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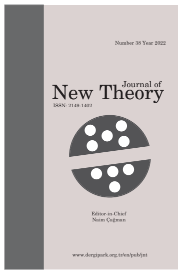

Anisotropic Conformal Model in $f(R, \phi)$ Theory

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Abstract — In this study, we examine conformal spherically symmetric space-time with anisotropic fluid in $f(R, \phi)$ theory. The exact solutions of field equations are obtained for $f(R, \phi) = (1 + \lambda\eta^2\phi^2)R$ model. All the quantities for anisotropic fluid are investigated through equation of state constant, ω. The models for three different selections of ω are represented for the constructed model. Moreover, string gas is the only condition that anisotropic fluid behaves as an isotropic fluid for the constructed model. Furthermore, the anisotropy parameter and causality conditions are examined. Lastly, the results for the solutions are concluded from the physical and geometrical viewpoint.

Keywords — *Conformal symmetry, $f(R, \phi)$ theory, extended theory, anisotropic fluid*

Mathematics Subject Classification (2020) — 83C05, 83C15

1. Introduction

Generalization of Einstein-Hilbert action is quite attractive topic in recent years. It is an alternative way to understand dynamical characteristic of universe. Especially, last observations such as supernova type Ia [1–3] and cosmic microwave background radiation [4, 5] lead to expansion universe with acceleration. Although studies indicate that the universe has exhibited different dynamic behaviors in different epochs, current time expansion of universe is correlated with exotic matter components called as "dark energy" on it. Matter form has negative pressure which causes to expansion. Within this framework, many researchers described great numbers of different dark energy models associated with scalar field. Although the existence of dark energy, which has such a repulsive effect cosmologically, is sufficient to explain the movement of late time universe, the origin and dynamic structure of this form of matter cannot be fully explained theoretically. Extended theories or generalizations of Einstein-Hilbert action give researchers to examine universe from beginning to current time as an alternative way. $f(R, \phi)$ theory is one of the most attractive generalization of Einstein-Hilbert action by way of a general function depending on Ricci curvature scalar, scalar field and it's terms. Einstein-Hilbert action for $f(R, \phi)$ theory is firstly studied by Hwang and his collaborators [6–9]. Existence of scalar field in action makes the theory quite interesting in order to study different epoch of universe correlated with scalar field. In this context, Myrzakulov et al. [10] examined possible inflation scenario for some models in $f(R, \phi)$ theory. Mathew et al. [11] obtained a possible exact inflationary model in $f(R, \phi)$ theory. They showed that an inflationary model with an exit is possible in theory. Stabile and Capozzielo [12] studied galaxy rotating curves without needing dark energy in $f(R, \phi)$ theory. They showed that Yukawa-like correction in theory could be explain problem of dark matter in spiral galaxies. In theory, many cosmological issue is studied by researchers [13–17].

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Conformal Killing vectors (CKVs) are quite important symmetrical property which could be used for General Relativity caused to simplification of space-time [18]. It is easier to find exact solutions for models by way of this isometries in field equations. Also, CKVs could be considered to explore conservation laws, as well. CKVs are defined by [19]

$$\mathfrak{L}_{\xi} g_{ik} = \psi g_{ik} \tag{1}$$

where $\mathfrak{L}_{\xi}$ represent the Lie derivative operator, ψ is conformal factor and ξ is vector field that generates conformal symmetry [19]. Classification of conformal Killing vectors such as Killing vectors, Homothetic Killing vectors, special conformal Killing vectors and non-special conformal Killing vectors depends on the structure of the conformal factor, ψ [20]. In literature, great number of cosmological models and issues in various theory are examined through conformal symmetry [21–24].

In this study, our main purpose to understand effect of conformal symmetry on anisotropic model in the framework of $f(R,\phi)$ theory. In this context, we investigated conformal spherically symmetric space-time filled with anisotropic fluid in $f(R,\phi)$ theory. Kinematic term of scalar field related with dynamic structure of theory is attained for constructed model.

