*Gazi University Journal of Science*
*GU J Sci*
*29(3):667-674 (2016)*

# New Operations over Generalized Interval Valued Intuitionistic Fuzzy Sets

Ezzatallah Baloui JAMKHANEH[1,♠]

[1]*Department of Statistics, Qaemshahr Branch, Islamic Azad University, Qaemshahr, Iran*

*Received:27/06/2016 Accepted: 27/07/2016*

**ABSTRACT**

The concept of intuitionistic fuzzy sets and its generalizations play a vital role in modeling uncertainty and vagueness involved in deferent field of science. Recently, generalized interval valued intuitionistic fuzzy sets ($\mathrm{GIVIFS_B}$s) were presented by Baloui Jamkhaneh (2015) and he defined some operations over it. In this paper, defined arithmetic mean operation and geometric mean operation over $\mathrm{GIVIFS_B}$swere proposed and few theorems were proved. In addition, some of the basic properties of the new operations were discussed. By using these new operations, a prioritization method for generalized interval valued intuitionistic fuzzy judgment matrix was proposed.

***Keywords:*** *Generalized interval valued intuitionistic fuzzy sets, generalized intuitionistic fuzzy sets, arithmetic mean operation, geometric mean operation.*

## 1. INTRODUCTION

After the introduction of fuzzy sets by Zadeh (1965), as an extension of crisp sets, a number of generalizations of this concept have come up. The intuitionistic fuzzy sets (IFSs) introduced by Atanassov (1986) is one among them. He defined some operations (as ∪, ∩ ,+, . ) over IFSs. IFSs are a powerful tool to deal with vagueness and use it most comprehensive than fuzzy sets. Atanassov (1994) introduced operations @ and $ over the IFSs. Later on, researcher defined different operations over the IFSs. For example: De et al. (2000) Riecan and Atanassov (2006) Liu et al. (2008) Vasilev (2008), Riecan and Atanassov (2010) Parvathi et al. (2012) Wang and Liu (2013). Following the definition IFS, Atanassov and Gargov (1989) introduced interval valued intuitionistic fuzzy sets (IVIFSs) with several properties on IVIFSs and shown applications of IVIFSs. The membership and non-membership degrees of IVIFSs are intervals instead of real numbers, then it can contain more information.So IVIFSs are more powerful in dealing with vagueness and uncertainty than IFSs. Zhang et al. (2011) introduced a generalized interval valued intuitionistic fuzzy sets. Hui (2013) introduced some operations on interval valued intuitionistic fuzzy sets. Broumi and Smarandache (2014) introduced operations @ and $ over interval valued intuitionistic hesitant fuzzy sets duo to Zhiming Zhang (2013) and studied several important properties. Zhao et al. (2016) studied derivative and differential operations on interval valued intuitionistic fuzzy.

Baloui Jamkhaneh (2015) considered a new generalized interval valued intuitionistic fuzzy sets ($\mathrm{GIVIFS_B}$s) and introduced some operators over $\mathrm{GIVIFS_B}$. He studied the various basic operations like union, intersection, subset, complement and etc..All operations, defined over IVIFS were transformed for the $\mathrm{GIVIFS_B}$ case. In this paper, our aim is to propose two new operations @ and $ over $\mathrm{GIVIFS_B}$s and we will discuss their properties and propose amulti-criteria group decision making method based on the new operations.

♠Corresponding author, e-mail:e_baloui2008@yahoo.com

## 2. PRELIMINARIES

For completeness, some operators and necessity definitions on $\mathrm{GIVIFS_B}$s are reviewed in this section. Let X be a non-empty set.

**Definition2.1***(Atanassov,1986) An IFS A in X is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ denote the degree of membership and non-membership functions of A, respectively and $0 \le \mu_A(x) + \nu_A(x) \le 1$ for each $x \in X$.*

**Definition2.2** *(Atanassov& Gargov, 1989) Let X be a non-empty set. Interval valued intuitionistic fuzzy sets (IVIFS) A in X, is defined as an object of the form $A = \{\langle x, M_A(x), N_A(x)\rangle | x \in X\}$ where the functions $M_A(x): X \to [I]$ and $N_A(x): X \to [I]$, denote the degree of membership and degree of non-membership of A respectively, where $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ , $N_A(x) = [N_{AL}(x), N_{AU}(x)]$, $0 \le M_{AU}(x) + N_{AU}(x) \le 1$ for each $x \in X$.*

**Definition2.3***Let$[I]$ be the set of all closed subintervals of the interval $[0,1]$ and $M_A(x) = [M_{AL}(x), M_{AU}(x)] \in [I]$ and $N_A(x) = [N_{AL}(x), N_{AU}(x)] \in [I]$ then $N_A(x) \le M_A(x)$ if and only if$N_{AL}(x) \le M_{AL}(x)$and $N_{AU}(x) \le M_{AU}(x)$.*

**Definition2.4***(Baloui Jamkhaneh & Nadarajah ,2015) Let X be a non-empty set. Generalized intuitionistic fuzzy sets ($GIFS_B$) A in X, is defined as an object of the form $A = \{\langle x, \mu_A(x), \nu_A(x)\rangle : x \in X\}$ where the functions $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$, denote the degree of membership and degree of non-membership functions of A respectively, and $0 \le \mu_A(x)^\delta + \nu_A(x)^\delta \le 1$ for each $x \in X$ and $\delta = n$ or $\frac{1}{n}, n = 1,2,\ldots,N$.*

**Definition2.5***(Baloui Jamkhaneh, 2015) Let X be a non-empty set. Generalized interval valued intuitionistic fuzzy sets ($GIVIFS_B$) A in X, is defined as an object of the form $A = \{\langle x, M_A(x), N_A(x)\rangle | x \in X\}$ where the functions $M_A(x): X \to [I]$ and $N_A(x): X \to [I]$, denote the degree of membership and degree of non-membership of A respectively, and $M_A(x) = [M_{AL}(x), M_{AU}(x)]$ , $N_A(x) = [N_{AL}(x), N_{AU}(x)]$ , where $0 \le M_{AU}(x)^\delta + N_{AU}(x)^\delta \le 1$, for each $x \in X$ and $\delta = n$ or $\frac{1}{n}, n = 1,2,\ldots,N$. The collection of all $GIVIFS_B(\delta)$ is denoted by$GIVIFS_B(\delta, X)$. A GIVIFS value is denoted by $A = ([M_{AL}(x), M_{AU}(x)], [N_{AL}(x), N_{AU}(x)])$ for convenience.*

**Definition2.6** The degree of non-determinacy (uncertainty) of an element $x \in X$ to the $GIVIFS_B$ A is defined by

$$\pi_A(x) = [\pi_{AL}(x), \pi_{AU}(x)] = \left[ \left(1 - M_{AU}(x)^\delta - N_{AU}(x)^\delta\right)^{\frac{1}{\delta}}, \left(1 - M_{AL}(x)^\delta - N_{AL}(x)^\delta\right)^{\frac{1}{\delta}} \right].$$

**Definition2.7** For every $G\mathrm{IVIFS_B} A = \{\langle x, \mathrm{M_A}(x), N_A(x)\rangle : x \in \mathrm{X}\}$ , we define the modal logic operators "necessity" and "possibility".

