

## A NEW MULTI-PARTY PRIVATE SET INTERSECTION PROTOCOL BASED on OPRFs

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#### Abstract

In many crucial real-world applications, parties must jointly perform some secure multi-party computation (MPC) while keeping their inputs hidden from other parties. Private Set Intersection (PSI), the specific area of Multi-Party Computation, let the parties learn the intersection of their private data sets without sharing their secret data with others. For instance, a smartphone user downloads a messaging application, naturally, he wants to discover who are the other contacts that are using the same application. The naive and insecure solution is to send all contacts to the server to discover them. However, the user does not want to share his contacts with the application for privacy issues. To handle this, in recent years, companies and organizations start to use PSI to enhance privacy and security with a little cost of communication and computation. In this paper, we introduce a novel method to compute Private Set Intersection with multi parties where there are at least three or more parties participating in the protocol. By employing the Zero-Secret Sharing scheme and Oblivious Pseudo-Random Functions (OPRFs), parties securely calculate the intersection with computational and communication complexities which are both linear in the number of parties.

Keywords: Private set intersection, multi-party private set intersection, multi-party computation, oblivious transfer, oblivious pseudorandom function, zero sharing

# OPRF'LERE DAYALI YENİ ÇOKLU KULLANICILI ÖZEL SET KESİŞİMİ PROTOKOLÜ

#### Özet

Birçok önemli gerçek dünya uygulamasında, taraflar girdilerini diğer taraflardan gizli tutarken bazı güvenli çok taraflı hesaplama (MPC) işlemlerini birlikte yapmalıdır. Çok Taraflı Hesaplamanın özel alanı olan Özel Set Kesişimi (PSI), tarafların gizli verilerini başkalarıyla paylaşmadan veri kümelerinin kesişimini öğrenmelerini sağlar. Örneğin, bir akıllı telefon kullanıcısı bir mesajlaşma uygulaması indirir, doğal olarak aynı uygulamayı kullanan diğer kişilerin kim olduğunu keşfetmek ister. Naif ve güvensiz çözüm, tüm kişileri, sunucuya göndermek ve kim olduklarını keşfetmektir. Ancak kullanıcı, gizlilik sorunları için uygulama ile temaslarını paylaşmak istemezler. Bunu halletmek için, son yıllarda şirketler ve kuruluşlar, küçük bir iletişim ve hesaplama maliyetiyle gizliliği ve güvenliği artırmak için PSI kullanmaya başladılar. Bu makalede, protokole en az üç veya daha fazla tarafın katıldığı çoklu taraflarla Özel Set Kesişimi hesaplamak için yeni bir yöntem tanıtıyoruz. Taraflar, Sıfır Gizli Paylaşım ve Habersiz Sözde Rastgele Fonksiyonları kullanarak, her ikisi de kullanıcı sayısı ile doğrusal olan hesaplama ve iletişim karmaşıklıklarıyla kesişimi güvenli bir şekilde hesaplar.

Anahtar Kelimeler: özel set kesişimi, çoklu kullanıcılı özel set kesişimi, çok taraflı hesaplama, habersiz transfer, habersiz sözde rastgele fonksiyonlar, sıfır gizli paylaşım.

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#### 1. Introduction

Private Set Intersection (PSI) is a way of finding the intersection of two secret data sets without disclosing their elements other than the intersection. In a formal

way, two parties  $P_1$  and  $P_2$  holding their data sets  $S_1$  and  $S_2$  are willing to compute the intersection  $S_1 \cap S_2$  without revealing their data sets. As our lives are getting digital day by day, as a consequence of this, people inevitably demand more secure applications from companies or organizations. Therefore, in the past years,

PSI has been enthusiastically researched and found various practical and fast applications [1-6].

Traditionally PSIs are designed for two parties. At certain setups, we can derive much more information than the two-party intersections and it is only possible to get these meaningful inferences with multiple parties simultaneously joined together. In this setting, there are more than two parties, say t parties  $P_1,...,P_t$  having their own sets  $S_1,...,S_t$  respectively, are interested in finding  $S_1 \cap ... \cap S_t$  without revealing any other information.

