Journal of Naval Sciences and Engineering
2022, Vol. 18, No. 1, pp. 121-141
Mathematics/Matematik

RESEARCH ARTICLE

**An ethical committee approval and/or legal/special permission has not been required within the scope of this study.*

A GOLDEN SECTION METHOD FOR THE MULTI-OBJECTIVE FRACTIONAL SOLID TRANSPORTATION PROBLEM USING THE EXPONENTIAL MEMBERSHIP FUNCTION*

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Received: 24.02.2022 **Accepted: 28.03.2022**

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ABSTRACT

The multi-objective Solid Transportation Problem (MSTP) is type of vector minimization (or maximization) problem with three parameters: source, destination, and mode of transport. It may have fractional objective functions in real-life applications to maximize the profitability ratio like profit/cost or profit/time. We refer to such transportation problems as the Multi-objective Fractional Solid Transportation Problem (MFSTP). In this article is presented a fuzzy approach that combines the usage of linear programming and the golden section algorithm with linear and exponential membership functions and a strongly efficient solution is obtained. Finally, a numerical example from the literature is solved to show the solution algorithm and a comparison is presented with the solution found by using a linear membership function.

Keywords: S*olid Transportation Problem, Fractional Programming, Exponential Membership Function, Fuzzy Programming, Golden Section Method.*

ÇOK AMAÇLI ÜÇ BOYUTLU KESİRLİ TAŞIMA PROBLEMİ İÇİN ÜSTEL ÜYELİK FONKSİYONU KULLANARAK ALTIN ORAN METODU

ÖZ

Çok amaçlı üç boyutlu taşıma problemi kaynak, varış yeri ve taşıma şekli parametrelerine sahip vektör minimizasyon (veya maksimizasyon) probleminin özel bir tipidir. Amaçları, kârlılık oranının- kâr/maliyet veya kâr/zaman- maksimizasyonu gibi iki lineer fonksiyonun oranı olabilir. Bu tür problemler, Çok Amaçlı Kesirli Üç Boyutlu Taşıma Problemi olarak adlandırılmaktadır. Bu çalışmada, lineer programlama ve altın oran yönteminin lineer ve üstel üyelik fonksiyonları ile kullanıldığı bulanık bir yaklaşım sunulmakta ve pareto-optimal bir çözüm elde edilmektedir. Son olarak, çözüm yöntemini göstermek için literatürden sayısal bir örnek çözülmüş ve doğrusal üyelik fonksiyonu kullanılarak elde edilen çözümle bir karşılaştırma yapılmıştır.

Anahtar Kelimeler: *Üç Boyutlu Taşıma Problemi, Kesirli Programlama, Üstel Üyelik Fonksiyonu, Bulanık Programlama, Altın Oran Metodu.*

1. INTRODUCTION

In the present day, transportation problems have extra conveyance constraints such as product type or mode of transportation. In this case, the Solid Transportation Problem (STP) is obtained as a type of single objective transportation problem.

There are many different situations due to uncertainty. To deal with such cases, fuzzy decision-making method should be used. Therefore, an adaptation of fuzzy set theory in the solution method increases the flexibility and effectiveness of the proposed approaches. This theory has been used for the development of the applications of solid transportation. Most research investigates MSTP under the fuzzy environment in two cases: (1) the costs, the supplies, the demands, and conveyances capacities are fuzzy numbers (2) All parameters are crisp while the fuzzy programming approach is used.

