,...,n)$ with respect to the criteria $c_j (j = 1,2,...,m)$ and determine the deviations based on pairwise comparisons:

$$d_j(x,y) = c_j(x) - c_j(y) \tag{2.9}$$

where $d_j(x,y)$ shows the distinction between the alternatives' the assessments x and y on the criterion c_j.

The decision maker first determines the assessment values of each alternative relative to different criteria and creates the paired preferences via the preference function, , which is also called as the general criterion in the classical PROMETHEE. There are six different types of generalized criteria. The preferences are limited in $[0,1]$ by the generalized criterion. If these preference functions are handled with a fuzzy set, membership functions and preference values are represented by $P_j(x,y)$. Besides preference values can be taken directly as fuzzy numbers as they are limited to $[0,1]$. However, with the help of the fuzzy set, only the preferred density defined by the membership function can be expressed. It is more convenient for the decision maker to use intuitionistic fuzzy set because it addresses all aspects of the criteria. Also with the help of intuitionistic fuzzy set, PROMETHEE method is more advantageous. Since; the intuitionistic fuzzy set could express not only the intensity of preferred but also the degrees of non-preferred and uncertain.

Definition 2.7. *[27] An intuitionistic fuzzy preference relation R on the set $X = x_1, x_2, ..., x_n$ is represented by a matrix $R = (r_{ik})_{n\times n}$, where $r_{ik} = \langle (x_i, x_k), \mu(x_i, x_k), \nu(x_i, x_k)\rangle$ for all $i, k = 1,2,...,n$. For convenience, we let $r_{ik} = (\mu_{ik}, \nu_{ik})$ where μ_{ik} denotes the degree to which the object x_i is preferred to the object x_k, ν_{ik} indicates the degree to which the object x_i is not preferred to the object x_k, and $\pi(x_i, x_k) = 1-\mu(x_i, x_k) - \nu(x_i, x_k)$ is interpreted as an indeterminacy degree or a hesitancy degree, with the condition:*

$$\mu_{ik},\ \nu_{ik} \in [0,1],\ \mu_{ik}+\nu_{ik} \le 1,\ \mu_{ik} = \nu_{ki},\ \mu_{ki} = \nu_{ik},\ \mu_{ii} = \nu_{ii} = 0.5,\ \pi_{ik} = 1-\mu_{ik}-\nu_{ik},\ for\ all\ i,k = 1,2,...,n. \tag{2.10}$$

2.2. Algorithm of intuitionistic fuzzy PROMETHEE

The preferences μ_{ik} between the alternatives x_i and x_k according to the criterion c_j could be calculated by Equations (2.9) and (2.8), and then the preference matrix according to the criterion c_j is obtained as follows [28]:

$$U^{(j)} = (\mu_{ik}^{(j)})_{n\times n} = \begin{bmatrix} - & \mu_{12}^{(j)} & \cdots & \mu_{1n}^{(j)} \\ \mu_{21}^{(j)} & - & \cdots & \mu_{2n}^{(j)} \\ \vdots & \vdots & - & \vdots \\ \mu_{n1}^{(j)} & \mu_{n2}^{(j)} & \cdots & - \end{bmatrix} \tag{2.11}$$

Using the equations $\nu_{ki} = \mu_{ik}$ and $\nu_{ik} = \mu_{ki}$, the nonmembership degree of an IFV could be obtained. Matrix of the intuitionistic fuzzy preference relation is as follows:

$$R^{(j)} = (r_{ik}^{(j)})_{n\times n} = \begin{bmatrix} - & (\mu_{12}^{(j)}, \nu_{12}^{(j)}) & \cdots & (\mu_{1n}^{(j)}, \nu_{1n}^{(j)}) \\ (\mu_{21}^{(j)}, \nu_{21}^{(j)}) & - & \cdots & (\mu_{2n}^{(j)}, \nu_{2n}^{(j)}) \\ \vdots & \vdots & - & \vdots \\ (\mu_{n1}^{(j)}, \nu_{n1}^{(j)}) & (\mu_{n2}^{(j)}, \nu_{n2}^{(j)}) & \cdots & - \end{bmatrix} \tag{2.12}$$

