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Set-Valued Prešić-Chatterjea Type Contractions and Fixed Point Theorems

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ABSTRACT

In this paper, we establish some common fixed point theorems for two mappings satisfying set-valued Prešić-Chatterjea type contractive conditions. The new theorems generalize and unify some well-known theorems of the literature. We also provide some examples to illustrate and confirm the usability of the obtained results.

Keywords: *Set-valued mapping, Prešić-Chatterjea type contraction, common fixed point, fixed point.*

1. INTRODUCTION AND PRELIMINARIES

Let X be a nonempty set and $x \in X$. Then x is called a fixed point of a mapping $T: X \to X$ if $Tx = x$. The existence of fixed points of self-mappings is considered by several authors in different spaces. Most of the results on fixed points are the generalizations of the famous Banach contraction principle which ensures the existence and uniqueness of the fixed point of self-mappings defined on complete metric spaces. It states that: if $T: X \to X$ is a Banach contraction on a complete metric space(X, d), that is, T satisfies the condition:

$$d(Tx, Ty) \leq \lambda\, d(x, y)$$

for all $x, y \in X$, where $\lambda \in [0,1)$, then T has a unique fixed point.

Kannan [3] extended the Banach's principle for the mappings $T: X \to X$ satisfying the following condition:

$$d(Tx, Ty) \leq \lambda[d(x, Tx) + d(y, Ty)]$$

for all $x, y \in X$, where $\lambda \in [0,1/2)$. We notice that, the conditions of Banach and Kannan are independent. Kannan [3] showed the existence and uniqueness of fixed point by using the above condition.

Another generalization of Banach's principle is due to Reich [8]. He unified the above two conditions for the mappings $T: X \to X$ by assuming the following condition:

$$d(Tx, Ty) \leq \lambda\, d(x, y) + \mu\, d(x, Tx) + \delta\, d(y, Ty)$$

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for all $x, y \in X$, where λ, μ, δ are nonnegative constants such that $\lambda + \mu + \delta < 1$. He also showed that the conclusions of Banach and Kannan about the existence and uniqueness of fixed point remain true for the above unified contractions.

Chatterjea [1] assumed the following condition:

$$d(Tx, Ty) \leq \lambda[d(x, Ty) + d(y, Tx)]$$

for all $x, y \in X$, where $\lambda \in [0,1/2)$, and proved the existence and uniqueness of the fixed point of mapping T.

Let $CB(X)$ be the class of all nonempty, closed and bounded subsets of X. A mapping $H: CB(X) \times CB(X) \to [0, \infty)$ defined by

$$H(A, B) = \max\left\{\sup_{a \in A}\left\{\inf_{b \in B} d(a, b)\right\}, \sup_{b \in B}\left\{\inf_{a \in A} d(b, a)\right\}\right\}$$

is called a Pompeiu-Hausdorff metric on $CB(X)$. The mapping H is indeed a metric on $CB(X)$.

The following lemma can be found in [5].

Lemma 1.1 ([5]). Let (X, d) be a metric space and $A, B \in CB(X)$. Then for all $\epsilon > 0$ and $a \in A$ there exists a point $b \in B$ such that $d(a, b) \leq H(A, B) + \epsilon$.

For a nonempty set X, the mapping $T: X \to CB(X)$ is called a set-valued mapping. A point $x \in X$ is called a fixed point of T if $x \in Tx$. Nadler [5] extended the Banach contraction principle from the setting of single-valued mapping to the setting of set-valued mappings and proved the following fixed point theorem.

Theorem 1.2 ([5]). Let (X, d) be a complete metric space and let T be a mapping from X into $CB(X)$ such that for all $x, y \in X$,

$$H(Tx, Ty) \leq \lambda d(x, y),$$

where $\lambda \in [0,1)$. Then T has a fixed point.

On the other hand, Prešić [6,7] extended the Banach contraction principle for the mappings defined from the product X^k (where k is a positive integer) into the space X and proved the following theorem.

