Some Modeling Features of Horizontally-Layered Ground's Return Parameters

T. Lazimov, and E. Saafan

***Abstract*—Possibility and conditions of modeling heterogeneous (horizontally-layered) earth as homogeneous medium in the Carson problem is considered in the present article. This problem consists of calculation the improper integral determines so called earth or ground return parameters i.e. earth's contribution to the some parameters of double-wired aerial system, such as intensity of electrical field and mutual impedance and also to the magnetic vector potential of a single aerial wire. As it was proven earlier the Carson integral for heterogeneous earth does not exist in the terms of general value. In the same time numerical computation of Carson integral for heterogeneous (horizontally-layered) earth may face with calculative difficulties. It is shown in the article that in the certain high frequency ranges horizontally-layered ground may be presented as a homogeneous medium having parameters of the upper layer, the corresponding differences and frequencies were evaluated for some earth's structures and parameters.**

***Index Terms*—Carson problem, horizontally-layered earth, modified linear impedance, depth of penetration.**

I. Introduction

It is well-known that the Carson integral appears in problems concerned to determination of long electrical lines' wave fields at taking into consideration influence of earth. It determines so called ground return parameters i.e. contribution of the earth into the some parameters such as electrical field intensity [1], magnetic vector potential [2], modified linear impedance [3, 4].

There have been obtained several solutions of the Carson integral since 1926 after it was stated in [1] and a modification of the Carson integral considered longitudinal displacement currents in the earth got by Wise in [5]. The most of the solutions either analytical (e.g. given in [1, 2, 3, 4, 5]) and approximate ones (e.g. given in [6, 7]) were got for the case of double-wired aerial line passed over homogeneous earth.

Obviously real structuring of the earth itself requires researching the ground return parameters especially for horizontally-layered ground. Unfortunately just the minority of works in the area under consideration is concerned to the heterogeneous ground's case. Some works may be noticed in this view. One of them is the article [8] which presented solution for the asymptotic frequencies band, another one is [9] in which the principal value of the Carson integral for horizontally-layered earth had been presented and the third one is [10] in which it is stated that the Carson integral for horizontally-layered ground does not exist in the terms of general value.

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Note that ground return parameters are the components of modified linear impedance which has great importance for the electric power systems electromagnetic compatibility problems [11] because that it is one of the two items of mutual impedance between single aerial wires at consideration the earth finite conductance [1, 12]. In other words the ground return parameters reflect the earth contribution in double-wired line's mutual impedance.

The purpose of the present research is determination conditions of representation horizontally-layered ground as homogeneous medium having parameters of the upper layer. Necessity of this will be cleared below (see section III).

II. Theoretical Ground

The basic formulae of the problem under consideration are the following ones.

$$Z = R + j\omega L$$

$$= j\frac{\omega\mu}{\pi}\int_0^\infty \frac{\exp[-(h_m + h_n)\lambda]\cos(a\lambda)}{\lambda + \sqrt{\lambda^2 + j\omega\mu\gamma}}\, d\lambda\,, \tag{1}$$

$$Z = j\frac{\omega\mu}{\pi}\int_0^\infty \frac{\exp[-(h_m + h_n)\lambda]\cos(a\lambda)}{\lambda + \sqrt{\lambda^2 + j[\omega\mu\gamma + j\omega^2\mu(\varepsilon - \varepsilon_o)]}}\, d\lambda\,, \tag{2}$$

$$Z = j\frac{\omega\mu}{\pi}\int_0^\infty \frac{\exp[-(h_m + h_n)\lambda]\cos(a\lambda)}{\lambda + \dfrac{\sqrt{\lambda^2 + j[\omega\mu\gamma_1 + j\omega^2\mu(\varepsilon_1 - \varepsilon_o)]}}{A}}\, d\lambda. \tag{3}$$

In the given formulae Z is modified linear impedance, Ohm/m; R is ground return resistance, Ohm/m; j is imaginary unit; ω is angular frequency, rad/s determined as $\omega=2\pi f$, where f is linear frequency, Hz; L is ground return inductance, H/m; μ is magnetic permeability, H/m; h_m and h_n are the mean highs of the two-wired system conductors with indexes m and n; a is the projection of distance between these conductors to the horizontal plane, m; ε is the dielectric permittivity, F/m; ε_o is the permittivity of free space, F/m.