This study organized as: In section 2, we recaptured field equations of $f(R,\phi)$ theory. After, we obtained field equations conformal spherically symmetric space-time with anisotropic fluid in $f(R,\phi)$ theory. Exact solutions of field equations are obtained. Matter distribution investigated through equation of state constant, ω. Anisotropy parameter and causality conditions are examined. In section 3, Results for solutions have been concluded in the point of view physical and geometrical.

2. Anisotropic Conformal Spherically Symmetric Model in $f(R,\phi)$ Theory

Action function of $f(R,\phi)$ gravity can be written as [25]

$$\mathcal{S} = \int d^4\sqrt{-g}\left[\frac{1}{\kappa^2}(f(R,\phi) + u(\phi)\phi_{;\ell}\phi^{;\ell})\right] + \mathcal{S}_m \tag{2}$$

where $\mathcal{S}_m$ is Lagrange density of matter field. Also, $f(R,\phi)$ is a general function connected with Ricci curvature scalar, R, and scalar field, ϕ [25]. $u(\phi)$ represents kinematic term of scalar field and g is determination of metric tensor $g_{\mu\nu}$ [26]. Variation of Eq.(2) leads to field equations of extended theory as follows:

$$f_R R_{ik} - \frac{1}{2}(f + u(\phi)\phi_{;\ell}\phi^{;\ell})g_{ik} - f_{R;ik} + u(\phi)\phi_{;i}\phi_{;k} + g_{ik}\Box f_R = \kappa T_{ik} \tag{3}$$

where $\Box$ corresponds to d'Alambertian operator ($\Box = \nabla_\ell \nabla^\ell$). Energy-momentum tensor is defined as $T_{ik} = -\frac{2}{\sqrt{-g}}\frac{\delta(\sqrt{-g}\mathcal{S}_m)}{\delta g^{ik}}$ [27]. From Eq.(2), the generalization of Klein-Gordon equation for $f(R,\phi)$ theory is attained as [26]

$$2u(\phi)\Box\phi + u_\phi(\phi)\phi_{;\ell}\phi^{;\ell} - f_\phi = 0 \tag{4}$$

Line element of static spherically symmetric metric is described by

$$ds^2 = e^{\mu(r)}dt^2 - e^{\nu(r)}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\Phi^2 \tag{5}$$

where $\mu(r)$ and $\nu(r)$ are metric potentials depending on radial coordinate. Also, Ricci curvature scalar for selected metric is attained as

$$R = e^{-\nu}\left[\frac{1}{2}(2\ddot{\mu} - \dot{\mu}\dot{\nu} + \dot{\mu}) - \frac{2}{r}(\dot{\nu} - \dot{\mu}) + \frac{2}{r^2}\right] - \frac{2}{r^2} \tag{6}$$

where dot signs partial derivative according to radial coordinate. On the other hand, energy-momentum tensor of anisotropic fluid is given by

$$T_{ik} = (\rho + p_t)u_i u_k - p_t g_{ik} + (p_r - p_t)x_i x_k \tag{7}$$

where $\rho(r)$ is energy density, $p_r(r)$ is radial pressure and $p_t(r)$ is tangential pressure of the fluid. u_i is four-velocity in co-moving coordinates and x_i is unit four-vector along the radial direction. By using Eqs.(3),(5) and (7), field equations for spherically symmetric anisotropic fluid in $f(R,\phi)$ theory are attained in the following form:

$$e^{-\nu}\left[\frac{1}{4}f_R\left(\dot{\mu}\dot{\nu}+\frac{4\dot{\nu}}{r}-2\ddot{\mu}-\dot{\mu}^2\right)+\frac{1}{2}e^{\nu}f+\frac{1}{2}u(\phi)\dot{\phi}^2+\partial_r f_R\left(\frac{2}{r}+\frac{\dot{\mu}}{2}\right)\right]=\kappa p_r \tag{8}$$

