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Cesàro Statistical Convergence on Neutrosophic Normed Spaces

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Abstract. Cesàro statistical convergence in neutrosophic normed spaces is investigated in this research. Additionally, in this study, we concentrate at several features of Cesàro statistical convergence in NNS such as concepts of Cesàro statistically Cauchy, Cesàro statistically convergent neutrosophic normed Cauchy.

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1. Introduction

Fuzzy sets are similar to sets whose components have different degrees of membership in mathematics. Lotfi A. Zadeh [12] was the first to propose fuzzy sets as an extension of the standard notion of set in 1965. After, many researchers have been interested in fuzzy sets. A membership function $\mu : X \to [0, 1]$ defines a fuzzy set description, in which a real number is assigned in the unit closed interval $[0, 1]$ to each object in the universe. Fuzzy set theory can be used in a wide range of fields, including bioinformatics, when knowledge is incomplete or inaccurate. Fuzzy topological spaces were established by Chang [3]. The intuitionistic fuzzy set (IFS) was proposed by Atanassov [1]- [2] as a direct extension of the fuzzy set, following the definition of membership grade (MG) and non-membership grade (NMG), with the limitation that the number of MG and NMG not exceed 1. The neutrosophic set (NS) is a modern variation of the classical set definition, according to Smarandache [11]. Fuzzy metric space (FMS) is a non-negative fuzzy number that indicates the distance between two places, according to Kaleva and Seikkala [5]. The intuitionistic fuzzy set (IFS) was defined and applied to all domains in which FS theory was studied. Park [10] created the concept of intuitionistic fuzzy metric space in addition to these. In [8], a new metric space called neutrosophic metric Spaces (NMS) was established based on the notion of NSs. Smarandache [11] expanded classical algebraic structures to neutron-algebraic structures with partially real, partially indeterminate, and partially false operations and axioms as extensions of partial algebra. Furthermore, for studying convergence difficulties, the concept of statistical convergence is a very valuable functional tool. To contribute to this idea, Fast [4] proposed the theory of statistical convergence of real-number sequences. Furthermore, Kirişçi [7] examined the statistical convergence of neutrosophic normed space. Recently, Khan et al. dealt with Fibonacci statistical convergence in (NNS) in [6]. In this research, Cesàro statistical convergence in neutrosophic normed spaces has been studied. This research was set up as follows:

(1) Introduction
(2) Preliminaries
(3) Main Results

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(4) Conclusion

2. Preliminaries

We will gather essential definitions for our further considerations in the sections that follow.

Definition 2.1. The infinite Cesàro operator $C = (c_{nk})$ is characterized with

$$c_{nk} = \begin{cases} \frac{1}{n+1}, & (0 \leq k \leq n) \\ 0, & \text{otherwise} \end{cases}$$

for all $n, k \in \mathbb{N}$. That is, if we show it in matrix form, the basic form of Cesàro matrix can be given as in the following:

$$C = \begin{pmatrix} 1 & 0 & 0 & \cdots \\ \frac{1}{2} & \frac{1}{2} & 0 & \cdots \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}.$$

Definition 2.2. A neutrosophic set (NS) $\mathcal{N}$ on X is defined as

$$\mathcal{N} = \{< x, D(x), E(x), F(x) >: x \in X\}.$$

Here, $D(x), E(x), F(x)$ are subsets of $[0, 1]$, where $D(x), E(x), F(x)$ show degree of membership, degree of indeterminacy and degree of non-membership of elements of X, respectively.

Menger [9] innovated the use of triangular norms (t-norms) (TN). He proposed utilizing probability distributions instead of values to solve the problem of estimating the distance between two items in space. The probability distribution of triangle inequality under metric space circumstances is generalized with the usage of TNs. On fuzzy operations, TNs and TCs are extremely important.

Definition 2.3. [9] A continuous t-norm is a function $\triangle : [0, 1] \times [0, 1] \to [0, 1]$ that satisfies the following properties:

- $\triangle$ is associative and commutative,
- $\triangle$ is continuous,
- If $w \leq u$ and $t \leq v$, then $w \triangle t \leq u \triangle v$,
- $w \triangle 1 = w$ for all $w \in [0, 1]$.

