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Approximate solution of a model describing biological species living together by Taylor collocation method

Elçin Gökmen[1] *and Mehmet Sezer*[2]

[1]Muğla Sıtkı Koçman University, Faculty of Science, Department of Mathematics, 48000, Muğla Turkey.
[2]Celal Bayar University, Faculty Science and Art,Department of Mathematics, 45140, Manisa Turkey

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Abstract: In this paper, a numerical method is presented to obtain approximate solutions for the system of nonlinear delay integro-differential equations derived from considering biological species living together. This method is essentially based on the truncated Taylor series and its matrix representations with collocation points. Also, to illustrate the pertinent features of the method examples are presented and results are compared to the Adomian decomposition method, the variational iteration method, pseudospectral Legendre method. All numerical computations have been performed on the computer algebraic system Maple 15.

Keywords: System of nonlinear integro-differential equations, Taylor polynomials and series, Biological species ,Collocation points.

1 Introduction

In science and engineering some important problems can usually be modeled to a system of integral and integro-differential equations (IDE). IDEs and their systems play significant role in biology, ecology, medicine, physics such as population growth, one dimensional viscoelasticity, electro magnetic theory and reactor dynamics [1]-[6]. Since few of these equations can be solved explicitly, it is needed to develop the numerical techniques to solve IDEs and their systems [7,8]. This paper is concerned with the dynamic of two interacting species which was first modeled by Volterra [9]. It is considered two separate species with numbers $y_1(t)$ and $y_2(t)$ at time t where first species increases and second one decreases. If they are put together, assuming that the second species will feed on the first, there will be increase in the rate of the second species $\frac{dy_2}{dt}$ which depends not only on the present population $y_1(t)$ but also on all previous values of the first species. When a steady-state condition or equilibrium is reached between these two species, it is described by the following system of nonlinear delay Volterra integro-differential equations:

$$\begin{cases} \frac{dy_1}{dt} = y_1(t)\left[h_1 - \gamma_1 y_2(t) - \int\limits_{t-T_0}^{t} f_1(t-\tau)y_2(\tau)\,d\tau\right] + g_1(t) \\ \frac{dy_2}{dt} = y_2(t)\left[-h_2 + \gamma_2 y_1(t) + \int\limits_{t-T_0}^{t} f_2(t-\tau)y_1(\tau)\,d\tau\right] + g_2(t) \end{cases} \tag{1}$$

where $h_1, \gamma_1, h_2, \gamma_2 > 0$, $0 \le t \le b$ with initial conditions

$$y_1(0) = \alpha_1,\ y_2(0) = \alpha_2 \tag{2}$$

* Corresponding author e-mail: egokmen@mu.edu.tr

where h_1 and $-h_2$ are coefficients of increase and decrease of the first and the second species, respectively. The parameters f_1, f_2, g_1 and g_2 are given functions while y_1 and y_2 are unknown functions and $T_0 \in \mathbb{R}$ is assumed to be the finite heredity duration of both species [10].

Up to now, Adomian decomposition method (ADM) [11], rational Chebyshev tau method (CTM)[12], variational iteration method (VIM), pseudospectal Legendre method (PLM) [13] and Legendre multiwavelet method (LVM) [14] have been used to solve the model (1)-(2).

The main purpose of this study is to solve equations (1)-(2) using the Taylor matrix method. Since the beginning of 1994, Taylor, Chebyshev, Legendre, Laguerre, Hermite and Bessel collocation and matrix methods have been used by Sezer et al.[15]-[25] to solve differential, difference, integral , integro-differential, delay differential equations and their systems. In this article, by modifying and developing matrix and collocation methods studied in [15,17,20], we will find the approximate solutions of the system (1)-(2) in the truncated Taylor series form

$$y_i(t) = \sum_{n=0}^{N} y_{i,n} t^n, \qquad y_{i,n} = \frac{y_i^{(n)}(0)}{n!}, \qquad i = 1,2, \qquad 0 \le t \le b \tag{3}$$

where $y_{i,n}$, $(n = 0,1,\ldots,N,\ i = 1,2)$ are unknown coefficients to be determined.

2 Fundamental relations

Let us consider the system of nonlinear Volterra integro-differential equations in the form (1) and find matrix representations of each term in the system. First we convert the solution defined by (3) and its first derivative, for $n = 0,1,\ldots,N$ to the following matrix forms:

$$y_i(t) = \mathbf{T}(t)\mathbf{Y}_i, \qquad i = 1,2 \tag{4}$$

$$y_i{}'(t) = \mathbf{T}(t)\mathbf{B}\mathbf{Y}_i, \qquad i = 1,2 \tag{5}$$

where

$$\mathbf{T}(t) = \begin{bmatrix} 1 & t & t^2 & \ldots & t^N \end{bmatrix}, \ \mathbf{B} = \begin{bmatrix} 0 & 1 & 0 & 0 & \ldots & 0 \\ 0 & 0 & 2 & 0 & \ldots & 0 \\ 0 & 0 & 0 & 3 & \ldots & 0 \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & 0 & \ldots & N \\ 0 & 0 & 0 & 0 & \ldots & 0 \end{bmatrix}.$$

$$\mathbf{Y}_i = \begin{bmatrix} y_{i,0} & y_{i,1} & y_{i,2} & \ldots & y_{i,N} \end{bmatrix}^T.$$

