Gazi University Journal of Science
GU J Sci
29(3):695-701 (2016)

The Smallest Dimension Submanifolds of Para $\beta-$ Kenmotsu Manifold

Aysel TURGUT VANLI[1,♠], Ramazan SARI[2]

[1]*Gazi Universty, Faculty of Arts and Sciences, Department of Mathematics, 06100, Ankara-TURKEY*
[2]*Amasya University, Merzifon Vocational School, Amasya, TURKEY*

Received:15/04/2016 Accepted: 30/05/2016

ABSTRACT

In this paper, we have studied the smallest dimensional submanifold of para β-Kenmotsu manifold. Necessary and sufficient conditions are given on 3-dimensional submanifolds of a 5-dimensional para β-Kenmotsu manifold to be a slant submanifold. After that, we have studied the 3-dimensional minimal slant submanifolds of para β-Kenmotsu manifold.

***Key words**: Para β-Kenmotsu manifold, smallest dimension, slant submanifold*

1. INTRODUCTION

As a generalization of invariant submanifold and anti-invariant submanifolds, B.Y. Chen introduced slant submanifolds of almost Hermitian manifold in 1990 [5], [6]. On the other hand A. Lotta introduced the notion of slant immersion of a Riemannian manifold into an almost contact manifold [9]. He also studied 3-dimensional slant submanifolds K-contact manifold [10] . Recently, Cabrerizo et al. [2] studied slant submanifold of Sasakian manifold and general view about slant immersions can be founds in [3]. Khan et al. studied slant submanifold of Kenmotsu manifold [7] , [8] .

In 1976, Sato defined the notion of an almost para contact Riemannian manifold [11]. After [12], Olszak introduced para β-Kenmotsu manifold. Many authors studied smallest dimension submanifolds [4], [8].

The purpose of present paper is to study slant submanifolds of para β-Kenmotsu manifolds with the smallest dimension. The paper organized as follows. In section 2, we give basic formula and defination of para β-Kenmotsu manifold. We review, in section 3, formulas and definitions for para β-Kenmotsu manifolds and their submanifolds, which we use later. In section 4, we obtain the smallet dimension slant submanifold of para β-

♠Corresponding author, e-mail: avanli@gazi.edu.tr

Kenmotsu manifold. Necessary and sufficient conditions are given on a 3-dimensional submanifolds of 5-dimensional para β-Kenmotsu manifold to be slant submanifold after studied 3-dimensional minimal submanifolds of para β-Kenmotsu manifold.

2. PRELIMINARIES

Let M be a *(2n+1)*-dimensional differentiable manifold endowed with a quadruplet (φ,ξ,η,g), where φ is (1,1)-tensor field, ξ is a vector field, η is a 1-form, and g is a pseudo-Riemannian such that

$$\varphi^2 X=\mu(X-\eta(X)\xi), \qquad \eta(\xi)=1 \tag{1}$$

$$g(\varphi x,\varphi Y)=-\mu(g(X,Y)-\varepsilon\eta(X)\eta(Y)) \tag{2}$$

for all $X,Y\epsilon\Gamma(TM)$, where $\mu,\epsilon=\pm 1$. In addition, we have

$$\varphi(\xi)=0, \quad \eta o\varphi=0, \; \eta(X)=\varepsilon g(X,\xi). \tag{3}$$

The manifold M will be called almost para contact metric, and the quadruplet (φ,ξ,η,g) will be called the almost para contact metric structure on M.

When $\mu=1$, then the manifold M is an almost contact metric manifold. In this case the metric g is assumed to be pseudo-Riemannian in general, including Riemannian. Thus, if "$\varepsilon=1$, the signature of g is equal to *2p*, where $0\le p\le n$ and if " $\varepsilon=1$, the signature of g is equal to 2p+1, where $0\le p\le n$.

When $\mu=1$, then the manifold M is an almost paracontact metric manifold. In this case, the metric g is pseudo-Riemannian, and its signature is equal to *n* when " $\varepsilon=1$, or n+1 when " $\varepsilon=-1$. One notes that in this case, the eigenspaces of the linear operator φ corresponding to the eigenvalues 1 and -1 are both *n*-dimensional at every point of the manifold [12].