The Necessity measure on A:

$$\Box A = \left\{\langle x, [M_{AL}(x), M_{AU}(x)], [(1 - M_{AU}(x)^\delta)^{\frac{1}{\delta}}, (1 - M_{AL}(x)^\delta)^{\frac{1}{\delta}}]\rangle : x \epsilon X\right\},$$

The Possibility measure on A:

$$\Diamond A = \{\langle x, \left[\left(1 - N_{AU}(x)^\delta\right)^{\frac{1}{\delta}}, \left(1 - N_{AL}(x)^\delta\right)^{\frac{1}{\delta}}\right], [N_{AL}(x), N_{AU}(x)]\rangle : x \epsilon X\}.$$

## 3. MAIN RESULTS

Here, we will introduce new operations over the $GIVIFS_B$ , which extend two operations in the research literature related to $IVIFS$. Let X is a non-empty finite set.

**Definition3.1**For every $\mathrm{GIVIFS_B}$ as $A = \{\langle x, M_A(x), N_A(x)\rangle : x \in X\}$ and $B = \{\langle x, M_B(x), N_B(x)\rangle : x \in X\}$ we define the arithmetic mean operation and geometric mean operation as follows:

i. $A@B = \{\langle x, M_{A@B}(x), N_{A@B}(x)\rangle | x \in X\}$,

$$M_{A@B}(x) = [\left(\frac{1}{2}\left(M_{AL}(x)^\delta + M_{BL}(x)^\delta\right)\right)^{\frac{1}{\delta}}, (\frac{1}{2}\left(M_{AU}(x)^\delta + M_{BU}(x)^\delta\right))^{\frac{1}{\delta}}],$$

$$N_{A@B}(x) = [\left(\frac{1}{2}\left(N_{AL}(x)^\delta + N_{BL}(x)^\delta\right)\right)^{\frac{1}{\delta}}, \left(\frac{1}{2}\left(N_{AU}(x)^\delta + N_{BU}(x)^\delta\right)\right)^{\frac{1}{\delta}}].$$

ii. $A\$B = \{\langle x, M_{A\$B}(x), N_{A\$B}(x)\rangle | x \in X\}$,

$$M_{A\$B}(x) = [\sqrt{M_{AL}(x).M_{BL}(x)}, \sqrt{M_{AU}(x).M_{BU}(x)}],$$

$$N_{A\$B}(x) = [\sqrt{N_{AL}(x).N_{BL}(x)}, \sqrt{N_{AU}(x).N_{BU}(x)}].$$

**Example3.1**If $A = \{\langle x, [0.2, 0.3], [0.3,0.4]\rangle : x \in X\}$ , $B = \{\langle x, [0.3,0.4], [0.2,0.4]\rangle : x \in X\}$ , $\delta = 2$ , then the arithmetic mean operation and geometric mean operation as follows:

$$A@B = \left\{\langle x, [(\tfrac{1}{2}(0.2^2 + 0.3^2))^{\frac{1}{2}}, (\tfrac{1}{2}(0.3^2 + 0.4^2))^{\frac{1}{2}}], [(\tfrac{1}{2}(0.3^2 + 0.2^2))^{\frac{1}{2}}, (\tfrac{1}{2}(0.4^2 + 0.4^2))^{\frac{1}{2}}], \rangle : x \epsilon X\right\} = \{\langle x, [0.2549, 0.3535], [0.2549,0.4]\rangle : x \epsilon X\},$$

$$A\$B = \left\{\langle x, [\sqrt{0.2 \times 0.3}, \sqrt{0.3 \times 0.4}], [\sqrt{0.3 \times 0.2}, \sqrt{0.4 \times 0.4}]\rangle : x \epsilon X\right\} = \{\langle x, [0.2449, 0.3446], [0.2449,0.4]\rangle : x \epsilon X\}.$$

**Theorem3.1**For every two $GIVIFS_B$s A and B, we have

i. $A@B$ is a $GIVIFS_B$,

ii. $A\$B$ is a $GIVIFS_B$.

**Proof.** (i)By using Definition2.4, we have

$M_{A@BU}(x)^\delta + N_{A@BU}(x)^\delta = \frac{1}{2}\left(M_{AU}(x)^\delta + M_{BU}(x)^\delta\right) + \frac{1}{2}\left(N_{AU}(x)^\delta + N_{BU}(x)^\delta\right),$

$= \frac{1}{2}(M_{AU}(x)^\delta + N_{AU}(x)^\delta) + \frac{1}{2}(M_{BU}(x)^\delta + N_{BU}(x)^\delta),$

$\le \frac{1}{2} + \frac{1}{2} = 1.$

Finally, it can be concluded that $A@B \in GIVIFS_B$s.

(ii)By using Definition2.4, and $\sqrt{a^\delta . b^\delta} \le \frac{1}{2}\ (a^\delta + b^\delta)$ , we have

$M_{A\$BU}(x)^\delta + N_{A\$BU}(x)^\delta = (\sqrt{M_{AU}(x).M_{BU}(x)})^\delta + (\sqrt{N_{AU}(x).N_{BU}(x)})^\delta,$

$\le \frac{1}{2}\ (M_{AU}(x)^\delta + M_{BU}(x)^\delta) + \frac{1}{2}(\ N_{AU}(x)^\delta + N_{BU}(x)^\delta),$

$\le \frac{1}{2}\ (M_{AU}(x)^\delta + N_{AU}(x)^\delta) + \frac{1}{2}\ (M_{BU}(x)^\delta + N_{BU}(x)^\delta) \le 1.$

Finally, it can be concluded that $A\$B \in GIVIFS_B$s.

