Gazi University Journal of Science
GU J Sci
29(2):459-466 (2016)

Total Graph and Complemented Graph of a Rough Semiring

B.PRABA[1], A. MANIMARAN[2,♠], V. M. CHANDRASEKARAN[2]

[1]*SSN College of Engineering, Kalavakkam, Chennai – 603110, India.*
[2]*Department of Mathematics, School of Advanced Science, VIT University, Vellore – 632014, India.*

Received:11/01/2016 *Accepted: 14/03/2016*

ABSTRACT

In this paper, we define the total graph and the complemented graph of a rough semiring (T, Δ, ∇). We prove that the existence theorem of the total graph and the complemented graph on (T, Δ, ∇) for $X \subseteq U$ where U is the finite universal set on the set of all rough sets for the given information system $I = (U, A)$ together with the operations $Praba\,\Delta$ and $Praba\,\nabla$. We illustrate these concepts through examples.

Keywords: *Semiring, Zero divisor, Zero divisor graph, Rough Semiring*

1. INTRODUCTION

The concept of semiring was first introduced by H. S. Vandiver in 1934. Z. Pawlak [11] introduced the concept of rough set theory in 1982 to process incomplete information in the information system and it is defined as a pair of sets called lower and upper approximation. Praba and Mohan [12] discussed the concept of rough lattice. In this paper the authors considered an information system $I = (U, A)$. A partial ordering relation was defined on $T = \{RS(X) \mid X \subseteq U\}$. The least upper bound and greatest lower bound were established using the operations $Praba\,\Delta$ and $Praba\,\nabla$. Praba et al. [13] discussed a commutative regular monoid on rough sets under the operation $Praba\,\Delta$ in 2013. In this paper the authors dealt the rough ideals on (T, Δ). Manimaran et al. [9] studied the notion of a regular rough ∇ monoid of idempotents under $Praba\,\nabla$ in 2014. Praba et al. [14] dealt semiring on the set of all rough sets also the authors discussed rough ideals on semirings in 2014. Zadeh [17] introduced the concept of fuzzy sets in his paper. Many authors contributed towards algebraic graph theory and one such work is in the direction of graph based on the characterization of rings and semirings. The concept of zero divisor graph of a ring was first introduced by beck [5] in 1988. In his paper, the author discussed the coloring of a commutative ring. In his work, all elements of the ring were assumed as vertices of the graph. D. D. Anderson et al. [4] discussed about coloring in his paper.

♠Corresponding author, e-mail:*marans2011@gmail.com*

D. F. Anderson et al. [2] dealt the zero divisor graph of a commutative ring in 1999. In his paper, the author considered only nonzero zero divisors as vertices of the zero divisor graph. S. Ebrahimi [7] studied the concepts of the zero divisor graph with respect to ideals of a commutative semiring in 2008 and in 2009 the same author [8] discussed an ideal based zero divisor graph of a commutative semiring. David et al. [6] discussed the zero divisor graphs of rings and semirings recently in 2012. The concepts total graph was introduced and discussed by D. F. Anderson and Badawi [3] in 2008. S. Visweswaran [16] discussed some properties of the complement of the zero divisor graph of a commutative semiring. In this paper, the author proved that $\overline{\Gamma(R)}$ is connected and the radius is equal to 2, girth is equal to 3 and the author determined the center of $\overline{\Gamma(R)}$. Akbari et al. [1] discussed about the edge chromatic number of the zero divisor graph of a commutative ring and the author proved that the edge chromatic number is equal to the maximum degree of $\Gamma(R)$ unless $\Gamma(R)$ is a complete graph of odd order. Praba et al. [15] dealt the concepts of the zero divisor graph of a rough semiring in 2015.

In this paper, we discuss the total graph and complemented graph of a rough semiring (T,Δ,∇) for the given information system $I=(U,A)$ where the information system is defined by using the universal set U and a nonempty set of fuzzy attributes A. The paper is organized as follows.

In section 2, we give the necessary definitions related to rough set theory, semiring theory and algebraic graph theory.

In section 3, we deal the total and complemented graph of a rough semiring (T,Δ,∇) and we illustrate these concepts through numerical examples.

Section 4 gives the conclusion.