On the other hand, we can write the matrix forms of the expressions $y_1(t)y_2(t)$, $y_1(t)y_2(\tau)$ and $y_1(\tau)y_2(t)$ as, respectively,

$$y_1(t)y_2(t) = \mathbf{T}(t)\mathbf{T}^*(t)\overline{\mathbf{Y}}_1 \\ = \mathbf{T}(t)\mathbf{T}^*(t)\overline{\mathbf{Y}}_2, \tag{6}$$

$$y_1(t)y_2(\tau) = \mathbf{T}(\tau)\mathbf{T}^*(t)\overline{\mathbf{Y}}_1 \tag{7}$$

and

$$y_1(\tau)y_2(t) = \mathbf{T}(\tau)\mathbf{T}^*(t)\overline{\mathbf{Y}}_2 \tag{8}$$

where

$$\mathbf{T}^*(t) = \begin{bmatrix} \mathbf{T}(t) & 0 & \dots & 0 \\ 0 & \mathbf{T}(t) & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & \mathbf{T}(t) \\ \end{bmatrix}_{(N+1)\times(N+1)^2},$$

$$\overline{\mathbf{Y}}_1 = \begin{bmatrix} y_{2,0}\mathbf{Y}_1 & y_{2,1}\mathbf{Y}_1 & \dots & y_{2,N}\mathbf{Y}_1 \end{bmatrix}^{\mathrm{T}}_{(N+1)^2\times 1},$$

$$\overline{\mathbf{Y}}_2 = \begin{bmatrix} y_{1,0}\mathbf{Y}_2 & y_{1,1}\mathbf{Y}_2 & \dots & y_{1,N}\mathbf{Y}_2 \end{bmatrix}^{\mathrm{T}}_{(N+1)^2\times 1}.$$

Now, we convert the kernel functions

$$K_1(t,\tau) = f_1(t-\tau) \quad \text{and} \quad K_2(t,\tau) = f_2(t-\tau)$$

to the matrix forms, by means of following procedure.
The functions $K_1 = f_1$ and $K_2 = f_2$ can be expressed by the truncated Taylor series, respectively,

$$K_1(t,\tau) = \sum_{m=0}^{N}\sum_{n=0}^{N} k^1_{m,n} t^m \tau^n \tag{9}$$

$$K_2(t,\tau) = \sum_{m=0}^{N}\sum_{n=0}^{N} k^2_{m,n} t^m \tau^n \tag{10}$$

where

$$k^i_{m,n} = \frac{1}{m!n!}\frac{\partial^{m+n}K_i(0,0)}{\partial t^m \partial \tau^n}, \qquad m,n = 0,1,\dots,N, \qquad i = 1,2.$$

The expressions (9) and (10) can be written in the matrix forms

$$\mathbf{K}_1(t,\tau) = \mathbf{T}(t)\mathbf{K}_1\mathbf{T}^T(\tau), \qquad \mathbf{K}_1 = \left[k^1_{m,n}\right], \qquad m,n = 0,1,\dots,N. \tag{11}$$

$$\mathbf{K}_2(t,\tau) = \mathbf{T}(t)\mathbf{K}_2\mathbf{T}^T(\tau), \qquad \mathbf{K}_2 = \left[k^2_{m,n}\right], \qquad m,n = 0,1,\dots,N. \tag{12}$$

where

$$\mathbf{T}(t) = \begin{bmatrix} 1 & t & t^2 & \dots & t^N \end{bmatrix}.$$

So that $\mathbf{K}_i = \left[k^i_{m,n}\right]$, $m,n = 0,1,\dots,N$, $i = 1,2$ are the Taylor coefficients matrices of functions $\mathbf{K}_i(t,\tau) = f_i(t-\tau)$, $i = 1,2$ at point $(0,0)$.

For example, in case $N = 2$, the matrix $\mathbf{K}_1$ becomes

$$\mathbf{K}_1 = \begin{bmatrix} f_1(0) & -f_1^{(1)}(0) & f_1^{(2)}(0) \\ f_1^{(1)}(0) & -f_1^{(2)}(0) & f_1^{(3)}(0) \\ f_1^{(2)}(0) & -f_1^{(3)}(0) & f_1^{(4)}(0) \end{bmatrix}$$

where

$$k^1_{0,0} = \frac{1}{0!0!}K_1(0,0) = f_1(0), \;\; k^1_{0,1} = \frac{1}{0!1!}\frac{\partial K_1(0,0)}{\partial \tau} = -f_1^{(1)}(0),$$
$$k^1_{0,2} = \frac{1}{0!2!}\frac{\partial^2 K_1(0,0)}{\partial \tau^2} = f_1^{(2)}(0), \;\; k^1_{1,0} = \frac{1}{1!0!}\frac{\partial K_1(0,0)}{\partial t} = f_1^{(1)}(0),$$
$$k^1_{1,1} = \frac{1}{1!1!}\frac{\partial^2 K_1(0,0)}{\partial t \partial \tau} = -f_1^{(2)}(0), \;\; k^1_{1,2} = \frac{1}{1!2!}\frac{\partial^3 K_1(0,0)}{\partial t \partial \tau^2} = f_1^{(3)}(0),$$
$$k^1_{2,0} = \frac{1}{2!0!}\frac{\partial^2 K_1(0,0)}{\partial t^2} = f_1^{(2)}(0), \;\; k^1_{2,1} = \frac{1}{2!1!}\frac{\partial^3 K_1(0,0)}{\partial t^2 \partial \tau} = -f_1^{(3)}(0),$$
$$k^1_{2,2} = \frac{1}{2!2!}\frac{\partial^4 K_1(0,0)}{\partial t^2 \partial \tau^2} = f_1^{(4)}(0).$$