**Theorem 3.2**For every two GIVIFSs A, B: (Idempotent laws)

i. $A@B = B@A,$

ii. $A\$B = B\$A.$

**Proof.** (i)By using Definition3.1, we have

$M_{A@B}(x) = [\left(\frac{1}{2}\left(M_{AL}(x)^\delta + M_{BL}(x)^\delta\right)\right)^{\frac{1}{\delta}}, \left(\frac{1}{2}\left(M_{AU}(x)^\delta + M_{BU}(x)^\delta\right)\right)^{\frac{1}{\delta}}],$

$= [\left(\frac{1}{2}\left(M_{BL}(x)^\delta + M_{AL}(x)^\delta\right)\right)^{\frac{1}{\delta}}, \left(\frac{1}{2}\left(M_{BU}(x)^\delta + M_{AU}(x)^\delta\right)\right)^{\frac{1}{\delta}}] = M_{B@A}(x),$

$N_{A@B}(x) = [\left(\frac{1}{2}\left(N_{AL}(x)^\delta + N_{BL}(x)^\delta\right)\right)^{\frac{1}{\delta}},$

$\left(\frac{1}{2}\left(N_{AU}(x)^\delta + N_{BU}(x)^\delta\right)\right)^{\frac{1}{\delta}}],$

$= [\left(\frac{1}{2}\left(N_{BL}(x)^\delta + N_{AL}(x)^\delta\right)\right)^{\frac{1}{\delta}},$

$\left(\frac{1}{2}\left(N_{BU}(x)^\delta + N_{AU}(x)^\delta\right)\right)^{\frac{1}{\delta}}] = N_{B@A}(x).$

The proof is completed. Proof (ii) is similar to that of (i).

**Theorem 3.3** For every two GIVIFSs A, B: (Commutative laws)

i. $A@A = A,$

ii. $A\$A = A.$

**Proof**. These also follow from Definition3.1.

**Theorem 3.4**For every two GIVIFSs A, B: (Complementary Law)

i. $\overline{\bar{A}@\bar{B}} = A@B,$

ii. $\overline{\bar{A}\$\bar{B}} = A\$B.$

**Proof.** (i) Since $\bar{A} = \{\langle x\ , N_A(x), M(x)\rangle | x\ \in X\}$ , $\bar{B} = \{\langle x\ , N_B(x), M_B(x)\rangle | x\ \in X\}$ we have

$\bar{A}@\bar{B} = \{\langle x\ , M_{\bar{A}@\bar{B}}(x), N_{\bar{A}@\bar{B}}(x)\rangle | x\ \in X\},$

$\overline{\bar{A}@\bar{B}} = \{\langle x\ , N_{\bar{A}@\bar{B}}(x), M_{\bar{A}@\bar{B}}(x)\rangle | x\ \in X\}.$

Since

$M_{\bar{A}@\bar{B}}(x) = [(\frac{1}{2}(N_{AL}(x)^\delta + N_{BL}(x)^\delta))^{\frac{1}{\delta}},$

$\left(\frac{1}{2}(N_{AU}(x)^\delta + N_{BU}(x)^\delta))^{\frac{1}{\delta}}\right] = N_{A@B}(x),$

$N_{\bar{A}@\bar{B}}(x) = [(\frac{1}{2}(M_{AL}(x)^\delta + M_{BL}(x)^\delta))^{\frac{1}{\delta}},$

$\left(\frac{1}{2}(M_{AU}(x)^\delta + M_{BU}(x)^\delta))^{\frac{1}{\delta}}\right] = M_{A@B}(x).$

Therefore $\overline{\bar{A}@\bar{B}} = A@B$.The proof is completed. Proof (ii) is similar to that of (i).

**Theorem3.5**For every three $GIVIFS_B$s A, B and C,(Associative Laws)

i. $(A@B)@C = (A@C)@(B@C),$

ii. $(A\$B)\$C = (A\$C)\$(B\$C).$

**Proof**. (i)

$M_{(A@B)@C}(x) = [(\frac{1}{2}(M_{A@BL}(x)^\delta + M_{CL}(x)^\delta))^{\frac{1}{\delta}},$

$(\frac{1}{2}(M_{A@BU}(x)^\delta + M_{CU}(x)^\delta))^{\frac{1}{\delta}}],$

$= [(\frac{1}{2}\left(\frac{1}{2}\left(M_{AL}(x)^\delta + M_{BL}(x)^\delta\right) + M_{CL}(x)^\delta\right))^{\frac{1}{\delta}},$

$(\frac{1}{2}\left(\frac{1}{2}\left(M_{AU}(x)^\delta + M_{BU}(x)^\delta\right) + M_{CU}(x)^\delta\right))^{\frac{1}{\delta}}],$

$= \left\{\langle x, (\frac{1}{2}\left(\frac{1}{2}\left(M_{AL}(x)^\delta + M_{CL}(x)^\delta\right) + \frac{1}{2}(M_{BL}(x)^\delta + M_{CL}(x)^\delta)\right))^{\frac{1}{\delta}}, (\frac{1}{2}\left(\frac{1}{2}\left(M_{AU}(x)^\delta + M_{CU}(x)^\delta\right) + \frac{1}{2}(M_{BU}(x)^\delta + M_{CU}(x)^\delta)\right))^{\frac{1}{\delta}}\rangle : x \epsilon X\right\},$

$= M_{(A@C)@(B@C)}(x).$

And

$N_{(A@B)@C}(x) = [(\frac{1}{2}(N_{A@BL}(x)^\delta + N_{CL}(x)^\delta))^{\frac{1}{\delta}},$

$(\frac{1}{2}(N_{A@BU}(x)^\delta + N_{CU}(x)^\delta))^{\frac{1}{\delta}}],$

$= [(\frac{1}{2}\left(\frac{1}{2}\left(N_{AL}(x)^\delta + N_{BL}(x)^\delta\right) + N_{CL}(x)^\delta\right))^{\frac{1}{\delta}},$

$(\frac{1}{2}\left(\frac{1}{2}\left(N_{AU}(x)^\delta + N_{BU}(x)^\delta\right) + N_{CU}(x)^\delta\right))^{\frac{1}{\delta}}],$

$= [\left(\frac{1}{2}\left(\frac{1}{2}\left(N_{AL}(x)^\delta + N_{CL}(x)^\delta\right) + \frac{1}{2}(N_{BL}(x)^\delta + N_{CL}(x)^\delta)\right)\right)^{\frac{1}{\delta}}, \left(\frac{1}{2}\left(\frac{1}{2}\left(N_{AU}(x)^\delta + N_{CU}(x)^\delta\right) + \frac{1}{2}(N_{BU}(x)^\delta + N_{CU}(x)^\delta)\right)\right)^{\frac{1}{\delta}}],$

$= N_{(A@C)@(B@C)}(x).$

Proof is complete.