Both, the usages of PSIs and Multi-party Private Set Intersections (MPSIs) are very broad in real-life applications and can be exemplified as follows:

**Contact Tracing:** In the ongoing COVID-19 pandemic, the researchers from UC Berkeley introduced a lightweight way for contact tracing. Users are alerted by an application if they contact any diagnosed people with the disease while protecting private information [7].

**Password Checkup:** Based on the numbers from the paper 1.5% of logins on the web involve breached credentials. Another use of PSI is checking whether your password is leaked or not. Users can compare their credentials with millions of entries in breached databases without revealing any part of it [8].

Ad Efficiency: Facebook, Datalogix, Epsilon, and Acxiom, consumer data collection companies measure how well an ad is performing. Rather than using naive hashing solutions to compare merchant's lists of customers and advertiser's lists, they are using PSI methods to increase security [9].

**Genome Discovery**: In the field of paternity and ancestry testing, it can be possible to perform these tasks without revealing any further individual genomic information [10].

## 1.1. Related Work

The naive solution of PSI is where all the parties deal with a hashing algorithm and then apply this algorithm to their sets. By comparing resulting hashes, parties easily come up with an intersection but the problem with this approach is when the input domains are small, the brute force attack can be applied. Early researches are based on public-key cryptography techniques that have computational challenges rather than symmetric key cryptography [11-13].

There are different types of techniques that are used to build PSIs and MPSIs. The well-known ones are permutation-based hashing [5], circuit-based computations [14-16], oblivious transfer [1,3,17], Bloom filters [18], cuckoo hashing [2], oblivious programmable hashing [19]. To the best of the author's knowledge, the fastest PSI protocols are designed by Pinkas et al. [5,19] where the former is based on oblivious transfer and permutation-based hashing and the latter uses a generic circuit-based multi-party computation.

The earliest MPSI protocol by Freedman et al. [20] is based on Oblivious Polynomial Evaluation (OPE) which makes use of a homomorphic encryption scheme. In this scheme, users' private data sets are represented as polynomials and the coefficients are encrypted by a homomorphic encryption scheme and obliviously evaluated on the other party's data sets. The same OPE technique is also used by Kissner et al. [21] which has a quadratic complexity in the number of parties. Another work by Hazay and Venkitasubramaniam is similar to work [22] which uses an additively homomorphic publickey encryption scheme with threshold decryption. The computational complexity of this protocol is linear in the input data sets. Recently, Kolesnikov et al. proposed an MPSI which is based on Oblivious Transfer which makes their protocol faster due to the usage of symmetric algorithms. The computational complexity is quadratic in the number of parties while it is independent of the size of the sets.

#### 1.2. Our Contribution

Inspired by Chase-Miao's PSI protocol [3] which is based on Oblivious Pseudorandom Functions (OPRFs), we design an efficient and secure multi-party PSI. In our design, we consider t parties each of which has private data sets of the same size n, and a Trusted Dealer D who has no clue about the secret key and data sets of the parties. The effectiveness of our protocol is that any party can compute the intersection, however, for the sake of simplicity, we consider the first party  $P_1$  as the server who outputs the intersection. Our scheme is based on again Oblivious Pseudorandom Functions and zero secret sharing to protect the privacy of the parties. Our protocol is efficient in terms of both computational and communication complexities which are both linear in the number of parties.

#### 2. Preliminaries

In this section, we will provide the necessary background for our protocol. We start with the notations, then we explain the security model of the protocol. Finally, we briefly explain the Chase-Miao PSI protocol that we inspired from [3].

## 2.1. Notations

We use  $\lambda$ ,  $\kappa$ ,  $\sigma$  and  $l_1$  to denote the computational and statistical security parameters.

- t: Number of parties.
- *m*: The number of rows in the matrices.
- *w*: The number of columns in the matrices.
- $l_1$ : The output length of the Hash function H.