Then a 2-form Φ is defined by $\Phi(X,Y)=g(X,\varphi Y)$, for any $X,Y\epsilon\Gamma(TM)$, called the *fundamental 2-form*. Moreover, an almost para contact metric manifold is *normal* if

$$[\varphi,\varphi]-2d\eta\otimes\xi=0.$$

where $[\varphi,\varphi]$ is denoting the Nijenhuis tensor field associated to φ [12] . A normal almost para contact metric manifold is called para contact metric manifold.

the almost para contact metric structure on M.

Proposition 1 *Let (M,φ,ξ,η,g) be an almost para contact manifold. Then , the Levi-Civita connection ∇ satisfies the following equality, for any $X,Y,Z\epsilon\Gamma(TM)$,*

$$\begin{aligned}2g\big((\nabla_X\varphi)Y,Z\big)&=3d\Phi(\mathrm{X},\varphi\mathrm{Y},\varphi\mathrm{Z})-3d\Phi(\mathrm{X},\mathrm{Y},\mathrm{Z})\\&\quad+\mathrm{g}(\mathrm{N}(\mathrm{Y},\mathrm{Z}),\varphi\mathrm{X})+\mu N^2(Y,Z)\eta(X)\\&\quad+2\mu d\eta(\varphi Y,X)\eta(Z)\\&\quad-2\mu d\eta(\varphi Z,X)\eta(Y)\end{aligned}$$

where $N^2(X,Y)=2d\eta(\varphi X,Y)-2d\eta(\varphi Y,X)$.

Definition 1 *Let M be an almost para contact metric manifold of dimension (2n+1), with (φ,ξ,η,g). M is said to be an almost para β-Kenmotsu manifold if 1-form η are closed and $d\Phi=2\beta\eta\wedge\Phi$. A normal almost para* β-Kenmotsu *manifold M is called a para β-Kenmotsu manifold.*

Theorem 1 *Let $(\bar{M},\varphi,\xi,\eta,g)$ be an almost para contact metric manifold. $\bar{M}$ is a para β-Kenmotsu manifold if and only if*

$$(\bar{\nabla}_X\varphi)Y=\beta\{g(\varphi X,Y)\xi-\eta(Y)\varphi X\} \tag{4}$$

for all $X,Y\epsilon\Gamma(T\bar{M})$ where $\bar{\nabla}$ is Levi-Civita connection on $\bar{M}$.

Proof. Let $\bar{M}$ be a para β-Kenmotsu manifold. From Proposition 1, $\forall\; X,Y\epsilon\Gamma(T\bar{M})$ we have

$$2g((\bar{\nabla}_X\varphi)Y,Z)=3d\Phi(\mathrm{X},\varphi\mathrm{Y},\varphi\mathrm{Z})-3d\Phi(\mathrm{X},\mathrm{Y},\mathrm{Z})).$$

Then, we have

$$\begin{aligned}g((\bar{\nabla}_X\varphi)Y,Z)&=-\beta\eta(X)g(\varphi Y,\varphi^2 Z)+\beta\eta(X)g(Y,\varphi Z)\\&\quad-\beta\eta(Y)g(Z,\varphi X)-\beta\eta(Z)g(X,\varphi Y)\\&=-\beta\eta(Y)g(Z,\varphi X)-\beta\eta(Z)g(X,\varphi Y)\\&=g(\beta\{g(\varphi X,Y)\xi-\eta(Y)\varphi X\},Z).\end{aligned}$$

Conversely, firstly, using (4) , we get

$$\varphi\bar{\nabla}_X\xi=\beta\{g(\varphi X,\xi)\xi-\eta(\xi)\varphi X\}$$

hence, we get

$$\bar{\nabla}_X\xi=\beta\varphi^2 X.$$

On the other hand, we have

$$d\eta(X,Y)=\frac{1}{2}\{g(Y,-\varphi^2X)-g(X,-\varphi^2Y)\}=0$$

for all $X,Y\epsilon\Gamma(T\overline{M})$. In addition, we know

$$3d\Phi(X,Y,Z)=g(Y,(\nabla_X\varphi)Z)-g(Z,(\nabla_Y\varphi)X) - g(X,(\nabla_Z\varphi)Y)$$