(ii)

$M_{(A\$B)\$C}(x) = [\sqrt{M_{A\$BL}(x)M_{CL}(x)}, \sqrt{M_{A\$BU}(x)M_{CU}(x)}],$

$= [\sqrt{\sqrt{M_{AL}(x).M_{BL}(x)}M_{CL}(x)}, \sqrt{\sqrt{M_{AU}(x).M_{BU}(x)}M_{CU}(x)}],$

$= [\sqrt{\sqrt{M_{AL}(x)M_{CL}(x)}.\sqrt{M_{BL}(x)M_{CL}(x)}},$

$\sqrt{\sqrt{M_{AU}(x)M_{CU}(x)}.\sqrt{M_{BU}(x)M_{CU}(x)}}],$

$= M_{(A\$C)\$(B\$C)}(x).$

$N_{(A\$B)\$C}(x) = [\sqrt{N_{A\$BL}(x)N_{CL}(x)}, \sqrt{N_{A\$BU}(x)N_{CU}(x)}],$

$= [\sqrt{\sqrt{N_{AL}(x).N_{BL}(x)}N_{CL}(x)}, \sqrt{\sqrt{N_{AU}(x).N_{BU}(x)}N_{CU}(x)}]$ ,

$= [\sqrt{\sqrt{N_{AL}(x)N_{CL}(x)}.\sqrt{N_{BL}(x)N_{CL}(x)}},$

$\sqrt{\sqrt{N_{AU}(x)N_{CU}(x)}.\sqrt{N_{BU}(x)N_{CU}(x)}}],$

$= N_{(A\$C)\$(B\$C)}(x).$

Proof is complete.

**Theorem 3.6** For every three GIVIFSs A, B and C: (Distributive laws)

i. $(A\cup B)@C = (A@C)\cup(B@C),$

ii. $(A\cap B)@C = (A@C)\cap(B@C),$

iii. $(A\cup B)\$C = (A\$C)\cup(B\$C),$

iv. $(A\cap B)\$C = (A\$C)\cap(B\$C).$

**Proof.**(i) Since $A\cup B = \{\langle x, \max(M_A(x), M_B(x)), \min(N_A(x), N_B(x))\rangle : x\epsilon X\}$, we have

$M_{(A\cup B)@C} = [(\frac{1}{2}(max\left(M_{AL}(x), M_{BL}(x)\right)^\delta + M_{CL}(x)^\delta))^{\frac{1}{\delta}},$

$(\frac{1}{2}(max\left(M_{AU}(x), M_{BU}(x)\right)^\delta + M_{CU}(x)^\delta))^{\frac{1}{\delta}}],$

$= [(\frac{1}{2}(\max\left(M_{AL}(x)^\delta, M_{BL}(x)^\delta\right) + M_{CL}(x)^\delta))^{\frac{1}{\delta}},$

$(\frac{1}{2}\left(\max\left(M_{AU}(x)^\delta, M_{BU}(x)^\delta\right) + M_{CU}(x)^\delta\right))^{\frac{1}{\delta}}],$

$= [\left(\max\left(\frac{1}{2}(M_{AL}(x)^\delta + M_{CL}(x)^\delta), \frac{1}{2}(M_{BL}(x)^\delta + M_{CL}(x)^\delta)\right)\right)^{\frac{1}{\delta}},$

$(\max\left(\frac{1}{2}(M_{AU}(x)^\delta + M_{CU}(x)^\delta), \frac{1}{2}(M_{BU}(x)^\delta + M_{CU}(x)^\delta)\right))^{\frac{1}{\delta}}],$

$= M_{(A@C)\cup(B@C)}.$

$N_{(A\cup B)@C}$
$= [(\frac{1}{2}(max\left(N_{AL}(x), N_{BL}(x)\right)^\delta + N_{CL}(x)^\delta))^{\frac{1}{\delta}}, (\frac{1}{2}(max\left(N_{AU}(x), N_{BU}(x)\right)^\delta + N_{CU}(x)^\delta))^{\frac{1}{\delta}}],$

$= [(\frac{1}{2}(\max(N_{AL}(x)^\delta, N_{BL}(x)^\delta) + N_{CL}(x)^\delta))^{\frac{1}{\delta}}, (\frac{1}{2}\left(\max\left(N_{AU}(x)^\delta, N_{BU}(x)^\delta\right) + N_{CU}(x)^\delta\right))^{\frac{1}{\delta}}],$

$= \left[\left(\max\left(\frac{1}{2}(N_{AL}(x)^\delta + N_{CL}(x)^\delta), \frac{1}{2}(N_{BL}(x)^\delta + N_{CL}(x)^\delta)\right)\right)^{\frac{1}{\delta}}, \left(\max\left(\frac{1}{2}(N_{AU}(x)^\delta + N_{CU}(x)^\delta), \frac{1}{2}(N_{BU}(x)^\delta + N_{CU}(x)^\delta)\right)\right)^{\frac{1}{\delta}}\right] = N_{(A@C)\cup(B@C)}.$

Proof is complete. Proof (ii) is similar to that of (i).

iii.

$M_{(A\cup B)\$C} = [\sqrt{\max\left(M_{AL}(x), M_{BL}(x)\right).M_{CL}(x)},$

$\sqrt{\max\left(M_{AU}(x), M_{BU}(x)\right).M_{CU}(x)}],$

$= [\sqrt{\max\left(M_{AL}(x)M_{CL}(x), M_{BL}(x)M_{CL}(x)\right)},$

$\sqrt{\max\left(M_{AU}(x)M_{CU}(x), M_{BU}(x)M_{CU}(x)\right)}],$

$= M_{(A\$C)\cup(B\$C)}$.

And

$N_{(A\cup B)\$C} = [\sqrt{\max\big(N_{AL}(x), N_{BL}(x)\big).N_{CL}(x)}$,

$\sqrt{\max\big(N_{AU}(x), N_{BU}(x)\big).N_{CU}(x)}$,

$= [\sqrt{\max\big(N_{AL}(x)N_{CL}(x), N_{BL}(x)N_{CL}(x)\big)}$,

$\sqrt{\max\big(N_{AU}(x)N_{CU}(x), N_{BU}(x)N_{CU}(x)\big)}]$,

$= N_{(A\$C)\cup(B\$C)}$.