*s*: A random string to be used in Oblivious Transfer operations.

**P**<sub>i</sub>: The i-th party, where **P**<sub>1</sub> is the server and the rest is the clients.

X<sub>i</sub>: The dataset of **P**<sub>i</sub>.

**D**: The Trusted Dealer.

**n**<sub>i</sub>: The size of the dataset of **P**<sub>i</sub>, we assume that the all the parties have data set of the same size.

 $sh_i^{a,b}$ : The i-th secret share part at index (a,b).

*C<sub>i</sub>*: The matrix constructed via OT operations of *P<sub>i</sub>*.

 $\pmb{\mathcal{C}}$  : The final matrix constructed by trusted dealer D

**PSI**: Private Set Intersection.

**MPSI**: Multi-Party Private Set Intersection.

**S**: The intersection of  $S_i$ 's,  $S = \bigcap_{i=1}^t S_i$ .

 $\pmb{\kappa}$  : The computational security parameter.

*r*: The length of secret shares in bits.

#### 2.2. Security Model

The security of the protocol is based on the semi-honest model with static adversaries. To explain briefly, the parties follow the protocol honestly without deviating from the flow of the protocol and the number of corrupted parties is determined before the start of the protocol.

As we will omit the proof, we refer the reader to the security definition of the semi-honest security for deterministic functionalities defined by Goldreich[23]. The protocol has a trusted Dealer D, who contacts with the clients through a secure communication channel. The security definition for our MPSI protocol matches up with that of Miyaji et al. [24] which can be explained as follows.

An MPSI is player-private secure under the existence of a trusted Dealer D if the following two conditions are satisfied:

- The clients can only learn the other client's elements only if the elements are in the intersection  $\cap_i S_i$ .
- The trusted dealer D cannot learn any information about the data sets of the clients.

Note that the trusted Dealer has no access to the data set of the clients, also he cannot obtain the mutual or full intersection client's data sets by trying the elements from the domain. The reason is that he does not know the secret key  $\mathbf{k}$  of PRF and cannot infer anything from the parties' matrices  $C_i$ 's which will be clarified in Sec. 3.

#### 2.3. Technical Background

The Zero-Secret Sharing Scheme: To extend two-party protocols to multiple parties, we usually make use of secret sharing schemes. The idea behind secret sharing schemes is as follows: we divide the secret into several shares in such a way that when a sufficient number of shares are available, which is greater than a threshold number, then the secret can be reconstructed. This means that if there is not enough shares, no one can be able to reconstruct the secret, nor any part of it. In this work, we make use of an additive XOR-based (t, t) secret sharing scheme [25] which can be briefly explained as follows: we create t - 1 shares of the same size as the secret s, the last share becomes the Exclusive OR (XOR) of all t - 1 shares and the secret *s*. To reconstruct the secret *s*, it is simply needed to XOR all *t* shares. In our case, we use a fixed secret as s = 0 which makes the scheme (t - 1, t) as *s* is already known by the parties.

**Oblivious Transfer:** Oblivious transfer (OT), proposed by Rabin [26], is a cryptographic protocol executed among two parties: the sender has two inputs  $m_0$  and  $m_1$  and the receiver has a bit b, at the end of the protocol, while the receiver learns  $m_b$  while no parties learn any additional information. When there are n OTs are needed, there is an efficient extension of this technique called *OT extension* which requires  $O(\kappa)$  public-key operations instead of O(n), where  $\kappa$  is a computational security parameter [27].

**Single-Point OPRF:** As proposed in Kolesnikov et. al. [4], the single point oblivious PRF (OPRF) is defined as follows: let the PRF key k be the two-bit strings  $q, s \in \{0,1\}^{\lambda}$ . Let H be a hash function and  $F(\cdot)$  be a pseudorandom code that produces a pseudorandom string. The oblivious pseudorandom function is defined as

$$OPRF_k(x) = H(q \oplus [F(x) \cdot s]) \tag{1}$$

where '.' is bitwise-AND and ' $\oplus$ ' is bitwise-XOR operands. Here, the OPRF is assumed to be pseudorandom if *H* is a collision-resistant hash function and *F* is a pseudorandom generator function, lastly *s* should be a randomly generated string.