From hypothesis, we have

$$\begin{aligned}3d\Phi(X,Y,Z)=\beta\{&g(\varphi X,Z)g(Y,\xi)-\eta(Z)g(Y,\varphi X)\\&-g(\varphi Y,Z)g(X,\xi)+\eta(Z)g(X,\varphi Y)\\&+g(\varphi Z,Y)g(X,\xi)-\eta(Y)g(X,\varphi Z)\}\\=2\beta\{&\Phi(Z,X)\eta(Y)+\Phi(X,Y)\eta(Z)\\&+\Phi(Y,Z)\eta(X).\end{aligned}$$

Then, we obtain

$$d\Phi=2\beta\eta\wedge\Phi.$$

Moreover, the Nijenhuis torsion of φ is obtained

$$\begin{aligned}N_\varphi(X,Y)=\varphi(-\beta\{&g(\varphi X,Y)\xi-\eta(Y)\varphi X\}\\&+\beta\{g(\varphi Y,X)\xi-\eta(X)\varphi Y\})\\&+\beta\{g(\varphi^2X,Y)\xi-\eta(Y)\varphi^2X\}\\&-\beta\{g(\varphi^2Y,X)\xi-\eta(X)\varphi^2Y\}\\=0.&\end{aligned}$$

Hence, we have

$$[\varphi,\varphi]-2d\eta\otimes\xi=0.$$

The proof is completed.

Corollary 1 *Let $\overline{M}$ be (2n+1)-dimensional a para β-Kenmotsu manifold with structure (φ,ξ,η,g). Then we have*

$$\overline{\nabla}_X\xi=\beta\varphi^2X \tag{5}$$

for all $X,Y\epsilon\Gamma(T\overline{M})$.

3 SUBMANIFOLDS OF PARA β-KENMOTSU MANIFOLD

Now, let M be a submanifold of the (2n+1) dimensional a para β-Kenmotsu manifold $\overline{M}$. Let ∇ be the Levi-Civita connection of M with respect to the induced metric g. Then Gauss and Weingarten formulas are given by

$$\overline{\nabla}_XY=\nabla_XY-h(X,Y) \tag{6}$$

$$\overline{\nabla}_XV=\nabla^{\perp}_XY-A_VX \tag{7}$$

for any $X,Y\epsilon\Gamma(TM)$ and $V\epsilon\Gamma(TM)^{\perp}$. $\nabla^{\perp}$ is the connection in the normal bundle, h is the second fundamental from of M and A_V is the Weingarten endomorphism associated with *V*. The second fundamental form *h* and the shape operator *A* related by

$$g(h(X,Y),V)=g(A_VX,Y). \tag{8}$$

The mean curvature tensor *H* is defined by

$$H=\frac{1}{m}\sum_{k=1}^{m}h(e_k,e_k)$$

where $\{e_1,\dots,e_m\}$ is a local orthonormal basis of *TM*. *M* said to be minimal if *H* vanishes identically.

Now, let $\{e_1,\dots,e_n,e_{n+1},\dots,e_m\}$ be local orthonormal basis of *TM* such that the vector fields $\{e_1,\dots,e_n\}$ are tanget to *M* and $\{e_{n+1},\dots,e_m\}$ are normal to m . Then for any $X\epsilon\Gamma(TM)$

$$\nabla_Xe_i=\textstyle\sum_{j=1}^{n}w_i^je_i+\sum_{k=n+1}^{m}w_i^ke_k \tag{9}$$

$$\nabla_Xe_r=\sum_{j=1}^{n}w_r^je_j+\sum_{k=n+1}^{m}w_r^ke_k$$

where *i=1,…,n* and *r=n+1,…,m* and $w_i^j=g(\nabla_{e_i},e_j)$. The 1-forms w_i^j,w_i^k and w_r^j can called connection forms of M.