Proof is complete. Proof (iv) is similar to that of (iii).

**Theorem3.7** For every two $GIVIFS_B$s A and B, we have (inclusion laws)

i. If $A \subset B$ then $A@B \subset B$,

ii. If $A \subset B$ then $A\$B \subset B$.

**Proof**. (i) if$a \leq b$ then $a \leq (\frac{1}{2}\big(a^\delta + b^\delta\big))^{\frac{1}{\delta}} \leq b$. In this case, proof is clear.

(ii) if$a \leq b$ then $a \leq \sqrt{ab} \leq b$. In this case, proof is clear.

**Corollary3.1** For every two $GIVIFS_B$s A and B, we have (Absorption laws)

i. $A@(A\cup B) \subset A\cup B$,

ii. $A\$(A\cup B) \subset A\cup B$,

iii. $A@(A\cap B) \subset A$,

iv. $A\$(A\cap B) \subset A$.

Proof. Since $A \subset A\cup B$ , $A\cap B \subset A$ , proofs are clearusing Theorem3.7.

**Theorem 3.8** For every three GIVIFSs A, B and C:

i. $\square(A@B) = \square A@\square B$,

ii. $\square(A\$B) \subset \square A\square\$B$,

iii. $\lozenge\,(A@B) = \lozenge\, A@\,\lozenge\, B$,

iv. $\lozenge\,(A\$B) \supset \lozenge\, A\$\,\lozenge\, B$.

**Proof.**(i)By using Definition3.1 and definition of Necessity measure, we have

$$\square(A@B) = \{\langle x, M_{\square(A@B)}, N_{\square(A@B)}\rangle : x\epsilon X\},$$

since

$$M_{\square(A@B)} = [(\frac{1}{2}(M_{AL}(x)^\delta + M_{BL}(x)^\delta))^{\frac{1}{\delta}}, (\frac{1}{2}(M_{AU}(x)^\delta + M_{BU}(x)^\delta))^{\frac{1}{\delta}}] = M_{\square A@\square B},$$

and

$N_{\square(A@B)} =[(1 - \frac{1}{2}\big(M_{AU}(x)^\delta + M_{BU}(x)^\delta\big))^{\frac{1}{\delta}}, (1 - \frac{1}{2}\big(M_{AL}(x)^\delta + N_{BL}(x)^\delta\big))^{\frac{1}{\delta}}]$,

$= [\left(\frac{1}{2}\Big(\big(1 - M_{AU}(x)^\delta\big) + \big(1 - M_{BU}(x)^\delta\big)\Big)\right)^{\frac{1}{\delta}}$,

$(\frac{1}{2}((1 - M_{AL}(x)^\delta) + (1 - M_{BL}(x)^\delta)))^{\frac{1}{\delta}}]$,

$= N_{\square A@\square B}$.

Therefore$\square(A@B)= \square A@\square B$.Proof is complete.

(ii)By using Definition3.1 and definition of Necessity measure, we have

$A\$B =\{\langle x, M_{A\$B}, N_{A\$B}\rangle : x\epsilon X\}$

$=\Big\{\langle x, [\sqrt{M_{AL}(x).M_{BL}(x)}\,, \sqrt{M_{AU}(x).M_{BU}(x)}], [(1 - \big(\sqrt{M_{AU}(x).M_{BU}(x)}\big)^\delta)^{\frac{1}{\delta}}, (1 - \big(\sqrt{M_{AL}(x).M_{BL}(x)}\big)^\delta)^{\frac{1}{\delta}}]\rangle : x\epsilon X\Big\}$,

$\square A\$\square B = \{\langle x, M_{\square A\$\square B}, N_{\square A\$\square B}\rangle : x\epsilon X\}=$

$\Big\{\langle x, \Big[\sqrt{M_{AL}(x).M_{BL}(x)}\,, \sqrt{M_{AU}(x).M_{BU}(x)}\Big]$,

$[\sqrt{(1 - M_{AU}(x)^\delta)^{\frac{1}{\delta}}(1 - M_{BU}(x)^\delta)^{\frac{1}{\delta}}}, \sqrt{(1 - M_{AL}(x)^\delta)^{\frac{1}{\delta}}(1 - M_{BL}(x)^\delta)^{\frac{1}{\delta}}})\rangle :$

$x\epsilon X\Big\}$,

To prove $\square(A\$B) \subset \square B\$\square A$, it is enough to prove $N_{\square(A\$B)} \geq N_{\square A\$\square B}$. It is clear

$0 \leq (\sqrt{a^\delta} + \sqrt{b^\delta})^2$, therefore

$0 \leq a^\delta + b^\delta - 2\big(\sqrt{a.b}\big)^\delta$,

$0 \leq a^\delta + b^\delta + \big(1 + (ab)^\delta\big) - (1 + (ab)^\delta) - 2\big(\sqrt{ab}\big)^\delta$,

$\big(1 - a^\delta\big)\big(1 - b^\delta\big) \leq 1 + (ab)^\delta - 2\big(\sqrt{ab}\big)^\delta$,

$\sqrt{(1 - a^\delta)(1 - b^\delta)} \leq 1 - \big(\sqrt{ab}\big)^\delta$.

Proof is complete.

(iii)By using the definition of Possibility measure, we have

$\lozenge\,(A@B) = \{\langle x, M_{\lozenge(A@B)}, N_{\lozenge(A@B)}\rangle : x\epsilon X\}$.

Since

$M_{\lozenge(A@B)} = [(1 - \frac{1}{2}\big(N_{AU}(x)^\delta + N_{BU}(x)^\delta\big))^{\frac{1}{\delta}}$,

$\Big(1 - \frac{1}{2}\big(N_{AL}(x)^\delta + N_{BL}(x)^\delta\big)\Big)^{\frac{1}{\delta}}]$,

$= [(\frac{1}{2}((1 - N_{AU}(x)^\delta) + (1 - N_{BU}(x)^\delta)))^{\frac{1}{\delta}}$,

$(\frac{1}{2}((1-N_{AL}(x)^{\delta})+(1-N_{BL}(x)^{\delta})))^{\frac{1}{\delta}}]$,

$= M_{\lozenge A@\lozenge B}$,

$$\lozenge A = \{\langle x, \left[\left(1- N_{AU}(x)^{\delta}\right)^{\frac{1}{\delta}}, \left(1 - N_{AL}(x)^{\delta}\right)^{\frac{1}{\delta}}\right], [N_{AL}(x), N_{AU}(x)]\rangle : x\epsilon X\}.$$

and

$$N_{\lozenge(A@B)} = [(\tfrac{1}{2}(N_{AL}(x)^{\delta} + N_{BL}(x)^{\delta}))^{\frac{1}{\delta}}, (\tfrac{1}{2}(N_{AU}(x)^{\delta} + N_{BU}(x)^{\delta}))^{\frac{1}{\delta}}] = N_{\lozenge A@\lozenge B},$$

Therefore $\square(A@B) = \square A@\square B$.