Let's see how we can evaluate the single-point OPRF on the receiver's input y. First of all, the receiver chooses a random  $r_0$  from  $\{0,1\}^{\lambda}$  and computes  $r_1 = r_0 \oplus F(y)$ . The sender then samples a random string *s* from  $\{0,1\}^{\lambda}$  where each bit of *s* is one of the  $\lambda$  choice bits for OT. Then, these two parties execute  $\lambda$  oblivious transfers where the sender acts as a receiver in the OT and inputs single bits  $s[1], s[2], ..., s[\lambda]$ . The receiver acts as a sender in the OT and inputs single bits  $\{r_0[i], r_1[i]\}_{i \in \{1,...,\lambda\}}$ . After all OTs are executed, the sender obtains a set of  $\lambda$  bits and sets q = $r_{s[1]}[1]|| \dots ||r_{s[\lambda]}[\lambda]|$ . The sender also sets the PRF key as k = (q, s). Then, he chooses x and computes  $q \oplus [F(x) \cdot$ s]. Therefore, the PRF value on y learned by the receiver is  $H(r_0)$  no matter *s* is chosen. That is, for x = y, we have  $H(q \oplus [s \cdot f(x)]) = H(r_0 \oplus [s \cdot (F(x) \oplus F(y)]) = H(r_0).$ Otherwise, the receiver has no clue about  $OPRF_k(x)$ .

**Multi-Point OPRF:** Instead of executing the single-point OPRF for every value of the receiver which is not efficient, the single-point OPRF is extended to Multi-Point OPRF [3][28]. According to Chase and Miao [3], instead of a vector PRF key k, here they define the key as  $m \times w$  matrix. Similarly, we use a hash function H and pseudorandom code  $F(\cdot)$  that produces a pseudorandom vector  $v \in [m]^w$ . The pseudorandom function is defined as

$$OPRF_{M}(x) = H(M_{1}[v[1]|| ... ||M_{w}[v[w]])$$
(2)

We evaluate OPRF on input *y*, the sender picks a random string *s* from  $\{0,1\}^w$ . The receiver prepares (in a way that it will be explained in the PSI protocol) two matrices *A* and *B*, where  $A_i \in \{0,1\}^m$  and  $B_i \in \{0,1\}^m$  are column matrices of *A* and *B* respectively, where  $1 \le i \le w$ . There will be *w* number of execution of OTs where the sender behaves as the receiver and the receiver behaves as the sender as in the single-point OPRF. After all, OTs are executed, the sender gets *w* columns vectors which will

be assigned to have the PRF key *M*. As for all  $x \in Y$ ,  $OPRF_M(x) = OPRF_A(x)$  is independent of chosen s.

#### 2.4. The Chase-Miao's PSI Protocol

In their protocol [3], Chase and Miao use the multi-point OPRF to find the intersection of two private data sets. Their protocol is as follows:

• **Input:** Let  $\lambda$ ,  $\sigma$  be security parameters,  $H_1: \{0,1\}^* \rightarrow \{0,1\}^{l_1}$  and  $H_2: \{0,1\}^* \rightarrow \{0,1\}^{l_2}$  and pseudorandom function  $F: \{0,1\}^{\lambda} || \{0,1\}^{l_1} \rightarrow \{0,1, \dots m-1\}^w$  agreed by two parties  $P_1$  and  $P_2$ .