(Cui & Sheng, 2013) defined a STP with expected constrained depending on fuzzy programming. A new procedure using based on the zero-point process is proposed to generate an optimal solution of STP by (Pandian & Anuradha, 2010). (Sobana & Anuradha, 2018) gave a procedure obtaining an optimal solution for STP using α-cut under an imprecise environment. Concerning MSTP, (Bit, Biswal, & Alam, 1993) and (Cadenas & Jimenez, 1994) presented some solution methods. (Jimenez & Verdegay, 1998; 1999) gave the solution method both interval and fuzzy STP. (Gen, Ida, Li, & Kubota, 1995) dealt with a genetic algorithm for the solution of the multi-criteria STP in which all constraints were fuzzy numbers. (Anitha, Venkateswarlu, & Akilbasha, 2021) gave an innovative procedure to solve fully rough interval integer STP. (Li, Ida & Gen, 1997) introduced a genetic algorithm to find a solution to the fuzzy MSTP. An interactive fuzzy satisfying method was given for MSTP by (Tao & Xu, 2012). (Dalman, 2016) gave a fuzzy approach to find a solution for interval MSTP. (Anuradha, Jayalakshmi, Deepa, & Sujatha, 2019) explained the procedure that finds to solve for the bi-objective STP using fuzzy linear membership functions. A general formulation of the MSTP with some random parameters is dealt with by (Singh, Pradhan, & Biswal, 2019). (Ojha, Das, Mondal & Maiti, 2009) dealt with a fully fuzzy version of MSTP using fuzzy numbers such as trapezoidal and triangular (Nagarajan, Jeyaraman, & Krishna, 2014) solved MSTP with interval cost in source and demand parameters. (Ammar & Khalifa, 2014) gave MSTP having fuzzy parameter. (Ida, Gen, & Li, 1995) solved multicriteria STP with fuzzy numbers by genetic algorithms. (Kumar & Dutta, 2015) proposed to base on expected value and the goal programming approach for solving fuzzy MSTP.

A STP with two or more fractional objective functions is referred to as a MFSTP. (Radhakrishnan & Anukokila, 2014) dealt with an interval STP applying fractional goal programming. A capacitated MSTP was defined as a constrained nonlinear problem and solved using Interactive Fuzzy Method and Gradient method by (Ojha, Mondal, & Maiti, 2014). (Jana & Jana, 2020) formed a solution method for STP with additional constraints and optimized through fuzzy and fractional programming methods. (Basu & Acharya, 2002) dealt with bi-criterion quadratic fractional STP and developed a method. In (Khalifa & Al-Shabi, 2018), a fully fuzzy multi-objective linear fractional programming is given for multi-product problems. (Khalifa, 2019) investigated a fractional multi-objective multi-product STP with interval costs, supply, demand, and conveyances. (Khalifa, Kumar, &Alharbi, 2021) presented fuzzy geometric programming approach by using membership function to obtain compromise solution of multi-objective fractional two-stage STP.

This paper is presented three-dimension MFSTP having fractional objectives and transportation constraints. In the proposed fuzzy solution method, after linear and exponential membership functions are constructed, the min operator model is obtained. This model is solved using a fuzzy method combining linear programming with the golden section algorithm. To show solution procedure a numerical example is applied.

This article is organized as follows. In Section 2 is presented the formulation of MFSTP with mixed constraints. Section 3 expresses the solution method of the problem using a golden section algorithm. A numerical example is given in Section 4. The last section ends some conclusions.

2. MULTI-OBJECTIVE LINEAR FRACTIONAL TRANSPORTATION PROBLEM WITH MIXED CONSTRAINTS

In fractional STPs, the constraints are not commonly presented with equalities. However, in many instances, it can be possible to encounter situations where the supply, demand and mode of transport constraints are "greater than or equal to" or "less than or equal to". Therefore, MFSTP with mixed constraints is discussed in this study in terms of being a more realistic model.

The aim of the problem is to obtain the minimum cost ratio for transporting a production from m supplies to n demands via K conveyances, whose capacities are a_i , $1 \le i \le m$; b_j , $1 \le j \le n$, and e_k , $1 \le k \le K$, respectively. $c_P = \left[c_{ijk}^p\right]_{mxn}$ and $d_p = \left[d_{ijk}^p\right]_{mxn}$ denote cost and profit matrix for the p-th objective functions, respectively. Also, the scalar c_0^p is constant cost and d_0^p is constant profit. The variable x_{ijk} expresses the unknown amount of the transported from source *i* to destination *j* through conveyance *k*, the mathematical model of MFSTP with mixed constraints is written following:

$$\min Z_p(\mathbf{x}) = \frac{N_p(x)}{D_p(x)} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} c_{ijk}^p\, x_{ijk} + c_0^p}{\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} d_{ijk}^p\, x_{ijk} + d_0^p},\ p = 1,..,P \qquad \textbf{(1)}$$

s.t.