On the other hand, the mix second fundamental form in the direction e_r is defined

$$h_{ij}^r=g(h(e_i,e_j),e_r)$$

For every tangent vector field *X* we write

$$\varphi X=TX+NX \tag{10}$$

where *TX* (resp. NX) denotes the tangential (resp. normal) component of φX and NX is the normal one. Moreover for every normal vector field V,

$$\varphi V=tV+nV \tag{11}$$

where *tV* in the tangential component and *nV* is the normal one.

Now, for later use, we establish proposition for submanifolds of para β-Kenmotsu manifold.

Proposition 2 *Let M be submanifold of para β-*

Kenmotsu *manifold* $\bar{M}$. *Then,*

$(\nabla_X T)Y = A_{NY}X + th(X,Y) + \beta\{g(TX,Y)\xi - \eta(Y)TX\}$ (12)

$(\nabla_X N)Y = nh(X,Y) - h(X,TY) - \beta\eta(Y)NX$ (13)

for all $X, Y\epsilon\Gamma(TM)$

Proof. For any $X, Y\epsilon\Gamma(TM)$

$$(\bar{\nabla}_X\varphi)Y = \bar{\nabla}_X\varphi Y - \varphi\bar{\nabla}_X Y.$$

Then, using (4), (6) and (7)

$$\beta\{g(TX+NX,Y)\xi - \eta(Y)(TX+NX)\}$$
$$= \bar{\nabla}_X(TY+NY) - \varphi(\nabla_X Y + h(X,Y))$$
$$= \nabla_X TY + h(X,TY) - A_{NY}X + \nabla_X^{\perp}NY - T\nabla_X Y - N\nabla_X Y - th(X,Y) - nh(X,Y)$$
$$= (\nabla_X T)Y + (\nabla_X N)Y + h(X,TY) - A_{NY}X - th(X,Y) - nh(X,Y)$$

or

$$(\nabla_X T)Y + (\nabla_X N)Y = \beta\{g(TX+NX,Y)\xi - \eta(Y)TX - \eta(Y)NX\} - h(X,TY) + A_{NY}X + th(X,Y) - nh(X,Y).$$

Proposition 3 *Let* M *be submanifold of para* β-Kenmotsu m*anifold* $\bar{M}$, *tanget to the structure vector field. Then,*

$$\nabla_X\xi = \beta\varphi^2 X$$

and

$$h(X,\xi) = 0$$

for any $X, Y\epsilon\Gamma(TM)$.

Now, we defined slant submanifold of para β-Kenmotsu manifold.

Definition 2 *Let* M *be a submanifold of a para* β-Kenmotsu *manifold* $\bar{M}$. M *is a slant submanifold if for any* $x\epsilon M$ *and* $X\epsilon T_x M$ *linearly independent of* $\{\xi\}$, *the angle between* φX *and* $T_x M$ *is a constant* $\theta\epsilon[0,\frac{\pi}{2}]$. *Then* θ *called the slant angle of* M *in* $\bar{M}$.

Theorem 2 *Let* M *be a submanifold of para* β-*Kenmotsu manifold* $\bar{M}$, *tanget to the structure vector fields. Then,* M *is a slant submanifold if and only if there exists a constant* $\lambda\epsilon[0,\frac{\pi}{2}]$. *such that*

$$T^2 = \lambda(I - \eta\otimes\xi) \quad (14)$$

Furthermore in such case, if θ is the slant angle of M it satisfies that $\lambda = cos^2\theta$.

Corollary 2 *Let* M *be a slant submanifold of para* β-*Kenmotsu manifold* $\bar{M}$, *with slant angle* θ. *Then, for any* $X, Y\epsilon\Gamma(TM)$ *we have*

$$g(TX,TY) = -cos^2\theta(g(X,Y) - \varepsilon\eta(X)\eta(Y))$$
$$g(TX,TY) = -sin^2\theta\big(g(X,Y) - \varepsilon\eta(X)\eta(Y)\big).$$

4 SUBMANIFOLDS OF SMALLEST DIMENSION IN PARA β-KENMOTSU MANIFOLD

Let M be 3-dimensional slant submanifold of 5-dimensional para contact manifold $\bar{M}$ and $\{e_1, e_2, e_3, e_4, \xi\}$ be local orthonormal basis of $T\bar{M}$. Let e_1 be unit vector field. $\tilde{\varphi}$ is para contact structure,

$$g(e_1, \tilde{\varphi}e_1) = 0.$$

Then, we can choice

$$e_2 = sec\theta Te_1.$$

Then

$$\{-sec\theta Te_2, -sec\theta Te_1, \xi\}$$

is a local orthonormal basis of *TM*.