2. PRELIMINARIES

In this section we present some preliminaries in rough semiring and zero divisor graph of a semiring.

2.1. Rough Semiring

Let $I=(U,A)$ be an information system, where U is a non empty set of finite objects, called the universe and A is a non empty finite fuzzy set of attributes. For $X\subseteq U$, let $RS(X)=(\underline{P}(X),\overline{P}(X))$ be the rough set of X and let $T=\{RS(X)\,|\,X\subseteq U\}$ be the set of all rough sets on U.

Theorem 2.1. [14] (T,Δ,∇) is a rough semiring.

Example 2.1. [14] Let us consider an information system $I=(U,A)$ where $U=\{x_1,x_2,x_3,x_4\}$ and $A=\{a_1,a_2,a_3,a_4\}$ where each $a_i(i=1\ to\ 4)$ is a fuzzy set of attributes whose membership values are shown in Table -1.

Table -1

A/U	a_1	a_2	a_3	a_4
x_1	0.2	0.3	1	0
x_2	0.8	0.4	0.1	0.9
x_3	0.2	0.3	1	0
x_4	0.8	0.4	0.1	0.9

Let $X=\{x_1,x_2,x_3,x_4\}\subseteq U$ then the equivalence classes induced by *IND(P)* are given below

$$X_1=[x_1]_p=\{x_1,x_3\}-----------------(1)$$

$$X_2=[x_2]_p=\{x_2,x_4\}-----------------(2)$$

and let $T=\{RS(X)\,|\,X\subseteq U\}$ be the set of all rough sets such that,

$$T=\left\{RS(\phi),RS(X_1),RS(X_2),RS(\{x_1\}),RS(\{x_2\}),RS(X_1\bigcup\{x_2\}),RS(\{x_1\}\bigcup X_2),RS(\{x_1\}\cup\{x_2\}),RS(U)\right\}-----(3)$$

(a) Cayley's table to prove (T,Δ) is commutative rough monoid.

Table 2:

Δ	$RS(\phi)$	$RS(U)$	$RS(X_1)$	$RS(X_2)$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(X_1 \cup \{x_2\})$	$RS(\{x_1\} \cup X_2)$	$RS(\{x_1\} \cup \{x_2\})$
$RS(\phi)$	$RS(\phi)$	$RS(U)$	$RS(X_1)$	$RS(X_2)$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(X_1 \cup \{x_2\})$	$RS(\{x_1\} \cup X_2)$	$RS(\{x_1\} \cup \{x_2\})$
$RS(U)$	$RS(U)$	$RS(U)$	$RS(U)$	$RS(U)$	$RS(U)$	$RS(U)$	$RS(U)$	$RS(U)$	$RS(U)$
$RS(X_1)$	$RS(X_1)$	$RS(U)$	$RS(X_1)$	$RS(U)$	$RS(X_1)$	$RS(X_1 \cup \{x_2\})$	$RS(X_1 \cup \{x_2\})$	$RS(U)$	$RS(X_1 \cup \{x_2\})$
$RS(X_2)$	$RS(X_2)$	$RS(U)$	$RS(U)$	$RS(X_2)$	$RS(\{x_1\} \cup X_2)$	$RS(X_2)$	$RS(U)$	$RS(\{x_1\} \cup X_2)$	$RS(\{x_1\} \cup X_2)$
$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(U)$	$RS(X_1)$	$RS(\{x_1\} \cup X_2)$	$RS(\{x_1\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(X_1 \cup \{x_2\})$	$RS(\{x_1\} \cup X_2)$	$RS(\{x_1\} \cup \{x_2\})$
$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(U)$	$RS(X_1 \cup \{x_2\})$	$RS(X_2)$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_2\})$	$RS(X_1 \cup \{x_2\})$	$RS(\{x_1\} \cup X_2)$	$RS(\{x_1\} \cup \{x_2\})$
$RS(X_1 \cup \{x_2\})$	$RS(X_1 \cup \{x_2\})$	$RS(U)$	$RS(X_1 \cup \{x_2\})$	$RS(U)$	$RS(X_1 \cup \{x_2\})$	$RS(X_1 \cup \{x_2\})$	$RS(X_1 \cup \{x_2\})$	$RS(U)$	$RS(X_1 \cup \{x_2\})$
$RS(\{x_1\} \cup X_2)$	$RS(\{x_1\} \cup X_2)$	$RS(U)$	$RS(U)$	$RS(\{x_1\} \cup X_2)$	$RS(\{x_1\} \cup X_2)$	$RS(\{x_1\} \cup X_2)$	$RS(U)$	$RS(\{x_1\} \cup X_2)$	$RS(\{x_1\} \cup X_2)$
$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(U)$	$RS(X_1 \cup \{x_2\})$	$RS(\{x_1\} \cup X_2)$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(X_1 \cup \{x_2\})$	$RS(\{x_1\} \cup X_2)$	$RS(\{x_1\} \cup \{x_2\})$