Theorem 1.3 Let (X, d) be a complete metric space, k a positive integer and $T: X^k \to X$ be a mapping satisfying the following contractive type condition:

$$d\big(T(x_1, x_2, \dots, x_k), T(x_2, x_3, \dots, x_{k+1})\big) \leq \sum_{i=1}^{k} q_i d(x_i, x_{i+1}) \qquad (1)$$

for every $x_1, x_2, \dots, x_k, x_{k+1} \in X$, where $q_1, q_2, \dots, q_k$ are nonnegative constants such that $q_1 + q_2 + \cdots + q_k < 1$. Then there exists a unique point $x \in X$ such that $T(x, x, \dots, x) = x$. Moreover if $x_1, x_2, \dots, x_k$ are arbitrary points in X and for $n \in \mathbb{N}$, $x_{n+k} = T(x_n, x_{n+1}, \dots, x_{n+k-1})$, then the sequence $\{x_n\}$ is convergent and

$$\lim x_n = T(\lim x_n, \lim x_n, \dots, \lim x_n).$$

A mapping satisfying (1) is called a Prešić type contraction. There are several applications of Prešić type contractions, e.g., in the convergence of sequences [6,7], in solving the nonlinear difference equations [2,14], in solving the nonlinear inclusion problems [9], in the convergence problems of nonlinear matrix difference equations [4] etc..

The mapping $T: X^k \to CB(X)$ is called a set-valued Prešić type contraction (see [11]) if it satisfies the following property: there exist nonnegative constants α_i such that $\sum_{i=1}^{k} \alpha_i < 1$ and

$$H\big(T(x_1, x_2, \dots, x_k), T(x_2, x_3, \dots, x_{k+1})\big) \leq \sum_{i=1}^{k} \alpha_i \, d(x_i, x_{i+1}) \qquad (2)$$

for all $x_1, x_2, \dots, x_k, x_{k+1} \in X$.

Shukla et al. [11] extended the result of Prešić for set-valued mappings and proved some fixed point results for set-valued Prešić type contraction. The results proved in [11] were the generalization and unification of the results of Nadler [5] and Prešić [6,7]. Shukla [10] unified the results of Assad-Kirk and Prešić and proved Assad-Kirk type theorems in product spaces. He also discussed some stability results concerning set-valued Prešić type mappings. Shukla and Sen [12] introduced the set-valued Prešić-Reich type contractions which extend and generalize the result of Reich [8] in product spaces. In the recent paper [9], Shazad and Shukla generalized the results of [11] in the spaces consisting graphical structure and applied the corresponding results to the difference inclusion problems. Another generalization for set-valued Prešić type mappings in the partial metric setting can be found in the recent paper of Shukla [13].

In this paper, we introduce the set-valued Prešić-Chatterjea mappings in metric spaces and prove some common fixed point results for a mapping $T: X^k \to X$ and a single-valued self-mapping of space X. Our results extend the results of Shukla et al. [11] for Chatterjea type contractive conditions and generalize and unify the results of Nadler [5], Prešić [6] and Chatterjea [1].

The following definitions and assumptions will be needed in the sequel.

Let k be a positive integer and $T: X^k \to CB(X)$ be a mapping. Then T is said to be a set-valued Prešić-Chatterjea type contraction if,

$$H\big(T(x_1, x_2, \ldots, x_k), T(x_2, x_3, \ldots, x_{k+1})\big) \leq \lambda \sum_{i=1}^{k+1} \sum_{\substack{j=1 \\ i \neq j}}^{k+1} d\left(x_i, T\big(x_j, x_j, \ldots, x_j\big)\right) \qquad (3)$$

for all $x_0, x_1, \ldots, x_k, x_{k+1} \in X$, where λ is a constant such that $0 \leq \lambda k^2(k+1) < 1$.

Note that the set-valued Prešić-Chatterjea type contraction in the case $k = 1$ reduces to set-valued Chatterjea type contraction. Therefore, the set-valued Prešić-Chatterjea type contractions are a generalization of set-valued Chatterjea type contractions.

Definition 1.4 ([11]). Let X be a nonempty set, k a positive integer, $T: X^k \to CB(X)$ and $g: X \to X$ be two mappings.

(a) If $x \in T(x, \ldots, x)$, then $x \in X$ is called a fixed point of T.