On the other hand,

$$\{csc\theta Ne_1, csc\theta Ne_2\}$$

is a local orthonormal basis of $TM^{\perp}$**.**

Proposition 4 *Let* M *be a 3-dimensional non-invariant slant submanifold of a 5-dimensional para contact manifold* $\bar{M}$. *Let* e_1 *be an unit vector field and tanget to* M. *If*

$$e_1 = -sec\theta Te_2, \quad e_2 = -sec\theta Te_1,$$
$$e_3 = csc\theta Ne_1, \quad e_4 = csc\theta Ne_2.$$

Then $\{e_1, e_2, e_3, e_4, \xi\}$ be a local orthonormal basis of $T\bar{M}$, where $\{e_1, e_2, \xi\}$ are tanget to M and $\{e_3, e_4\}$ are normal to M. Moreover, we have

$$te_3 = -sin\theta e_1, \quad ne_3 = -cos\theta e_4,$$

$$te_4 = -sin\theta e_2, \quad ne_4 = -cos\theta e_3.$$

Proof. It is easy that $\{e_1, e_2, e_3, e_4, \xi\}$ is local orthonormal basis off $T\bar{M}$. We only show that last section

$\varphi e_3 = \varphi\{csc\theta N e_1\}$

$te_3 + ne_3 = csc\theta\{\varphi(\varphi e_1 - Te_1)\}$

$= csc\theta\{e_1 - \varphi(cos\theta e_2)\}$

$= csc\theta\{e_1 - cos\theta(Te_2 + Ne_2)\}$

$= csc\theta\{e_1 - cos\theta(cos\theta e_1 + sin\theta e_4)$

$= \dfrac{1}{sin\theta} e_1 - \dfrac{cos^2\theta}{sin\theta} e_1 - cos\theta e_4.$

Then

$$te_3 = sin\theta e_1$$

and

$$ne_3 = -cos\theta e_4.$$

Similarly

$$te_4 = -sin\theta e_2, \quad ne_4 = -cos\theta e_3 \ .$$

Theorem 3 *Let M be 3-dimensional submanifold of para β-Kenmotsu manifold $\bar{M}$ Then M is slant submanifold if and only if*

$$(\nabla_X T)Y = \beta\{g(TX, Y)\xi - \eta(Y)TX\} \qquad (15)$$

(15)

for all $X, Y \epsilon \Gamma(TM)$.

Proof. Let M be slant submanifold. We can choose local orthonormal basis $\{e_1, e_2, \xi\}$ of TM, where $e_1 = sec\theta Te_2$ and $e_2 = sec\theta Te_1$. Then $\forall \ X, Y \epsilon \Gamma(TM)$

$(\nabla_X T)e_1 = \nabla_X Te_1 - T\nabla_X e_1$

$= \nabla_X T(sec\theta Te_2) - T\nabla_X e_1$

$= sec\theta \nabla_X T^2 e_2 - T\nabla_X e_1$

from (14)

$$(\nabla_X T)e_1 = cos\theta \nabla_X e_2 - T\nabla_X e_1.$$

Then using (9)

$$(\nabla_X T)e_1 = cos\theta \sum_{i=1}^{3} \beta\, g(TX, e_2)\xi_i$$

$$= \sum_{i=1}^{3} \beta\, g(X, T^2 e_1)\xi_i$$

$$= -cos^2\theta \textstyle\sum_{i=1}^{3} \beta\, g(X, e_1)\xi_i \ .$$

(16)