The formula (1) is the classic one obtained and presented by J.R. Carson in [1]. As it is seen from the formula (1) there are not considered dielectric properties of ground. In other words there are not considered longitudinal displacement currents so formula (1) may be more or less adequate just in limited frequency ranges.

The formula (2) obtained and presented by W.H. Wise in [5] let us take into consideration the dielectric properties of ground via dielectric permittivity appeared in the expression (2). Note that the Carson and Wise formulae concern just to the case of homogeneous ground.

The formula (3) is given in numerous works e.g. [9]. This formula has the most general character because that so called ground impedance *(A)* included in the formula (3) let us take into consideration structure, geometrical and electrical parameters of horizontally-layered ground. We can also note that the formula (3) is a generalization of the Carson formula for the case of heterogeneous earth presented as horizontally-layered medium.

For the two-layered ground in accordance with [2, 9] we can write

$$A = \coth\left[-jk_1 d_1 + \coth^{-1}\sqrt{K_1/K_2}\right], \qquad (4)$$

where K_1 and K_2 are the wave coefficients of the upper and lower layers equaled respectively

$$K_1 = \sqrt{-(\gamma_1 + j\omega\varepsilon_1)(j\omega\mu_1)}, \qquad (5)$$

$$K_2 = \sqrt{-(\gamma_2 + j\omega\varepsilon_2)(j\omega\mu_2)}. \qquad (6)$$

Notice that indexes 1 and 2 in the formulae (3) – (6) are concerned to the parameters of the upper and lower layers respectively.

Also note that the problem is considered for the case of non-magnetic earth i.e. it is accepted that $\mu_1=\mu_2=\mu=\mu_o$, where μ_o is the permeability of free space, H/m.

We had carried out numerous computations of two-layered ground frequency-dependent return parameters. A plan of the research consisted on the following stages:

- determination the two-layered ground return parameters for different thicknesses of the upper layer using the formula (3);
- determination the ground return parameters for the homogeneous earth having parameters of the upper layer of the two-layered ground using the formula (2);
- calculation the relative differences between the values of parameters obtained at use the formulae (2) and (3) for different thicknesses of the upper layer and determination the frequencies at which the relative differences become negligible and the two-layered ground may be considered as homogeneous media.

The computations were carried out for two types of the two-layered earth. The first one was assumed consisted of fresh water as the upper layer with thicknesses 16, 11 and 9 meters and the granite rocks as the lower layer. The second type of structure consists of the clay-type soils as the upper layer with thicknesses 15, and 8 meters and the granite rocks as the lower layer.

All the results presented below were got for the double-wired aerial line with $h_m = h_n = 8$ meters and $a = 3$ meters.

All the computations of the improper integrals (2) and (3) had been done using the MATLAB–2013 set.

III. Results Obtained and Discussion

The graphs of the two-layered ground (fresh water-granite rocks) return parameters and earth modified linear impedance against frequency for the different thicknesses of the upper layer are presented below in the Figures 1, 2 and 3.

As it is seen from the curves given in the Figures 1, 2, and 3 at increasing frequency there has taken place convergence of ground return parameters *(R, L)* and modified linear impedance *(Z)* to ones for the homogeneous medium having parameters of the upper layer (the graphs for homogeneous earth denoted with mark d_1 = infinity).

To demonstrate a rate of the minded convergence in dependence on frequency the relative differences between parameters under consideration for homogeneous and two-layered earth (fresh water-granite rocks, thickness of the upper layer 16 meters) are presented in the table I.

We can see from the table I that at frequencies about 500 kHz and more the difference between parameters under consideration calculated for homogeneous and structured earth becomes negligible. It means that in certain half-infinity range of high frequencies horizontally-layered ground can be presented as homogeneous earth with parameters of upper layer.