$$e^{-\nu}\left[\frac{1}{2r}f_R\left(\dot{\nu}-\dot{\mu}+\frac{2e^{\nu}}{r}-\frac{2}{r}\right)+\frac{1}{2}e^{\nu}f-\frac{1}{2}u(\phi)\dot{\phi}^2+\frac{1}{2}\partial_r f_R\left(\frac{2}{r}+\dot{\mu}-\dot{\nu}\right)+\partial_{rr}f_R\right]=\kappa p_t \tag{9}$$

and

$$e^{-\nu}\left[\frac{1}{4}f_R\left(\dot{\mu}^2-\dot{\mu}\dot{\nu}+2\ddot{\mu}+\frac{4\dot{\mu}}{r}\right)-\frac{1}{2}e^{\nu}f+\frac{1}{2}u(\phi)\dot{\phi}^2+\frac{1}{2}\partial_r f_R\left(\dot{\nu}-\frac{4}{r}\right)-\partial_{rr}f_R\right]=\kappa\rho \tag{10}$$

It is clearly seen that constructed field equations have eight unknown components. In consequence, we have considered a viable model in $f(R,\phi)$ theory. The model is [25]

$$f(R,\phi)=(1+\lambda\eta^2\phi^2)R \tag{11}$$

Also, kinematic term of scalar field is considered as power-law form as $u(\phi)=u_0\phi^m$. In this study, we assumed constant as $m=1$. Conformal symmetry gives us to an opportunity to simplify space-time via vector field, ξ. Conformal symmetry for static spherically symmetric metric is studied by Herrera and Ponce de Leon [28]. They obtained homothetic vector fields and metric potentials in the following form:

$$\xi^a=k_1\delta_4^a+\left(\frac{\psi r}{2}\delta_1^a\right) \tag{12}$$

$$e^{\mu}=k_2^2r^2 \tag{13}$$

and

$$e^{\nu}=\frac{k_3^2}{\psi^2} \tag{14}$$

Line element of conformal spherically symmetric metric could be defined as

$$ds^2=k_2^2r^2dt^2-\frac{k_3^2}{\psi^2}dr^2-r^2d\theta^2+r^2\sin^2\theta d\Phi^2 \tag{15}$$

Under all consideration, it is possible to rewrite field equations of constructed model in $f(R,\phi)$ theory in the following form:

$$\left(\frac{1}{r^2}+\frac{\lambda\eta^2\phi^2}{r^2}\right)\left(\frac{3\psi}{k_3^2}-1\right)+\frac{1}{2}\frac{\psi^2}{k_3^2}\phi\dot{\phi}\left(\omega\phi+12\frac{\lambda\eta^2}{r}\right)=\kappa p_r \tag{16}$$

$$2\frac{\psi}{rk_3^2}\left(\dot{\psi}+\frac{\psi}{2r}\right)+2\frac{\psi\dot{\psi}}{k_3^2}\lambda\eta^2\phi\left(\frac{\phi}{r}+\dot{\phi}\right)+2\frac{\psi^2}{k_3^2}\lambda\eta^2\dot{\phi}\left(2\frac{\phi}{r}+\dot{\phi}\right)+\frac{\psi^2}{k_3^2}\lambda\eta^2\phi\left(\frac{\phi}{r^2}+2\ddot{\phi}\right)-\frac{1}{2}\frac{\psi^2}{k_3^2}u_0\phi\dot{\phi}^2=\kappa p_t \tag{17}$$

and

$$-2\frac{\psi}{rk_3^2}\left(\dot{\psi}+\frac{\psi}{2r}\right)+\frac{1}{r^2}\left(1+\lambda\eta^2\phi^2\right)-2\frac{\psi\dot{\psi}}{k_3^2}\lambda\eta^2\phi\left(\frac{\phi}{r}+\dot{\phi}\right)-2\frac{\psi^2}{k_3^2}\lambda\eta^2\dot{\phi}\left(2\frac{\phi}{r}+\dot{\phi}\right)$$
$$-2\frac{\psi^2}{k_3^2}\lambda\eta^2\phi\left(\frac{\phi}{2r^2}+\ddot{\phi}\right)+\frac{1}{2}\frac{\psi^2}{k_3^2}u_0\phi\dot{\phi}^2=\kappa\rho \tag{18}$$