From Table 2, it is clear that (T,Δ) is a commutative rough monoid of idempotents.

(b) Cayley's table to prove (T,∇) is a commutative rough ∇ monoid

Table 3:

∇	$RS(\phi)$	$RS(U)$	$RS(X_1)$	$RS(X_2)$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(X_1 \cup \{x_2\})$	$RS(\{x_1\} \cup X_2)$	$RS(\{x_1\} \cup \{x_2\})$
$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$	$RS(\phi)$
$RS(U)$	$RS(\phi)$	$RS(U)$	$RS(X_1)$	$RS(X_2)$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(X_1 \cup \{x_2\})$	$RS(\{x_1\} \cup X_2)$	$RS(\{x_1\} \cup \{x_2\})$
$RS(X_1)$	$RS(\phi)$	$RS(X_1)$	$RS(X_1)$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\phi)$	$RS(X_1)$	$RS(\{x_1\})$	$RS(\{x_1\})$
$RS(X_2)$	$RS(\phi)$	$RS(X_2)$	$RS(\phi)$	$RS(X_2)$	$RS(\phi)$	$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(X_2)$	$RS(\{x_2\})$
$RS(\{x_1\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\phi)$	$RS(\{x_1\})$	$RS(\{x_1\})$	$RS(\{x_1\})$
$RS(\{x_2\})$	$RS(\phi)$	$RS(\{x_2\})$	$RS(\phi)$	$RS(\{x_2\})$	$RS(\phi)$	$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(\{x_2\})$	$RS(\{x_2\})$
$RS(X_1 \cup \{x_2\})$	$RS(\phi)$	$RS(X_1 \cup \{x_2\})$	$RS(X_1)$	$RS(\{x_2\})$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(X_1 \cup \{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$
$RS(\{x_1\} \cup X_2)$	$RS(\phi)$	$RS(\{x_1\} \cup X_2)$	$RS(\{x_1\})$	$RS(X_2)$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\} \cup X_2)$	$RS(\{x_1\} \cup \{x_2\})$
$RS(\{x_1\} \cup \{x_2\})$	$RS(\phi)$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(\{x_1\})$	$RS(\{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$	$RS(\{x_1\} \cup \{x_2\})$

From Table 3, it is clear that (T,∇) is a commutative rough ∇ monoid of idempotents.

(c) (Distributive law) for $RS(X_1), RS(X_2), RS(\{x_1\}) \in T$,

To prove: (i) $RS(X_1\Delta(X_2\nabla\{x_1\})) = RS((X_1\Delta X_2)\nabla(X_1\Delta\{x_1\}))$

(ii) $RS(X_1\nabla(X_2\Delta\{x_1\})) = RS((X_1\nabla X_2)\Delta(X_1\nabla\{x_1\}))$

Case(i):

$RS(X_1\Delta(X_2\nabla\{x_1\})) = RS(X_1)\Delta(RS(X_2)\nabla RS(\{x_1\})) = RS(X_1)\Delta RS(\phi) = RS(X_1)$

$RS((X_1\Delta X_2)\nabla(X_1\Delta\{x_1\})) = (RS(X_1)\Delta RS(X_2))\nabla(RS(X_1)\Delta RS(\{x_1\})) = RS(U)\nabla RS(X_1) = RS(X_1)$

Thus, $RS(X_1\Delta(X_2\nabla\{x_1\})) = RS((X_1\Delta X_2)\nabla(X_1\Delta\{x_1\}))$

Case(ii)