$$\sum_j\sum_k x_{ijk} \ge a_i\ , i \in I_1;\ \sum_j\sum_k x_{ijk} = a_i\ , i \in I_2;\ \sum_j\sum_k x_{ijk} \le a_i\ , i \in I_3 \qquad \textbf{(1.a)}$$

$$i \in I_1 \cup I_2 \cup I_3 = I$$

$$\sum_k\sum_i x_{ijk} \ge b_j\ , j \in J_1;\ \sum_k\sum_i x_{ijk} = b_j\ , j \in J_2;\ \sum_k\sum_i x_{ijk} \le b_j\ , j \in J_3 \qquad \textbf{(1.b)}$$

$$j \in J_1 \cup J_2 \cup J_3 = J$$

$$\sum_i\sum_j x_{ijk} \ge e_k\ , k \in K_1;\ \sum_i\sum_j x_{ijk} = e_k\ , k \in K_2;\ \sum_i\sum_j x_{ijk} \le e_k\ , k \in K_3 \qquad \textbf{(1.c)}$$

$$k \in K = K_1 \cup K_2 \cup K_3$$

$$x_{ijk} \ge 0$$

where the subscripts $Z_p(x)$ and superscript c_{ijk}^p denote the p-th objective function, and $a_i, b_j, e_k > 0$, $c_{ijk}^p \ge 0$, $\forall i, j, k$. (1.a)-(1.c) are defined as supply, demand and conveyance constraints, respectively. Furthermore, I_1, I_2, and I_3 correspond to “greater than or equal to”, “equal to”, and “less than or equal to” mode of constraints, respectively. $J_j,\ j = 1,2,3$ and $K_k,\ k = 1,2,3$ are also denoted in this way.

A set S that is compact and convex is called the feasible set of Problem (1).

In a multi-objective context, we often encounter the descriptions of efficient or non-dominated or strongly efficient solutions different from the optimal solution concepts.

Concerning the multi-objective linear fractional programming Pareto-optimal solution that is the standard definition of efficient solution is insufficient and thus weakly efficient concept is paid attention to. Although obtaining the strongly efficient solutions is claimed in theory, solution approach tent to generate weakly efficient solutions, since the vertexes of E^W (Weakly efficient solutions set) design a connected graph. For MFSTP with mixed constraints will be got the definition of strongly efficient, weakly Pareto-optimal, compromise, compensatory compromise solution concepts.

Definition 2.1. $\mathbf{x}^* \in S$ is a strongly efficient solution iff there does not exist another feasible point $\mathbf{x}$ such that $Z_p(\mathbf{x}) \le Z_p(\mathbf{x}^*)$ $\forall p$, and $Z_p(\mathbf{x}) \ne Z_p(\mathbf{x}^*)$ for at least one p.

Definition 2.2. $\mathbf{x}^* \in S$ is a weakly pareto-optimal solution iff there does not exist another $\mathbf{x} \in S$ such that $Z_p(\mathbf{x}) < Z_p(\mathbf{x}^*)$, $\forall p$.

Under these definitions, $E^w \supset E^s$, here E^w indicates the weakly pareto-optimal solution set and E^s determines the set of strongly efficient solutions.

Definition 2.3. A feasible point $\mathbf{x}^* \in S$ is *compensatory compromise solution* if $\mathbf{x}^* \in E^s$ and $\mathbf{Z}(\mathbf{x}^*) \le \min\limits_{\mathbf{x}\in S} (\mathbf{Z}_1(\mathbf{x}), \mathbf{Z}_2(\mathbf{x}), ..., \mathbf{Z}_p(\mathbf{x})) + \mathbf{I} \cdot \lambda_\bullet$, $\lambda_\bullet \ge 0$, the Q-dimensional column vector with $\bullet$ th element 1 and others elements 0 is $\mathbf{I}$.

Definition 2.4. A feasible point $\mathbf{x}^* \in S$ is a *compromise solution* iff $\mathbf{x}^* \in E^w$ and $\mathbf{Z}(\mathbf{x}^*) \le \min\limits_{\mathbf{x}\in S} (\mathbf{Z}_1(\mathbf{x}), \mathbf{Z}_2(\mathbf{x}), ..., \mathbf{Z}_p(\mathbf{x}))$.