Definition 2.4. [9] A continuous t-conorm is a function $\triangledown : [0, 1] \times [0, 1] \to [0, 1]$ that satisfies the following properties:

- $\triangledown$ is associative and commutative,
- $\triangledown$ is continuous,
- If $w \leq u$ and $t \leq v$, then $w \triangledown t \leq u \triangledown v$,
- $w \triangledown 0 = w$ for all $w \in [0, 1]$.

Definition 2.5. [8] Assume that X is a vector space.

$$\mathcal{N} = \{< x, D(x), E(x), F(x) >: x \in X\}.$$

be a normed space (NS) such that $\mathcal{N} : X \times \mathbb{R} \to [0, 1]$. Let $\triangle$ and $\triangledown$ represent continuous TN and TC, respectively. When the following statements are fulfilled, $(X, N, \triangle, \triangledown)$ is referred to be NNS for all $x, y \in X, \alpha \neq 0$ and $\gamma, \theta > 0$:

- $0 \leq D(x, \gamma) \leq 1, 0 \leq E(x, \gamma) \leq 1, 0 \leq F(x, \gamma) \leq 1$ for all $\gamma \in \mathbb{R}$,
- $D(x, \gamma) + E(x, \gamma) + F(x, \gamma) \leq 3, \gamma \in \mathbb{R}$,
- $D(x, \gamma) = 1, iff \ \ x = 0$,
- $D(\alpha x, \gamma) = D(x, \frac{\gamma}{|\alpha|})$,
- $D(x, \theta) \triangle T(y, \gamma) \leq D(x + y, \gamma + \theta)$,
- $D(x, \triangle)$ is continuous non decreasing function,
- $\lim_{\gamma \to \infty} D(x, \gamma) = 1$,
- $E(x, \gamma) = 0$ for $\gamma > 0$ iff $x = 0$,
- $E(\alpha x, \gamma) = E(x, \frac{\gamma}{|\alpha|})$,
- $E(x, \theta) \triangledown H(y, \gamma) \geq E(x + y, \theta + \gamma)$,

- $E(x,\cdot)$ is continuous non increasing function,
- $\lim_{\gamma\to\infty} E(x,\gamma) = 0$,
- $F(x,\gamma) = 0$ for $\gamma > 0$ iff $x = 0$,
- $F(\alpha x,\gamma) = F(x, \frac{\gamma}{|\alpha|})$,
- $F(x,\theta) \triangledown F(y,\gamma) \geq F(x+y, \theta+\gamma)$,
- $F(x,\cdot)$ is continuous non increasing function,
- $\lim_{\gamma\to\infty} F(x,\gamma) = 0$,
- If $\gamma \leq 0$, then $D(x,\gamma) = 0, E(x,\gamma) = 1, F(x,\gamma) = 1$.

We can see that, if we take $0 < e_1, e_2 < 1$ with $e_1 > e_2$, then there is $0 < e_3, e_4 < 1$ such that $e_1 \vartriangle e_3 \geq e_2, e_1 \geq e_4 \triangledown e_2$. Moreover, if we choose $e_5 \in (0,1)$, then there exist $e_6, e_7 \in (0,1)$ such that $e_6 \vartriangle e_6 \geq e_5$ and $e_7 \triangledown e_7 \leq e_5$.

Definition 2.6. [7] A neutrosophic normed space is $(X, \mathcal{N}, \vartriangle, \triangledown)$. A sequence $x = (x_k)$ is supposed to be convergent to t in relation to $\mathcal{N}$, if for all $0 < \epsilon < 1$ and $\gamma > 0$, there occurs $p \in \mathbb{N} \ni D(x_k - t, \gamma) > 1-\epsilon, E(x_k - t,\gamma) < \epsilon, F(x_k - t, \gamma) < \epsilon$. Namely, for all $\gamma > 0$ we have

$$\lim_{k\to\infty} D(x_k - t,\gamma) = 1, \lim_{k\to\infty} E(x_k - t,\gamma) = 0, \lim_{k\to\infty} F(x_k - t,\gamma) = 0.$$

This convergence in $(X, \mathcal{N}, \vartriangle, \triangledown)$ is represented by $\mathcal{N} - \lim x_k = t$.

3. Main Results

We generalized and examined the ideas presented by Kirişçi [7] in (NNS) in this part, and we came up with some remarkable conclusions.