$RS(X_1\nabla(X_2\Delta\{x_1\})) = RS(X_1)\nabla(RS(X_2)\Delta RS(\{x_1\})) = RS(X_1)\nabla RS(X_2 \bigcup \{x_1\}) = RS(\{x_1\})$

$RS((X_1\nabla X_2)\Delta(X_1\nabla\{x_1\})) = (RS(X_1)\nabla RS(X_2))\Delta(RS(X_1)\nabla RS(X_2 \cup \{x_1\})) = RS(\phi)\Delta RS(\{x_1\}) = RS(\{x_1\})$

Thus, $RS(X_1\nabla(X_2\Delta\{x_1\})) = RS((X_1\nabla X_2)\Delta(X_1\nabla\{x_1\}))$

from case(i) and case(ii), distributive law holds. Therefore (T,Δ,∇) is a rough semiring.

Lemma 2.1. [10] Let $\mathscr{X}$ be the set of equivalence classes and P_x be the set of representatives of the equivalence classes whose cardinality is greater than 1 and let $|\mathscr{X}| = n$ and $|P_x| = m$ with $1 \le m \le n$ then the order of the rough semiring is $2^{n-m}3^m$.

Example 2.2. From example 2.1, $|\mathscr{X}| = 2$ and $|P_x| = 2$ then $|T| = 9$.

Definition 2.1. [15] A subset X of U is said to be dominant if $X \cap X_i \ne \phi$ for $i = 1,2,3...n$.

Definition 2.2. [15] Let (T,Δ,∇) be a commutative rough semiring. An element $RS(X) \ne RS(\phi)$ of T is said to be a zero divisor of T if there exist $RS(Y) \ne RS(\phi)$ in T such that $RS(X)\nabla RS(Y) = RS(\phi)$ *i.e.*, $RS(X \nabla Y) = RS(\phi)$

Example 2.3. From [15] Let

$$T = \left\{ \begin{array}{l} RS(X_1), RS(X_2), RS(X_3), RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(\phi), RS(U), \\ RS(\{x_1\}), RS(\{x_1\} \cup X_2), RS(\{x_1\} \cup X_3), RS(\{x_2\}), RS(\{x_2\} \cup X_1), RS(\{x_2\} \cup X_3), \\ RS(\{x_1\} \cup \{x_2\}), RS(\{x_1\} \cup X_2 \cup X_3), RS(X_1 \cup \{x_2\} \cup X_3), RS(\{x_1\} \cup \{x_2\} \cup X_3) \end{array} \right\}$$ be the set of all rough sets and from [14], (T,Δ,∇) be the rough semiring then the set of nonzero zero divisors of the rough semiring *T* is denoted by $Z(T^*)$

where $$Z(T^*) = \left\{ \begin{array}{l} RS(X_1), RS(X_2), RS(X_3), RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(\{x_1\}), RS(\{x_1\} \cup X_2), \\ RS(\{x_1\} \cup X_3), RS(\{x_2\}), RS(X_1 \cup \{x_2\}), RS(\{x_2\} \cup X_3), RS(\{x_1\} \cup \{x_2\}) \end{array} \right\}$$

Definition 2.3. The zero divisor graph of a semiring *S* is denoted by $\Gamma(S)$. The vertex set $V(\Gamma(S))$ of $\Gamma(S)$ is the set of elements in $Z(S^*) = Z(S) \setminus 0$ and an unordered pair of vertices $x, y \in V(\Gamma(S)), x \ne y$ is an edge $x - y$ in $\Gamma(S)$ if $xy = 0$.

Theorem 2.2. [15] Let $I = (U, A)$ be an information system. If a subset X of U is not dominant then $RS(X)$ is a zero divisor of the rough semiring (T,Δ,∇) .

Theorem 2.3. [15] Let (T,Δ,∇) be a rough semiring. If a subset X of U is dominant then $RS(X)$ is not a zero divisor in *T.*

2.2. Total and Complemented Graph of a Semiring

Definition 2.4. The total graph of a semiring *S* is denoted by $T(\Gamma(S))$ is the undirected graph with all elements of *S* as vertices and for distinct $x, y \in S$, the vertices *x* and *y* are adjacent if and only if $x + y \in Z(S^*)$.

