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Some characterizations of curves of AW(k)-type according to the Bishop frame

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Abstract: In this paper, we consider curves of AW(k)-type with harmonic curvatures in the Bishop frame in Euclidean space. Moreover, we obtain slant helices of AW(k)-type according to the Bishop frame in Euclidean space.

Keywords: Bishop frame, Frenet frame, AW(k)-type, harmonic curvature, slant helices.

1 Introduction

In the the study of the fundamental theory of curves, getting new and interesting characterizations of space curves are a field of research. One of these characterizations is related to curves of $AW(k)$-type. There are many interesting results obtained about curves of $AW(k)$-type in both Euclidean and Lorentzian spaces (see [2], [8],[9], [10]).

The Frenet frame is constructed for the curve of 3-time continuously differentiable non-degenerate curves. Curvature of the curve may vanish at some points on the curve, that is, second derivative of the curve may be zero. In this situation, we need an alternative frame in $\mathbb{E}^3$. Therefore Bishop defined a new frame for a curve and called it Bishop frame which is well defined even when the curve has vanishing second derivative in Euclidean space $\mathbb{E}^3$ [4]. We can take parallel transport of an orthonormal frame along a curve simply by parallel transporting each component of the frame into consideration.

The natural curvatures of Bishop frame were studied in [7]. Also, the same authors considered slant helix according to Bishop frame in Euclidean 3-Space [5]. In [11], the spinor formulations of curves according to Bishop frame were given in $\mathbb{E}^3$.

The notion of $AW(k)$-type submanifolds was introduced by Arslan and West in [1]. In particular, many works related to curves of $AW(k)$-type have been done by several authors. For example, in [2], [10], the authors gave curvature conditions and characterizations related to these curves in $\mathbb{E}^m$. Curves and surfaces of $AW(k)$ ($k = 1,2$ or 3)-type were considered in [8]. They also give related examples of curves and surfaces satisfying $AW(k)$-type conditions. Also, the characterizations of curves of $AW(k)$-type were given in [9].

In this paper, we give some characterizations of curves of $AW(k)$-type in terms of harmonic curvatures according to the Bishop frame in Euclidean space $\mathbb{E}^3$. Also, we obtain slant helices of $AW(k)$-type according to the Bishop frame in Euclidean space $\mathbb{E}^3$.

2 Preliminaries

The parallel transport frame is established on the observation that, while $T(s)$ for a given curve model is unique, we may choose any convenient arbitrary basis $(N_1(s), N_2(s))$ for the remainder of the frame, so long as it is in the normal plane perpendicular to $T(s)$ at each point. If the derivatives of $(N_1(s), N_2(s))$ depend only on $T(s)$ and not each other we

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can make $N_1(s)$ and $N_2(s)$ variable smoothly throughout the path regardless of the curvature [4]. Therefore, we have the alternative frame equations

$$\begin{aligned} T' &= k_1N_1 + k_2N_2 \\ N_1' &= -k_1T \\ N_2' &= -k_2T \end{aligned}$$

One can show [4] that

$$\kappa(s) = \sqrt{k_1^2 + k_2^2},\ \tau(s) = -\frac{d\theta(s)}{ds},\ \theta(s) = \arctan\left(\frac{k_2}{k_1}\right),\ k_1 \neq 0$$

so that k_1 and k_2 effectively correspond to a Cartesian coordinate system for the polar coordinates κ, θ with $\theta = -\int \tau(s)ds$.

Definition 2.1. [3] Let γ be a unit speed curve of osculating order d. The functions $H_i : I \to E$, $1 \leq j \leq d-2$ defined by

$$H_1(s) = \frac{k_1(s)}{k_2(s)},\ \ H_j = \{D_{v_1}H_{j-1} + H_{j-2}\kappa_j\}\frac{1}{\kappa_{j+1}},\ 2 \leq j \leq d-2 \tag{1}$$

are called the harmonic curvatures of γ where κ_1, κ_2,..., κ_{d-1} are Frenet curvatures γ which are not necessarily constant.