Definition 3.1. If there is a number $t \in X$ for every $\epsilon > 0$ and $K_\epsilon(C) = \{k \leq n : |Cx_k - t| \geq \epsilon\}$ has a natural density zero, i.e., $\delta(K_\epsilon(C)) = 0$ after the sequence $x = (x_k)$ is said to be Cesàro statistically convergent (or CX_k-statistically convergent). It means that

$$\lim_{n\to\infty} \frac{1}{n} |\{k \leq n : |Cx_k - t| \geq \epsilon\}| = 0.$$

$\delta(C) - \lim x_k = t$ is written in this situation. Additionally, x is termed as Cesàro statistically Cauchy if there is a $p \in \mathbb{N}$ such that for each $\epsilon > 0$

$$\lim_{n\to\infty} \frac{1}{n} |\{k \leq n : |Cx_k - Cx_p| \geq \epsilon\}| = 0.$$

Definition 3.2. A neutrosophic normed space is $(X, \mathcal{N}, \vartriangle, \triangledown)$. A sequence $x = (x_k)$ is thought to be Cesàro statistical (CS)- convergent to $t \in X$ with respect $\mathcal{N}$ if for each $\epsilon, t > 0$

$$\begin{aligned}\delta(\{k \in \mathbb{N} : D(Cx_k - t,\gamma) \leq 1-\epsilon, E(Cx_k - t,\gamma) \geq \epsilon \text{ and}\\ F(Cx_k - t,\gamma) \geq \epsilon\}) = 0.\end{aligned}$$

Equivalently

$$\begin{aligned}\lim_{n\to\infty} \frac{1}{n} |\{k \leq n : D(Cx_k - t,\gamma) \leq 1-\epsilon, E(Cx_k - t,\gamma) \geq \epsilon \text{ and}\\ F(Cx_k - t,\gamma) \geq \epsilon\}| = 0.\end{aligned}$$

We shall write in this situation that $\delta(C)_\mathcal{N} - \lim x_k = t$. The set of all Cesàro statistical convergent sequences is represented by $\mathcal{N}(CS)_\mathcal{N}$.

Definition 3.3. A neutrosophic normed space is $(X, \mathcal{N}, \vartriangle, \triangledown)$. If there exists $p = p(\epsilon)$ for every $\epsilon, \gamma > 0$, a sequence $x = (x_k)$ is claimed to be Cesàro statistically Cauchy with attention to norm $\mathcal{N}$ with following condition:

$$\begin{aligned}\delta(\{k \in \mathbb{N} : D(Cx_k - Cy_p,\gamma) \leq 1-\epsilon \text{ or } E(Cx_k - Cy_p,\gamma) \geq \epsilon,\\ F(Cx_k - Cy_p,\gamma) \geq \epsilon\}) = 0.\end{aligned}$$

Lemma 3.4. *Let's call the neutrosophic normed space as $(X, \mathcal{N}, \vartriangle, \triangledown)$. Then, the following sentences are equal for every $\epsilon, \gamma > 0$:*

(1) $\delta(C)_\mathcal{N} - \lim x_k = t$

(2) $\lim_{n\to\infty}\frac{1}{n}|\{k \leq n : D(Cx_k - t,\gamma) \leq 1-\epsilon\}| = \lim_{n\to\infty}\frac{1}{n}|\{k \leq n : E(Cx_k - t,\gamma) \geq \epsilon\}| = \lim_{n\to\infty}\frac{1}{n}|\{k \leq n : F(Cx_k - t,\gamma) \geq \epsilon\}| = 0$
(3) $\lim_{n\to\infty}\frac{1}{n}|\{k \leq n : D(Cx_k - t,\gamma) > 1-\epsilon\}|, E(Cx_k - t,\gamma) < \epsilon, F(Cx_k - t,\gamma) < \epsilon\}| = 1$
(4) $\lim_{n\to\infty}\frac{1}{n}|\{k \leq n : D(Cx_k - t,\gamma) > 1-\epsilon\}| = \lim_{n\to\infty}\frac{1}{n}|\{k \leq n : E(Cx_k - t,\gamma) < \epsilon\}| = \lim_{n\to\infty}\frac{1}{n}|\{k \leq n : F(Cx_k - t,\gamma) < \epsilon\}| = 1$
(5) $\mathcal{N} - \lim D(Cx_k - t,\gamma) = 1$ and $\mathcal{N} - \lim E(Cx_k - t,\gamma) = 0, \mathcal{N} - \lim F(Cx_k - t,\gamma) = 0$.