## • Precomputation:

- 1.  $P_1$  chooses a random string  $s \in_R \{0,1\}^w$ .
- 2.  $P_2$  constructs a matrix D which have all entries are set to 1. Let  $D_1, \dots, D_w$  be column vectors of D.
- 3.  $P_2$  chooses a random key k of length  $\lambda$  for  $F, k \in_R \{0,1\}^{\lambda}$ .
- 4.  $P_2$  computes  $v = F_k(H_1(y))$  for each element  $y \in Y$  and update D as  $D_i[v[i]] = 0$  for all  $i \in \{1, ..., w\}$ .
- 5.  $P_2$  constructs a random matrix A of size  $m \times w$ and computes another matrix B by  $B = A \oplus D$ .
- **Oblivious Transfer:** *P*<sub>1</sub> as the receiver with input choice bits *s*[*i*]'s and *P*<sub>2</sub> as the sender with inputs {*A*<sub>*i*</sub>, *B*<sub>*i*</sub>}<sub>*i*</sub> execute *w* oblivious transfers (OTs) and *P*<sub>1</sub> obtains an *m* × *w* matrix *C* at the end.

## • Computing Intersections with OPRF:

- 1.  $P_2$  sends k to  $P_1$ .
- 2.  $P_1$  computes  $v = F_k(H_1(x))$  for each element x in his data set.
- 3. Then,  $P_1$  computes OPRF value of each element  $X \in X$  by  $\psi = H_2(C_1[v[1]]|| ... ||C_w[v[w]])$  constructs the set of  $\psi$ 's as  $\Psi$  and send  $\Psi$  to  $P_2$ .
- 4.  $P_2$  computes  $v = F_k(H_1(y))$  for all  $y \in Y$ , and their OPRF values  $OPRF_M(y) =$  $H_2(A_1[v[1]|| ... ||A_w[v[w]]])$ . For each  $y \in \Psi$ , outputs y.

## 3. Our MPSI Protocol

Assume that there are *t* parties as  $P = \{P_1, ..., P_t\}$  each of which has a private data set  $X_i$  of size  $n_i$ , respectively, and a trusted Dealer *D* who helps parties to compute the intersection. Although any party can compute the intersection, for simplicity, we consider  $P_1$  as the server who computes and outputs the intersection.

They agree on parameters  $\lambda$ ,  $\sigma$ ,  $m, w, n, l_1$  and a hash function  $H_1: \{0,1\}^* \to \{0,1\}^{l_1}$ , a pseudorandom function  $F: \{0,1\}^{\lambda} || \{0,1\}^{l_1} \to \{0,1,\ldots,m-1\}^w$ . The parties generate a secret  $k = \{0,1\}^{\lambda}$  and keep it hidden from the dealer.

## 1. Precomputation

- Each  $P_i$  samples t 1 random strings  $s = \{0, 1\}^w$ .
- Each *P<sub>i</sub>* chooses a random matrix *A<sup>i</sup>* of size  $m \times w$ , each entry of *A<sup>i</sup>* is a random number of

sizes r, where  $A_j^i$  denotes the j-th column of  $A^i$ , where  $1 \le j \le w$ .

- Each  $P_i$ , for each  $x_{i,l} \in X_i$ , computes  $v = F_k(H_1(x_{i,l}))$ , where  $1 \le l \le n_i$ , updates  $A_c[v[c]] = 0$  for all  $c \in [w]$  by keeping rest of the entries the same.
- Each party generates another random matrix  $B^i$ , where  $B^i$  shares zero entries with  $A^i$  (that is, each zero entry in  $A^i$  appears in the same place  $B^i$ ). The rest of the entries in  $B^i$  are chosen at random.

## 2. Zero Sharing

- Each  $P_i$  generates t 1 matrices of size  $m \times w$  called  $A^{i,r^*}$ ,  $1 \le r < t 1$ , and fills with random numbers, does the same thing again, and is named as  $B^{i,r^*}$ .
- For each 0 appears in  $A^i$  matrix at index (a, b), the party *i* does the following:
  - The party *i* generates *t* shares that satisfy:

$$0 = sh_1^{a,b} \oplus sh_2^{a,b} \dots \oplus sh_t^{a,b}$$
(3)

 $-P_i$  keeps the first share to itself by updating  $A_a^i[b] = sh_1^{a,b}$  and  $B_a^i[b] = sh_1^{a,b}$ .