Proof is complete.Proof (iv) are similar to that of (ii).

**Theorem3.9**For every two GIVIFS$_{\text{B}}$s A and B,(Distributive laws):

i. $(A@B) + C = (A + C)@(B + C)$,

ii. $(A@B).C = (A.C)@(B.C)$.

**Proof**. (i)

$$M_{(A@B)+C} = \Big[\frac{1}{2}\left(M_{AL}(x)^{\delta} + M_{BL}(x)^{\delta}\right) + M_{CL}(x)^{\delta} - \frac{1}{2}\left(M_{AL}(x)^{\delta} + M_{BL}(x)^{\delta}\right)M_{CL}(x)^{\delta}, \frac{1}{2}\left(M_{AU}(x)^{\delta} + M_{BU}(x)^{\delta}\right) + M_{CU}(x)^{\delta} - \frac{1}{2}\left(M_{AU}(x)^{\delta} + M_{BU}(x)^{\delta}\right)M_{CU}(x)^{\delta}\Big].$$

Since

$$f_{(A@B)+C} = \tfrac{1}{2}\left(a^{\delta} + b^{\delta}\right) + c^{\delta} - \tfrac{1}{2}\left(a^{\delta} + b^{\delta}\right)c^{\delta},$$

$$= \tfrac{1}{2}(\left(a^{\delta} + c^{\delta}\right) + (b^{\delta} + c^{\delta})) - \tfrac{1}{2}(a^{\delta}c^{\delta} + b^{\delta}c^{\delta}),$$

$$= \tfrac{1}{2}\left(\left(a^{\delta} + c^{\delta} - a^{\delta}c^{\delta}\right) + \left(b^{\delta} + c^{\delta} - b^{\delta}c^{\delta}\right)\right) = f_{(A+C)@(B+C)}.$$

Therefore $M_{(A@B)+C} = M_{(A+C)@(B+C)}$.

$$N_{(A@B)+C} = [\frac{1}{2}(N_{AL}(x)^{\delta} + N_{BL}(x)^{\delta})N_{CL}(x)^{\delta}, \frac{1}{2}(N_{AU}(x)^{\delta} + N_{BU}(x)^{\delta})N_{CU}(x)^{\delta}],$$

Since

$$g_{(A@B)+C} = \tfrac{1}{2}(a^{\delta} + b^{\delta})c^{\delta} = \tfrac{1}{2}\left(a^{\delta}c^{\delta} + b^{\delta}c^{\delta}\right) = g_{(A+C)@(B+C)}.$$

Therefore $N_{(A@B)+C} = N_{(A+C)@(B+C)}$.Proof (ii) is similar to that of (i).

**Definition3.2**For every GIVIFS$_{\text{B}}$s as $A_i = \left\{\langle x, M_{A_i}(x), N_{A_i}(x)\rangle : x \in X\right\}, i = 1, \dots, n$ we define the arithmetic mean operation and geometric mean operation of $A_i$, $i = 1, \dots, n$ as follows:

(i) $@_{i=1}^{n} A_i = \left\{\langle x, M_{@_{i=1}^{n} A_i}(x), N_{@_{i=1}^{n} A_i}(x)\rangle : x\epsilon X\right\}$,

$M_{@_{i=1}^{n} A_i}(x)=$
$[(\frac{1}{\mathrm{n}}\left(\sum_{i=1}^{n} M_{A_iL}(x)^{\delta}\right))^{\frac{1}{\delta}}, (\frac{1}{\mathrm{n}}\left(\sum_{i=1}^{n} M_{A_iU}(x)^{\delta}\right))^{\frac{1}{\delta}}]$,

$N_{@_{i=1}^{n} A_i}(x)=$
$[(\frac{1}{\mathrm{n}}\left(\sum_{i=1}^{n} N_{A_iL}(x)^{\delta}\right))^{\frac{1}{\delta}}, (\frac{1}{\mathrm{n}}\left(\sum_{i=1}^{n} N_{A_iU}(x)^{\delta}\right))^{\frac{1}{\delta}}]$.

(ii) $\$_{i=1}^{n} A_i = \left\{\langle x, M_{\$_{i=1}^{n} A_i}(x), N_{\$_{i=1}^{n} A_i}(x)\rangle : x\epsilon X\right\}$,

$$M_{\$_{i=1}^{n} A_i}(x) = [\sqrt[n]{\Pi_{\mathrm{i=1}}^{\mathrm{n}} M_{A_iL}(x)}, \sqrt[n]{\Pi_{\mathrm{i=1}}^{\mathrm{n}} M_{A_iU}(x)}],$$

$$\mathrm{N}_{\$_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{A_i}}(\mathrm{x}) = [\sqrt[\mathrm{n}]{\Pi_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{N}_{\mathrm{A_iL}}(\mathrm{x})}, \sqrt[\mathrm{n}]{\Pi_{\mathrm{i=1}}^{\mathrm{n}} \mathrm{N}_{\mathrm{A_iU}}(\mathrm{x})}],$$

**Corollary3.2**For every GVIIFS$_{\text{B}}$s as $A_i = \left\{\langle x, M_{A_i}(x), N_{A_i}(x)\rangle : x \in X\right\}, i = 1, \dots, n$ and C, we have

i. $\$_{i=1}^{n} A_i \in$ GIVIFS$_{\text{B}}$,

ii. $\$_{i=1}^{n} k A_i = k\$_{i=1}^{n} A_i$,

iii. $\overline{\$_{i=1}^{n} \overline{A_i}} = \$_{i=1}^{n} A_i$,

iv. $(\$_{i=1}^{n} A_i)\$\mathrm{C} = \$_{i=1}^{n} (A_i\$\mathrm{C})$.

Using previous theorems simply proved to be these relations.