Then; considering the $c_j (j = 1,2,\dots,m)$ criteria, we must establish the general preference index for each alternative. We can get what we want by using weighted aggregation operators. There are a number of aggregation operators for intuitionistic fuzzy sets, such as the IFWA, IFWG, IFOWA, IFOWG, IFHA and IFHG operators [17, 18]. We will use the IFWA operator in this paper. The all intuitionistic fuzzy preference index of the alternative x_i to x_k on all criteria can be derived as:

$$r(x_i,x_k) = r_{ik} = \bigoplus_{j=1}^{m}\left(\tilde{w}_j \bigotimes r_{ik}^{(j)}\right) \tag{2.13}$$

where $r(x_i,x_k) = r_{ik}$ shows the degree to which the alternative x_i is preferred to the alternative x_k all criteria. Also, r_{ik} is an IFV. $\tilde{w}_j = (\mu_{\tilde{w}j}, \nu_{\tilde{w}j})$, then according to Equation (2.2), (2.3):

$$\tilde{w}_j \bigotimes r_{ik}^{(j)} = \left(\mu_{ik}^{(j)}\mu_{\tilde{w}j}, \nu_{ik}^{(j)} + \nu_{\tilde{w}j} - \nu_{ik}^{(j)}\nu_{\tilde{w}j}\right) \tag{2.14}$$

If Equations (2.4), (2.13) and (2.14) are combined;

$$\begin{aligned} r(x_i,x_k) &= \bigoplus_{j=1}^{m}\left(\tilde{w}_j \bigotimes r_{ik}^{(j)}\right) \\ &= \left(1-\prod_{j=1}^{m}(1-\mu_{ik}^{(j)}\mu_{\tilde{w}j}), \prod_{j=1}^{m}(\nu_{ik}^{(j)} + \nu_{\tilde{w}j} - \nu_{ik}^{(j)}\nu_{\tilde{w}j})\right) \end{aligned}$$

Overall intuitionistic fuzzy preference relationship is established as follows:

$$R = (r_{ik})_{n\times n} = \begin{bmatrix} - & (\mu_{12},\nu_{12}) & \dots & (\mu_{1n},\nu_{1n}) \\ (\mu_{21},\nu_{21}) & - & \dots & (\mu_{2n},\nu_{2n}) \\ \vdots & \vdots & - & \vdots \\ (\mu_{n1},\nu_{n1}) & (\mu_{n2},\nu_{n2}) & \dots & - \end{bmatrix} \tag{2.15}$$

Every alternative is compared to option $(n-1)$. As a result of intuitionistic fuzzy positive and negative outranking flow can be achieved as follows:

(1) The intuitionistic fuzzy positive outranking flow:

$$\tilde{\varphi}^{+}(x_i) = \frac{1}{n-1}\bigoplus_{k=1,k\neq i}^{n} r(x_i,x_k) = \frac{1}{n-1}\bigoplus_{k=1,k\neq i}^{n} r_{ik} \tag{2.16}$$

(2) The intuitionistic fuzzy negative outranking flow:

$$\tilde{\varphi}^{-}(x_i) = \frac{1}{n-1}\bigoplus_{k=1,k\neq i}^{n} r(x_k,x_i) = \frac{1}{n-1}\bigoplus_{k=1,k\neq i}^{n} r_{ki} \tag{2.17}$$

The relationship between $\tilde{\varphi}^{+}(x_i)$ and $\tilde{\varphi}^{-}(x_i)$ can be explained with the help of Equation (2.6). The intuitionistic fuzzy net cannot be obtained by directly subtraction the outranking flow. The difference between the intuitionistic fuzzy positive and negative outranking flow can be calculated using the function defined by Szmidt and Kacprzyk.

$$\rho(\varphi(x_i)) = \rho(\tilde{\varphi}^{+}(x_i)) - \rho(\tilde{\varphi}^{-}(x_i)) \tag{2.18}$$