Proposition 2.1. [3] Let γ be a Frenet curve of osculating order 3 in $\mathbb{E}^3$, then we have

$$\begin{gathered} \gamma'(s) = T(s) \\ \gamma''(s) = T'(s) = \kappa(s)N(s) \\ \gamma'''(s) = -\kappa^2(s)T(s) + \kappa'(s)N(s) + \frac{\kappa^2(s)}{H_1(s)}B(s) \\ \gamma^{(4)}(s) = -3\kappa(s)\kappa'(s)T(s) + \{\kappa''(s) - \kappa^3(s) - \frac{\kappa^3(s)}{H_1^2(s)}\}N(s) + \{\frac{3\kappa'(s)\kappa(s)H_1(s) - \kappa^2(s)H_1'(s)}{H_1^2(s)}\}B(s). \end{gathered}$$

Notation 2.1. [3] Let us write the base vectors as follows:

$$\begin{gathered} N_1(s) = \kappa(s)N(s) \\ N_2(s) = \kappa'(s)N(s) + \frac{\kappa^2(s)}{H_1(s)}B(s) \\ N_3(s) = \{\kappa''(s) - \kappa^3(s)(1 + \frac{1}{H_1^2(s)})\}N(s) + \{\frac{3\kappa'(s)\kappa(s)H_1(s) - \kappa^2(s)H_1'(s)}{H_1^2(s)}\}B(s). \end{gathered}$$

Definition 2.2. [2] The Frenet curves of order 3 are
(i) of weak $AW(2)$-type if they satisfy

$$N_3(s) = \langle N_3(s),\ N_2^*(s)\rangle N_2^*(s),$$

(ii) of weak $AW(3)$-type if they satisfy

$$N_3(s) = \langle N_3(s),\ N_1^*(s)\rangle N_1^*(s)$$

where

$$\begin{gathered} N_1^*(s) = \frac{N_1(s)}{|N_1(s)|} \\ N_2^*(s) = \frac{N_2(s) - \langle N_2(s),\ N_1^*(s)\rangle N_1^*(s)}{\left|N_2(s) - \langle N_2(s),\ N_1^*(s)\rangle N_1^*(s)\right|}. \end{gathered}$$

Definition 2.3. [1] The Frenet curves of order 3 are
(i) of $AW(1)$-type if they satisfy

$$N_3(s) = 0,$$

(ii) of $AW(2)$-type if they satisfy

$$\|N_2(s)\|^2 N_3(s) = \langle N_3(s),\, N_2(s)\rangle N_2(s),$$

(iii) of $AW(3)$-type if they satisfy

$$\|N_1(s)\|^2 N_3(s) = \langle N_3(s),\, N_1(s)\rangle N_1(s).$$

Proposition 2.2. [2] Let γ be a Frenet curve of osculating order 3. Then γ is of $AW(1)$-type if and only if

$$\kappa''(s) - \kappa^3(s) - \kappa(s)\tau^2(s) = 0,\ \tau(s) = \frac{c}{\kappa^2(s)},\ c \in \mathbb{R}.$$

Proposition 2.3. [2] Let γ be a Frenet curve of osculating order 3. Then γ is of $AW(2)$-type if and only if

$$2(\kappa'(s))^2\tau(s) + \kappa(s)\kappa'(s)\tau'(s) = \kappa''(s)\kappa(s)\tau(s) - \kappa^4(s)\tau(s) - \kappa^2(s)\tau^3(s).$$

Proposition 2.4. [2] Let γ be a Frenet curve of osculating order 3. Then γ is of $AW(3)$-type if and only if

$$2\kappa'(s)\tau(s) + \kappa(s)\tau'(s) = 0,$$

the solution of this differential equation is $\tau(s) = \dfrac{c}{\kappa^2(s)}$, $c \in \mathbb{R}$.

