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On Left Primary and Weakly Left Primary Ideals in Γ-LA- Rings

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ABSTRACT

In this paper, we study left ideals, left primary and weakly left primary ideals in Γ-LA-rings. Some characterizations of left primary and weakly left primary ideals are obtained. Moreover, we investigate relationships left primary and weakly left primary ideals in Γ-LA-rings. Finally, we obtain necessary and sufficient conditions of a weakly left primary ideal to be a left primary ideals in Γ-LA- rings.

Keywords: *Γ-LA-ring, left primary ideal, weakly left primary ideal, left ideal.*

1. INTRODUCTION

Abel-Grassmann's groupoid (AG-groupoid) is the generalization of semigroup theory with the wide range of usages in theory of flocks [6]. The fundamentals of this non-associative algebraic structure were the first discovered by Kazim and Naseeruddin [1]. AG-groupoid is a non-associative algebraic structure mid way between a groupoid and a commutative semigroup. It is interesting to note that an AG-groupoid with right identity becomes a commutative monoid [4]. This structure is closely related with a commutative semigroup because if an AG-groupoid contains a right identity, then it becomes a commutative monoid [4]. A left identity in an AG-groupoid is unique. Ideals in AG-groupoids have been discussed by Mushtaq and Yousuf [4, 5].

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In 1981, the notion of Γ -semigroups was introduced by Sen. Let S and Γ be any nonempty sets. If there exists a mapping $S \times\Gamma\times S \to S$ written (a,α,c) by $a\alpha c$, S is called a Γ -semigroup if S satisfies the identity:

$$(a\alpha b)\beta\, c = a\alpha(b\beta c)$$

for all $a,b,c\in S$ and $\alpha,\beta \in\Gamma$. A Γ -AG-groupoids analogous to Γ -semigroups. A groupoid S is called a Γ -AG-groupoid if it satisfies the left invertive law:

$$(a\gamma b)\delta c = (c\gamma b)\delta a$$

for all $a,b,c,d \in S$ and $\gamma ,\delta \in\Gamma$ [2]. This structure is also known as left almost semigroup (LA-semigroup).

S.M. Yusuf in [18] introduces the concept of a left almost ring (LA-ring). That is, a non-empty set R with two binary operations "+" and " · " is called a left almost ring, if $(R,+)$ is a LA-group, $(R,\cdot)$ is a LA-semigroup and distributive laws of "·" over "+" holds. Further in [12] T. Shah and I. Rehman generalize the notions of commutative semigroup rings into LA-semigroup LA-rings. However T. Shah and Fazal ur Rehman in [12] generalize the notion of a LA-ring into an nLA-ring. A near left almost ring (nLA-ring) N is a LAgroup under "+", a LA-semigroup under "·" and left distributive property of "·" over "+" holds.

T. Shah, Fazal ur Rehman and M. Raees asserted that a commutative ring $(R,+,\cdot)$, we can always obtain a LA-ring $(R,\oplus,\cdot)$ by defining, for $a,b,c\in R, a\oplus b = b-a$ and $a\cdot$b is same as in the ring. Furthermore, in this paper we characterize the left primary and weakly left primary ideals in Γ -LA-rings. Moreover, we investigate relationships left primary and weakly left primary ideals in Γ -LA-rings. Finally, we obtain necessary and sufficient conditions of a weakly left primary ideal to be a left primary ideals in Γ -LArings.

2. IDEALS IN Γ -LA-RINGS

The results of the following lemmas seem play an important role to study Γ -LA-ring; these facts will be used so frequently that normally we shall make no reference to this lemma.

Definition 2.1. Let $(R,+)$ and $(\Gamma,+)$ be a two LA-groups, R is called a Γ - left almost ring (Γ -LA-ring) if there exists a mapping $R\times\Gamma\times R\to R$ by $(a,\alpha,b)\mapsto a\alpha b$, for all $a,b\in R$ and $\alpha \in\Gamma$ satisfying the following conditions

1. $a\alpha(b +c) = a\alpha b + a\alpha c$
2. $(a +b)\alpha c = a\alpha c +b\alpha c$
3. $a(\alpha +\beta)b = a\alpha b+ a\beta b$
4. $(a\alpha b)\beta\, c = (c\alpha b)\beta\, a$, for all $a,b,c\in$R and $\alpha,\beta \in\Gamma$.