(b) An element $x \in X$ is said to be a coincidence point of T and g if $gx \in T(x, \ldots, x)$.

(c) If $w = gx \in T(x, \ldots, x)$, then w is called a point of coincidence of T and g.

(d) If $x = gx \in T(x, \ldots, x)$, then x is called a common fixed point of T and g.

(e) Mappings T and g are said to be weakly compatible if whenever $gx \in T(x, \ldots, x)$ we have $g\big(T(x, \ldots, x)\big) \subseteq T(gx, \ldots, gx)$.

We denote the set of all fixed points of the mapping T by $\mathrm{Fix}(T)$.

Now we can state the main results of this paper.

2. MAIN RESULTS

The following theorem is a coincidence point result for a mapping on product space and a self-mapping of space, and a generalization of fixed point result for set-valued Prešić-Chatterjea type contractions.

Theorem 2.1. Let (X, d) be any complete metric space and k be a positive integer. Let $T: X^k \to CB(X)$ and $g: X \to X$ be two mappings such that $g(X)$ is a closed subspace of X and $T(\Delta) \subset g(X)$, where $\Delta = \{(x, x, \ldots, x): x \in X\}$ is the diagonal of Cartesian product X^k. Suppose the following condition holds:

$$H\big(T(x_1, x_2, \ldots, x_k), T(x_2, x_3, \ldots, x_{k+1})\big) \leq \lambda \sum_{i=1}^{k+1} \sum_{\substack{j=1 \\ i \neq j}}^{k+1} d\left(gx_i, T\big(x_j, x_j, \ldots, x_j\big)\right) \qquad (4)$$

for all $x_1, x_2, \ldots, x_k, x_{k+1} \in X$, where λ is a constant such that $0 \leq \lambda k^2(k+1) < 1$. Then T and g have a point of coincidence $v \in X$.

Proof. Let $x_0 \in X$ be arbitrary. As $T(x_0, \ldots, x_0) \in CB(X)$ and $T(\Delta) \subset g(X)$ we have $T(x_0, \ldots, x_0) \subset g(X)$, let $y_1 = gx_1 \in T(x_0, \ldots, x_0)$ for some $x_1 \in X$, so by Lemma 1.1 there exists $y_2 = gx_2 \in T(x_1, \ldots, x_1)$, $x_2 \in X$ such that

$$d(gx_1, gx_2) \leq H(T(x_0, \ldots, x_0), T(x_1, \ldots, x_1)) + \theta,$$

where $\theta > 0$ is arbitrary. Similarly, there exists $y_3 = gx_3 \in T(x_2, \ldots, x_2)$ such that

$$d(gx_2, gx_3) \leq H\big(T(x_1, \ldots, x_1), T(x_2, \ldots, x_2)\big) + \theta^2.$$

Continuing this procedure we obtain $y_{n+1} = gx_{n+1} \in T(x_n, \ldots, x_n)$ and

$$d(gx_n, gx_{n+1}) \leq H\big(T(x_{n-1}, \ldots, x_{n-1}), T(x_n, \ldots, x_n)\big) + \theta^n \qquad (5)$$

for all $n \in \mathbb{N}$. For simplicity, set

$d_i = d(y_i, y_{i+1})$, $D_{i,j} = d\left(gx_i, T\big(x_j, x_j, \ldots, x_j\big)\right) \forall\ i, j \in \{1, 2, \ldots, k\}$.

As H is a metric on $CB(X)$, for any $n \in \mathbb{N}$ it follows from (5) that

$$\begin{aligned} d_n &= d(y_n, y_{n+1}) = d(gx_n, gx_{n+1}) \\ &\leq H(T(x_{n-1}, \ldots, x_{n-1}), T(x_n, \ldots, x_n)) + \theta^n \\ &\leq H(T(x_{n-1}, \ldots, x_{n-1}), T(x_{n-1}, \ldots, x_{n-1}, x_n)) \\ &\quad + H(T(x_{n-1}, \ldots, x_{n-1}, x_n), T(x_{n-1}, \ldots, x_{n-1}, x_n, x_n)) \\ &\quad + \cdots + H\big(T(x_{n-1}, x_n, \ldots, x_n), T(x_n, \ldots, x_n)\big) + \theta^n. \end{aligned}$$