**Corollary3.3**For every GIVIFS$_{\text{B}}$ as $A_i = \left\{\langle x, M_{A_i}(x), N_{A_i}(x)\rangle : x \in X\right\}$, $i = 1, \dots, n$ and C, we have

i. $@_{i=1}^{n} A_i \in$ GIVIFS$_{\text{B}}$,

ii. $@_{i=1}^{n} k A_i = k@_{i=1}^{n} A_i$,

iii. $\overline{@_{i=1}^{n} \overline{A_i}} = @_{i=1}^{n} A_i$,

iv. $(@_{i=1}^{n} A_i)@\mathrm{C} = @_{i=1}^{n} (A_i@\mathrm{C})$,

v. $\square(@_{i=1}^{n} A_i) = @_{i=1}^{n} \square A_i$,

vi. $\lozenge(@_{i=1}^{n} A_i) = @_{i=1}^{n} \lozenge A_i$.

Using previous theorems simply proved to be these relations.

## 4. PRIORITIZATION METHOD FOR GENERALIZED INTERVAL VALUED INTUITIONISTIC FUZZY JUDGMENT MATRIX

In a multi-criteria decision making problem, suppose that there exists a set of criteria $U = \{U_1, U_2, \dots, U_n\}$. Preference $\mathrm{U_i}$ than $\mathrm{U_j}$ can be represented by a $GIVIFS_B$ value as $r_{ij} = ([a_{ij}, b_{ij}], [c_{ij}, d_{ij}])$, which can represent the membership degree and non-membership degree of the preference $\mathrm{U_i}$ than $\mathrm{U_j}$ for the interval valued intuitionistic fuzzy concept “excellence”.The decision makers provide intuitionistic fuzzy preference for each pair of criteria, and construct a generalized interval valued intuitionistic judgment matrix. A generalized interval valued intuitionistic fuzzy judgment matrix of $l^{th}$ decision maker is defined as follows.

$$r^{(l)} = \begin{bmatrix} r_{11}^{(l)} r_{12}^{(l)} \dots r_{1n}^{(l)} \\ r_{21}^{(l)} r_{22}^{(l)} \cdots r_{2n}^{(l)} \\ \vdots \quad \vdots \quad \cdots \quad \vdots \\ r_{n1}^{(l)} r_{n2}^{(l)} \ddots r_{nn}^{(l)} \end{bmatrix},$$

where $r_{ij}^{(l)} = ([M_{ijL}^{(l)}, M_{ijU}^{(l)}], [N_{ijL}^{(l)}, N_{ijU}^{(l)}])$, ${M_{ijU}^{(l)}}^{\delta} + {N_{ijU}^{(l)}}^{\delta} \leq 1, M_{ijL}^{(l)} = N_{jiL}^{(l)}, M_{ijU}^{(l)} = N_{jiU}^{(l)}, r_{ii}^{(l)} = ([0.5, 0.5], [0.5, 0.5]), i = 1, \dots, n, j = 1, \dots, n, l = 1, \dots, k,$

The next four steps can summarize the criteria ranking.

Step1. Calculate the finally judgment matrix $r = (r_{ij})_{n \times n}$ using combine interval-valued intuitionistic fuzzy judgment matrix of decision makers,

$$r_{ij} = ([\$_{l=1}^{k} M_{ijL}^{(l)}, \$_{l=1}^{k} M_{ijU}^{(l)}], [\$_{l=1}^{k} N_{ijL}^{(l)}, \$_{l=1}^{k} N_{ijU}^{(l)}]), i = 1, \dots, n, j = 1, \dots, n,$$

Step2.The collective overall preference values of $i^{th}$ criteria calculate as follows

$$r_i = ([@_{j=1}^{n} \$_{l=1}^{k} M_{ijL}^{(l)}, @_{j=1}^{n} \$_{l=1}^{k} M_{ijU}^{(l)}],$$

$$[@_{j=1}^{n} \$_{l=1}^{k} N_{ijL}^{(l)}, @_{j=1}^{n} \$_{l=1}^{k} N_{ijU}^{(l)}]), i = 1, \dots, n,$$

Step3. Calculate score function $S(r_i)$ and accuracy function $H(r_i)$

$$\Delta = (@_{j=1}^{n} \$_{l=1}^{k} M_{ijL}^{(l)})^{\delta} + (@_{j=1}^{n} \$_{l=1}^{k} M_{ijU}^{(l)})^{\delta} -$$

$$\left(@_{j=1}^{n} \$_{l=1}^{k} N_{ijL}^{(l)}\right)^{\delta} - \left(@_{j=1}^{n} \$_{l=1}^{k} N_{ijU}^{(l)}\right)^{\delta},$$

$$S(r_i) = \frac{|\Delta|^{\frac{1}{\delta}} sgn(\Delta)}{2},$$

$$H(r_i) = \frac{((@_{j=1}^{n} \$_{l=1}^{k} M_{ijL}^{(l)})^{\delta} + (@_{j=1}^{n} \$_{l=1}^{k} M_{ijU}^{(l)})^{\delta} + \left(@_{j=1}^{n} \$_{l=1}^{k} N_{ijL}^{(l)}\right)^{\delta} + (@_{j=1}^{n} \$_{l=1}^{k} N_{ijU}^{(l)})^{\delta})^{\frac{1}{\delta}}}{2}$$

.

Step4. Ranking order of the criteria is as follows

(i) if $S(r_{i_1}) < S(r_{i_2})$ then $r_{i_1}$ is smaller than $r_{i_2}$, denoted by $r_{i_1} < r_{i_2}$, that is, $U_{i_2}$ has higher priority than $U_{i_1}$.

(ii) if $S(r_{i_1}) > S(r_{i_2})$ then $r_{i_1}$ is biger than $r_{i_2}$, denoted by $r_{i_1} > r_{i_2}$, that is, $U_{i_1}$ has higher priority than $U_{i_2}$.

(iii) if $S(r_{i_1}) = S(r_{i_2})$ and $H(r_{i_1}) < H(r_{i_2})$ then $r_{i_1}$ is smaller than $r_{i_2}$,

(iv) if $S(r_{i_1}) = S(r_{i_2})$ and $H(r_{i_1}) > H(r_{i_2})$ then $r_{i_1}$ is biger than $r_{i_2}$,

(v) if $S(r_{i_1}) = S(r_{i_2})$ and $H(r_{i_1}) = H(r_{i_2})$ then $r_{i_1}$ and $r_{i_2}$ represent the same information, denoted by $r_{i_1} = r_{i_2}$, that is, priority $r_{i_1}$ and $r_{i_2}$ are same.