3 Fundamental matrix equations for the system

We now ready to construct the fundamental matrix equations for the system of nonlinear delay integro-differential equations (1). For this purpose, substituting the matrix relations (4)-(8), (11) and (12) into system (1) and simplifying, we obtain the two matrix equations.

$$\mathbf{T}(t)\mathrm{B}\mathbf{Y}_1 = h_1\mathbf{T}(t)\mathbf{Y}_1 - \gamma_1\mathbf{T}(t)\mathbf{T}^*(t)\overline{\mathbf{Y}}_1 - \int_{t-T_0}^{t} \mathbf{T}(t)\mathbf{K}_1\mathbf{T}^T(\tau)\mathbf{T}(\tau)\mathbf{T}^*(t)\overline{\mathbf{Y}}_1\,d\tau + g_1(t)$$

and

$$\mathbf{T}(t)\mathbf{B}\mathbf{Y}_2 = -h_2\mathbf{T}(t)\mathbf{Y}_2 + \gamma_2\mathbf{T}(t)\mathbf{T}^*(t)\overline{\mathbf{Y}}_2 + \int_{t-T_0}^{t} \mathbf{T}(t)\mathbf{K}_2\mathbf{T}^T(\tau)\mathbf{T}(\tau)\mathbf{T}^*(t)\overline{\mathbf{Y}}_2\,d\tau + g_2(t)$$

or the system

$$\begin{cases} T(t)B - h_1T(t))Y_1 + (\gamma_1T(t)T^*(t) + T(t)K_1Q(t)T^*(t))\overline{\mathbf{Y}}_1 = g_1(t) \\ T(t)B + h_2T(t))Y_2 + (-\gamma_2T(t)T^*(t) - T(t)K_2Q(t)T^*(t))\overline{\mathbf{Y}}_2 = g_2(t) \end{cases} \tag{13}$$

where

$$\mathbf{Q}(t) = \int_{t-T_0}^{t} \mathbf{T}^T(\tau)\mathbf{T}(\tau)\,d\tau = [q_{m,n}(t)], \; m,n = 0,1,\dots,N,$$

$$[q_{m,n}(t)] = \frac{t^{m+n+1} - (t-T_0)^{m+n+1}}{m+n+1}, \ \ m,n = 0,1,...,N.$$

Therefore, we can write the matrix representation of the system (13) in the form

$$\begin{cases} \mathbf{D}_1(t)\mathbf{Y}_1 + \mathbf{A}_1(t)\overline{\mathbf{Y}}_1 = g_1(t) \\ \mathbf{D}_2(t)\mathbf{Y}_2 + \mathbf{A}_2(t)\overline{\mathbf{Y}}_2 = g_2(t) \end{cases} \tag{14}$$

where

$$\mathbf{D}_1(t) = \mathbf{T}(t)\mathbf{B} - h_1\mathbf{T}(t),$$

$$\mathbf{A}_1(t) = \gamma_1\mathbf{T}(t)\mathbf{T}^*(t) + \mathbf{T}(t)\mathbf{K}_1\mathbf{Q}(t)\mathbf{T}^*(t),$$

$$\mathbf{D}_2(t) = \mathbf{T}(t)\mathbf{B} + h_2\mathbf{T}(t)$$

and

$$\mathbf{A}_2(t) = -\gamma_2\mathbf{T}(t)\mathbf{T}^*(t) - \mathbf{T}(t)\mathbf{K}_2\mathbf{Q}(t)\mathbf{T}^*(t).$$

Consequently, the fundamental matrix equations of the system (14) can be written in the following compact form

$$\mathbf{D}(t)\mathbf{Y} + \mathbf{A}(t)\overline{\mathbf{Y}} = \mathbf{G}(t) \tag{15}$$

where

$$\mathbf{D}(t) = \begin{bmatrix} \mathbf{D}_1(t) & 0 \\ 0 & \mathbf{D}_2(t) \end{bmatrix}_{2\times 2(N+1)}, \ \mathbf{Y} = \begin{bmatrix} \mathbf{Y}_1 \\ \mathbf{Y}_2 \end{bmatrix}_{2(N+1)\times 1}, \ \mathbf{G}(t) = \begin{bmatrix} g_1(t) \\ g_2(t) \end{bmatrix}_{2\times 1},$$

$$\mathbf{A}(t) = \begin{bmatrix} \mathbf{A}_1(t) & 0 \\ 0 & \mathbf{A}_2(t) \end{bmatrix}_{2\times 2(N+1)^2}, \ \overline{\mathbf{Y}} = \begin{bmatrix} \overline{\mathbf{Y}}_1 \\ \overline{\mathbf{Y}}_2 \end{bmatrix}_{2(N+1)^2\times 1}.$$