Using (4) in above inequality we obtain

$$\begin{aligned} d_n &\leq \lambda k(k-1) D_{n-1,n-1} + \lambda k D_{n-1,n} \\ &\quad + \lambda k D_{n,n-1} + \lambda(k-1)(k-2) D_{n-1,n-1} \\ &\quad + 2\lambda(k-1) D_{n-1,n} + 2\lambda(k-1) D_{n,n-1} \\ &\quad + 2 \cdot 1\lambda D_{n,n} + \cdots + 2 \cdot 1\lambda D_{n-1,n-1} \\ &\quad + 2\lambda(k-1) D_{n-1,n} + 2\lambda(k-1) D_{n,n-1} \\ &\quad + \lambda(k-1)(k-2) D_{n,n} + \lambda k D_{n-1,n} + \lambda k D_{n,n-1} \\ &\quad + \lambda k(k-1) D_{n,n} + \theta^n \\ &= \lambda[\mathrm{k}(\mathrm{k}-1) + (\mathrm{k}-1)(\mathrm{k}-2) \cdots + 2 \cdot 1] \\ &\quad \times \left[D_{n-1,n-1} + D_{n,n}\right] + \lambda[\mathrm{k} + 2(\mathrm{k}-1) + \cdots + \mathrm{k}] \\ &\quad \times \left[D_{n-1,n} + D_{n,n-1}\right] + \theta^{\mathrm{n}} \end{aligned}$$

that is,

$$\begin{aligned} d_n &\leq \frac{\lambda k(k^2-1)}{3}\left[D_{n-1,n-1} + D_{n,n}\right] \\ &\quad + \frac{\lambda k(k+1)(k+2)}{6}\left[D_{n-1,n} + D_{n,n-1}\right] \\ &\quad + \theta^n. \end{aligned} \qquad (6)$$

Since $y_n = gx_n \in T(x_{n-1}, \ldots, x_{n-1})$ for all $n \in \mathbb{N}$, by definition we have

$$
\begin{aligned}
D_{n,n} &= d\big(gx_n, T(x_n, \dots, x_n)\big) \\
&\leq d(y_n, y_{n+1}) = d_n, \\
D_{n,n-1} &= d\big(gx_n, T(x_{n-1}, \dots, x_{n-1})\big) \\
&\leq d(y_n, y_n) = 0
\end{aligned}
$$

and $D_{n-1,n} = d\big(gx_{n-1}, T(x_n, \dots, x_n)\big) \leq d(y_{n-1}, y_{n+1})$, therefore we obtain from (6) that

$$
\begin{aligned}
d_n \leq\ & \frac{\lambda k(k^2-1)}{3}[d_{n-1} + d_n] \\
&+ \frac{\lambda k(k+1)(k+2)}{6}[d(y_{n-1}, y_{n+1})] + \theta^n \\
\leq\ & \frac{\lambda k(k^2-1)}{3}[d_{n-1} + d_n] \\
&+ \frac{\lambda k(k+1)(k+2)}{6}[d_{n-1} + d_n] + \theta^n
\end{aligned}
$$

that is,

$$
d_n \leq \frac{\lambda k^2(k+1)}{2 - \lambda k^2(k+1)} d_{n-1} + \frac{2\theta^n}{2 - \lambda k^2(k+1)}.
$$

For simplicity, set $\mu = \lambda k^2(k+1)$ and $\alpha = \frac{\mu}{2-\mu}$, then by assumption we have $0 \leq \mu < 1$ and $0 \leq \alpha < 1$. Since $\theta > 0$ was arbitrary, choose $\theta = \alpha$, then from (7) we have

$$
d_n \leq \alpha\, d_{n-1} + \frac{2\alpha^n}{2-\mu}. \qquad (8)
$$

It follows from successive application of (8) that

$$
\begin{aligned}
d_n &= \alpha\left[\alpha d_{n-2} + \frac{2\alpha^{n-1}}{2-\mu}\right] + \frac{2\alpha^n}{2-\mu} \\
&= \alpha^2 d_{n-2} + \frac{4\alpha^n}{2-\mu} \\
&\leq \alpha^2\left[\alpha d_{n-3} + \frac{2\alpha^{n-2}}{2-\mu}\right] + \frac{4\alpha^n}{2-\mu} \\
&= \alpha^3 d_{n-3} + \frac{6\alpha^n}{2-\mu}.
\end{aligned}
$$