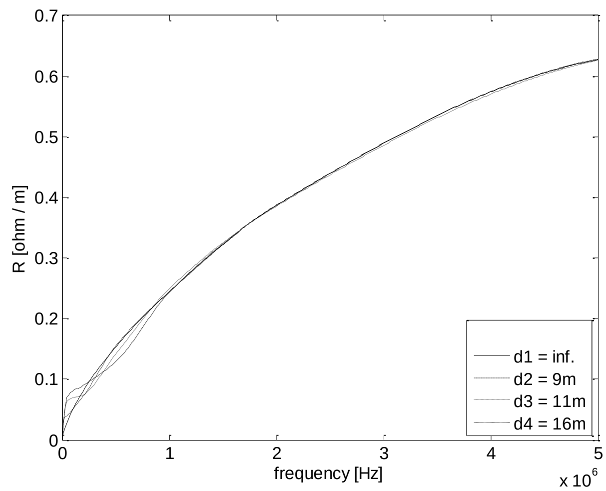

Fig.1. Ground returns resistance against frequency for the two-layered earth (fresh water-granite rocks)

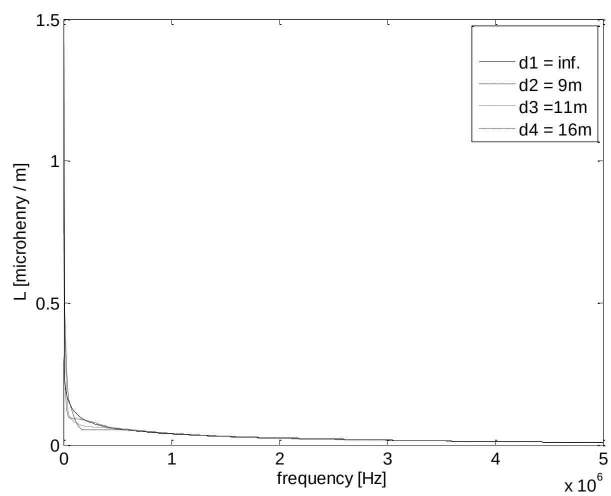

Fig.2. Ground returns inductance against frequency for the two-layered earth (fresh water-granite rocks)

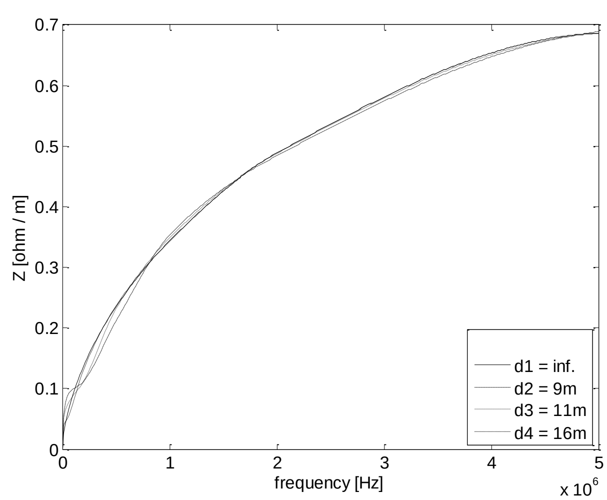

Fig.3. Two-layered earth (fresh water-granite rocks) modified linear impedance against frequency

The option of fresh water as the upper layer of the horizontally-layered earth is conditioned by two reasons. The first one is that this case is the most difficult from the computational point of view because of water's high relative permittivity. As a result the ratio ($\omega\varepsilon/\gamma$) can get notable value at relatively little frequencies. The second reason is a practical importance of this case for the overhead lines passing over fresh water areas.

We had carried out researches also for other kind of two-layered earth structures. Some results obtained for the case with clay-type upper layer presented below (see table II).

As it is seen from the last table there takes place satisfactory convergence of parameters under consideration for two-layered and homogeneous ground at frequencies less than 400 kHz (about 300 kHz). Our calculations showed that the same effect for the case with thickness of clay layer of 15 m is reached even at frequencies about 100 kHz.