-  $P_i$  sets rest of the shares to  $A^{i,r^*}$  and  $B^{i,r^*}$  at index (a, b) respectively to be used in Oblivous Transfer later.

## 3. Oblivious Transfer

- Each  $P_i$  does OT with the rest  $P \setminus P_i$ 's.
- The OT interaction, while  $P_i$  will be taking the role of the sender, the rest  $P_r \in P \setminus P_i$  takes the role of the receiver, happens as follows:

-Party  $P_r$  uses  $s_{i,c}[1], \ldots, s_{i,c}[w]$ , while  $P_i$  has the column vectors  $(A^{i,r^*}, B^{i,r^*})$  as inputs. Here, we have  $(A^{i,r^*}, B^{i,r^*})$ , these matrices are composed of the shares of entries of  $(A^i, B^i)$ . Of course every  $(A^{i,r^*}, B^{i,r^*})$  is different for different  $P_r$ 's (which denotes the shares of entries to be sent to  $P_r$ .

-Now, each party has a matrix  $C_{i,r}$  from their OT interactions. Namely,  $P_i$  has  $C_{i,r}$  whose columns are either from  $A^{i,r^*}$  or  $B^{i,r^*}$ , where  $r \neq i$  and has  $C_{i,i} = B^{i,i^*}$  consisting of the shares holding for himself. That is, for example,  $P_1$  keeps  $sh_1^{a,b}$  for himself, sends other sh's to the other parties where

$$0 = sh_1^{a,b} \oplus sh_2^{a,b} \dots \oplus sh_t^{a,b}$$

$$\tag{4}$$

• At the end each  $P_i$  element-wise XOR their obtained matrices and  $A^i$  to construct  $C_i$ , after computation each  $P_i$  sends  $C_i$  to the Dealer, where

$$C_i = A^i \oplus C_{i,j} \oplus C_{i,j+1} \oplus \dots \oplus C_{i,t-1}$$
(5)

## 4. Dealer Side and Requests

• The trusted dealer gathers all *C*'s and XOR them to construct the final C by

$$C = C_1 \oplus C_2 \oplus \dots \oplus C_t. \tag{6}$$

- Any party can act as a server and check an item is whether inside the intersection or not by sending previously calculated  $v = F_k(H_1(x_{i,l}))$ values to the Dealer. For simplicity, we think  $P_1$ acts like a server.
- The Dealer *D* checks  $C_c[v[c]] = 0$  for all  $c \in [w]$ , if it encounters an entry other than 0 it means the item is not inside the intersection. On the other hand, if Dealer *D* checks that all items are 0, that means the item is inside the intersection. In the end, Dealer *D* sends the output vector  $(d_1, ..., d_n)$  with the same order to Server  $P_1$ where bits represent an item whether inside the intersection or not.



Figure 1: Our Multi-party private set intersection protocol when only  $P_1$  outputs the intersection.

#### 4. Complexity Analysis and Security of Our Mpsi Protocol

## 4.1 Communication Complexity

There are two major contributions to the communication complexity which are one from OT and the other from communications with the server.

-There is t(t - 1) number of OT interactions executed during this protocol. Each party receives a matrix from all other members. These interactions include m \* w matrix and matrix and if there the shares have length of r bits, the communication cost from OT will be (t - 1) \* m \* w \* r.

-Each party sends its matrix  $C_i$  of size m \* w to the server to construct the final C. Same as before, if there are the shares have a length of r bits, the communication cost from communications with the server will be m \* w \* r. In the case that all the parties compute the intersection,

then for each party, the communicational complexity will be t \* m \* w \* r.

Choosing m and w play a crucial role in our construction in the sense of security leaks and communication overhead. As in [3], we set row size(m) to number of data(n). Also, we fix column size(w) to statistical parameter  $\lambda$  for all parties. We refer to reader the Miao's statistical explanations of how to choose m and w [3,30]. This makes our complexity  $t * n * \lambda * r$ .