Lemma 2.2. If R is a Γ -LA-ring with left identity, then $a\gamma b = a\beta b$, for all $a,b\in R$ and $\gamma ,\beta \in\Gamma$.

Proof. Let R is a Γ -LA-ring and e be the left identity of $a,b\in R$ and let $\gamma ,\beta \in\Gamma$ therefore we have

$$\begin{aligned} a\gamma\, b &= a\gamma\, (e\beta b) \\ &= e\gamma\, (a\beta b) \\ &= a\beta\, b. \end{aligned}$$

Hence $a\gamma\, b = a\beta b$.

Lemma 2.3. Let R be a Γ -LA-ring with left identity e. Then $R\Gamma R = R$ and $R = e\Gamma R = R\Gamma e$.

Proof. Let R be a Γ -LA-ring with left identity e and let $r\in R$ then $r= e\alpha r\in R\Gamma R$, for all $\alpha \in\Gamma$, so that $R \subseteq R\Gamma R$. Since R is a Γ -LA-ring, we have $R\Gamma R \subseteq R$. Thus $R\Gamma R = R$. Now as e is a left identity in $R, e\alpha a = a$, for all $a\in R$ and $\alpha \in\Gamma$. Then $R = e\Gamma R$. Since $(a\alpha b)\beta\, c = (c\alpha b)\beta\, a$, for all $a,b,c\in R$ and $\alpha,\beta \in\Gamma$, we have $(R\Gamma R)\Gamma e = (e\Gamma R)\Gamma R$. Now,

$$R\Gamma e = (R\Gamma R)\Gamma e = (e\Gamma R)\Gamma R = R\Gamma R = R.$$

Hence $R = e\Gamma R = R\Gamma e$.

Definition 2.4. A nonempty subset I of a Γ -LA-ring R is a subring of R if under the binary operations in R, form a Γ -LA-ring.

Definition 2.5. A subring I of R is called a left (right) ideal of R if $R\Gamma I \subseteq I$ ($I\Gamma R \subseteq I$) and is called ideal if it is left as well as right ideal.

Lemma 2.6. If R is a Γ -LA-ring with left identity, then every right ideal is a left ideal.

Proof. Let R be a Γ -LA-ring with left identity and let A be a right ideal of R. Then for $a\in A$, $r\in R$ and $\alpha \in\Gamma$, consider

$$\begin{aligned} r\alpha a &= (e\beta\, r)\alpha a \\ &= (a\beta\, r)\alpha e \\ &\in (A\Gamma R)\Gamma R \\ &\subseteq A\Gamma R \\ &\subseteq A, \end{aligned}$$

where e is a left identity and $\beta \in\Gamma$, that is $r\alpha a\in A$. Therefore A is left ideal of R.

Lemma 2.7. If I is a left ideal of a Γ -LA-ring R with left identity, and if for any $a\in R$, $\gamma \in\Gamma$, then $a\gamma\, I$ is a left ideal of R.

Proof. Let I be a left ideal of R, consider

$$\begin{aligned} s\gamma\, (a\gamma\, i) &= (e\gamma\, s)\gamma\, (a\gamma\, i) \\ &= (e\gamma\, a)\gamma\, (s\gamma\, i) \\ &= a\gamma\, (s\gamma\, i)\in a\gamma\, I \end{aligned}$$

and $(a\gamma\, i) +(a\gamma\, j) = a\gamma\, (i + j)\in a\gamma\, I$. Hence $a\gamma\, I$ is a left ideal of R.

Lemma 2.8. Let R be a Γ -LA-ring with left identity, and $a\in R$, $\gamma \in\Gamma$. Then $R\gamma a$ is a left ideal of R.