Repeating in similar manner we obtain:

$$
d_n \leq \alpha^n d_0 + \frac{2n\alpha^n}{2-\mu}.
$$

As $0 \leq \alpha < 1$, therefore $\sum_{n=0}^{\infty} \alpha^n < \infty$ and $\sum_{n=0}^{\infty} n\alpha^n < \infty$, we have

$$
\begin{aligned}
\sum_{n=0}^{\infty} d(y_n, y_{n+1}) &= \sum_{n=0}^{\infty} d_n \\
&\leq d_0 \sum_{n=0}^{\infty} \alpha^n + \frac{2}{2-\mu} \sum_{n=0}^{\infty} n\alpha^n \\
&< \infty \qquad (9)
\end{aligned}
$$

It follows from (9) that the series $\sum_{n=0}^{\infty} d(y_n, y_{n+1})$ is a convergent series, and so, the sequence of its partial sums must be a Cauchy sequence. Suppose, $\varepsilon > 0$ be given, then there exists $n_0 \in \mathbb{N}$ such that $\sum_{i=n}^{m-1} d(y_i, y_{i+1}) < \varepsilon$ for all $n, m > n_0$.

Now, for $n, m \in \mathbb{N}$ with $m > n$ we have

$$
\begin{aligned}
d(y_n, y_m) &\leq d(y_n, y_{n+1}) + d(y_{n+1}, y_{n+2}) \\
&\quad + \dots + d(y_{m-1}, y_m) \\
&= \sum_{i=n}^{m-1} d(y_i, y_{i+1}) \\
&< \varepsilon
\end{aligned}
$$

for all $n, m > n_0$. Therefore $\{y_n\} = \{gx_n\}$ is a Cauchy sequence in $g(X)$. As $g(X)$ is closed and X is complete, there exists $u, v \in X$ such that $v = gu$ and

$$
\lim_{n\to\infty} d(y_n, v) = \lim_{n\to\infty} d(gx_n, gu) = 0. \quad (10)
$$

We shall show that u is a coincidence point of T and g.

As $y_{n+1} = gx_{n+1} \in T(x_n, \dots, x_n)$ for all $n \in \mathbb{N}$, we have

$$
\begin{aligned}
& d\big(v, T(u, \dots, u)\big) \\
&\leq d(v, y_{n+1}) + d\big(y_{n+1}, T(u, \dots, u)\big) \\
&\leq d(v, y_{n+1}) + H\big(T(x_n, \dots, x_n), T(u, \dots, u)\big) \\
&\leq d(v, y_{n+1}) + H\big(T(x_n, \dots, x_n), T(x_n, \dots, x_n, u)\big) \\
&\quad + H\big(T(x_n, \dots, x_n, u), T(x_n, \dots, x_n, u, u)\big) \\
&\quad + \dots + H\big(T(x_n, u, \dots, u), T(u, \dots, u)\big).
\end{aligned}
$$

Using (4) in the above inequality we obtain

$$
\begin{aligned}
& d\big(v, T(u, \dots, u)\big) \\
&\leq d(v, y_{n+1}) + \lambda[k(k-1) + \dots + 2 \cdot 1] \\
&\quad \times \big[d\big(gx_n, T(x_n, \dots, x_n)\big) + d\big(gu, T(u, \dots, u)\big)\big] \\
&\quad + \lambda[k + 2(k-1) + \dots + 2(k-1) + k] \\
&\quad \times \big[d\big(gx_n, T(u, \dots, u)\big) + d\big(gu, T(x_n, \dots, x_n)\big)\big] \\
&\leq d(v, y_{n+1}) \\
&\quad + \frac{\lambda k(k^2-1)}{3}\big[d(y_n, y_{n+1}) + d\big(v, T(u, \dots, u)\big)\big] \\
&\quad + \frac{\lambda k(k+1)(k+2)}{6} \\
&\qquad \times \big[d\big(y_n, T(u, \dots, u)\big) + d(v, y_{n+1})\big]
\end{aligned}
$$