Three different ranking for $\tilde{\varphi}^{+}(x_i)$ and $\tilde{\varphi}^{-}(x_i)$ can be achieved [28]:

(1) Partial ranking: x_i outranks x_k if $\tilde{\varphi}^{+}(x_i) \geq \tilde{\varphi}^{+}(x_k)$ and $\tilde{\varphi}^{-}(x_i) \leq \tilde{\varphi}^{-}(x_k)$;
(2) Equality: $\tilde{\varphi}^{+}(x_i) = \tilde{\varphi}^{+}(x_k)$ and $\tilde{\varphi}^{-}(x_i) = \tilde{\varphi}^{-}(x_k)$ hold at the same time involves indifference between two options.
(3) Incomparability: This takes if $\tilde{\varphi}^{+}(x_i) > \tilde{\varphi}^{+}(x_k)$ and $\tilde{\varphi}^{-}(x_i) > \tilde{\varphi}^{-}(x_k)$ or $\tilde{\varphi}^{+}(x_i) < \tilde{\varphi}^{+}(x_k)$ and $\tilde{\varphi}^{-}(x_i) < \tilde{\varphi}^{+}(x_k)$.

If incomparability happens, intuitionistic fuzzy net outranking flow can be obtained by the help of Equation (2.18).

2.3. General algorithm

In this section, the steps of the algorithm that can be applied in every field are explained in detail. The algorithm we need to use when we want the best, the most accurate and the smartest solution, where the importance of the criteria are different, multiple criteria and alternatives are together, is given below by explaining all the steps. This algorithm is an algorithm that has an application area in all areas of multi-criteria decision making and will give us the most accurate result in any situation. At the same time, by using this algorithm in our study, we have shown its application and usefulness in the field of education. When we want to choose among alternatives with more than one criteria, not only in the field of education, this algorithm is used to make the best choice and the most accurate ranking among the alternatives. The general algorithm for the intuitionistic fuzzy PROMETHEE have been created as follows [16]:
Step 1: Alternatives $X = x_1, x_2, \dots, x_n$ to be evaluated are determined. And criteria $C = c_1, c_2, \dots, c_m$ that are important to evaluate alternatives are determined.
Step 2: The importance degree of the criteria is determined $\tilde{w}_j (j = 1,2,\dots,m)$ where $\mu_{\tilde{w}_j} + \nu_{\tilde{w}_j} \leq 1, \mu_{\tilde{w}_j} \in [0,1], \quad \nu_{\tilde{w}_j} \in [0,1]$.
Step 3: The parameters q as an indifference threshold and p as a strict preference threshold are determined. Deviations $d_j(x,y)$ using

Equation (2.9), the preferences $\mu_{ik}^{(j)}$ for the alternative x_i against alternative x_k with respect to the criterion c_j by using the V-shape with indifference criterion are calculated. Then, the preference matrix $U^{(j)}(j=1,2,\ldots,m)$ are created.
Step 4: The intuitionistic fuzzy preference relation $R^{(j)}=(r_{ik}^{(j)})_{n\times n}$ is created.
Step 5: The overall intuitionistic fuzzy preference relation $R=(r_{ik})_{n\times n}$ using Equation (2.13) is obtained.
Step 6: The IF positive outranking flow $\tilde{\varphi}^{+}(x_i)$ and the intuitionistic fuzzy negative outranking flow $\tilde{\varphi}^{-}(x_i)$ by using Equation (2.15) and (2.15) are obtained.
Step 7: The relationship between $\tilde{\varphi}^{+}(x_i)$ and $\tilde{\varphi}^{-}(x_i)$ is determined. According to this relationship is made a ranking.