Proof. Let R be a Γ -LA-ring with left identity, and $a\in R,\gamma \in\Gamma$. Then

$$\begin{aligned} R\gamma\,(R\gamma\, a) &= (R\gamma\, R)\gamma\,(R\gamma\, a) \\ &= (a\gamma\, R)\gamma\,(R\gamma\, R) \\ &= (a\gamma\, R)\gamma\, R \\ &= (R\gamma\, R)\gamma\, a \\ &= R\gamma\, a \end{aligned}$$

and $(r\gamma\, a) + (s\gamma\, a) = (r + s)\gamma\, a \in R\gamma a$. Hence $R\gamma a$ is a left ideal of R.

Lemma 2.9. If I is an ideal of a Γ -LA-ring R with left identity, and if for any $a\in R, \gamma \in \Gamma$, then $a^2\gamma I$ is an ideal of R.

Proof. By Lemma 2.7, we have $a^2\gamma I$ is a left ideal of R. Now consider

$$\begin{aligned} (a^2\gamma r)\gamma s &= ((a\gamma\, a)\gamma\, r)\gamma\, s \\ &= ((r\gamma\, a)\gamma\, a)\gamma\, s \\ &= [e\gamma\,((r\gamma\, a)\gamma\, a)]\gamma\, s \\ &= [s\gamma\,((r\gamma\, a)\gamma\, a)]\gamma\, e \\ &= [(r\gamma\, a)\gamma\,(s\gamma\, a)]\gamma\, e \\ &= [((s\gamma\, a)\gamma\, a)\gamma\, r]\gamma\, e \\ &= [((a\gamma\, a)\gamma\, s)\gamma\, r]\gamma\, e \\ &= [(r\gamma\, s)\gamma\,(a\gamma\, a)]\gamma\, e \\ &= [e\gamma\,(a\gamma\, a)]\gamma\,(r\gamma\, s) \\ &= (a\gamma\, a)\gamma\,(r\gamma\, s) \\ &= a^2\gamma\left(r\gamma s\right) \in a^2\gamma I. \end{aligned}$$

Hence $a^2\gamma I$ is an ideal of R.

Lemma 2.10. Let R be a Γ -LA-ring with left identity, and $a\in R$, $\gamma \in \Gamma$. Then $R\gamma a^2$ is an ideal of R.

Proof. Let R be a Γ -LA-ring with left identity, and $a\in R$, $\gamma \in \Gamma$. Now consider

$$\begin{aligned} R\gamma a^2 &= \left(R\Gamma R\right)\gamma a^2 \\ &= a^2\gamma\left(R\Gamma R\right) \\ &= a^2\gamma R \end{aligned}$$

By Lemma 2.9, we have $R\gamma a^2$ is an ideal of R.

Lemma 2.11. Let R be a Γ -LA-ring with left identity, and let A,B be left ideals of R. Then $(A{:}\Gamma{:}\, B)$ is a left ideal in R, where $(A{:}\Gamma{:}\, B) = \{r\in R{:}\, B\Gamma r \subseteq A\}$.

Proof. Suppose that R is a Γ -LA-ring. Let $s\in R$ and let $a,b\in (A{:}\Gamma{:}\, B)$. Then $B\Gamma a \subseteq A$ and $B\Gamma b \subseteq A$ so that

$$\begin{aligned} B\Gamma(a + b) &= (B\Gamma a) + (B\Gamma b) \\ &\subseteq A + A \\ &= \mathrm{A} \end{aligned}$$

and

$$\begin{aligned} B\Gamma(s\gamma\, a) &= s\Gamma(B\gamma\, a) \\ &= s\Gamma A \\ &= \mathrm{A}. \end{aligned}$$

Therefore $a + b\in (A{:}\Gamma{:}\, B)$ and $s\gamma a\in (A{:}\Gamma{:}B)$ so that $R\Gamma(A{:}\Gamma{:}\, B) \subseteq (A{:}\Gamma{:}\, B)$. Hence $(A{:}\Gamma{:}\, B)$ is a left ideal in R.

Corollary 2.12. Let R be a Γ -LA-ring with left identity, and let A be left ideals of R Then $(A{:}\gamma\, {:}b)$ is a left ideal in R, where $(A{:}\gamma\, {:}b) = \{r\in R{:}b\gamma\, r\in A\}$.