Using (10) in the above inequality we obtain

$$
\begin{aligned}
& d\big(v, T(u, \dots, u)\big) \\
&\leq \frac{\lambda k(k^2-1)}{3} d\big(v, T(u, \dots, u)\big) \\
&\quad + \frac{\lambda k(k+1)(k+2)}{6} d\big(v, T(u, \dots, u)\big) \\
&= \frac{\lambda k^2(k+1)}{2} d\big(v, T(u, \dots, u)\big) \\
&= \frac{\mu}{2} d\big(v, T(u, \dots, u)\big)
\end{aligned}
$$

Since $0 \leq \mu < 1$, therefore it follows from the above inequality that $d(v, T(u, \dots, u)) = 0$, that is, $v = gu \in T(u, \dots, u)$. Thus u is a coincidence point and v is a point of coincidence of T and g

□

Taking $g = I_X$ in the above theorem we obtain the following fixed point result for a set-valued Prešić-Chatterjea type contraction.

Corollary 2.2. Let (X, d) be any complete metric space and k be a positive integer. Let $T: X^k \to CB(X)$ be a set-valued Prešić-Chatterjea type contraction. Then T has a fixed point $v \in X$.

Now we give two examples which show that the conditions (2) and (3) are independent of each other, that is, a set-valued Prešić type contraction need not be a set-valued Prešić-Chatterjea type contraction and a set-valued Prešić-Chatterjea type contraction need not be a set-valued Prešić type contraction, and therefore the results of this paper are proper extensions of the result of Shukla et al. [11].

Our first example shows that a set-valued Prešić type contraction may not be a set-valued Prešić-Chatterjea type contraction.

Example 2.3. Let $X = [0,1]$, then (X, d) is a complete metric space, where d is the usual metric on X. Let b be a positive number such that $1 < b < 7$. For $k = 2$, define $T: X^2 \to CB(X)$ by

$$T(x, y) = \left\{0, \frac{b-1}{6b}(x+y) + \frac{1}{b}\right\}$$

for all $x, y \in X$. Then, at points $x = y = 0, z = 1$ we have

$$H\big(T(x,y), T(y,z)\big) = H\left(\left\{0, \frac{1}{b}\right\}, \left\{0, \frac{b+5}{6b}\right\}\right) = \frac{b-1}{6b},$$

$$d\big(x, T(y,y)\big) + d\big(x, T(z,z)\big) + d\big(y, T(x,x)\big) + d\big(y, T(z,z)\big) + d\big(z, T(x,x)\big) + d\big(z, T(y,y)\big) = \frac{2(b-1)}{b}.$$

Therefore, if (3) is satisfied then we have

$$\frac{b-1}{6b} \leq \frac{2\lambda(b-1)}{b}$$

that is, $\frac{1}{12} \leq \lambda$. But then $\lambda k^2(k+1) = 12\lambda \geq 1$. Therefore, T is not a set-valued Prešić-Chatterjea type contraction. On the other hand, since $b > 1$, taking

$$\alpha_1 = \alpha_2 = \frac{b-1}{6b}$$

we have

$$0 < \alpha_1 + \alpha_2 = \frac{b-1}{3b} < 1.$$

Now it is easy to see that T is a set-valued Prešić type contraction with $\alpha_1 = \alpha_2 = \frac{b-1}{6b}$.

Next example shows that a set-valued Prešić-Chatterjea type contraction may not be a set-valued Prešić type contraction, also that the fixed point of a set-valued Prešić-Chatterjea type contraction may not be unique.