Theorem 3.5. *A neutrosophic normed space is $(X, \mathcal{N}, \triangle, \triangledown)$. $\delta(C)_{\mathcal{N}}-$ limit is unique if a sequence $x = (x_k)$ is Cesàro statistically convergent with regard to the norm $\mathcal{N}$.*

Proof. Presume that $\delta(C)_{\mathcal{N}} - \lim x_k = t_1, \delta(C)_{\mathcal{N}} - \lim x_k = t_2$ and $t_1 \neq t_2$. Assume $\epsilon, w > 0 \ni w \triangle w < \epsilon$ and $(1-w) \triangledown (1-w) > 1-\epsilon$. After that, for everyone $\gamma > 0$ the following statements hold:

$$\begin{aligned}
\mathscr{K}_{T,1}(w,\gamma) &= \{k \leq n : D\Big(Cx_k - t_1, \frac{\gamma}{2}\Big) \leq 1-w\},\\
\mathscr{K}_{T,2}(w,\gamma) &= \{k \leq n : D\Big(Cx_k - t_2, \frac{\gamma}{2}\Big) \leq 1-w\},\\
\mathscr{K}_{H,1}(w,\gamma) &= \{k \leq n : E\Big(Cx_k - t_1, \frac{\gamma}{2}\Big) \geq w\},\\
\mathscr{K}_{H,2}(w,\gamma) &= \{k \leq n : E\Big(Cx_k - t_2, \frac{\gamma}{2}\Big) \geq w\},\\
\mathscr{K}_{F,1}(w,\gamma) &= \{k \leq n : F\Big(Cx_k - t_1, \frac{\gamma}{2}\Big) \geq w\},\\
\mathscr{K}_{F,2}(w,\gamma) &= \{k \leq n : F\Big(Cx_k - t_2, \frac{\gamma}{2}\Big) \geq w\}.
\end{aligned}$$

Because of the fact that $\delta(C)_{\mathcal{N}} - \lim x_k = t_1$. Then,

$$\delta\left(\mathscr{K}_{D,1}(\epsilon,\gamma)\right) = \delta\left(\mathscr{K}_{E,1}(\epsilon,\gamma)\right) = \delta\left(\mathscr{K}_{F,1}(\epsilon,\gamma)\right) = 0.$$

Additionally, by utilizing $\delta(C)_{\mathcal{N}} - \lim x_k = t_2$ someone obtains

$$\delta\left(\mathscr{K}_{D,2}(\epsilon,\gamma)\right) = \delta\left(\mathscr{K}_{E,2}(\epsilon,\gamma)\right) = \delta\left(\mathscr{K}_{F,2}(\epsilon,\gamma)\right) = 0.$$

Now, let make a distinction

$$\begin{aligned}
\mathscr{K}_{\mathcal{N},\gamma} = \Big[\mathscr{K}_{D,1}(w,\gamma) \bigcup \mathscr{K}_{D,2}(w,\gamma)\Big] \bigcap \Big[\mathscr{K}_{E,1}(w,\gamma) \bigcup \mathscr{K}_{E,2}(w,\gamma)\Big]\\
\bigcap \Big[\mathscr{K}_{F,1}(w,\gamma) \bigcup \mathscr{K}_{F,2}(w,\gamma)\Big].
\end{aligned}$$

It is clear to observe that $\delta\left(\mathscr{K}_{(\mathcal{N},\delta)}\right) = 0$ implying $\delta\left(\mathbb{N} \setminus \mathscr{K}_{(\mathcal{N},\delta)}\right) = 1$. If $k \in \mathbb{N} \setminus \mathscr{K}_{(\mathcal{N},\delta)}$, then we have three possible cases:

(1) $\left(\{k \in \mathbb{N} \setminus \mathscr{K}_{D,1}(\epsilon,\gamma) \bigcup \mathscr{K}_{D,2}(\epsilon,\gamma)\}\right)$
(2) $\left(\{k \in \mathbb{N} \setminus \mathscr{K}_{E,1}(\epsilon,\gamma) \bigcup \mathscr{K}_{E,2}(\epsilon,\gamma)\}\right)$
(3) $\left(\{k \in \mathbb{N} \setminus \mathscr{K}_{F,1}(\epsilon,\gamma) \bigcup \mathscr{K}_{F,2}(\epsilon,\gamma)\}\right)$.