4 Taylor matrix-collocation method

In this section, by substituting the collocation points defined by

$$t_s = \frac{b}{N}s, \ \ s = 0,1,...,N,$$

into the fundamental matrix equation (15), we obtain the new system

$$\mathbf{D}(t_s)\mathbf{Y} + \mathbf{A}(t_s)\overline{\mathbf{Y}} = \mathbf{G}(t_s), \ \ s = 0,1,\ldots,N \tag{16}$$

and therefore, the new fundamental matrix equation

$$\mathbf{D}\mathbf{Y}^* + \mathbf{A}\overline{\overline{\mathbf{Y}}} = \mathbf{G} \tag{17}$$

where

$$\mathbf{D}=\begin{bmatrix}\mathbf{D}(t_0) & 0 & \dots & 0\\ 0 & \mathbf{D}(t_1) & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \dots & \mathbf{D}(t_N)\end{bmatrix}_{2(N+1)\times 2(N+1)^2},\quad \mathbf{Y}^*=\begin{bmatrix}\mathrm{Y}\\ \mathrm{Y}\\ \vdots\\ \mathrm{Y}\end{bmatrix}_{2(N+1)^2\times 1},$$

$$\mathbf{A}=\begin{bmatrix}\mathbf{A}(t_0) & 0 & \dots & 0\\ 0 & \mathbf{A}(t_1) & \dots & 0\\ \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & \dots & \mathbf{A}(t_N)\end{bmatrix}_{2(N+1)\times 2(N+1)^3},\quad \overline{\overline{\mathbf{Y}}}=\begin{bmatrix}\overline{\mathrm{Y}}\\ \overline{\mathrm{Y}}\\ \vdots\\ \overline{\mathrm{Y}}\end{bmatrix}_{2(N+1)^3\times 1}.$$

By using equation(4), we obtain the matrix forms of the conditions as

$$\mathbf{T}(0)\mathbf{Y}_1=\alpha_1 \tag{18}$$

and

$$\mathbf{T}(0)\mathbf{Y}_2=\alpha_2. \tag{19}$$

By replacing the row matrices (18) and (19) by two rows of the matrix equation (17), we have the matrix equation based on the conditions. Thus, the fundamental matrix equation of the system (1) under initial conditions (2) corresponds to a system of $2(N+1)$ nonlinear algebraic equations with the unknown coefficients $y_{1,n}$ and $y_{2,n},\ (n=0,1,\dots,N)$.
Finally, the unknown coefficients are computed by solving this system and they are substituted in equation (3). Hence, the Taylor polynomial solutions

$$y_{i,N}(t)=\sum_{n=0}^{N}y_{i,n}t^n,\quad i=1,2 \tag{20}$$

can be obtained.

5 Accuracy of solutions

Accuracy of the approximate solutions are checked by substituting this solutions into the system (1); that is, for $t\in[0,1]$,

$$\begin{aligned}E_{1,N}(t)&=\left|\frac{dy_{1,N}(t)}{dt}-y_{1,N}(t)\left[h_1-\gamma_1 y_{2,N}(t)-\int_{t-T_0}^{t}f_1(t-\tau)y_{2,N}(\tau)\,d\tau\right]-g_1(t)\right|\\ E_{2,N}(t)&=\left|\frac{dy_{2,N}(t)}{dt}-y_{2,N}(t)\left[-h_2+\gamma_2 y_{1,N}(t)+\int_{t-T_0}^{t}f_2(t-\tau)y_{2,N}(\tau)\,d\tau\right]-g_2(t)\right|\end{aligned} \tag{21}$$

We expect that $E_{i,N}(t)=0$ on collocation points. The closer $y_i(t)\cong y_{i,N}(t)$ the closer $E_{i,N}(t)\cong 0$ for $t\in[0,1]$.

6 Numerical examples

In this section, some numerical examples are given to illustrate the accuracy and efficiency of the presented method (PM). The values of exact solutions $y_i(t),\ i=1,2$, and the absolute error functions $e_{i,N}(t)=|y_i(t)-y_{i,N}(t)|,\ i=1,2$, are

	Exact value	The absolute error obtained by the VIM	The absolute error obtained by the ADM	The absolute error obtained by the PLM	The absolute error obtained by the Present Method for $N=5$
t_i	$y_1(t_i)=-3t_i+1$				$e_{1,5}(t_i)$
0.1	0.7	0.31518853698e-3	0.1091753814113e-3	0.1986261648238e-12	0.1e-14
0.2	0.4	0.42728911642e-3	0.1788920829385e-3	0.3231153999959e-12	0
0.3	0.1	0.47331318020e-3	0.1083617853644e-3	0.3615806499154e-12	0.17e-14
0.4	-0.2	0.48554023324e-3	0.3094825605990e-3	0.3213172783951e-12	0.1e-14
0.5	-0.5	0.47436316821e-3	0.00135621442726	0.2096206492477e-12	0.2e-14
0.6	-0.8	0.44598126050e-3	0.00135621442726	0.3378612628621e-13	0.2e-14
0.7	-1.1	0.43682261124e-3	0.00637023644866	0.1988909266767e-12	0
0.8	-1.4	0.53581434761e-3	0.01046626665686	0.4811151458281e-12	0.1e-13
0.9	-1.7	0.91000161606e-3	0.01522647918089	0.8055911673554e-12	0.1e-13
1.0	-2.0	0.001829471578	0.01994495324168	0.1165023627445e-11	0

Table 1: Comparison of the absolute errors obtained by the VIM , the ADM, the PLM method and the present method for $y_1(t)$ in Example 2.

