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Application of simplest equation method to the (2+1)-dimensional nonlinear evolution equations

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Abstract : In this paper, the simplest equation method has been used for finding the general exact solutions of nonlinear evolution equations that namely (2+1)-dimensional Calogero-Bogoyavlenskii-Schiff(CBS) equation and (2+1)-dimensional breaking soliton equation (BS) when the simplest equation is the equation of Riccati.

Keywords: Simplest equation method, Calogero-Bogoyavlenskii- Schiff, breaking soliton equation, Riccati equation.

1 Introduction

The research area of nonlinear equations has been very active for the past few decades. There are various kinds of nonlinear equations that appear in various areas of physical and mathematics sciences. Much effort has been made on the construction of exact solutions of nonlinear equations, for their important role in the study of nonlinear physical phenomena [3], [7]. Nonlinear wave phenomena appears in various scientific and engineering fields, such as fluid mechanics, plasma physics, optimal fiber, biology, oceanology [10], solid state physics, chemical physics and geometry. In recent years, the powerful and efficient methods to find analytic solutions of nonlinear equation have drawn a lot of interest by a diverse group of scientists, such as Tanh-function method, extended Tanh-function method [4], [15], Sine-cosine method [16], (G'/G)-expansion method[1], [6], [11]. In this paper we obtain the exact solutions of CBS equation by using the simplest equation method. The simplest equation method was developed by Kudryashov [8], [9] on the basis of a procedure analogous to the first step of the test for the Painleve property [5]. The paper is organized as follows:

In Section 2, we explain the main steps of the simplest equation method. In Section 3, we apply this method to the (2 + 1)-dimensional CBS equation and BS equation. concluding remarks are summarized in Section 4.

2 Discussion

In this section we recall the basic idea of the simplest equation method. Let we have a partial differential equation and by means of an appropriate transformation this equation is reduced to a nonlinear ordinary differential equation as follow:

$$P(u, u', u'', u''', \dots) = 0 \tag{1}$$

Exact solution of this equation can be constructed as finite series

$$u(x) = \sum_{i=0}^{n} a_i (G(x))^i \tag{2}$$

Where G(x) is a solution of some ordinary differential equation referred to as the simplest equation, and $a_0, a_1, a_2, \dots, a_M$ are parameters to be determined. In this paper we use the equation of Riccati, as the simplest equation

$$G'(x) = c\, G(x) + d\, G(x)^2. \tag{3}$$

This equation is well-known nonlinear ordinary differential equation which process exact solution constructed by elementary function. In this paper we work with the following solutions of the Riccati equation

$$G(x)=\frac{c\exp[c(x+x_0)]}{1-d\exp[c(x+x_0)]}, \tag{4}$$

for case $d < 0$, $c > 0$, here x_0 is a constant of integration. and

$$G(x)=-\frac{c\exp[c(x+x_0)]}{1+d\exp[c(x+x_0)]}. \tag{5}$$

for case $d > 0$, $c < 0$, similar above x_0 is a constant of integration.

The simplest equation has two properties: (i)The order of simplest equation is lesser than equation (1), (ii) we know the general solution of the simplest equation or we know at least exact analytical particular solution(s) of the simplest Eq(3). Now $u(x)$ can be determined explicitly by using the following three steps:

• Step (1). By considering the homogeneous balance between the highest nonlinear terms and the highest order derivatives of $u(x)$ in Eq. (1), the positive integer n in (2) is determined.

• Step (2). By substituting Eq. (2) with Eq. (3) into Eq. (1) and collecting all terms with the same powers of G together, the left hand side of Eq. (1) is converted into a polynomial. After setting each coefficient of this polynomial to zero, we obtain a set of algebraic equations in terms of a_i $(i = 0,1,2, \dots, n), c,\ d$.

• Step (3). Solving the system of algebraic equations and then substituting the results and the general solutions of (4) or (5) into (2) gives solutions of (1). Now, we will demonstrate the simplest equation method on CBS and BS equations.

3.1 Apllication of simplest equation method to CBS equation

The CBS equation is

$$u_{xxxy} + 2u_y u_{xx} + 4u_x u_{xy} + u_{xt} = 0. \tag{6}$$

There are many efforts to solve Eq. (6). some new traveling wave solutions for the (2+1)-dimensional CBS equation are obtained in [12].

Wazwaz considered the (2+1)-dimensional CBS equation [13]. He employed the Cole-Hopf transformation and the Hirota's bilinear method to derive multiple-soliton solutions and multiple singular soliton solutions for the equation. Also he employed the Hirota's bilinear method to the (2+1)-dimensional CBS equation [14]. In this paper we obtain exact solutions of this equation using simplest equation method. By the transformation $\xi = x + y + z - \nu t$, Eq. (6) becomes:

$$u^{(4)} + 3[(u')^2]' - \nu u'' = 0. \tag{7}$$

Integrating Eq. (7) once with respect to ξ and setting the integration constant as zero yields

$$u''' + 3(u')^2 - \nu u' = 0. \tag{8}$$

Setting $u' = v$, Eq. (8) becomes

$$v'' + 3v^2 - \nu v = 0. \tag{9}$$

With balancing according procedure that be described, we get $n = 2$, therefore the solution of Eq. (9) can be expressed as follows:

$$v(\xi) = \sum_{i=0}^{2} A_i(G(\xi))^i. \tag{10}$$

where $G(\xi)$ satisfies the Riccati equation and A_0, A_1, A_2 are parameters to be determined. solutions of Riccati equation are given in (4), (5). With substituting (10) into (9) and use of (3) and then equating all coefficients of the functions G^i to zero, we obtain A_0, A_1 and A_2:

Case1:

$$A_0 = -\frac{c^2}{3}, \quad A_1 = -2cd, \quad A_2 = -2d^2; \quad cd \neq 0, \tag{11}$$

Case2:

$$A_0 = 0, \quad A_1 = -2cd, \quad A_2 = -2d^2; \quad cd \neq 0. \tag{12}$$

Recall $u'(x,y,z,t) = v(x,y,z,t)$, therefore when $d < 0$ and $c > 0$ the solution of Eq. (6) with using Case 1 (11) is given by

$$u_1(x,y,z,t) = -\frac{c^2(x+y+z-\nu t)}{3} - \frac{2c}{1-d\exp[c(x+y+z-\nu t+x_1)]}, \tag{13}$$

where $\nu = -c^2$ and x_1 is a constant of integration. Also solution of Eq. (6) with using Case 2 (12) is given by

$$u_2(x,y,z,t) = \frac{-2c}{1-d\exp[c(x+y+z-\nu t+x_2)]}, \tag{14}$$

where $\nu = c^2$ and x_2 is a constant of integration. And when $d > 0$ and $c < 0$ the solution of Eq. (6) with using Case 1 (11) is given by

$$u_3(x,y,z,t) = -\frac{c^2(x+y+z-\nu t)}{3} - \frac{2c}{1+d\exp[c(x+y+z-\nu t+x_3)]}, \tag{15}$$

where $\nu = -c^2$ and x_3 is a constant of integration. also solution of Eq. (6) with using Case 2 (12) is

given by

$$u_4(x,y,z,t) = -\frac{2c}{1+d\exp[c(x+y+z-\nu t+x_4)]}. \tag{16}$$

where $\nu = c^2$ and x_4 is a constant of integration.

3.2 Application of simplest equation method to BS equation

The BS equation is [2]

$$u_{xxxy} - 2u_y u_{xx} - 4u_x u_{xy} + u_{xt} = 0. \tag{17}$$

Here we obtain the exact solutions of this equation using simplest equation method. By the transformation $\xi = x + y + z - \nu t$, Eq.(17) becomes:

$$u^{(4)} - 3[(u')^2]' - \nu u'' = 0. \tag{18}$$

Integrating Eq.(18) once with respect to ξ and setting the integration constant as zero yields

$$u''' - 3(u')^2 - \nu u' = 0. \tag{19}$$

Setting $u' = v$, Eq.(19) becomes

$$v'' - 3v^2 - \nu v = 0. \tag{20}$$

With balancing according procedure that be described, we get $n = 2$, therefore the solution of Eq.(20) can be expressed as follows:

$$v(\xi) = \sum_{i=0}^{2} A_i(G(\xi))^i. \quad (21)$$

where $G(\xi)$ satisfies the Riccati equation and A_0, A_1, A_2 are parameters to be determined. solutions of Riccati equation are given in (4), (5). With substituting (21) into (20) and use of (3) and then equating all coefficients of the functions G^i to zero, we obtain A_0, A_1 and A_2:

Case1:

$$A_0 = \frac{c^2}{3}, \quad A_1 = 2cd, \quad A_2 = 2d^2; \quad cd \neq 0, \quad (22)$$

Case2:

$$A_0 = 0, \quad A_1 = 2cd, \quad A_2 = 2d^2; \quad cd \neq 0. \quad (23)$$

Recall $u'(x, y, z, t) = v(x, y, z, t)$, therefore when $d < 0$ and $c > 0$ the solution of Eq.(17) with using Case 1 (22) is given by

$$u_1(x, y, z, t) = \frac{c^2(x+y+z-\nu t)}{3} + \frac{2c}{1-d\exp[c(x+y+z-\nu t+x_1)]} \quad (24)$$

where $\nu = -c^2$ and x_1 is a constant of integration. also solution of Eq.(17) with using Case 2 (23) is given by

$$u_2(x, y, z, t) = \frac{2c}{1\text{-}d\exp[c(x+y+z\text{-}\nu t+x_2)]}, \quad (25)$$

where $\nu = c^2$ and x_2 is a constant of integration. And when $d > 0$ and $c < 0$ the solution of Eq.(17) with using Case 1 (22) is given by

$$u_3(x, y, z, t) = \frac{c^2(x+y+z-\nu t)}{3} + \frac{2c}{1+d\exp[c(x+y+z-\nu t+x_3)]}, \quad (26)$$

where $\nu = -c^2$ and x_3 is a constant of integration. also solution of Eq.(17) with using Case 2 (23) is

given by

$$u_4(x, y, z, t) = \frac{2c}{1+d\exp[c(x+y+z-\nu t+x_4)]}, \quad (27)$$

where $\nu = c^2$ and x_2 is a constant of integration.

4 Conclusions

In this paper, the simplest equation method has been successfully used to obtain exact solutions of the CBS and BS equations. As the simplest equations, we have used the equation of Riccati. For this simplest equation, we have obtained a balance equation. By means of balance equations, we obtained exact solutions of the studied class nonlinear PDEs. We have also verified that these solutions we have found are indeed solutions to the original equations.

5. References

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