3. A Current Application of Intuitionistic Fuzzy PROMETHEE

The aim of this application; it is to create an algorithm that takes into account not only exam scores but also other factors affecting success of students. According to this algorithm, the factors affecting student achievement and exam scores have been evaluated together and the evaluation scores of the students have been calculated with the intuitionistic fuzzy PROMETHEE method. Each school points has been calculated depending on student examination score. While investigating the factors affecting the success, interviews were made with experts in the field and the resources related to this field were examined. As a result of these researches, interviews were made with school guidance teacher. In this process, the results of the exams applied to the students at school and private courses were examined. In addition, the method of individual interview was used to evaluate other factors affecting exam success. During the individual interview, the guidance teachers gave points to the students according to some criteria. Some of the interview questions asked are sub criteria, some are basic criteria. The weights of the criteria were determined in line with expert opinion. The basic lessons in the exam for students are Mathematics, Science, Social and Turkish. Other factors that affect success are; the economic situation of the family, the health status of the student, the study order of the student, the study discipline, the student's self-confidence, his relationship with his friends, his relationship with his family, and his private working environment. The guidance teachers were asked to give points in the range of 0-10 to these criteria for each student. The success of the students for each course was determined according to the points of the exam that the students took regularly every month. Afterwards, an evaluation score was calculated for each student by considering all the lessons and factors at the same time. As a result of this evaluation score, the success graph of the students was obtained and the success ranking was made. The personal information form have been prepared for the students and have been filled out with the support of guidance teacher for each student.
The set of criteria is as follows:

- K_1 :Turkish
- K_2 :Social
- K_3 :Mathematics
- K_4 :Science
- K_5 :Family factor- relationship to family
- K_6 :Study factor
  - Study system of the student
  - Study discipline
  - Private study environment
- K_7 :Self-confidence factor
  - Student's self-confidence
  - Relationship with friends, sociability
- K_8 :Demographic factors
  - Economic status of the family
- K_9 :Other factors
  - Health status of the student
  - Socio Cultural factors

Step 1: $\{A_1,A_2,A_3,A_4,A_5,A_6,A_7,A_8,A_9,A_{10},A_{11},A_{12},A_{13},A_{14},A_{15}\}$ be set of alternatives (students).
$\{K_1,K_2,K_3,K_4,K_5,K_6,K_7,K_8,K_9\}$ be set of criteria. Table 1 was created according to the net numbers of the students according to the results of the trial exams they took and the personal information forms filled in by the guidance teacher:

	C_1	C_2	C_3	C_4	C_5	C_6	C_7	C_8	C_9
A_1	23.5	12	27	14	9	10	5.5	8	9
A_2	27.5	12	10.25	0	9	9.66	8.5	7	4
A_3	20.75	6.5	20.75	0	9	9	7	8	7
A_4	17.5	12.5	14.25	0	3	5.33	5.5	7	6
A_5	19.5	8.5	10.75	8.25	9	8.33	7.5	8.	9
A_6	19	0	21	5.25	4.	7.33	9	7	9
A_7	26.5	0	11	0	3	7.66	6.5	5	9
A_8	20	3.75	9.5	2	6	7	4	8	3
A_9	17	13	1	0	9	8.33	7.5	10	10
A_{10}	21	8.5	0	0	3	7	7.5	5	9
A_{11}	15	7	1.75	0	9	9.33	8.5	7	10
A_{12}	9.25	5.25	0	0	9	9.33	8	8	10
A_{13}	8.25	5.25	0	0	3.5	8	5.5	10	10
A_{14}	7	0.5	0	1	6	9	6	9	6
A_{15}	29	0.5	24.75	2	10	8.66	8.5	8	8

Table 1: Exam results of students

Step 2: The weights of the criteria were calculated with the help of controlled sets. Weights of criteria $\mu_{\tilde{w}_j}, \nu_{\tilde{w}_j}$ determined with 3.1 in Table 2 according to the following situations:

Weights	$\mu_{\tilde{w}_j}$	$\nu_{\tilde{w}_j}$
$\tilde{w}_1$	1	0
$\tilde{w}_2$	0.5015	0.3
$\tilde{w}_3$	1	0
$\tilde{w}_4$	0.5015	0.3
$\tilde{w}_5$	0.3	0.5015
$\tilde{w}_6$	0.1126	0.5015
$\tilde{w}_7$	0.15	0.5015
$\tilde{w}_8$	0.1126	0.5015
$\tilde{w}_9$	0.075	0.5015