In order to simplify our solution, it is considered constants in $f(R,\phi)$ model as $\lambda=-1$ and $\eta=1$. Also, equation of state between radial pressure and density of fluid is given by

$$p_r=\omega\rho \tag{19}$$

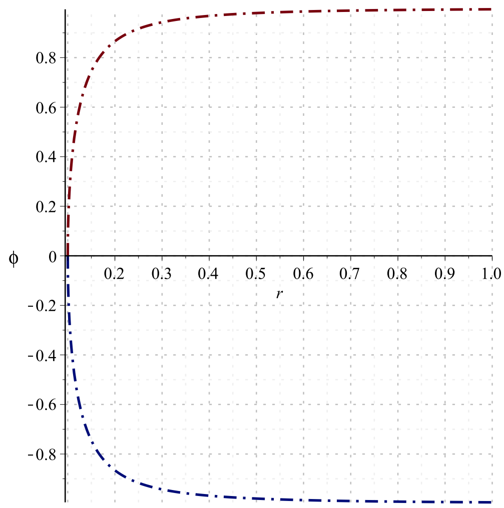

Fig. 1. Evolution of scalar field with r. Positive scalar field (red line) and positive scalar field (red line) are represented for $k_4 = -0.01$.

By using Eqs.(16)-(19), we obtained conformal factor as

$$\psi(r) = \frac{\sqrt{2k_3^2 r(\omega^2-1)\sqrt{r^2+k_4}(6r\sqrt{r^2+k_4}-k_4u_0)}}{6(\omega-1)(\frac{1}{6}k_4u_0 - r\sqrt{r^2+k_4})} \tag{20}$$

Scalar field of theory is obtained as:

$$\phi(r) = \pm\frac{\sqrt{r^2+k_4}}{r} \tag{21}$$

Evolution of scalar field is represented with respect to radial coordinate in Fig. 1. The scalar field is estranged from x-axis for bigger value of radial coordinate. Also, constructed model refers to both complex scalar field and complex conformal factor depending on value of k_4. In order to avoid that condition, it is a way to define critical radius for constructed model. It could be given by $r_{cri}^2 > -k_4$. Also, matter components such as density, radial pressure and tangential pressure are attained as

$$\rho(r) = \frac{2k_4}{\kappa r^4(\omega-1)} \tag{22}$$

$$p_r(r) = \frac{2\omega k_4}{\kappa r^4(\omega-1)} \tag{23}$$

and

$$p_t(r) = -\frac{k_4(\omega+1)}{\kappa r^4(\omega-1)} \tag{24}$$

Line element of conformal spherically symmetric space-time in the presence of anisotropic fluid in $f(R,\phi)$ theory is rewritten as follows:

$$ds^2 = k_2^2 r^2 dt^2 - \frac{(\omega-1)(6r\sqrt{r^2+k_4}-k_4u_0)}{2r(\omega+1)\sqrt{r^2+k_4}}dr^2 - r^2 d\theta^2 + r^2\sin^2\theta d\Phi^2 \tag{25}$$

One can get a physically meaningful density in the case of $\rho > 0$. In Eq.(22), it depends on arbitrary constant, k_4, and ω. k_4 must be negative when equation of state parameter is selected as $\omega < 1$. On the other hand, k_4 can be selected as positive in the case of $\omega > 1$ which isn't expressive because this condition refers to sound speed bigger than the speed of light. In order to get physically meaningful matter distribution, one must select arbitrary constant as negative for constructed model. Also, it is obviously seen that constructed model does not allow to value of $\omega = 1$. In Eq.(24), tangential