3. A SOLUTION PROCEDURE FOR MFSTP WITH MIXED CONSTRAINTS

In the proposed solution procedure, both linear and exponential membership functions will be used. To present the method clearly, first, the linear membership function will be discussed in the following subsection.

3.1. Constructing the Linear Membership Functions

$$\mu_p(Z_P(\mathbf{x}))=\begin{cases}1, & L_p > Z_P \\ \dfrac{Z_p(\mathbf{x})-U_P}{L_p-U_p}, & L_p \le Z_p < U_p \\ 0, & U_p < Z_p\end{cases} \tag{2}$$

where $\max\limits_{\mathbf{x}\in S} Z_p(\mathbf{x})=U_p$ and $\min\limits_{\mathbf{x}\in S} Z_p(\mathbf{x})=L_p$, $\forall p=1,..,P$.

By defining a new auxiliary variable $\lambda=\min\ \mu_p(Z_p(\mathbf{x}))$, problem (1) can be transformed into using Zimmermann's "min" operator model:

$$\begin{aligned} &\max \quad \lambda \\ \text{s.t.} \quad &\mu_p(Z_p(\mathbf{x}))\ge\lambda, \\ &\mathbf{x}\in S,\ \forall p. \end{aligned} \tag{3}$$

The cooperative satisfactory degree of all objectives is represented by λ. Here, the "cooperative" refers to the lowest degree of satisfaction obtained for each objective of (1).

3.2. Constructing the Exponential Membership Function

Usage of the exponential membership function would give a more realistic conclusion than the linear ones in many real-life problems. Also, the satisfaction rate of exponential one is not always constant, as with linear one. Therefore, this non-linear membership function is preferred in this article.

The exponential membership function for an objective can be defined as

$$\mu_p^E(Z_p(\mathbf{x}))=\begin{cases}1-\exp\left(\dfrac{\alpha_p(Z_p(\mathbf{x})-L_p)}{U_p-L_p}\right), & U_P \ge Z_p(\mathbf{x}) \ge L_p \\ 1, & Z_p(\mathbf{x}) < L_p \\ 0, & U_p < Z_p(\mathbf{x})\end{cases} \tag{4}$$

where α_p is a shape parameter which is generally assumed as $\alpha_p = 3$.
Using (4), the fuzzy model can be written as:

$$\max \lambda \tag{5}$$

s.t. $\quad \mu_p^E(Z_p(\mathbf{x})) = 1 - \exp\left(\dfrac{\alpha_p(Z_p(\mathbf{x}) - L_p)}{U_p - L_p}\right) \geq \lambda \ , \ \forall p$

$\mathbf{x} \in S$.

Problem (5) is rewritten by making mathematical arrangements as:

$$\max \lambda \tag{6}$$

s.t. $\quad U_p + \ln(1-\lambda) \cdot \left(\dfrac{L_p - U_p}{-\alpha_p}\right) \geq Z_p(\mathbf{x})$,

$\mathbf{x} \in S$, $\forall p$.

We note that, in the problem (6), provided the value of λ is constant, it could be transformed to a set of linear inequalities. Finding the optimal solution λ^* to the problem is equivalent to have an acceptable set that satisfies the constraints of the problem because of $0 \leq \lambda \leq 1$.

The proposed solution method for this problem can be given as follows.

3.3. Golden Section Method to MFSTP with Mixed Constraints

(Sakawa & Yumine, 1983) and (Sakawa & Yano, 1988) gave an interactive fuzzy procedure for solving MFTP, which is a combination of linear programming and the golden section algorithm.

In our paper, an algorithm based on this method is applied to MFSTP with mixed constraints. The non-linear (3) problem could be transformed into a set of linear inequalities if the value of λ is fixed. Finding the optimal solution λ^* to the problem is equivalent to obtain the maximum value of λ in order that there is an admissible set satisfying the constraints of (3). Because λ satisfies $\mu_{\min} \leq \lambda \leq \mu_{\min} + 1$, where $\mu_{\min}$ means the minimum value of μ_p, $\forall p$.

Following are the steps for Golden Section Method:

Step 1: Set $\lambda = \mu_{\min} = 0$ to test whether an admissible set satisfying the constraints of (3) exists or not. Provided an admissible set exists, go to Step 2. Otherwise, $\lambda^* = \mu_{\min}$. and STOP.