Proposition 2.5. [9] Let γ be a unit speed curve of osculating order 3 in $\mathbb{E}^3$ and $\{T,\, M_1,\, M_2\}$ be the Bishop frame. We have

$$\begin{aligned}
\gamma'(s) &= T(s)\\
\gamma''(s) &= k_1(s)M_1(s) + k_2(s)M_2(s)\\
\gamma'''(s) &= \left\{-k_1^2(s) - k_2^2(s)\right\} T(s) + k_1'(s)M_1(s) + k_2'(s)M_2(s)\\
\gamma^{(4)}(s) &= \{\left(-k_1^2(s) - k_2^2(s)\right)' + \left(-k_1(s)k_1'(s) - k_2(s)k_2'(s)\right)\}T(s) + \{k_1''(s) - k_1^3(s) - k_1(s)k_2^2(s)\}M_1(s)\\
&\quad + \{k_2''(s) - k_2^3(s) - k_1^2(s)k_2(s)\}M_2(s).
\end{aligned}$$

Notation 2.2. [9] Let us write the base vectors as follows:

$$\overline{N}_1(s) = k_1(s)M_1(s) + k_2(s)M_2(s) \tag{2}$$

$$\overline{N}_2(s) = k_1'(s)M_1(s) + k_2'(s)M_2(s) \tag{3}$$

$$\begin{aligned}\overline{N}_3(s) = \{k_1''(s) - k_1^3(s) - k_1(s)k_2^2(s)\}M_1(s)\\ + \{k_2''(s) - k_2^3(s) - k_1^2(s)k_2(s)\}M_2(s).\end{aligned} \tag{4}$$

Corollary 2.1. [9] Let $\gamma'(s)$, $\gamma''(s)$, $\gamma'''(s)$, $\gamma^{iv}(s)$ are linearly dependent if and only if $\overline{N}_1(s)$, $\overline{N}_2(s)$, $\overline{N}_3(s)$ are linearly dependent.

Theorem 2.1. [9] Let γ be a curve of order 3, then

$$\overline{N}_3(s) = \langle \overline{N}_3(s),\, \overline{N}_1^*(s)\rangle \overline{N}_1^*(s) + \langle \overline{N}_3(s),\, \overline{N}_2^*(s)\rangle \overline{N}_2^*(s) \tag{5}$$

where

$$\overline{N}_1^*(s) = \frac{\overline{N}_1(s)}{|\overline{N}_1(s)|} \tag{6}$$

$$\overline{N}_2^*(s) = \frac{\overline{N}_2(s) - \langle \overline{N}_2(s),\, \overline{N}_1^*(s)\rangle \overline{N}_1^*(s)}{|\overline{N}_2(s) - \langle \overline{N}_2(s),\, \overline{N}_1^*(s)\rangle \overline{N}_1^*(s)|}. \tag{7}$$

Definition 2.4. [9] The unit speed curves of order 3 are

(i) of weak Bishop $AW(2)$-type if they satisfy

$$\overline{N}_3(s) = \langle \overline{N}_3(s),\, \overline{N}_2^*(s)\rangle \overline{N}_2^*(s), \tag{8}$$

(ii) of weak Bishop $AW(3)$-type if they satisfy

$$\overline{N}_3(s) = \langle \overline{N}_3(s),\, \overline{N}_1^*(s)\rangle \overline{N}_1^*(s). \tag{9}$$

Proposition 2.6. [9] Let γ be a curve of order 3. If γ is

(i) of the Bishop $AW(2)$-type, then the curvature equation is

$$k_2'(s)(k_1''(s) - k_1^3(s) - k_1(s)k_2^2(s)) = k_1'(s)(k_2''(s) - k_2^3(s) - k_1^2(s)k_2(s)), \tag{10}$$

(ii) of the Bishop $AW(3)$-type, then the curvature equation is

$$k_2(s)(k_1''(s) - k_1^3(s) - k_1(s)k_2^2(s)) = k_1(s)(k_2''(s) - k_2^3(s) - k_1^2(s)k_2(s)). \tag{11}$$

Definition 2.5. [5] A regular curve $\gamma : I \to E^3$ is called a slant helix provided the unit vector $N_1(s)$ of γ has constant angle θ with some fixed unit vector u; that is, $\langle N_1(s),\, u\rangle = \cos\theta$ for all $s \in I$.

Slant helices can be identified by simple condition on natural curvatures as follows:

Theorem 2.2. [5] Let $\gamma : I \to E^3$ be a unit speed curve with nonzero natural curvatures. Then γ is a slant helix if and only if $\dfrac{k_1}{k_2}$ is constant.