Similarly,

$(\nabla_X T)e_2 = \nabla_X Te_2 - T\nabla_X e_2$

$$= -cos\theta \sum_{i=1}^{3} w_1^i(X)\xi_i$$

$$= -cos^2\theta \textstyle\sum_{i=1}^{s} g(X, e_2)\, \xi_i \qquad (17)$$

and

$(\nabla_X T)\xi = -T\big(cos^2\theta\beta(T^2X)\big)$

$= -cos^2\theta\beta(TX). \qquad (18)$

On the other hand, for any $Y \epsilon \Gamma(TM)$ writing

$$Y = c_1 e_1 + c_2 e_2 + \eta(Y)\xi.$$

Then

$$\nabla_X TY = c_1 \nabla_X Te_1 + c_2 \nabla_X Te_2 + g(Y, \xi)\nabla_X T\xi \qquad (19)$$

and

$$T\nabla_X Y = c_1 T\nabla_X e_1 + c_2 T\nabla_X e_2 + g(Y, \xi) T\nabla_X \xi. \qquad (20)$$

Finally, using (19) and (20)

$$(\nabla_X T)Y = c_1(\nabla_X T)e_1 + c_2(\nabla_X T)e_2 + \eta(Y)(\nabla_X T)\xi. \ (21)$$

(21)

Then, using (16), (17) and (18) into (21) it follows that

$$(\nabla_X T)Y = \beta\{g(TX, Y)\xi - \eta(Y)TX\}.$$

Corollary 3 *Let M be 3-dimensional submanifold of para β-Kenmotsu manifold $\bar{M}$ Then M is slant submanifold if and only if*

$$A_{NY}X = A_{NX}Y$$

for all $X, Y \epsilon \Gamma(TM)$.

Proposition 5 *Let M be 3-dimensional proper slant submanifold of 5-dimensional para β −Kenmotsu manifold $\bar{M}$ and let $\{e_1, e_2, e_3, e_4, e_5 = \xi\}$ be basis of $T\bar{M}$. Then*

$$h_{12}^3 = h_{11}^4, \ h_{22}^3 = h_{12}^4 \qquad (22)$$

and the other mixed second fundamental forms are zero.

Proof. Firstly,

$$h_{12}^3 = g(h(e_1,e_2),e_3)$$
$$= g(h(e_1,e_2),csc\theta Ne_1)$$
$$= csc\theta g(h(e_1,e_2),Ne_1)$$

using (8),

$$h_{12}^3 = csc\theta g(A_{Ne_1}e_2,e_1)$$

from Corollary 3,

$$h_{12}^3 = csc\theta g(A_{Ne_2}e_1,e_1)$$
$$= csc\theta g(h(e_1,e_1),Ne_2)$$
$$= g(h(e_1,e_1),e_4)$$
$$= h_{11}^4.$$

Similary

$$h_{22}^3 = h_{12}^4.$$

Theorem 4 *Let M be 3-dimensional submanifold of 5-dimensional para β-Kenmotsu manifold $\bar{M}$ Then M proper slant submanifold of para β-Kenmotsu manifold $\bar{M}$ if and only if*

$$(\nabla_X N)Y = -\beta\eta(Y)NX.$$

Proof. Let $\{e_1,e_2,e_3,e_4,e_5=\xi\}$ be basis of $T\bar{M}$. Using (13)

$$(\nabla_X N)Y = nh(X,Y) - h(X,TY) - \beta\eta(Y)NX$$

and from (22),

$$(\nabla_X N)Y = -\beta\eta(Y)NX.$$

Conversely, let (23) hold. Then, $\forall\ X,Y\epsilon\Gamma(TM)$

$$nh(X,Y) = h(X,TY).$$

On the other hand, from (8)

$$g(A_{Ne_1}e_2,X) = g(h(e_2,X),Ne_1).$$

Then

$$g(A_{Ne_1}e_2,X) = g(h(sec\theta Te_1,X),sin\theta e_3)$$
$$= sin\theta g(h(e_1,X),e_4)$$
$$= g(h(e_1,X),sin\theta e_4)$$
$$= g(h(e_1,X),Ne_2)$$
$$= g(A_{Ne_2}e_1,X).$$

On the other hand,

$$g(A_{Ne_1}e_5,X) = g(h(e_5,X),Ne_1) = 0.$$

In that case, M is slant submanifold of corollary 2.