Step 2: Set $\lambda = \mu_{\min} + 1$, then test whether an admissible set satisfying the constraints of (3) exists or not. Provided an admissible set exists, set $\lambda^* = \mu_{\min} + 1$. Else, go to the next step since the maximum λ satisfying the constraints of (3) exist between $\mu_{\min}$ and $\mu_{\min} + 1$.

Step 3: Take $\lambda_1 = \mu_{\min} + \frac{\sqrt{5}-1}{2}$ as an initial value, and update the value of λ using the golden section algorithm as follows:

$$\begin{cases} \lambda_{n+1} = \dfrac{\sqrt{5}-1}{2^{n+1}} + \lambda_n, \ \text{if admissible set exists for } \lambda_n \\ \lambda_{n+1} = \dfrac{1-\sqrt{5}}{2^{n+1}} + \lambda_n, \ \text{if no admissible set exists for } \lambda_n. \end{cases} \quad \textbf{(7)}$$

That is, for $\forall \lambda$, test whether an admissible set of (3) exists or not, solve linear inequalities and obtain the maximum value of λ satisfying the constraints of (3).

4. AN ILLUSTRATIVE EXAMPLE

Considering the following numerical example which is adapted from (Radhakrishnan & Anukokila, 2014).

$$\min\ Z_p(\mathbf{x}) = \frac{\left(\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2} c_{ijk}^p \cdot x_{ijk}\right) + c_0^p}{\left(\sum_{i=1}^{2}\sum_{j=1}^{2}\sum_{k=1}^{2} d_{ijk}^p \cdot x_{ijk}\right) + d_0^p} = \frac{N_p(\mathbf{x})}{D_p(\mathbf{x})}\ , \quad p = 1,2 \qquad \textbf{(8)}$$

s.t.

$$\sum_{j=1}^{2}\sum_{k=1}^{2} x_{1jk} \le 5, \qquad \sum_{j=1}^{2}\sum_{k=1}^{2} x_{2jk} \ge 9,$$

$$\sum_{k=1}^{2}\sum_{i=1}^{2} x_{i1k} \ge 7, \qquad \sum_{k=1}^{2}\sum_{i=1}^{2} x_{i2k} \le 5,$$

$$\sum_{i=1}^{2}\sum_{j=1}^{2} x_{ij1} \le 6, \qquad \sum_{i=1}^{2}\sum_{j=1}^{2} x_{ij2} \ge 5,$$

$$x_{ijk} \ge 0 \quad \forall i,\ j,\ k$$

where

$$c_{ij1}^1 = \begin{bmatrix} 5 & 7 \\ 3 & 12 \end{bmatrix} \quad c_{ij2}^1 = \begin{bmatrix} 2 & 1 \\ 8 & 1 \end{bmatrix} \quad c_0^1 = 10 \quad d_{ij1}^1 = \begin{bmatrix} 2 & 3 \\ 4 & 8 \end{bmatrix} \quad d_{ij2}^1 = \begin{bmatrix} 1 & 2 \\ 7 & 1 \end{bmatrix} \quad d_0^1 = 4$$

$$c_{ij1}^2 = \begin{bmatrix} 6 & 5 \\ 6 & 15 \end{bmatrix} \quad c_{ij2}^2 = \begin{bmatrix} 3 & 7 \\ 10 & 3 \end{bmatrix} \quad c_0^2 = 5 \quad d_{ij1}^2 = \begin{bmatrix} 1 & 5 \\ 1 & 9 \end{bmatrix} \quad d_{ij2}^2 = \begin{bmatrix} 2 & 3 \\ 6 & 2 \end{bmatrix} \quad d_0^2 = 6$$

S' is the feasible region of (8). Also, observe that $\varnothing = I_1, \varnothing \ne I_2 = I_3$; $\varnothing = J_1, \varnothing \ne J_2 = J_3$; $\varnothing = K_1, \varnothing \ne K_2 = K_3$