Proof. This follows from Lemma 2.11.

Remark.1. Let R be a Γ -LA-ring and let A be a left ideal of R. It is easy to verify that $A\subseteq (A{:}\gamma : r)$.

2. Let R be a Γ -LA-ring with left identity e, and let A be a proper left (right) ideal of R. By Corollary 2.12 , we have $e\notin (A{:}\gamma : r)$, where $r\in R- A$.

3. Let R be a Γ -LA-ring and let A,B,C be left ideals of R. It is easy to verify that $(A{:}\Gamma{:}C) \subseteq (A{:}\Gamma{:} B)$,where $B \subseteq C$.

3. LEFT PRIMARY AND WEAKLY LEFT PRIMARY IDEAL IN Γ -LA-RINGS

We start with the following theorem that gives a relation between left primary and weakly left primary ideal in Γ -LA-ring. Our starting points is the following definition:

Definition 3.1. A left ideal P is called left primary if $A\Gamma B \subseteq P$ implies that $(((A\Gamma A)\ \Gamma)\ldots A\ \Gamma A) = A^n \subseteq P$ or $B \subseteq P$ for some positive integer n, where A, B is a left ideals of R.

Definition 3.2. A left ideal P is called weakly left primary if $0 \neq A\Gamma B \subseteq P$ implies that $(((A\Gamma A)\ \Gamma)\ldots A\ \Gamma A) = A^n \subseteq P$ or $B \subseteq P$ for some positive integer n, where A, B is a left ideals of R.

Remark. It is easy to see that every left primary ideal is weakly left primary.

Lemma 3.3. If R is a Γ -LA-ring with left identity, then a left ideal P of R is left primary if and only if $a\gamma b\in P$ implies that $a^n \in P$ or $b\in P$ for some positive integer n, where $\gamma \in \Gamma$ and $a,b\in R$.

Proof. Let P be a left ideal of Γ -LA-ring R with left identity. Now suppose that $a\gamma b\in P$. Then by Definition of left ideal, we get

$$\begin{aligned} (R\gamma a)\beta\,(R\alpha b) &= (R\gamma R)\beta(a\alpha b) \\ &= R\beta(a\alpha b) \\ &\subseteq R\beta P \\ &\subseteq P. \end{aligned}$$

Then $a = (e\gamma\, a)^n \in (R\gamma\, a)^n \subseteq P$ or $b = e\alpha b \in R\alpha b \subseteq P$, for some positive integer n. Conversely, the proof is easy.

Corollary 3.4. If R is a Γ -LA-ring with left identity, then a left ideal P of R is weakly left primary if and only if $0 \neq a\gamma b \in P$ implies that $a^n \in P$ or $b \in P$ for some positive integer n, where $\gamma \in \Gamma$ and $a,b \in R$.

Proof. This follows from Lemma 3.3.

Let R be a Γ -LA-ring and A be a subset of R. We write

$$\sqrt{A} = \{a \in R: a^k \in A, \text{ for some positive integer } k\}.$$

Theorem 3.5. Let R be a Γ -LA-ring with left identity, and let P be an ideal of R. If P is a weakly left primary ideal that is not let primary. Then $\sqrt{P} = \sqrt{0}$.

Proof. Let R be a Γ -LA-ring with left identity. First, we prove that $P^2 = 0$. Suppose that $P^2 \neq 0$ we show that P is weakly left primary. Let $a\gamma b \in P$, where $a,b \in R$, $\gamma \in \Gamma$. If $a\gamma\, b \neq 0$, then either

$$a \in \sqrt{P} \text{ or } b \in P$$

since P is weakly left primary ideal. So suppose that $a\gamma b = 0$. If $P\gamma b \neq 0$, then there is an element p' of P such that $p'\gamma b \neq 0$, so that

$$0 \neq p'\gamma\, b = (\,p' + a)\gamma b \in P$$

and hence P weakly left primary ideal gives either $p' + a \in \sqrt{P}$ or $b \in P$. As $p' + a \in \sqrt{P}$ and $p' \in P \subseteq \sqrt{P}$ we have either $a \in \sqrt{P}$ or $b \in P$. So we can assume that $P\gamma b = 0$. Similarly, we can assume that $P\gamma a = 0$. Since $P^2 \neq 0$, there exist $c, d \in P$ such that $c\gamma d \neq 0$. Then