	Exact value	The absolute error obtained by the VIM	The absolute error obtained by the ADM	The absolute error obtained by the PLM	The absolute error obtained by the Present Method for $N=5$
t_i	$y_2(t_i)=t_i^2-t_i$				$e_{2,5}(t_i)$
0.1	-0.09	0.33411928037255e-4	0.75832265307973e-5	0.44685408802023e-13	0
0.2	-0.16	0.85452915051187e-4	0.13695925555615e-3	0.37454199426623e-13	0
0.3	-0.21	0.13335169483697e-3	0.76561773005442e-3	0.11098050333474e-13	0.1e-14
0.4	-0.24	0.17989583110958e-3	0.00275335330117	0.90378079241246e-13	0.1e-14
0.5	-0.25	0.22278054776309e-3	0.00763820604833	0.18979262605967e-12	0.2e-14
0.6	-0.24	0.23711636589643e-3	0.01773821505593	0.29874842955172e-12	0.1e-14
0.7	-0.21	0.16268898580501e-3	0.03615084131132	0.40665222848038e-12	0.3e-14
0.8	-0.16	0.10708347556534e-3	0.06669525870549	0.50291076160862e-12	0.3e-14
0.9	-0.09	0.73102416289476e-3	0.11388830112804	0.57693076769942e-12	0.37e-14
1.0	0	0.00190775600163	0.18307114572060	0.61811898551576e-12	0.4592130396e-14

Table 2: Comparison of the absolute errors obtained by the VIM , the ADM, the PLM method and the present method for $y_2(t)$ in Example 2.

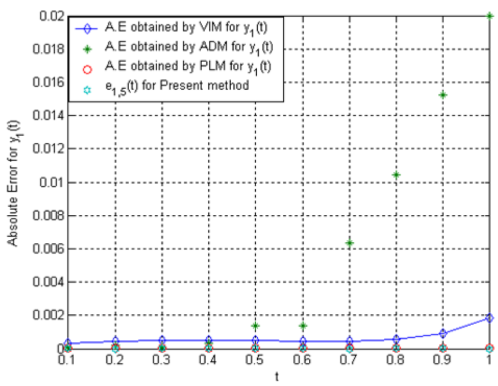
presented at selected points of the given interval. Results are shown with tables and figures. All of them were performed on the computer using a program written in Maple 15.

Example 1. As the first example, consider the system (1)-(2) with $f_1(t)=f_2(t)=1, h_1=h_2=1, \gamma_1=\gamma_2=1, T_0=1, \alpha_1=0,\ \alpha_2=1,\quad g_1(t)=1+t$ and $g_2(t)=\frac{3}{2}-2t$. The exact solutions of this system are in the following form $y_1(t)=t,\quad y_2(t)=1$. By applying the presented method in Section 4 for $N=2$, we have the solutions $\hat{y}_1(t)=t,\quad \hat{y}_2(t)=1$ which are the exact solutions.

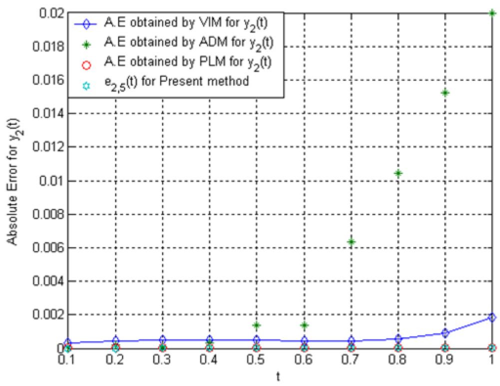
Example 2. [11], [13], [14] Now we consider the system (1)-(2) with $f_1(t)=1,\quad f_2(t)=t-1, h_1=1,\ h_2=2, \gamma_1=\frac{1}{3},\ \gamma_2=1, T_0=\frac{1}{2},\quad \alpha_1=1,\ \alpha_2=0,\ g_1(t)=-\frac{5}{2}t^3+\frac{49}{12}t^2+\frac{17}{2}t-\frac{23}{6}$, and $g_2(t)=\frac{15}{8}t^3-\frac{1}{4}t^2+\frac{3}{8}t-1$. The exact solutions of this system are in the following form

$$y_1(t)=-3t+1,\quad y_2(t)=t^2-t.$$

We obtain the approximate solutions by Taylor polynomials of the problem for $N=5,6,7$. In Tables 1-2 and Figure 1, the absolute errors obtained by the present method for $N=5$ are compared with the results obtained by variational iteration method [13], Adomian decomposition method and pseudospectral Legendre method [11]. It is seen from this tables and figures that the present method is closer to exact solution than the other methods. Additionally in Tables 3-4 and Figure 2, the absolute errors for the present method are shown for different values of N. These datas show that as N increases, than the absolute errors decrease more rapidly.