After these, we can conclude that

$$D(t_1 - t_2,\gamma) \geq D\Big(Cx_k - t_1, \frac{\gamma}{2}\Big) \triangle D\Big(Cx_k - t_2, \frac{\gamma}{2}\Big) > (1-w) \triangle (1-w) > 1-\epsilon.$$

Since $\epsilon > 0$ was chosen at random, for all $\gamma > 0$ we have $T(t_1 - t_2,\gamma) = 1$, which gives us $t_1 = t_2$. Now, take (2), if $k \in \mathbb{N} \setminus \mathscr{K}_{E,1}(\epsilon,\gamma) \bigcup \mathscr{K}_{E,2}(\epsilon,\gamma)$. After that, one composes the follows:

$$E(t_1 - t_2,\gamma) \leq E\Big(Cx_k - t_1, \frac{\gamma}{2}\Big) \triangledown E\Big(Cx_k - t_2, \frac{\gamma}{2}\Big) < w \triangledown w < \epsilon.$$

As a result, we obtain $H(t_1 - t_2,\gamma) = 0$ for all $\gamma > 0$, implying that $t_1 = t_2$, and we evaluate (3) as follows. If $k \in \mathbb{N} \setminus \mathscr{K}_{F,1}(\epsilon,\gamma) \bigcup \mathscr{K}_{F,2}(\epsilon,\gamma)$, consequently

$$F(t_1 - t_2,\gamma) \leq F\Big(Cx_k - t_1, \frac{\gamma}{2}\Big) \triangledown F\Big(Cx_k - t_2, \frac{\gamma}{2}\Big) < w \triangledown w < \epsilon.$$

For any $\gamma > 0$, $F(t_1 - t_2,\gamma) = 0$, meaning that $t_1 = t_2$. The theorem is now well established. □

Theorem 3.6. *A neutrosophic normed space is* $(X, \mathcal{N}, \triangle, \triangledown)$. *If* $\mathcal{N} - \lim x_k = t$ *then* $\delta(C)_{\mathcal{N}} - \lim x_k = t$.

Proof. Allow that $\mathcal{N} - \lim x_k = t$. After, each of $\epsilon > 0$ and $\gamma > 0$ there exists $n_0 \in \mathbb{N}$ in order for $D(Cx_k - t, \gamma) > 1 - \epsilon$ or $E(Cx_k - t, \gamma) < \epsilon, F(Cx_k - t_1, \gamma) < \epsilon$ for all $k \geq n_0$. From here,

$$\{k \leq n : D(Cx_k - t, \gamma) \leq 1 - \epsilon \text{ or } E(Cx_k - t, \gamma) \geq \epsilon, F(Cx_k - t, \gamma) \geq \epsilon\}$$

can only have a certain amount of terms. That's because the natural density of any finite subset of $\mathbb{N}$ is 0, it derives that

$$\lim_{n\to\infty} \frac{1}{n} |\{k \leq n : D(Cx_k - t, \gamma) \leq 1 - \epsilon \text{ or } E(Cx_k - t, \gamma) \geq \epsilon, \\ F(Cx_k - t, \gamma) \geq \epsilon\}| = 0.$$

It means that $\delta(C)_{\mathcal{N}} - \lim x_k = t$. □

3.1. Cesàro Statistically Cauchy NNS.

Theorem 3.7. *Every Cesàro statistically convergent sequence in* $(X, \mathcal{N}, \triangle, \triangledown)$ *is Cesàro statistically convergent neutrosophic normed Cauchy sequence.*