The lower and upper bounds for objectives are found as: $U_1 = 1.5$, $L_1 = 0.953; U_2 = 2.609$, $L_2 = 1.412$. Then, using (2) and (4), linear and exponential membership functions are defined, respectively, as follows:

$$\mu_1(Z_1(\mathbf{x})) = \frac{1.537 - Z_1(\mathbf{x})}{0.584} \Rightarrow \mu_1(Z_1(\mathbf{x})) = \frac{1.537 \cdot D_1(\mathbf{x}) - N_1(\mathbf{x})}{0.584 \cdot D_1(\mathbf{x})}$$

$$\mu_2(Z_2(\mathbf{x})) = \frac{2.609 - Z_2(\mathbf{x})}{1.197} \Rightarrow \mu_2(Z_2(\mathbf{x})) = \frac{2.609 \cdot D_2(\mathbf{x}) - N_2(\mathbf{x})}{1.197 \cdot D_2(\mathbf{x})},$$

$$\mu_1(Z_1(\mathbf{x})) = 1 - \exp\left(\frac{\alpha_1(L_1 - Z_1(\mathbf{x}))}{L_1 - U_1}\right) \Rightarrow \mu_1(Z_1(\mathbf{x})) = 1 - \exp\left(\frac{3 \cdot (Z_1(\mathbf{x}) - 0.953)}{0.584}\right)$$

$$\Rightarrow \mu_1(Z_1(\mathbf{x})) = 1 - \exp\left(\frac{3 \cdot (N_1(\mathbf{x}) - 0.953 \cdot D_1(\mathbf{x}))}{0.584 \cdot D_1(\mathbf{x})}\right)$$

$$\mu_2(Z_2(\mathbf{x})) = 1 - \exp\left(\frac{\alpha_2(L_2 - Z_2(\mathbf{x}))}{L_2 - U_2}\right) \Rightarrow \mu_2(Z_2(\mathbf{x})) = 1 - \exp\left(\frac{3 \cdot (Z_2(\mathbf{x}) - 1.412)}{1.197}\right)$$

$$\Rightarrow \mu_2(Z_2(\mathbf{x})) = 1 - \exp\left(\frac{3 \cdot (N_2(\mathbf{x}) - 1.412 \cdot D_2(\mathbf{x}))}{1.197 \cdot D_2(\mathbf{x})}\right).$$

Then, the nonlinear problems obtained using the form (3) can be rewritten as:

$$\begin{aligned} & \max \lambda \\ \text{s.t.} \quad & 1.537 \cdot D_1(\mathbf{x}) - N_1(\mathbf{x}) \geq 0.584 \cdot \bar{\lambda} \cdot D_1(\mathbf{x}) \\ & 2.609 \cdot D_2(\mathbf{x}) - N_2(\mathbf{x}) \geq 1.197 \cdot \bar{\lambda} \cdot D_2(\mathbf{x}) \\ & \mathbf{x} \in S' \end{aligned} \tag{9}$$

and

$$\begin{aligned} & \max \lambda \\ \text{s.t.} \quad & N_1(\mathbf{x}) \leq \left(1.537 + 0.19467 \cdot \ln\left(1 - \bar{\lambda}\right)\right) \cdot D_1(\mathbf{x}) \\ & N_2(\mathbf{x}) \leq \left(2.609 + 0.399 \cdot \ln\left(1 - \bar{\lambda}\right)\right) \cdot D_2(\mathbf{x}) \\ & \mathbf{x} \in S', \end{aligned} \tag{10}$$

respectively.

Start applying golden section method proposing for solving (8).

Step 1: Set $\lambda = \mu_{\min} = 0$, then test whether an admissible set satisfying the constraints of (9) exists or not solving linear inequalities. Solving (9) problem, $\mathbf{x}^{(0)} = (0,0,0,0,0,9,0,0)$ is found. That is, an admissible $\phi \neq S'$ set exists.

Step 2: Set $\lambda = \mu_{\min} + 1$, then test whether an admissible set satisfying the constraints of (9) exists or not solving linear inequalities. An admissible set does not exist. That is, $S' = \phi$.

Step 3: For the onset value of $\lambda_1 = \mu_{\min} + \frac{\sqrt{5}-1}{2} = 0 + 0.61803 = 0.61803$, the problem (9) is solved again. And $\mathbf{x}^{(1)} = (0,0,0,0,0,9,0,0)$ is obtained. That is, an admissible $\phi \neq S'$ set exists.