(a) For $y_1(t)$



(b) For $y_2(t)$

Fig. 1: Comparison of the absolute errors obtained by different methods for $y_1(t)$ and $y_2(t)$ in Example 2.

		Present Method	
t_i	$e_{1,5}(t_i)$	$e_{1,6}(t_i)$	$e_{1,7}(t_i)$
0.1	0.1e-14	0.9e-15	0.2e-16
0.2	0	0.17e-14	0.1e-16
0.3	0.17e-14	0.24e-14	0.14e-16
0.4	0.1e-14	0.32e-14	0.1e-16
0.5	0.2e-14	0.39e-14	0.7e-16
0.6	0.2e-14	0.46e-14	0.6e-16
0.7	0	0.5e-14	0
0.8	0.1e-13	0.5e-14	0
0.9	0.1e-13	0.6e-14	0.1e-15
1.0	0	0.8e-14	0.1e-15

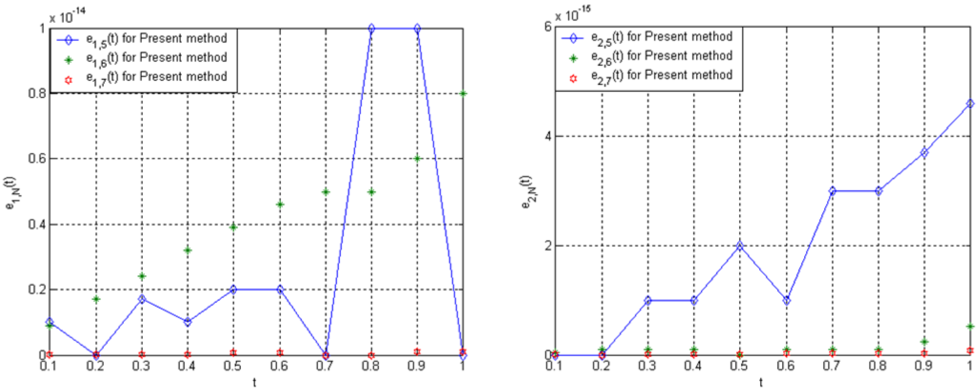
Table 3: The comparison of the absolute error functions $e_{1,N}(t)$ obtained by the present method for $N = 5, 6, 7$ in Example 2.

		Present Method	
t_i	$e_{2,5}(t_i)$	$e_{2,6}(t_i)$	$e_{2,7}(t_i)$
0.1	0	0.5e-16	0.2e-17
0.2	0	0.1e-15	0
0.3	0.1e-14	0.1e-15	0.1e-16
0.4	0.1e-14	0.1e-15	0.1e-16
0.5	0.2e-14	0	0.1e-16
0.6	0.1e-14	0.1e-15	0.2e-16
0.7	0.3e-14	0.1e-15	0.2e-16
0.8	0.3e-14	0.1e-15	0.3e-16
0.9	0.37e-14	0.23e-15	0.32e-16
1.0	0.4592130396e-14	0.524200101921e-15	0.824048308268e-16

Table 4: The comparison of the absolute error functions $e_{2,N}(t)$ obtained by the present method for $N = 5, 6, 7$ in Example 2.

Example 3. [13] In this example, we solve the system (1)-(2) with
$f_1(t) = 2t - 3,\ f_2(t) = t,\ h_1 = h_2 = 2,\ \gamma_1 = \gamma_2 = 1,\ T_0 = \frac{1}{3}$,
$\alpha_1 = \alpha_2 = 0,\ g_1(t) = t^2\left(2 - 3te^{-t} - \frac{7}{2}e^{-t} + \frac{13}{6}te^{\frac{1}{3}-t} + \frac{22}{9}e^{\frac{1}{3}-t}\right) - 2t$
and $g_2(t) = \frac{1}{648}e^{-t}\left(324t^3 - 8t^2 + 325t + 324\right)$. $y_1(t) = t^2,\ y_2(t) = \frac{1}{2}te^{-t}$ are the exact solutions of this system.

Using the procedure in section 4, we calculate the approximate solutions $y_{1,N}(t)$ and $y_{2,N}(t)$ for $N = 7, 8, 9$. The exact solutions, absolute errors obtained by other methods and the present method are given in Tables 5-6 and Figure 3. On the other hand, in Tables 7-8 and Figure 4, the absolute errors for the present method are shown for different values of N.



(a) Error function $e_{1,N}(t)$ **(b)** Error function $e_{2,N}(t)$

Fig. 2: Comparison of absolute error functions of Example 2 for $N = 5, 6, 7$.