Proof. Suppose that the sequence $x = (x_k)$, with regard to the norm $\mathcal{N}$, is Cesàro statistically convergent to t. It means that, $\delta(C)_{\mathcal{N}} - \lim x_k = t$. For $\epsilon > 0$ prefer $\sigma > 0$ in the manner $(1 - \epsilon) \triangle (1 - \epsilon) > 1 - \sigma$ and $\epsilon \triangledown \epsilon < \sigma$ are both true. As a result, after one gets for $\gamma > 0$ the followings:

$$\delta\left(B(\epsilon, \gamma)\right) = \delta\left(\{k \leq n : D\left(Cx_k - t, \frac{\gamma}{2}\right) \leq 1 - \epsilon \text{ or } E\left(Cx_k - t, \frac{\gamma}{2}\right) \geq \epsilon, F\left(Cx_k - t, \frac{\gamma}{2}\right) \geq \epsilon\}\right) = 0,$$

which implies

$$\delta\left(B^c(\epsilon, \gamma)\right) = \delta\left(\{k \leq n : D\left(Cx_k - t, \frac{\gamma}{2}\right) > 1 - \epsilon \text{ and } E\left(Cx_k - t, \frac{\gamma}{2}\right) < \epsilon, F\left(Cx_k - t, \frac{\gamma}{2}\right) < \epsilon\}\right) = 1.$$

Assume that $j \in B^c(\epsilon, \gamma)$. Then, $D(Cx_j - t, \gamma) > 1 - \epsilon, E(Cx_j - t, \gamma) < \epsilon$ and $F(Cx_j - t, \gamma) < \epsilon$. Consider the following situation:

$$M(\epsilon, \gamma) = \{k \leq n : D(Cx_k - Cx_j, \gamma) \leq 1 - \sigma \text{ or } E(Cx_k - Cx_j, \gamma) \geq \sigma, F(Cx_k - Cx_j, \gamma) \geq \sigma\}.$$

We must demonstrate this; $M(\epsilon, \gamma) \subset B(\epsilon, \gamma)$. Suppose $i \in M(\epsilon, \gamma) - B(\epsilon, \gamma)$. Then,

$$D(Cx_i - Cx_j, \gamma) \leq 1 - \sigma \text{ and } D(Cx_j - t, \frac{\gamma}{2}) > 1 - \sigma. \tag{3.1}$$

Specifically, $D(Cx_j - t, \frac{\gamma}{2}) > 1 - \epsilon$. Then

$$1 - \sigma \geq D(Cx_i - Cx_j, \gamma) \geq D(Cx_i - t, \frac{\gamma}{2}) \triangle \\ D(Cx_j - t, \frac{\gamma}{2}) > (1 - \epsilon) \triangle (1 - \epsilon) > 1 - \sigma.$$

That is unimaginable. Similarly,

$$E(Cx_i - Cx_j, \gamma) \geq \sigma \text{ and } E(Cx_j - t, \frac{\gamma}{2}) < \sigma.$$

Specifically, $E(Cx_j - t, \frac{\gamma}{2}) < \epsilon$. After, we can conclude;

$$\sigma \leq E(Cx_i - Cx_j, \gamma) \leq E(Cx_i - t, \frac{\gamma}{2}) \triangle E(Cx_j - t, \frac{\gamma}{2}) < \epsilon \triangledown \epsilon < \sigma.$$

It is not possible. In the same way,

$$F(Cx_i - Cx_j, \gamma) \geq \sigma \text{ and } F(Cx_i - t, \frac{\gamma}{2}) < \sigma.$$

Especially, $F(Cx_j - t, \frac{\gamma}{2}) < \epsilon$. Then,

$$\sigma \leq F(Cx_i - Cx_j, \gamma) \leq F(Cx_i - t, \frac{\gamma}{2}) \triangle F(Cx_j - t, \frac{\gamma}{2}) < \epsilon \triangledown \epsilon < \sigma.$$

That situation is also not possible. Thus, $M(\epsilon, \gamma) \subset B(\epsilon, \gamma)$. As a result of formula (3.1), we can write $\delta(M(\epsilon, \gamma)) = 0$. Then, $x = (x_k)$ is a Cesàro statistically Cauchy sequence with respect to norm $\mathcal{N}$. □

4. Conclusion

In this work, Ces'aro statistically convergence and Ces'aro statistically convergence in NNS were defined. Further, the relationship between the notions Cesàro statistically convergence and Cesàro statistically convergence in NNS sets is given and a few essential features are investigated.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this article.

Authors Contribution Statement

The author has read and agreed to the published version of the manuscript.

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