$$0 \neq (a + c)\gamma\, (b + d\,) \in P,$$

so either $a + c \in \sqrt{P}$ or $b + d \in P$, and hence either $a \in \sqrt{P}$ or $b \in P$. Thus P is left primary ideal. Clearly, $\sqrt{0} \subseteq \sqrt{P}$. As $P^2 = 0$, we get $\sqrt{P} \subseteq \sqrt{0}$, hence $\sqrt{P} = \sqrt{0}$, P as required.

Corollary 3.6. Let R be a Γ -LA-ring with left identity, and let P an ideal of R. If $\sqrt{P} \neq \sqrt{0}$, then P is left primary if and only if P is weakly left primary.

Proof. This follows from Theorem 3.5.

Lemma 3.7. Let R be a Γ -LA-ring with left identity, and let P be a proper ideal of R. If P is a weakly left primary ideal of R, then

$$(P{:}\Gamma{:}\ R\Gamma a) = P \cup (0{:}\Gamma{:}\ R\Gamma a),$$

where $a \in R - \sqrt{P}$.

Proof. Let R be a Γ -LA-ring with left identity, and let P be a weakly left primary ideal of R. Clearly,

$$P \cup (0{:}\Gamma{:}\ R\Gamma a) \subseteq (P{:}\Gamma{:}\ R\Gamma a).$$

For the other inclusion, suppose that $m \in (P{:}\Gamma{:}\ R\Gamma a)$, so that

$$\begin{aligned}(R\Gamma a)\Gamma(R\Gamma m) &= (m\Gamma R)\Gamma(a\Gamma R)\\ &= (m\Gamma a)\Gamma(R\Gamma R)\\ &= (m\Gamma a)\Gamma R\\ &= (R\Gamma a)\Gamma m\\ &\subseteq P.\end{aligned}$$

If $0 \neq (R\Gamma a)\Gamma m$, then $m = e\gamma\, m \in R\Gamma m \subseteq P$ since P is weakly left primary. If $0 = (R\Gamma a)\Gamma m$, then $m \in (0{:}\Gamma{:}\ R\Gamma a)$ so we have the equality.

Corollary 3.8. Let R be a Γ -LA-ring with left identity, and let P be a proper ideal of R. If P is a weakly left primary ideal of R, then

$$(P{:}\Gamma{:}\ a) = P \cup (0{:}\Gamma{:}\ a),$$

where $a \in R - \sqrt{P}$.

Proof. This follows from Lemma 3.7.

Corollary 3.9. Let R be a Γ -LA-ring with left identity, and let P be a proper ideal of R. If $(P{:}\Gamma{:}\ R\Gamma a) = P \cup (0{:}\Gamma{:}\ R\Gamma a)$, then

$$(P{:}\Gamma{:}\ R\Gamma a) = P \text{ or } (P{:}\Gamma{:}\ R\Gamma a) = (0{:}\Gamma{:}\ R\Gamma a),$$

where $a \in R - \sqrt{P}$.

Proof. This follows from Lemma 3.7.

Theorem 3.10. Let R be a Γ - LA-ring with left identity, and let P be a proper ideal of R. If $(P{:}\Gamma{:}\ n) = P$ or $(P{:}\Gamma{:}\ n) = (0{:}\Gamma{:}\ n)$, then P is a weakly left primary ideal of R, where $n \in R - \sqrt{P}$.

Proof. Let R be a Γ -LA-ring with left identity, and let P be a proper ideal of R. Suppose that Let $0 \neq m\gamma n \in P$, where $m \in R - \sqrt{P}$, $\gamma \in \Gamma$. Then

$$m \in (P{:}\Gamma{:}\ n) = P \cup (0\ {:}\Gamma{:}\ n)$$

by Corollary 3.9 hence $m \in P$ since $m\gamma n \neq 0$, as required.