Table 2: Weights of criteria

- The contribution rates of the courses in the exam are as follows: (% 80)
  - K_1 :Turkish: %33,33
  - K_2 :Social: %16,67
  - K_3 :Mathematics: %33,33
  - K_4 :Science: %16,67
- Contribution rates of other factors that affect success are as follows:(%20) (These contribution rates have been determined with the opinions of experts in their field.)
  - K_5 :Family factor- relationship to family: %40
  - K_6 :Study factor: %15
  - K_7 :Self-confidence factor: %20
  - K_8 :Demographic factors: %15
  - K_9 :Other factors: %10

Step 3: Deviation $d_j(x_i,x_k)$ for each criterion; V-shape was calculated using generalization criteria. For the V-shape generalization criterion, the q indifference threshold is 40 for the K_1 and K_3 criteria, 20 for the K_2 and K_4 criteria, 10 for the criteria K_5, K_6, K_7, K_8, K_9, and the absolute preference threshold is p, and is 0 for all criteria. This is the threshold set by the researcher or decision maker. Threshold calculations for each study may vary.

$$d_j(x_i,x_k) = c_j(x_i) - c_j(x_k)$$

$$\mu_{ik}^{(j)} = \begin{cases} 0, & d_j(x_i,x_k) \leq q \\ \frac{d_j(x_i,x_k)-q}{p-q}, & q < d_j(x_i,x_k) \leq p \\ 1, & d_j(x_i,x_k) > p \end{cases}$$

Step 4: The intuitionistic fuzzy preference matrices $R^{(j)} = (r_{ik}^{(j)})_{n\times n}$ are determined by benefitted from equation $\nu_{ik} = \mu_{ki}$ and $\nu_{ki} = \mu_{ik}$.
Step 5: Overall intuitionistic fuzzy preference relation $R = (r_{ik})_{n\times n}$ is created using Equation (2.15).

Step 6: The intuitionistic fuzzy positive outranking flows and intuitionistic fuzzy negative outranking flows are calculated using Equation (2.16), (2.17).
Step 7: The values are ranked in Table 3:

$\rho(\tilde{\varphi}^{+}(x_1)) =$	0.895408163	$\rho(\tilde{\varphi}^{-}(x_1)) =$	0.939320674
$\rho(\tilde{\varphi}^{+}(x_2)) =$	0.895408163	$\rho(\tilde{\varphi}^{-}(x_2)) =$	0.898759017
$\rho(\tilde{\varphi}^{+}(x_3)) =$	0.89541498	$\rho(\tilde{\varphi}^{-}(x_3)) =$	0.900289988
$\rho(\tilde{\varphi}^{+}(x_4)) =$	0.895547851	$\rho(\tilde{\varphi}^{-}(x_4)) =$	0.895508379
$\rho(\tilde{\varphi}^{+}(x_5)) =$	0.895424014	$\rho(\tilde{\varphi}^{-}(x_5)) =$	0.898117312
$\rho(\tilde{\varphi}^{+}(x_6)) =$	0.895418008	$\rho(\tilde{\varphi}^{-}(x_6)) =$	0.895566097
$\rho(\tilde{\varphi}^{+}(x_7)) =$	0.895700692	$\rho(\tilde{\varphi}^{-}(x_7)) =$	0.895439819
$\rho(\tilde{\varphi}^{+}(x_8)) =$	0.896767967	$\rho(\tilde{\varphi}^{-}(x_8)) =$	0.895470838
$\rho(\tilde{\varphi}^{+}(x_9)) =$	0.895709231	$\rho(\tilde{\varphi}^{-}(x_9)) =$	0.895446384
$\rho(\tilde{\varphi}^{+}(x_{10})) =$	0.899104761	$\rho(\tilde{\varphi}^{-}(x_{10})) =$	0.895412326
$\rho(\tilde{\varphi}^{+}(x_{11})) =$	0.898075022	$\rho(\tilde{\varphi}^{-}(x_{11})) =$	0.895419782
$\rho(\tilde{\varphi}^{+}(x_{12})) =$	0.903031948	$\rho(\tilde{\varphi}^{-}(x_{12})) =$	0.89540828
$\rho(\tilde{\varphi}^{+}(x_{13})) =$	0.921333364	$\rho(\tilde{\varphi}^{-}(x_{13})) =$	0.895408168
$\rho(\tilde{\varphi}^{+}(x_{14})) =$	0.927293663	$\rho(\tilde{\varphi}^{-}(x_{14})) =$	0.895408162
$\rho(\tilde{\varphi}^{+}(x_{15})) =$	0.895408181	$\rho(\tilde{\varphi}^{-}(x_{15})) =$	0.89873238