Definition 2.5. The complemented graph of a semiring S is denoted by $\overline{\Gamma(S)}$ is the undirected graph with all elements of $Z(S^*)$ as vertices and for distinct vertices $x, y \in Z(S^*)$ the vertices *x* and *y* are adjacent if and only if $xy \in Z(S^*)$.

In the following section, we discuss the Total and Complemented graph of a rough semiring (T,Δ,∇) .

3. TOTAL AND COMPLEMENTED GRAPH OF A ROUGH SEMIRING

In this section, we consider an information system $I = (U, A)$. Now for any $X \subseteq U$, $RS(X) = (\underline{P}(X), \overline{P}(X))$ be the rough set of *X* and let $T = \{RS(X) \mid X \subseteq U\}$ be the set of all rough sets and let $X_1, X_2, ..., X_n$ be the equivalence classes induced by *IND(P)*.

Definition 3.1. Total graph $\Gamma(T)$ of a rough semiring *T* is defined as $\Gamma(T) = (V, E)$ where *V* be the set of elements of *T* and two distinct vertices $RS(X), RS(Y) \in V$ are adjacent if and only if $RS(X)\Delta RS(Y) \in Z(T)$ where $Z(T)$ be the set of all zero divisors of *T.*

Definition 3.2. The complemented graph of a rough semiring (T,Δ,∇) is $\overline{\Gamma(Z(T^*))} = (V, E)$ where *V* is the set of vertices in $Z(T^*)$ that is, $V = \left\{ RS(X) \mid RS(X) \in Z(T^*) \right\}$ and *E* is the set of edges connecting the elements of *V* such that there exist an edge connecting $RS(X), RS(Y) \in V$ if and only if $RS(X) \nabla RS(Y) \neq RS(\phi)$. This graph $\overline{\Gamma(Z(T^*))}$ is called a complemented rough zero divisor of a rough semiring *T*.

Let us consider, Number of dominant sets = 2^m where *m* denotes the number of pivot elements with $|X_i| > 1$, D is a set of dominant sets and P_x be the set of all pivot elements and consider a map $f : P(P_x) \to D$ such that $f(B) = B \cup \left\{ U - \bigcup_{x_i \in B} [x_i]_p \right\}$

Theorem 3.1. If $P(P_x)$ be the power set of pivot elements and *D* be the set of all dominant sets in *U* then there exists a bijection from $P(P_x)$ to *D*.

Proof. Let $|D| = 2^m$ and let us define a map from $f : P(P_x) \to D$ by $f(B) = B \cup \left\{ U - \bigcup_{x_i \in B} [x_i]_p \right\}$. It is enough to prove that *f* is one to one and onto. Let $f(B_1) = f(B_2)$ implies that $B_1 \cup \left\{ U - \bigcup_{x_i \in B_1} [x_i]_p \right\} = B_2 \cup \left\{ U - \bigcup_{x_i \in B_2} [x_i]_p \right\}$. If $B_1 \neq B_2$ then $B_1 \cup \left\{ U - \bigcup_{x_i \in B_1} [x_i]_p \right\} \neq B_2 \cup \left\{ U - \bigcup_{x_i \in B_2} [x_i]_p \right\}$, which is a contradiction. Therefore $B_1 = B_2$ and let $X \subseteq U$ be a dominant set implies that $X \cap X_i \neq \phi$ for all *i*. Let $B \subset X$ containing only the pivot elements in *X* then $B \in P(P_x)$ implies $f(B) = B \cup \left\{ U - \bigcup_{x_i \in B} [x_i]_p \right\} = X$.

Remark 3.1. From the above theorem, the number of dominant sets $= |D| = 2^{|P_x|}$. If $|P_x| = m$ then the number of zero divisors in $T = |T| - |D| = 2^{n-m}3^m - 2^m$. Note that there are $2^{n-m}3^m - 2^m$ zero divisors (including $RS(\phi)$) in the rough semiring (T, Δ, ∇) . Hence $2^{n-m}3^m - 2^m - 1$ elements are there in $Z(T^*)$. Therefore the number of vertices in the zero divisor graph as well as in the complemented zero divisor graph is $2^{n-m}3^m - 2^m - 1$. From example 2.3, we have $n = 3, m = 2$ and $|T| = 18$, $|D| = 4$ then $|Z(T^*)| = 18 - 4 - 1 = 13$.