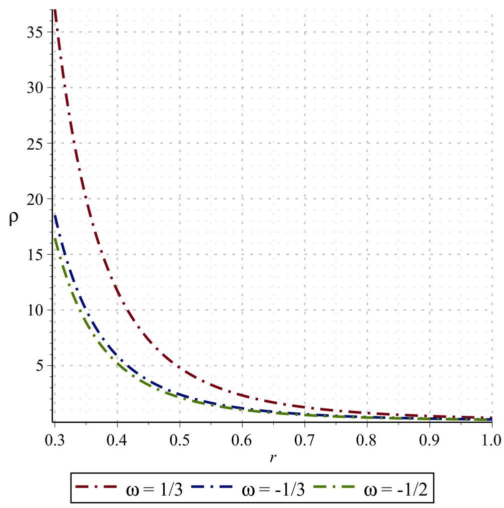

Fig. 2. Evolution of energy density with r for different selection of ω. ($k_4 = -0.1$ and $\kappa = 1$)

pressure and radial pressure could be attractive or repulsive depending on value of ω. In the case of $-1 < \omega < 1$, tangential pressure has repulsive effect. Also, radial pressure has same effect for $\omega < 0$ or $\omega > 1$, as well. At the same time, tangential pressure given by Eq.(24) vanishes for selection of $\omega = -1$. Under this assumption, it could be said that cosmological constant case breaks structure of anisotropy for constructed model in $f(R, \phi)$ theory. In Fig. 2, graphical representation of energy density for different selection of ω is represented with respect to radial coordinate. Energy density for all selections is decreasing for bigger value of radial coordinate. In Fig. 3 and Fig. 4, graphical representations of radial and tangential pressures are represented with respect to radial coordinate. Both figure shows that pressure components of fluid are approaching x-axis with bigger value of radial coordinate. For $\omega = 1/3$, radial pressure has attractive effect, while tangential pressure is repulsive effect for constructed model. In addition to this, cases of $\omega = -1/3$ and $\omega = -1/2$, both pressure components show repulsive behavior for constructed model.

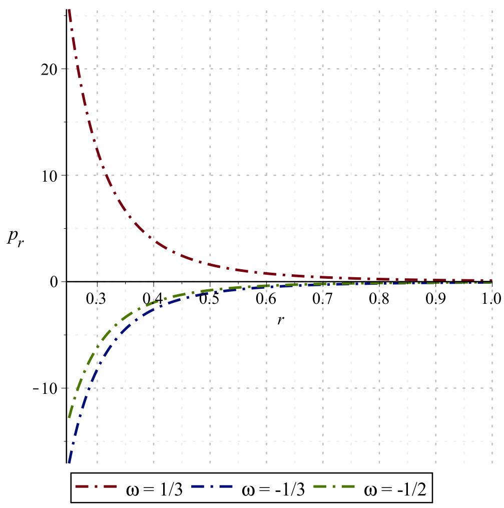

Fig. 3. Evolution of radial pressure with r for different selection of ω. ($k_4 = -0.1$ and $\kappa = 1$)

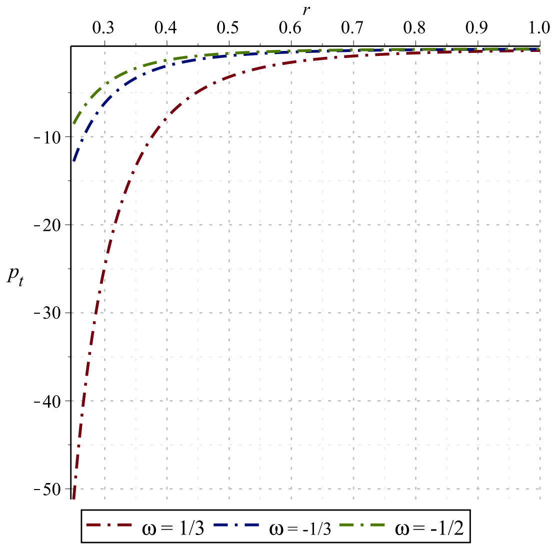

Fig. 4. Evolution of tangential pressure with r for different selection of ω . ($k_4 = -0.1$ and $\kappa = 1$)