Theorem 2.3. [6] γ is a slant helix if and only if the geodesic curvature of the spherical image of principal normal indicatrix N of γ

$$\sigma(s) = \left(\frac{\kappa^2}{c(\kappa^2+\tau^2)^{\frac{3}{2}}}\left(\frac{\tau}{\kappa}\right)'\right)(s)$$

is a constant function.

Remark 2.1. [10] If γ is a slant helix of osculating order 3 in E^3, then the curvature equation is

$$k_2'(s)k_1(s) - k_1'(s)k_2(s) = c(k_1^2(s) + k_2^2(s))^{\frac{3}{2}};\ c \in \mathbb{R}. \tag{12}$$

3 Characterizations of curves of $AW(k)$-type with harmonic curvatures according to the Bishop frame

In this section, we consider curves of $AW(k)$-type according to the Bishop Frame by using harmonic curvatures.

Proposition 3.1. Let γ be a unit speed curve of osculating order 3 in $\mathbb{E}^3$ and $\{T,\, M_1,\, M_2\}$ be the Bishop frame of the curve with harmonic curvature. We have

$$\begin{aligned}
\gamma'(s) &= T(s)\\
\gamma''(s) &= k_1(s)M_1(s) + \frac{k_1(s)}{H_1(s)}M_2(s)\\
\gamma'''(s) &= \{-k_1^2(s) - \frac{k_1^2(s)}{H_1^2(s)}\}T(s) + k_1^{'}(s)M_1(s) + \frac{k_1^{'}(s)H_1(s) - k_1(s)H_1^{'}(s)}{H_1^2(s)}M_2(s)\\
\gamma^{(4)}(s) &= \{(-k_1^2(s) - \frac{k_1^2(s)}{H_1^2(s)}) + (-k_1(s)k_1'(s) - \frac{k_1(s)k_1^{'}(s)H_1(s) - k_1^2(s)H_1^{'}(s)}{H_1^2(s)})\}T(s)\\
&\quad + \{k_1^{''}(s) - k_1^3(s)(1 + \frac{1}{H_1^2(s)})\}M_1(s) + \{k_2^{''}(s) - k_2^3(s)\left(1 + H_1^2(s)\right)\}M_2(s)).
\end{aligned}$$

Notation 3.1. Let us write the base vectors as follows:

$$\overline{N}_1(s) = k_1(s)M_1(s) + \frac{k_1(s)}{H_1(s)}M_2(s) \tag{13}$$

$$\overline{N}_2(s) = k_1^{'}(s)M_1(s) + \frac{k_1^{'}(s)H_1(s) - k_1(s)H_1^{'}(s)}{H_1^2(s)}M_2(s) \tag{14}$$

$$\overline{N}_3(s) = \{k_1^{''}(s) - k_1^3(s)(1 + \frac{1}{H_1^2(s)})\}M_1(s) + \{k_2^{''}(s) - k_2^3(s)\left(1 + H_1^2(s)\right)\}M_2(s). \tag{15}$$

Proposition 3.2. Let γ be a unit speed curve of osculating order 3 in $\mathbb{E}^3$. If γ is a weak harmonic curve of the Bishop $AW(2)$-type, then the equations reduce to the following form:

$$\begin{aligned}
&(k_1^{''}(s)H_1^2(s) - k_1^3(s)H_1^2(s) - k_1^3(s) - k_2^{''}(s)H_1^3(s) + k_2^3(s)H_1^3(s) + k_2^3(s)H_1^5(s))\\
&= \left(H_1^2(s) + 1\right)(k_1^{''}(s)H_1^2(s) - k_1^3(s)H_1^2(s) - k_1^3(s))
\end{aligned} \tag{16}$$

and

$$\begin{aligned}
&(-k_1^{''}(s)H_1^3(s) + k_1^3(s)H_1^3(s) + k_1^3(s)H_1(s) + H_1^4(s)k_2^{''}(s) + k_2^3(s)H_1^4(s) + k_2^3(s)H_1^6(s))\\
&= H_1^2(s)\left(H_1^2(s) + 1\right)(k_2^{''}(s) - k_2^3(s) - k_2^3(s)H_1^2(s)).
\end{aligned} \tag{17}$$

Proof. With the use of equations (2), (3), (6) and (7), we get

$$\overline{N}_1^*(s) = \frac{H_1(s)M_1(s) + M_2(s)}{\sqrt{H_1^2(s) + 1}} \tag{18}$$

and

$$\overline{N}_2^*(s) = \frac{M_1(s) - M_2(s)H_1(s)}{\sqrt{H_1^2(s) + 1}}. \tag{19}$$

Substituting the equation (18) into (9), we obtain (16) and (17).