Theorem 3.2. For $RS(X), RS(Y)(\neq RS(\phi)) \in T$, If $RS(X \Delta Y) \in Z(T^*)$ then $RS(X)$ and $RS(Y) \in Z(T^*)$ but the converse is not true.

Proof. Let $RS(X \Delta Y) \in Z(T^*)$ which implies $X \Delta Y$ is not dominant. To prove: $RS(X)$ and $RS(Y) \in Z(T^*)$. Let either $RS(X)$ *or* $RS(Y) \in Z(T^*)$ implies X *or* Y is dominant. That is $X_i \cap X \neq \phi$ or $X_i \cap Y \neq \phi$ for all *i*. If $X_i \cap X \neq \phi$ for all *i* then $X_i \cap (X \Delta Y) \neq \phi$ for all *i* which implies $X \Delta Y$ is dominant. Which is a contradiction. Hence $RS(X)$ and $RS(Y) \in Z(T^*)$ but the converse is not true. That is, Let *T* be the set of all rough sets and let $RS(\{x_1\} \cup \{x_2\})$ and $RS(X_3) \in Z(T^*)$ then $RS(\{x_1\} \cup \{x_2\}) \Delta RS(X_3) = RS(\{x_1\} \cup \{x_2\} \cup X_3) \notin Z(T^*)$.

Remark 3.2. From the above theorem 3.2, $Z(T^*)$ need not be closed under Δ .

Theorem 3.3. For $RS(X), RS(Y)(\neq RS(\phi)) \in T$, $RS(X)$ and $RS(Y) \notin Z(T^*)$ if and only if $RS(X \nabla Y) \notin Z(T^*)$.

Proof. Let $RS(X)$ and $RS(Y)\notin Z(T^*)$ implies X and Y are dominant then $X\cap X_i\neq\phi$ and $Y\cap X_i\neq\phi$ for all i implies that $X_i\cap(X\nabla Y)\neq\phi$ for all i implies $X\nabla Y$ is dominant. Hence $RS(X\nabla Y)\notin Z(T^*)$. Conversely, Let $RS(X\nabla Y)\notin Z(T^*)$ implies $X\nabla Y$ is dominant implies that $X_i\cap(X\nabla Y)\neq\phi$ for all $i$implies $X_i\cap\left(\left\{x\,|\,[x]_p\subseteq X\cap Y\right\}\cup P_{X\cap Y}\right)\neq\phi$ for all $i$implies $\left(X_i\cap\left\{x\,|\,[x]_p\subseteq X\cap Y\right\}\right)\cup\left(X_i\cap P_{X\cap Y}\right)\neq\phi$ for all i implies $\left(X_i\cap\left\{x\,|\,[x]_p\subseteq X\cap Y\right\}\right)\neq\phi$ *or* $\left(X_i\cap P_{X\cap Y}\right)\neq\phi$ for all *i*.

Case: 1 $X_i\cap\left\{x\,|\,[x]_p\subseteq X\cap Y\right\}\neq\phi$ for all i implies $X\cap Y=U$ implies $X=Y=U$ implies that X and Y are dominant.

Case: 2 Let $X_i\cap P_{X\cap Y}\neq\phi$ for all $i$implies $X_i\cap X\neq\phi$ $X_i\cap Y\neq\phi$ for all i implies X and Y are dominant hence $RS(X)$ and $RS(Y)\notin Z(T^*)$.

Example 3.1. From [15] Let

$$T=\begin{Bmatrix} RS(X_1),RS(X_2),RS(X_3),RS(X_1\cup X_2),RS(X_1\cup X_3),RS(X_2\cup X_3),RS(\phi),RS(U),\\ RS(\{x_1\}),RS(\{x_1\}\cup X_2),RS(\{x_1\}\cup X_3),RS(\{x_2\}),RS(\{x_2\}\cup X_1),RS(\{x_2\}\cup X_3),\\ RS(\{x_1\}\cup\{x_2\}),RS(\{x_1\}\cup X_2\cup X_3),RS(X_1\cup\{x_2\}\cup X_3),RS(\{x_1\}\cup\{x_2\}\cup X_3)\end{Bmatrix}$$ be the set of all rough sets and from [14] (T,Δ,∇) be the rough semiring then the set of nonzero zero divisors of the rough semiring$T$ is denoted by $Z(T^*)$ where