Also, spherically symmetric anisotropic models give a opportunity to investigate behavior of radial structures by way of anisotropy measurement indicated by $\triangle$. $\triangle$ is represented as $\triangle = \kappa(p_t - p_r)$. Positive values of $\triangle$ are correlated with outward pressure, otherwise the opposite occurs on it. For constructed model, anisotropy is

$$\triangle = \kappa p_t - p_r = -\frac{k_4(3\omega+1)}{r^4(\omega-1)} \tag{26}$$

Considering that arbitrary constant, k_4, is negative, as we studied earlier, constructed model in $f(R,\phi)$ theory indicates outward pressure in the cases of $\omega > 1$ which is physically meaningless and $\omega < -1/3$ which corresponds to dark energy. At the same time, isotropic case ($p_t = p_r = p$) for fluid is possible in the case of $\omega = -1/3$. Under that condition, fluid behaves as string cloud. Also constructed model could be practised for radial objects under some conditions. So, it is good to examine radial and transverse sound velocities for constructed model.

$$\upsilon_{sr}^2 = \frac{dp_r}{d\rho} = \omega \tag{27}$$

and

$$\upsilon_{st}^2 = \frac{dp_t}{d\rho} = -\frac{1}{2}(\omega+1) \tag{28}$$

Both velocities must be satisfied for condition given by $0 < \upsilon_{sd,st}^2 \leq 1$. From Eqs.(27) and (28), both velocities could not be satisfied because $0 < \omega < 1$ case allows valid radial sound, while $-3 \leq \omega < -1$ case is possible condition in order to get valid transverse sound. Even thought both condition can not be satisfied for same region for ω, one can examine the stabile region in the case of $-1 \leq \upsilon_{st}^2 - \upsilon_{sr}^2 \leq 0$ offered by Abreu *et al.* [29] for constructed model. Stabile region for constructed model is described as $-1/3 \leq \omega \leq 1/3$.

3. Conclusion

In this study, we investigated conformal spherically symmetric space-time in the presence of anisotropic fluid in $f(R,\phi)$ theory. Firstly, we get field equation for field equations of spherically symmetric space-time with anisotropic fluid in $f(R,\phi)$ theory. After that, we considered conformal symmetry for spherically symmetric metric offered by Herrera and Ponce de Leon [28] and field equations of anisotropic conformal symmetric model are examined in $f(R,\phi)$ theory. Exact solution of field equation for constructed model are obtained to take notice of equation of state for $f(R,\phi) = (1 + \lambda\eta^2\phi^2)R$ model.

We defined line element of conformal spherically symmetric metric with anisotropic fluid in $f(R, \phi)$ theory. Also, it is shown that constructed model allow complex scalar field. In order to avoid complex scalar field, critical radius is defined for constructed model. At the same time, effect of scalar field is increasing with bigger value of radial coordinate in theory. All matter components are investigated via arbitrary constants and equation of state parameter. Constructed model allows negative value of arbitrary constant, k_4. All possible cases for repulsive or attractive behaviors of radial and tangential pressure are examined by favour of ω. Anisotropic conformal spherically symmetric model in $f(R, \phi)$ theory for selected model has singularity for $\omega = 1$. Also, cosmological constant case ($\omega = -1$) breaks structure of anisotropy for constructed model in $f(R, \phi)$ theory. For different selection of ω, behaviors of matter components with respect to radial coordinate are investigated via graphical representations of them. All quantities approaches to zero for bigger values of radial coordinate. Anisotropy parameter is investigated for conformal spherically symmetric model. The parameter shows that outward pressure is possible when $\omega < -1/3$. $\omega < -1/3$ corresponds to dark energy. For constructed model, $f(R, \phi)$ theory designates dark energy for outward pressure. Also, $\omega = -1/3$ case which corresponds to string gas is a only condition that anisotropic fluid behaves as isotropic fluid. Lastly, constructed spherical model could be practised for radial objects under some conditions. For this reason, we investigated causality condition and we defined stabile region for constructed model.

Author Contributions

The author read and approved the last version of the manuscript.

Conflicts of Interest

The author declares no conflict of interest.

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