Proposition 3.3. Let γ be a unit speed curve of osculating order 3 in $\mathbb{E}^3$. If γ is a weak harmonic curve of the Bishop

$AW(3)$-type, then the equations are

$$\begin{aligned}(k_1''(s)H_1^4(s) - k_1^3(s)H_1^4(s) - k_1^3(s)H_1^2(s) + k_2''(s)H_1^3(s) - k_2^3(s)H_1^3(s) + k_2^3(s)H_1^5(s)) \\ = \left(H_1^2(s)+1\right)(k_1''(s)H_1^2(s) - k_1^3(s)H_1^2(s) - k_1^3(s)),\end{aligned} \tag{20}$$

and

$$\begin{aligned}(k_1''(s)H_1^3(s) - k_1^3(s)H_1^3(s) - k_1^3(s)H_1(s) + k_2''(s)H_1^2(s) - k_2^3(s)H_1^2(s) - k_2^3(s)H_1^4(s)) \\ = H_1^2(s)\left(H_1^2(s)+1\right)(k_2''(s) - k_2^3(s) - k_2^3(s)H_1^2(s)).\end{aligned} \tag{21}$$

Proof. Substituting the equation (19) into (8), we obtain (20) and (21).

Definition 3.1. The unit speed curves of order 3 are

(i) of the Bishop $AW(1)$-type with harmonic curvature if they satisfy

$$\overline{N}_3(s) = 0, \tag{22}$$

(ii) of the Bishop $AW(2)$-type with harmonic curvature if they satisfy

$$\left\|\overline{N}_2(s)\right\|^2 \overline{N}_3(s) = \left\langle \overline{N}_3(s), \overline{N}_2(s)\right\rangle \overline{N}_2(s), \tag{23}$$

(iii) of the Bishop $AW(3)$-type with harmonic curvature if they satisfy

$$\left\|\overline{N}_1(s)\right\|^2 \overline{N}_3(s) = \left\langle \overline{N}_3(s), \overline{N}_1(s)\right\rangle \overline{N}_1(s). \tag{24}$$

Proposition 3.4. Let γ be a unit speed curve of osculating order 3 in $\mathbb{E}^3$. If γ is of the Bishop $AW(1)$-type with harmonic curvature, then the curvature equations are

$$k_1''(s) - k_1^3(s)(1 + \frac{1}{H_1^2(s)}) = 0 \tag{25}$$

$$k_2''(s) - k_2^3(s)\left(1 + H_1^2(s)\right) = 0, \tag{26}$$

and a solution of this differential equation system is

$$k_1(s) = k_2(s) = \mp\frac{1}{s+c}, \; c \in \mathbb{R}. \tag{27}$$

Proof. Using the equations (15) and (22), we get the equations (25) and (26). If we solve (25) and (26), we obtain $k_1(s) = k_2(s)$.

Proposition 3.5. Let γ be a unit speed curve of osculating order 3 in $\mathbb{E}^3$. There is not circular and general helix of the Bishop $AW(1)$-type with harmonic curvature.

Proof. Let γ be a unit speed curve of osculating order 3 in $\mathbb{E}^3$. Then since $H_1(s)$ is constant, $H_1'(s) = 0$. Also since γ is a curve of $AW(1)$-type, we get

$$\begin{aligned}\{k_1''(s) - k_1^3(s)(1 + \frac{1}{H_1^2(s)})\}M_1(s) \\ + \{k_2''(s) - k_2^3(s)\left(1 + H_1^2(s)\right)\}M_2(s) = 0.\end{aligned} \tag{28}$$

$H_1(s) \neq 0$ because γ is a space curve. We also obtain $k_2(s) \neq 0$ from the equation (1). Therefore $k_2^{'}(s)$ and $k_2^{''}(s)$ vanish as well as $k_1^{'}(s)$ and $k_1^{''}(s)$. For there are not solutions of differential equations in (28), the proof is completed.