$$Z(T^*)=\begin{Bmatrix} RS(X_1),RS(X_2),RS(X_3),RS(X_1\cup X_2),RS(X_1\cup X_3),RS(X_2\cup X_3),RS(\{x_1\}),RS(\{x_1\}\cup X_2),\\ RS(\{x_1\}\cup X_3),RS(\{x_2\}),RS(X_1\cup\{x_2\}),RS(\{x_2\}\cup X_3),RS(\{x_1\}\cup\{x_2\})\end{Bmatrix}$$

Then for $RS(\{x_1\}\cup\{x_2\}\cup X_3)\notin Z(T^*)$ and $RS(X_1\cup\{x_2\}\cup X_3)\notin Z(T^*)$ if and only if

$$RS(\{x_1\}\cup\{x_2\}\cup X_3)\nabla RS(X_1\cup\{x_2\}\cup X_3)=RS\left((\{x_1\}\cup\{x_2\}\cup X_3)\nabla(X_1\cup\{x_2\}\cup X_3)\right)=RS(\{x_1\}\cup\{x_2\}\cup X_3)\notin Z(T^*)$$

Theorem 3.4. If for given $RS(X)\in Z(T^*)$ there exist $RS(Y)\in T$ such that $X\nabla Y\neq\phi$ then $RS(X\nabla Y)\in Z(T^*)$

Proof. Let $RS(X)\in Z(T^*)$, if $RS(Y)\in T$ such that $X\Delta Y\neq\phi$. As $RS(X)\in Z(T^*)$ there exist $RS(Z)\in Z(T)$ such that $RS(X)\nabla RS(Z)=RS(X\nabla Z)=RS(\phi)$. Hence $RS((X\nabla Y)\nabla Z)=RS(Y\nabla(X\nabla Z))=RS(Y)\nabla RS(X\nabla Z)=RS(Y)\nabla RS(\phi)=RS(\phi)$. Hence $RS(X\nabla Y)\in Z(T^*)$.

Example 3.2. From [15] Let

$$T=\begin{Bmatrix} RS(X_1),RS(X_2),RS(X_3),RS(X_1\cup X_2),RS(X_1\cup X_3),RS(X_2\cup X_3),RS(\phi),RS(U),\\ RS(\{x_1\}),RS(\{x_1\}\cup X_2),RS(\{x_1\}\cup X_3),RS(\{x_2\}),RS(\{x_2\}\cup X_1),RS(\{x_2\}\cup X_3),\\ RS(\{x_1\}\cup\{x_2\}),RS(\{x_1\}\cup X_2\cup X_3),RS(X_1\cup\{x_2\}\cup X_3),RS(\{x_1\}\cup\{x_2\}\cup X_3)\end{Bmatrix}$$ be the set of all rough sets and from [14] (T,Δ,∇) be the rough semiring then set of nonzero zero divisors of the rough semiring T is denoted by $Z(T^*)$ where

$$Z(T^*)=\begin{Bmatrix} RS(X_1),RS(X_2),RS(X_3),RS(X_1\cup X_2),RS(X_1\cup X_3),RS(X_2\cup X_3),RS(\{x_1\}),RS(\{x_1\}\cup X_2),\\ RS(\{x_1\}\cup X_3),RS(\{x_2\}),RS(X_1\cup\{x_2\}),RS(\{x_2\}\cup X_3),RS(\{x_1\}\cup\{x_2\})\end{Bmatrix}.$$