Table 3: The intuitionistic fuzzy positive and negative outranking flows

The graphs of the alternatives in the positive, negative and net outranking flow are shown in Graph 1, Graph 2, Graph 3:

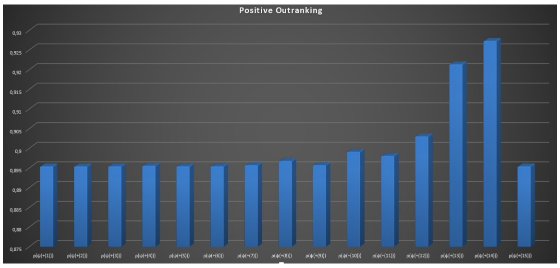

Figure 3.1: Graph of positive outranking flow

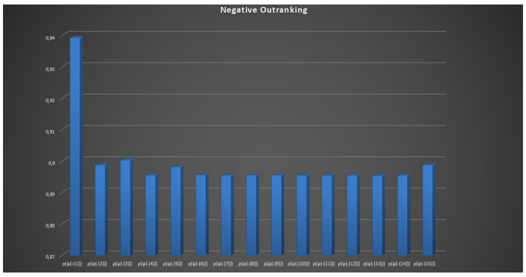

Figure 3.2: Graph of negative outranking flow

When the graphs are investigated, the ranking obtained between alternatives is as follows:

- Positive ranking:
  A_1, A_2, A_{15}, A_3, A_6, A_5, A_4, A_7, A_9, A_8, A_{11}, A_{10}, A_{12}, A_{13}, A_{14}
- Negative ranking:
  A_{14}, A_{13}, A_{12}, A_{10}, A_{11}, A_7, A_9, A_8, A_4, A_6, A_5, A_{15}, A_2, A_3, A_1

- According to these rankings, the students whose order is changed positively and negatively are $A_{15}, A_2, A_3, A_6, A_5, A_7, A_8$ students. When the scores of these students according to the criteria are examined, the following judgments can be reached:
  - When all the criteria of these students were examined one by one, it was seen that the scores of some criteria were very low. In a positive sense, although we see a successful picture in general, the low scores of some criteria have attracted these students to the front ranks negatively.
  - According to the data obtained in this study; It has been observed that ranking only positively while making the decision process can lead to wrong results.
  - The method that allows us to make the right decision is the method that takes into account both positive and negative ranking at the same time.

In order to make a general ranking, a net outranking is needed since positive outranking and negative outranking are not the same. The net outranking flow in Table 4 is calculated with the help of the equation (2.18) and its graph is as follows:

$\rho(\tilde{\varphi}(x_1)) =$	-0.043912511
$\rho(\tilde{\varphi}(x_2)) =$	-0.003350854
$\rho(\tilde{\varphi}(x_3)) =$	-0.004875008
$\rho(\tilde{\varphi}(x_4)) =$	0.0000394712
$\rho(\tilde{\varphi}(x_5)) =$	-0.002693298
$\rho(\tilde{\varphi}(x_6)) =$	-0.000148089
$\rho(\tilde{\varphi}(x_7)) =$	0.000260873
$\rho(\tilde{\varphi}(x_8)) =$	0.001297128
$\rho(\tilde{\varphi}(x_9)) =$	0.000262847
$\rho(\tilde{\varphi}(x_{10})) =$	0.003692435
$\rho(\tilde{\varphi}(x_{11})) =$	0.00265524
$\rho(\tilde{\varphi}(x_{12})) =$	0.007623668
$\rho(\tilde{\varphi}(x_{13})) =$	0.025925196
$\rho(\tilde{\varphi}(x_{14})) =$	0.031885501
$\rho(\tilde{\varphi}(x_{15})) =$	-0.003324199