Proposition 3.6. Let γ be a unit speed curve of osculating order 3 in $\mathbb{E}^3$. If γ is of the Bishop $AW(2)$-type with harmonic curvature, then the curvature equation is

$$\left(\frac{k_1^{'}(s)H_1(s) - k_1(s)H_1^{'}(s)}{H_1^2(s)}\right)\left(k_1^{''}(s) - k_1^3(s)\left(1+\frac{1}{H_1^2(s)}\right)\right) = k_1^{'}(s)\left(k_2^{''}(s) - k_2^3(s)\left(1+H_1^2(s)\right)\right). \tag{29}$$

Proof. If we use the equations (14) and (15) in (23), we get the equation (29).

Proposition 3.7. Let γ be a unit speed curve of osculating order 3 in $\mathbb{E}^3$. If γ is of the Bishop $AW(3)$-type with harmonic curvature, then the curvature equation is

$$\frac{k_1(s)}{H_1(s)}\left(k_1^{''}(s) - k_1^3(s)\left(1+\frac{1}{H_1^2(s)}\right)\right) = k_1(s)\left(k_2^{''}(s) - k_2^3(s)\left(1+H_1^2(s)\right)\right). \tag{30}$$

Proof. If we use the equations (13) and (15) in (24), we get the equation (30).

3.1 Slant helices of $AW(k)$-type according to the Bishop frame

Proposition 3.1.1. Let γ is a slant helix of osculating order 3 in E^3. If γ is of the Bishop $AW(1)$-type, then the equations

$$k_1^{''}(s) - k_1^3(s) - k_1(s)k_2^2(s) = 0 \tag{31}$$

$$k_2^{''}(s) - k_2^3(s) - k_1^2(s)k_2(s) = 0, \tag{32}$$

and a solution of this differential equation system is

$$k_1(s) = k_2(s) = \mp\left(\frac{1}{s+c_1}\right), c_1 \in \mathbb{R}. \tag{33}$$

Proof. We get the differential equation system in (31) and (32). By solving this system, we get $k_1(s) = k_2(s)$ and substituting it into (31) and (32), we obtain the result. Differentiating (33) we have

$$k_1'(s)k_2(s) = k_1(s)k_2'(s). \tag{34}$$

Using (34) in (12), we get

$$0 = c(k_1^2(s)+k_2^2(s))^{\frac{3}{2}}.$$

By (33), we have

$$0 = \mp 2\sqrt{2}c\left(\frac{1}{s+c_1}\right)^3 \tag{35}$$

when $c = 0$. Where c, c_1 are constants. The equation (35) satisfies the condition to become slant helices of the Bishop $AW(1)$-type.

Proposition 3.1.2. Let γ is a slant helix of osculating order 3 in E^3. If γ is of the Bishop $AW(2)$-type, then the equation is

$$\begin{aligned}&(c(k_1^2(s)+k_2^2(s))^{\frac{3}{2}}k_1''(s) - k_1(s)c(k_1^2(s)+k_2^2(s))^{\frac{5}{2}})k_1^2 \\ &= k_1'^2(s)(6ck_2(s)k_2'(s)(k_1^2(s)+k_2^2(s))^{\frac{1}{2}}k_1^2(s) + k_2'(s)k_1(s) - c(k_1^2(s)+k_2^2(s)) - k_1'(s)k_2(s)).\end{aligned} \tag{36}$$

Proof. Using (12) in (10), we obtain the equation (36).

Proposition 3.1.3. Let γ is a slant helix of osculating order 3 in E^3. If γ is of the Bishop $AW(3)$-type, then the equation is

$$3c(k_1(s)k_1'(s)+k_2(s)k_2'(s)) = 0. \tag{37}$$

Proof. Differentiating (12) and using it in (11), we have the equation (37).

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