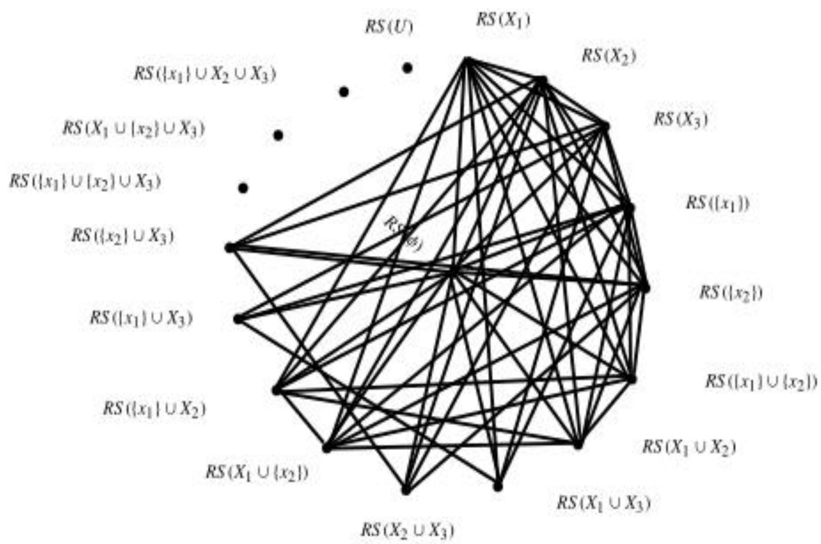
For $RS(X_1)\in Z(T^*)$ if $RS(\{x_1\})\in T$ such that $X_1\nabla\{x_1\}\neq\phi$. As $RS(X_1)\in Z(T^*)$ there exist an $RS(\{x_2\})\in T$ such that $RS((X_1\nabla\{x_1\})\nabla\{x_2\})=RS((\{x_1\}\nabla X_1)\nabla\{x_2\})=RS(\{x_1\}\nabla(X_1\nabla\{x_2\}))=RS(\phi)$. Hence $RS(X_1\nabla\{x_1\})=RS(\{x_1\})\in Z(T^*)$.

Example 3.3. From [15] Let $T = \left\{\begin{matrix} RS(X_1), RS(X_2), RS(X_3), RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(\phi), RS(U), \\ RS(\{x_1\}), RS(\{x_1\} \cup X_2), RS(\{x_1\} \cup X_3), RS(\{x_2\}), RS(\{x_2\} \cup X_1), RS(\{x_2\} \cup X_3), \\ RS(\{x_1\} \cup \{x_2\}), RS(\{x_1\} \cup X_2 \cup X_3), RS(X_1 \cup \{x_2\} \cup X_3), RS(\{x_1\} \cup \{x_2\} \cup X_3) \end{matrix}\right\}$

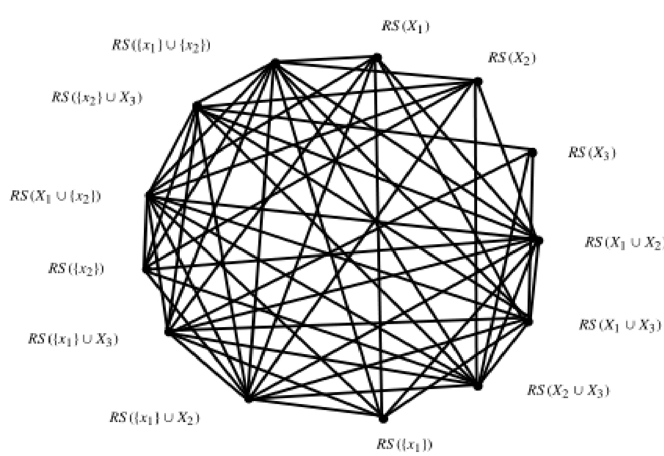
be the set of all rough sets and from [14] (T, Δ, ∇) be the rough semiring then set of nonzero zero divisors of the rough semiring T is denoted by $Z(T^*)$ where

$$Z(T^*) = \left\{\begin{matrix} RS(X_1), RS(X_2), RS(X_3), RS(X_1 \cup X_2), RS(X_1 \cup X_3), RS(X_2 \cup X_3), RS(\{x_1\}), RS(\{x_1\} \cup X_2), \\ RS(\{x_1\} \cup X_3), RS(\{x_2\}), RS(X_1 \cup \{x_2\}), RS(\{x_2\} \cup X_3), RS(\{x_1\} \cup \{x_2\}) \end{matrix}\right\}.$$

then the total graph of $\Gamma(T)$ and complemented graph $\overline{\Gamma(T)}$ are given below.



Total Graph of T



Complemented Graph of T

4. CONCLUSION

In this paper, we proved an existence theorem for total graph and complemented graph of the rough semiring (T, Δ, ∇) for a given information system $I = (U, A)$. The concepts are illustrated with examples.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

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