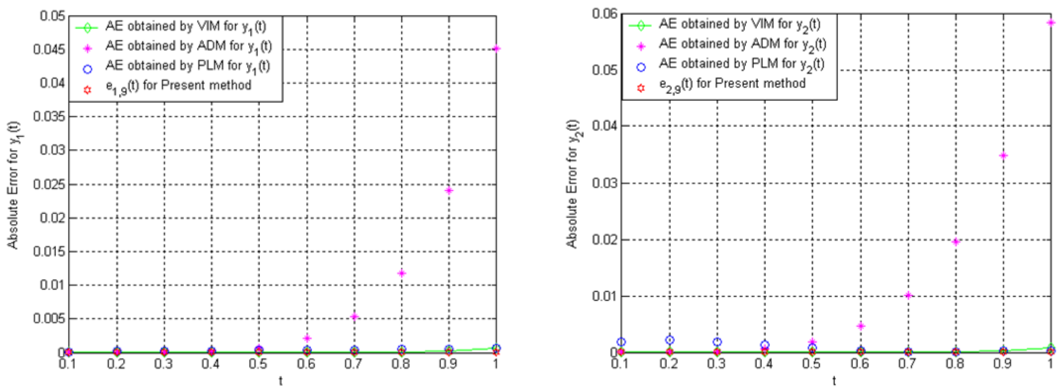
	The absolute error obtained by the VIM	The absolute error obtained by the ADM	The absolute error obtained by the PLM	The absolute error obtained by the Present Method for $N = 9$
t_i				$e_{1,9}(t_i)$
0.1	0.45022739981e-9	0.1685348539268161e-5	0.102301896075768e-3	0.54621e-12
0.2	0.40721536988e-8	0.2561087051100586e-5	0.176153769775657e-3	0.103513e-11
0.3	0.47234385000e-7	0.3700245898130561e-4	0.229322565360404e-3	0.130287e-11
0.4	0.36479857182e-6	0.1882102487513748e-3	0.269575227090752e-3	0.15896e-11
0.5	0.20359558284e-5	0.6922368720978511e-3	0.304678699227443e-3	0.19494e-11
0.6	0.88059904333e-5	0.00207699931636	0.342399926031219e-3	0.23863e-11
0.7	0.31211009285e-4	0.00527714088699	0.390505851762824e-3	0.29375e-11
0.8	0.94446707722e-4	0.01179548204175	0.456763420682999e-3	0.35725e-11
0.9	0.25161861726e-3	0.02398306778480	0.548939577052489e-3	0.45035e-11
1.0	0.60400227331e-3	0.04515276541565	0.674801265132035e-3	0.8311e-11

Table 5: The comparison of the absolute errors obtained by the VIM , the ADM, the PLM method and the present method for $y_1(t)$ in Example 3.

	The absolute error obtained by the VIM	The absolute error obtained by the ADM	The absolute error obtained by the PLM	The absolute error obtained by the Present Method for $N = 9$
t_i				$e_{2,9}(t_i)$
0.1	0.98098325907e-7	0.2531305741406212e-5	0.00176027644054	0.594778e-11
0.2	0.69336692832e-7	0.2128400908629757e-4	0.00220320442481	0.382128e-11
0.3	0.26970838007e-6	0.1502891420450681e-3	0.00191611272021	0.34042e-11
0.4	0.35540733734e-6	0.6290715880985509e-3	0.00133037513186	0.26107e-11
0.5	0.24947044039e-5	0.00189973140060	0.7399618486387920e-3	0.21271e-11
0.6	0.10872453004e-4	0.00469004735611	0.3178723165631683e-3	0.16516e-11
0.7	0.38522813611e-4	0.01007382241455	0.1306848384973053e-3	0.11407e-11
0.8	0.11488099653e-3	0.01950960742570	0.1514325199644534e-3	0.15007e-11
0.9	0.30092661745e-3	0.03487342272136	0.2709923394895036e-3	0.44028e-11
1.0	0.71128431455e-3	0.05840750457072	0.3081537298062764e-3	0.2518797e-9

Table 6: The comparison of the absolute errors obtained by the VIM , the ADM, the PLM method and the present method for $y_2(t)$ in Example 3.

Additionally, in Table 9, the accuracy of solutions are stated. These results show that present method is closer to exact solution than the other methods and if N increases, than the absolute errors decrease more rapidly.



(a) For $y_1(t)$ **(b)** For $y_2(t)$

Fig. 3: Comparison of the absolute errors obtained by different methods for $y_1(t)$ and $y_2(t)$ in Example 3.

	Exact Solution		Present method	
t_i	$y_1(t_i) = -t_i^2$	$e_{1,7}(t_i)$	$e_{1,8}(t_i)$	$e_{1,9}(t_i)$
0.1	-0.01	0.5461790e-10	0.709735e-11	0.54621e-12
0.2	-0.04	0.14796565e-9	0.1432477e-10	0.103513e-11
0.3	-0.09	0.20549045e-9	0.1815547e-10	0.130287e-11
0.4	-0.16	0.2411574e-9	0.220830e-10	0.15896e-11
0.5	-0.25	0.3069964e-9	0.274994e-10	0.19494e-11
0.6	-0.36	0.4059928e-9	0.337797e-10	0.23863e-11
0.7	-0.49	0.4782768e-9	0.416153e-10	0.29375e-11
0.8	-0.64	0.5705779e-9	0.527492e-10	0.35725e-11
0.9	-0.81	0.12988953e-8	0.584030e-10	0.45035e-11
1.0	-1	0.4714350e-8	0.7797e-11	0.8311e-11