For the initial value of $\lambda_2 = \lambda_1 + \frac{\sqrt{5}-1}{2^2} = 0.92705$, by solving (9), an admissible set cannot be found. ($S' = \phi$). Since there is no admissible set of S', $\lambda_3 = \lambda_2 - \frac{\sqrt{5}-1}{2^3} \Rightarrow \lambda_3 = 0.77254$ is updated from (7).

Continuing in this way, the sixth iteration, the solution is $\mathbf{x}^{(6)} = (0,0,0,5,1.973,22.138,0,0)$ solution. Because the same point is found at the end of two consecutive iterations, that is, the value cannot be updated after that, the iteration ends and $\lambda^* = 0.72425$ is found. The corresponding $\mu_1 = 0.71460$ and $\mu_2 = 0.71$ membership function values for this point are obtained.

As all steps of the algorithm are repeated for problem (10), the solutions are also obtained in this way. Also, the results of models corresponding to linear and exponential membership functions are presented in Table 1.

In the proposed method, when the linear one is used, both objective functions have the same satisfaction degree. However, when the exponential one is used, the objectives Z_1 and Z_2 have higher than degree of satisfaction compared to the linear one. Also, the satisfaction degree of Z_2 objective function is better than that of Z_1. Moreover, the satisfaction degree of Z_2 objective function is better than that of Z_1. Each of the solutions obtained by using both membership functions is pareto-optimal. However, average satisfaction values of exponential and linear membership functions are 0.7252 and 0.7123, respectively. That is, the first membership function has achieved a higher degree of satisfaction.

Table 1. The comparison of membership functions.

Solution	**Linear**		**Exponential**	
$\mathbf{x}^{(0)}=(0,0,0,0,0,9,0,0)$	$\mu_1=0.53616$	$\mu_2=0.86$	$\mu_1=0.53616$	$\mu_2=0.86$
$\mathbf{x}^{(1)}=(0,0,0,0,0,22.778,0,0)$	$\mu_1=0.61803$	$\mu_2=0.82$		
$\mathbf{x}^{(4)}=(0,0,0,0,4.805,22.156,0,0)$	$\mu_1=0.69528$	$\mu_2=0.70$		
$\mathbf{x}^{(6)}=(0,0,0,5,1.973,22.138,0,0)$	$\mu_1=0.71460$	$\mu_2=0.71$		
$\mathbf{x}^{(1)}=\mathbf{x}^{(3)}=(0,0,0,0,0,9,0,0)$			$\mu_1=0.53616$	$\mu_2=0.86$
$\mathbf{x}^{(4)}=(0,0,0,0,0,30.322,0,0)$			$\mu_1=0.63192$	$\mu_2=0.81$
$\mathbf{x}^{(6)}=(0,0,0,0,3.561,11.669,0,0)$			$\mu_1=0.677$	$\mu_2=0.68$
$\mathbf{x}^{(7)}=(0,0,0,0,4.736,19.522,0,5)$			$\mu_1=0.70328$	$\mu_2=0.79$
$\mathbf{x}^{(9)}=(0,0,0,0.780,6,34.298,0,4.22)$			$\mu_1=0.70995$	$\mu_2=0.71$
$\mathbf{x}^{(10)}=(0,0,0,5,1.078,14.540,0,0)$			$\mu_1=0.72031$	$\mu_2=0.73$

5. CONCLUSION

This paper presents an iterative method to solve this MFSTP with mixed constraints. In proposed solution procedure, Firstly linear and exponential membership functions for all objectives has been defined and then obtained nonlinear problem using Zimmermann's minimum operator. Secondly, this nonlinear problems have been convert into a linear inequality problem by means of golden section method. This transformation enabled mathematically nonlinear problem to be solved easily. Also, a comparison is provided for the solutions of linear and exponential membership functions. The reason for choosing the exponential membership function is that it is versatile and generates better solutions than the linear one. In other words, since the solution obtained using the exponential one provides the highest level of satisfaction among the objectives. Therefore, a more qualified solution is presented to the decision-maker. As a result, it can be concluded that the use of the exponential membership function can generate solutions that fulfill the decision-maker's expectations at a better degree.

ACKNOWLEDGEMENT

This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors.

CONFLICT OF INTEREST STATEMENT

The authors declare no conflict of interest.

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