Moreover,

$$h_{12}^3 = g(h(e_1,e_1),e_3)$$
$$= -g(h(e_1,e_2),e_4)$$
$$= sec\theta g(h(Te_2,e_2),e_4)$$
$$= -g(h(e_2,e_2),e_3)$$
$$= -h_{22}^3.$$

Similarly

$$h_{11}^4 = -h_{22}^4.$$

Then M is minimal slant submanifold.

Example 1 *In what follows,* $(\mathbb{R}^{2n+1},\varphi,\xi,\eta,g)$ *will denote the manifold* $\mathbb{R}^{2n+1}$ *with its usual β- Kenmotsu structure given by*

$$\varphi(X_1,\dots,X_n,Y_1,\dots,Y_n,\xi) = (Y_1,\dots,Y_n,-X_1,\dots,-X_n)$$
$$\xi = \frac{\partial}{\partial z}, \qquad \eta = dz$$
$$g = e^{-2z}\sum_{i=1}^{n}[dx_i\otimes dx_i + dy_i\otimes dy_i] - \varepsilon dz\otimes dz$$

where $\beta = e^{-2z}$. The consider a submanifold of $\mathbb{R}^5$ defined by

$$M = X(u,v,t) = (ucos\theta, usin\theta, v, 0, t).$$

Then the local frame of *TM*

$$e_1 = cos\theta\frac{\partial}{\partial x_1} + sin\theta\frac{\partial}{\partial x_2}, \qquad e_2 = \frac{\partial}{\partial y_1}, \qquad e_3 = \xi = \frac{\partial}{\partial t}.$$

On the other hand

$$(\nabla_X N)e_1 = 0, \qquad (\nabla_X N)e_2 = 0, \qquad (\nabla_X N)e_3 = -\beta NX.$$

For any $Y\epsilon\Gamma(TM)$ writing

$$Y = c_1e_1 + c_2e_2 + \eta(Y)\ e_3.$$

In that case,

$$(\nabla_X N)Y = c_1(\nabla_X N)e_1 + c_2(\nabla_X N)e_2 + \eta(Y)(\nabla_X N)e_3.$$

Then M is a minimal slant submanifold.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

REFERENCES

[1] D. E. Blair, Riemannian Geometry of Contact and Symplectic Manifolds, Progress in MathematicsVol. 203, Birkhauser, Boston, 2002.

[2] J. L.Cabrerizo, A. Carriazo and L. M. Fernandez, Slant submanifolds in Sasakian manifolds. Glasgow Math. J. 42 (2000), 125-138.

[3] A. Carriazo, New developments in slant submanifolds theory, In Applicable Mathematics in the Golden Age (2002), 339–356.

[4] A. Carriazo, L. M. Fernandez and M. B. Hans-Uber, Minimal slant submanifolds of the smallest dimension in S-manifold, Rev.Mat. Iberoamericana 21 (2005), 47-66.

[5] B.Y. Chen, Geometry of Slant Submanifolds, Katholieke Universiteit Leuven, 1990.

[6] B.Y. Chen, Slant Immersions, Bull, Australian Math. Soc. 41 (1990), 135-147.

[7] R. S. Gupta, and P. K. Pandey, Structure on a slant submanifold of a Kenmotsu manifold. Differential Geometry-Dynamical Systems 10 (2008), 139-147.

[8] R. S. Gupta, S. M. K. Haider and M. H. Shaid, Slant submanifolds of Kenmotsu manifolds, Radovi Math., 12 (2004), 205-214.

[9] A. Lotta, Slant submanifolds in contact geometry, Bull. Math. Soc. Roumanie. 39 (1996), 183-198.

[10] A.Lotta, Three dimensional slant submanifolds of K-contact manifolds, Balkan J. Geom. Appl., 3 (1) (1998), 37-51.

[11] I. Sato, On a structure similar to the almost contact structure, Tensor (N.S.), 30 (1976), 219-224.

[12] Z. Olszak, The Schouten-Van Kampen Affine Connection Adapted to an Almost Para Contact Metric Structure, Pub. de Linst. Math., 94(108), (2013), 31-42.