Example 2.4. Let $X = [0,1]$, then (X, d) is a complete metric space, where d is usual metric on X. Let a, b, c are positive numbers such that $b + c < 1, c < a$ and $7a < b$. For $k = 2$, define $T: X^2 \to CB(X)$ by

$$T(x,y) = \begin{cases} \{0, a\}, & \text{if } x, y \in [0,b] \times [0,b] \cup (b,1] \times (b,1] \\ \{0\}, & otherwise. \end{cases}$$

Then, at points $x = y = b, z = b + c$ we have $H\big(T(x,y), T(y,z)\big) = a$ and $d(x,y) = 0, d(y,z) = c$. Because $c < a$, therefore we cannot find the nonnegative constants α_1, α_2 such that $\alpha_1 + \alpha_2 < 1$ and

$$H\big(T(x,y), T(y,z)\big) \leq \alpha_1 d(x,y) + \alpha_2 d(y,z).$$

Therefore T is not a set-valued Prešić type contraction.

On the other hand, since $7a < b$ for $\lambda = \frac{a}{2(b-a)}$ we have

$$\lambda k^2(k+1) = \frac{12a}{2(b-a)} = \frac{14a - 2a}{2(b-a)} < \frac{2b - 2a}{2(b-a)} = 1.$$

Now it is easy to see that T is a set-valued Prešić-Chatterjea type contraction with $\lambda = \frac{a}{2(b-a)}$. Thus, all the conditions of Corollary 2.2 are satisfied and T has two fixed points, namely, Fix(T)=$\{0, a\}$.

The following theorem provides a sufficient condition for the uniqueness of common fixed point of mappings.

Theorem 2.5. Let (X, d) be any complete metric space and k be a positive integer. Let $T: X^k \to CB(X)$ and $g: X \to X$ be two mappings such that, all the conditions of Theorem 2.1 are satisfied. Suppose in addition that T and g are weakly compatible in such a way that, for any coincidence point u of T and g we have $T(u, \dots, u) = \{gu\}$, then T and g have a unique common fixed point.

Proof. The existence of coincidence point u and point of coincidence v of the mappings T and g follows from Theorem 2.1. Suppose T and g are weakly compatible in such a way that, for any coincidence point u of T and g we have $T(u, \dots, u) = \{gu\} = \{v\}$. We shall show that the point of coincidence v is unique.

If v' is another point of coincidence with coincidence point u' of T and g, then $T(u',\ldots,u') = \{gu'\} = \{v'\}$.

As H is metric, we obtain

$$\begin{aligned} d(v,v') &= H(\{v\},\{v'\}) \\ &= H\big(T(u,\ldots,u),T(u,\ldots,u)\big) \\ &\leq H\big(T(u,\ldots,u),T(u,\ldots,u,u')\big) \\ &\quad +H\big(T(u,\ldots,u,u'),T(u,\ldots,u,u',u')\big) \\ &\quad +\cdots+H\big(T(u,u',\ldots,u'),T(u',\ldots,u')\big). \end{aligned}$$

Using (4) and the process as used several times before, we obtain

$$\begin{aligned} d(v,v') &\leq \lambda[k(k-1)+(k-1)(k-2)+\cdots+2\cdot 1] \\ &\quad \times \big[d\big(gu,T(u,\ldots,u)\big)+d\big(gu',T(u',\ldots,u')\big)\big] \\ &\quad +\lambda[k+2(k-1)+\cdots+2(k-1)+k] \\ &\quad \times \big[d\big(gu,T(u',\ldots,u')\big)+d\big(gu',T(u,\ldots,u)\big)\big] \\ &= \frac{\lambda k(k+1)(k+2)}{6}[d(v,v')+d(v',v)] \\ &= \frac{\lambda k(k+1)(k+2)}{3}d(v,v'). \end{aligned}$$

Since $\lambda k^2(k+1) < 1$, therefore

$$\frac{\lambda k(k+1)(k+2)}{3} < \frac{k+2}{3k} \leq 1$$

and so, it follows from above inequality that $d(v,v') = 0$, that is, $v = v'$. Thus the point of coincidence of T and g is unique.

Since T and g are weakly compatible, therefore we have

$g\big(T(u,\ldots,u)\big) \subseteq T(gu,\ldots,gu) = T(v,\ldots,v)$, that is, $\{gv\} \subseteq T(v,\ldots,v)$.

Therefore, $gv \in T(v,\ldots,v)$, which shows that gv is another point of coincidence of T and g and by uniqueness we have $v = gv \in T(v,\ldots,v)$. Thus, v is the unique common fixed point of T and g. □

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CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

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