## 4.2. Computational Complexity

In the precomputation phase, for every element x in each data set, each party has to compute  $v = F_k(H_1(x))$  which requires n calls of F function, where  $n = \max\{n_i\}$ . Each party creates shares for at most m \* w entries. Then, each party has to do 2(t - 1) OT interactions. Finally, each party does (t - 1) matrix addition. The server makes t matrix addition.

## 4.3. Security

Our protocol is secure according to the following Theorem.

**Theorem 5.1:** If *F* is a *PRF*, *H* is modelled as a random oracle, and the underlying OT protocol is secure, for any

coalition of fewer than t clients including the server, our MPSI has semi-honest security against an honest-butcurious adversary.

The security of the protocol is due to the use of OT and the zero-secret sharing protocol, as long as the parties and the Dealer *D* follow the instruction of the protocol which is aligned with the semi-honest security, no one gets any information rather than the intersection. The full proof will be provided later in the full version of the paper.

#### 5. Final Remarks and Comparison with Other Protocols

## 5.1. Remarks

**Remark 1:** Our multi-party private set intersection protocol relies on computationally fast primitives such as hashing, oblivious transfers based on symmetric keys, and bitwise-XOR operations.

**Remark 2:** As the number of parties participating in the protocol increases, the communication and computational complexity increases linearly for each party. This makes our protocol scale well particularly when there is a large number of participants in the setup.

**Remark 3:** The bottleneck of our protocol is the mesh topology that arises from its design. To execute the oblivious transfer section, every party needs to communicate with the rest of the parties. The authors in [29] proposed a multi-party PSI that is an extension of the work of Chase and Miao's [3] private set intersection protocol. They employed Garbled Bloom Filters to their protocol and this way parties interact with path-like communication. Naturally, from the communication perspective path-like topology is better than mesh topology.

## 5.2. Comparison

Protocol	Communication		Computation	
	#bits		#operations	
	Server	Client	Server	Client
[30]	$O(\lambda t^2 n)$		O(tn)	
[22]	$O(tn\lambda)$	$\mathcal{O}(n\lambda)$	O(tnlog(n))	$\mathcal{O}(n)$
[2]	$O(tn\lambda)$	$O(nl\lambda)$	$\mathcal{O}(t\kappa)$	$O(l\kappa)$
[17]	O(tnĸk)	O(tnĸk)	O(tnκk)	O(tnĸk)
Sec. 3	$O(tn\lambda r)$	$O(tn\lambda r)$	Negl	$\mathcal{O}(t\lambda)^*$

\* In our construction, computational complexity heavily depends on OT interactions. Oblivious transfers are cryptographic primitives introduced by Rabin[26]. It relies on public-key operations and this makes OTs computationally expensive. However, Ishai [31] proposed a method (OT-extension) that you can do a number of oblivious transfers by using a few public-key operations.

## 6. Implementation

We implemented and tested our protocol in Python in a primary way. Our implementation confirms our

theoretical approach logically works. In order to do a benchmark comparison with other protocols which have already been implemented and are publicly available, we plan to implement our code in C++ in the near future. In this way, we can use the Oblivious Transfer Protocol developed by Peter Rindal [32] where symmetric primitives are used. For concrete analysis, the full code will be available later.

#### 7. Conclusion

In this paper, a novel MPSI protocol based on OPRFs and the zero-secret sharing scheme is proposed. The key idea of the construction is to extend OPRF-based Chase-Miao's PSI protocol [3] to multi parties. That is to say, we take OPRFs idea that is used in two-party Chase-Miao's PSI protocol and use it in a multi-party set intersection while the zero-secret sharing protects the privacy of the parties. Also, the developed construction uses fast primitives such as hashing, oblivious transfers based on symmetric keys, and bitwise-XOR operations. These primitives make the computational and communication complexities of our protocol efficient which are both linear in the number of parties (t $\lambda$  and t $n\lambda r$  respectively). As future work, we plan to do benchmark implementation of the scheme in C++ and remove the Trusted Dealer D safely by keeping the parties' input secure.

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