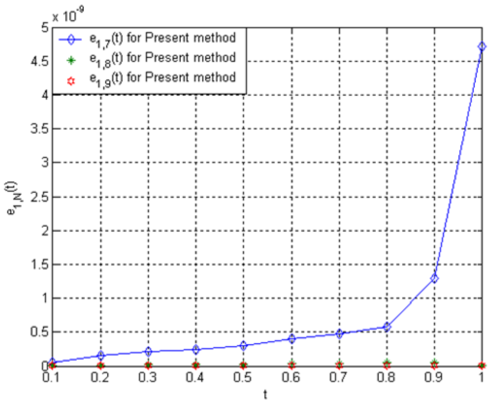
Table 7: The comparison of the absolute error functions $e_{1,N}(t)$ obtained by the present method for $N = 7, 8, 9$ in Example 3.

	Exact Solution		Present method	
t_i	$y_2(t) = \frac{1}{2}te^{-t}$	$e_{2,7}(t_i)$	$e_{2,8}(t_i)$	$e_{2,9}(t_i)$
0.1	0.04524187090180	0.434756143e-8	0.16058984e-9	0.594778e-11
0.2	0.08187307530780	0.349666241e-8	0.11001843e-9	0.382128e-11
0.3	0.11112273310226	0.21396338e-8	0.872505e-10	0.34042e-11
0.4	0.13406400920713	0.24005294e-8	0.784700e-10	0.26107e-11
0.5	0.15163266492816	0.15742853e-8	0.512789e-10	0.21271e-11
0.6	0.16464349082821	0.9457157e-9	0.553099e-10	0.16516e-11
0.7	0.17380485632699	0.24452969e-8	0.220715e-10	0.11407e-11
0.8	0.17973158564689	0.45216675e-8	0.405498e-10	0.15007e-11
0.9	0.18295634688327	0.408948078e-7	0.902790e-10	0.44028e-11
1.0	0.18393972058572	0.748574986e-7	0.60301322e-8	0.2518797e-9

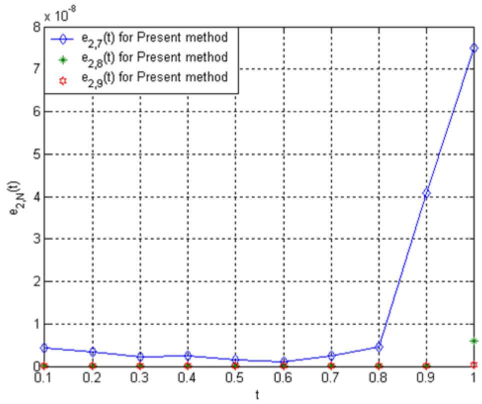
Table 8: The comparison of the absolute error functions $e_{2,N}(t)$ obtained by the present method for $N = 7, 8, 9$ in Example 3.

			Present method	
t_i	$E_{1,8}(t_i)$	$E_{1,9}(t_i)$	$E_{2,8}(t_i)$	$E_{2,9}(t_i)$
0	0	0	0	
0.1	0.806584461e-10	0.255e-10	0.86000006e-9	0.6000517404e-10
0.2	0.719508141e-10	0.532e-10	0.47999997e-9	0.1999601427e-10
0.3	0.343505686e-9	0.220e-9	0.18000124e-9	0.2007735870e-10
0.4	0.364979109e-9	0.15e-9	0.5998097e-10	0.6059128040e-10
0.5	0.446603860e-9	0.16e-9	0.10015322e-9	0.1028384404e-9
0.6	0.48728944e-9	0.10e-9	0.26081783e-9	0.5009288648e-10
0.7	0.6897561e-9	0.39e-9	0.2767199e-9	0.905070594e-11
0.8	0.2655416e-9	0.21e-9	0.18906488e-8	0.1915105741e-9
0.9	0.449297e-9	0.2e-9	0.68306403e-8	0.1838402061e-9
1.0	0.781939e-8	0.7e-9	0.183328948e-6	0.8389655597e-8

Table 9: Accuracies of the solutions of Example 3 for $N = 8, 9$.



(a) Error function $e_{1,N}(t)$



(b) Error function $e_{2,N}(t)$

Fig. 4: Comparison of the absolute errors functions in Example 3 for $N = 7, 8, 9$.

7 Conclusion

In this study, a new Taylor matrix-collocation method is proposed for numerical solutions of a model describing biological species living together. It is observed from Figures and Tables that the method is a simple and powerful tool to obtain the approximate solution. When the numerical experiments are analyzed and the results are compared, it is seen that, the present method is quite effective. Additionally, if is increased, it can be seen that approximate solutions obtained by the mentioned method are closed to the exact solutions. One of the considerable advantage of the method is founding the approximate solutions very easily by using the computer program written in Maple 15. Shorter computation time and lower operation count results in a reduction of cumulative truncation errors and improvement of overall accuracy. In addition, the method can also be extended to other models in the future.

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