**Example4.1** A customer intends to buy a car. The customer takes into account the following threecriteria: design ($S_1$), price ($S_2$), level of after-sale service ($S_3$). Generalized interval-valued intuitionistic fuzzy judgment matrix of decision makers ($\delta = 2$) are as follows.

$$r^{(1)} = \begin{bmatrix} ([0.5,0.5],[0.5,0.5])([0.3,0.6],[0.1,0.3])([0.5,0.7],[0.1,0.2]) \\ ([0.1,0.3],[0.3,0.6])([0.5,0.5],[0.5,0.5])([0.4,0.5],[0.2,0.3]) \\ ([0.1,0.2],[0.5,0.7])([0.2,0.3],[0.4,0.5])([0.5,0.5],[0.5,0.5]) \end{bmatrix},$$

$$r^{(2)} = \begin{bmatrix} ([0.5,0.5],[0.5,0.5])([0.2,0.5],[0.1,0.2])([0.6,0.7],[0.1,0.2]) \\ ([0.1,0.2],[0.2,0.5])([0.5,0.5],[0.5,0.5])([0.5,0.7],[0.2,0.4]) \\ ([0.1,0.2],[0.6,0.7])([0.2,0.4],[0.5,0.7])([0.5,0.5],[0.5,0.5]) \end{bmatrix}.$$

The finally judgment matrix is as follows

$$r = \begin{bmatrix} [0.5,0.5],[0.5,0.5] & [0.244,0.547],[0.1,0.244] & [0.547,0.7],[0.1,0.2] \\ [0.1,0.244],[0.244,0.547] & [0.5,0.5],[0.5,0.5] & [0.447,0.591],[0.2,0.346] \\ [0.1,0.2],[0.547,0.7] & [0.2,0.346],[0.447,0.591] & [0.5,0.5],[0.5,0.5] \end{bmatrix}.$$

The collective overall preference values of criteria and them score function are as follows

$r_1 = [0.4503,0.5886],[0.3,0.3413]$ , $S(r_1) = 0.2927$,

$r_2 = [0.3915,0.4686],[0.3413,0.4722]$,

$S(r_2) = 0.0914$,

$r_3 = [0.3162,0.3696],[0.4997,0.6026]$,

$S(r_3) = -0.3065$.

Therefore, the ranking order of the three criteria is $S_1$, $S_2$ and $S_3$.

## 5. CONCLUSIONS

In this paper, we have defined two new operations over generalized interval valued intuitionistic fuzzy sets and their relationships are proved. We have studied some desirable properties of the proposed operations, such as idempotent laws, complementary law, commutative laws, distributive laws and etc. and applied the these operations to decision making with generalized interval valued intuitionistic fuzzy information.

## CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

**REFERENCES**

[1] Atanassov, K.T. (1986) Intuitionistic fuzzy sets, Fuzzy Sets and Systems, 20(1), 87-96.

[2]Atanassov, K. T., & Gargov, G. (1989) Interval valued intuitionistic fuzzy sets. Fuzzy Sets and Systems, 31(3), 343-349.

[3] Atanassov, K.T. (1994) New operations defined over the intuitionistic fuzzy sets, Fuzzy Sets & Systems, 61, 37-42.

[4] Atanassov, K.T. (2001) Remark on two operations over intuitionistic fuzzy sets, Int. J. of Uncertainty, Fuzziness and Knowledge Syst., 9(1), 71-75.

[5] Baloui Jamkhaneh E., Nadarajah S. (2015) A new generalized intuitionistic fuzzy sets, Hacettepe Journal of Mathematics and Statistics, 44 (6), 1537 – 1551.

[6] Baloui Jamkhaneh E. (2015) New generalized interval value intuitionistic fuzzy sets, Research and Communications in Mathematics and Mathematical Sciences, 5(1), 33-46.

[7] Broumi S., Smarandache F. (2014) New operations over interval valued intuitionistic hesitant fuzzy set, Mathematics and Statistics, 2(2): 62-71.

[8] De, S. K., Biswas R. and Roy A. R. (2000) Some operations on intuitionistic fuzzy sets, Fuzzy Sets and Systems, 114 (4), 477–484.

[9] Hui W. (2013) Some operations on interval-valued intuitionistic fuzzy sets", 2013 Fifth International Conference on Computational and Information Sciences (ICCIS), 832-834, doi:10.1109/ICCIS.2013.223.

[10] Liu Q., Ma C. and Zhou X. (2008) On properties of some IFS operators and operations, Notes on Intuitionistic Fuzzy Sets, 14(3), 17-24.

[11] Riecan B. and Atanassov K. T.(2006) n-extraction operation over intuitionistic fuzzy sets, Notes in Intuitionistic Fuzzy Sets, 12(4), 9–11.

[`12] Riecan B. and Atanassov K. T.(2010) Operation division by n over intuitionistic fuzzy sets, Notes in Intuitionistic Fuzzy Sets, 16 (4), 1–4.

[13] Parvathi R., Riecan B. and Atanassov K. T.(2012) Properties of some operations defined over intuitionistic fuzzy sets, Notes on Intuitionistic Fuzzy Sets, 18 (1), 1–4.

[14] Vasilev, T. (2008) Four equalities connected with intuitionistic fuzzy sets, Notes on Intuitionistic Fuzzy Sets, 14(3), 1-4.

[15] Wang W., Liu X. (2013) Some operations over Atanassovs intutionestic fuzzy sets based on Einstein t-norm and t-conorm, Int. J. Unc. Fuzz. Knowl. Based Syst. 21, 263. DOI: http://dx.doi.org/10.1142/S0218488513500141.

[16] Zadeh L.A. (1965) Fuzzy sets, Information Control, 8, 338-353.

[17] Zhang Z. (2013), Interval-valued intuitionistic hesitant fuzzy aggregation operators and their application in group decision-making, Journal of Applied Mathematics, Vol. 2013, Article ID 670285, http://dx.doi.org/10.1155/2013/670285.

[18] Zhang Z., Yang J., Ye Y., Zhang Q. S. (2011) A generalized interval valued intuitionistic fuzzy sets theory, Procedia Engineering 15, 2037 – 2041.

[19] Zhao H., Xu Z.&Yao Z. (2016) Interval-valued intuitionistic fuzzy derivative and differential operations, International Journal of Computational Intelligence Systems, 9(1), 35-56.