TABLE I
RELATIVE DIFFERENCES BETWEEN CALCULATED R, L, Z PARAMETERS FOR TWO-LAYERED AND HOMOGENEOUS EARTH (FRESH WATER-GRANITE ROCKS) AT D = 16 M

f, Hz	Parameters		
	ΔR, %	ΔL, %	ΔZ, %
50	36.6964	81.8586	80.2312
250	12.4647	97.5083	93.4503
500	-10.5391	108.1827	101.9877
1000	-46.4123	120.4246	111.2562
5000	126.8123	106.7629	109.5916
20000	156.2737	-10.2259	39.6080
50000	40.1858	-37.7118	-11.4632
200000	-10.5208	-0.9029	-4.1175
500000	1.0877	0.6317	0.8252
1000000	-0.1578	-0.0647	-0.1118
2000000	-0.0128	-0.0188	-0.0150
5000000	0.0021	-0.0020	0.0014

TABLE II
RELATIVE DIFFERENCES BETWEEN CALCULATED R, L, Z PARAMETERS FOR TWO-LAYERED AND HOMOGENEOUS EARTH (CLAY-TYPE SOIL-GRANITE ROCKS) AT D = 8 M

f, Hz	Parameters		
	ΔR, %	ΔL, %	ΔZ, %
20000	97.9708	-39.7100	17.9020
60000	2.3377	-26.0963	-15.0061
100000	-10.0992	-11.1107	-10.7273
200000	-4.5379	1.4653	-0.9414
400000	0.4857	0.5182	0.5042
600000	0.1590	-0.0856	0.0232
800000	-0.0095	-0.0505	-0.0320
1000000	-0.0187	-0.0062	-0.0119
2000000	0.0003	-0.0002	0.0000
3000000	0.0000	0.0000	0.0000

To obtain some quantitative criterion of expediency of modeling the horizontally-layered earth as homogeneous medium we analyzed depths of penetration the electromagnetic field *(δ)* in the ground having parameters of the upper layer. The formula

$$\delta = \omega^{-1}\left[\sqrt{\frac{\mu_1\varepsilon_1}{2}\left(\sqrt{1+\frac{{\gamma_1}^2}{\omega^2{\varepsilon_1}^2}}-1\right)}\right]^{-1} \qquad (7)$$

given in [13]was used.

Comparison of penetration depths calculated by the formula (7) with the results of integration by (2) and (3) did let us to discover certain correspondence between the penetration depth and upper layer's thickness at satisfactory convergence of frequency characteristics of homogeneous and horizontally-layered earth (see the second conclusion below).

IV. Conclusions

For the frequencies more than some frequency determined by ground parameters and double-wired line's geometrical parameters horizontally-layered ground may be presented as a homogeneous medium having parameters of the upper layer. The differences at such a presentation decrease at the frequency increasing. The differences about and less than 1% take place for frequencies no less than 100 kHz – 1 MHz and thicknesses of the earth upper layer about dozens and less meters for different kinds of soils.

The best convergence of horizontally-layered earth parameters with ones for the homogeneous medium having parameters of the upper layer takes place at values of the *(d/δ)* ratio (where *d* is the upper layer's thickness, *δ* is depth of penetration of electromagnetic field in the upper layer) about (1.0 – 1.25)*e* where *e* is the base of natural logarithms. Appearance in this evaluation the number *e* is conditioned by the exponential character of attenuation the electromagnetic fields in ground.

There may have taken place rare deviations of ground return reactance curve from its monotonous character. It may occur at very high frequencies (about a few MHz) for the grounds with upper layer of relatively high dielectric permittivity and thickness about or less than a dozen of meters. Our evaluations showed that at relatively high values of the *(ωε/γ)* ratio an adequacy of computation the horizontally-layered ground return frequency-dependent parameters is worsened.

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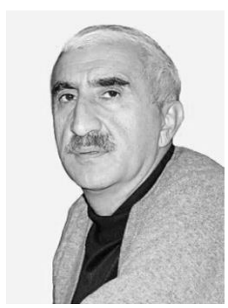
Biographies



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