Table 4: Values of net outranking flow

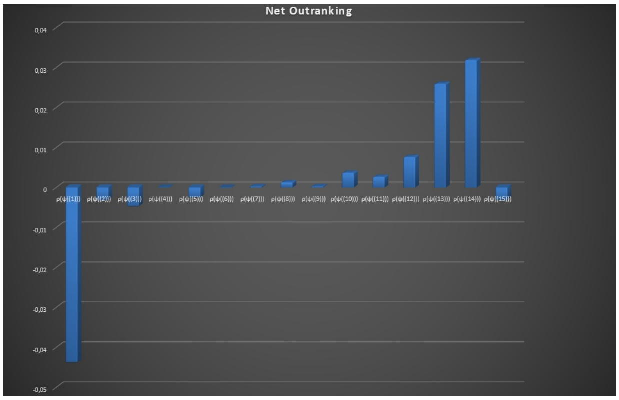

Figure 3.3: Graph of net outranking flow

As a result, the ranking from the best alternative to the worst alternative is as follows:

$$A_1,\ A_3,\ A_2,\ A_{15},\ A_5,\ A_6,\ A_4,\ A_7,\ A_9,\ A_8,\ A_{11},\ A_{10},\ A_{12},\ A_{13},\ A_{14}$$

With net ranking, only the difference between positive and negative rankings is clearly visible. When choosing, ranking and evaluating, not only positive sense but also positive and negative sense alternatives are compared and a net ranking is obtained, the most accurate result is achieved. The ranking of the students was made, starting with the most successful according to the net ranking.

4. Conclusion

In this study, it is aimed to find the most rational solution to evaluate the youth of a country in education. Thanks to the method have been created an evaluation system that takes into account not only the exam scores of the students, but also other factors that affect the success of the students. In our previous studies, we developed and analyzed different methods that calculate students' success based on exam scores. One of the deficiencies we see while evaluating in education is that evaluation is made only according to the exam results. Based on these reasons, an intuitionistic fuzzy logic-based evaluation system was created for the first time in education with this method, which will add an innovation to the education system. More objective and rational results will be obtained thanks to this new method that we present to the evaluation system in education. The general algorithm is detailed to show the applicability of this new method in all areas. This algorithm can be applied in any situation where it is desired to be preferred among multi-criteria alternatives. New research can be done by following the steps in the algorithm given in this study. This study will lead to innovations not only in the field of education but also in many applications. Thanks to this system that we will create that will have the ability to apply not only in the field of education, but also in many fields, many factors will be evaluated at the same time and different degrees of importance will be determined for each factor.

Intuitionistic fuzzy PROMETHEE is a method that attracts our attention thanks to its benefits such as allowing the researcher to observe the positive and negative rankings simultaneously, expressing the degree of hesitation, changing the significance weights of the criteria, and using different methods when identifying the significance levels for each criterion, putting the degree of hesitation of significance weights into action, and enabling decision-makers, who are given the opportunity to use different criteria types and different criteria types for alternatives and criteria, to establish a unique system; moreover, the method also provides us numerous advantages while using it in our application area. Because of these advantages, the intuitionistic fuzzy PROMETHEE method continues to be applied in many decision-making processes.

In the last part of our study, our method is explained, visualized and interpreted with graphics. Factors affecting student achievement and exam scores were evaluated together with the intuitive fuzzy PROMETHEE method and the evaluation scores of the students were calculated with the determined algorithm. The accuracy, suitability and rationality of the developed method has been proven with the given application. This method will provide rational, useful, profitable results in many areas and will facilitate the work of many decision makers. Thanks to our work, we shed light on many areas where our method will be applied.

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Author's contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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