Lemma 3.11. Let $R = R_1 \times R_2$, where each R_i is a Γ - LA-ring with left identity. Then the following hold:

(i) If A is a left ideal of R_1, then $\sqrt{A \times R_2} = \sqrt{A} \times R_2$.

(ii) If A is a left ideal of R_2, then $\sqrt{R_1 \times A} = R_1 \times \sqrt{A}$.

Proof. The proof is straightforward.

Theorem 3.12. Let $R = R_1 \times R_2$, where each R_i is a Γ-LA-ring with left identity. If P is a weakly left primary (left primary) ideal of R_1, then $P \times R_2$ is a weakly left primary (left primary) ideal of *R*.

Proof. Suppose that $R = R_1 \times R_2$, where each R_i is a Γ-LA-ring with left identity and *P* is a weakly left primary ideal of R_1. Let

$$0 \neq (a, b)\gamma(c, d) = (a\gamma c, b\gamma d) \in P \times R,$$

where $(a,b),(c,d) \in R$, $\gamma \in \Gamma$ so either $a \in \sqrt{P}$ or $c \in P$ since *P* is weakly left primary. It follows that either

$$(a, b) \in \sqrt{P} \times R = \sqrt{P \times R_2} \text{ or } (c, d) \in P \times R .$$

By Definition of weakly left primary ideal, we have $P \times R_2$ is a weakly left primary ideal of *R*.

Corollary 3.13. Let $R = R_1 \times R_2$, where each R_i is a Γ-LA-ring with left identity. If *P* is a weakly left primary (left primary) ideal of R_2, then $R_1 \times P$ is a weakly left primary (left primary) ideal of *R*.

Proof. This follows from Theorem 3.12.

Corollary 3.14. Let $R = \prod_{i=1}^{n} R_i$, where each R_i is a Γ-LA-ring with left identity. If P is a weakly left primary (left primary) ideal of R_j, then

$$R_1 \times R_2 \times \ldots \times P_{j-1} \times P_j \times R_{j+1} \times \ldots \times R_n$$

is a weakly left primary (left primary) ideal of R.

Proof. This follows from Theorem 3.12 and Corollary 3.13.

Theorem 3.15. Let $R = R_1 \times R_2$, where each R_i is a Γ-LA-ring with left identity. If P is a weakly left primary ideal of *R*, then either $P = 0$ or *P* is left primary.

Proof. Let $R = R_1 \times R_2$, where each R_i is a Γ-LA-ring with identity and let $P = P_1 \times R_2$ be a weakly left primary ideal of *R*. We can assume that $P \neq 0$. So there is an element (a, b) of *P* with $(a,b) \neq (0,0)$. Then

$$(0,0) \neq (a,e)\gamma\, (e, b) \in P,$$

where $\gamma \in \Gamma$, gives either

$$(a, e) \in \sqrt{P} = \sqrt{P_1 \times R_2} = \sqrt{P_1} \times R_2 \text{ or } (e, b) \in P$$

If $(e, b) \in P$, then $P = R_1 \times P_2$. We will show that P_2 is left primary hence *P* is weakly left primary by Corollary 3.13. Let $c\gamma d \in P_2$, where $c,d \in R_2$. Then

$$(0, 0) \neq (e, c)\gamma\, (e, d) = (e, c\gamma) \in P,$$

so either $(e, c) \in \sqrt{P} = \sqrt{R_1 \times P_2} = R_1 \times \sqrt{P_2}$ or $(e, d) \in P$ and hence either $c \in \sqrt{P_2}$ or $d \in P_2$. By a similar argument, $P = R_1 \times P_2$ is left primary.

Proposition 3.16. Let $A \subseteq P$ be proper ideals of a Γ-LA-ring *R*. Then the following hold:

(i) If *P* is weakly left primary (left primary), then *P* /*A* is weakly left primary (left primary).

(ii) If *A* and *P*/*A* are weakly left primary (left primary), then P is weakly left primary (left primary).

Proof. (i) Let $0 \neq (a + A)\gamma\, (b + A) = a\gamma\, b + A \in P/A$, where $a,b \in R$, $\gamma \in \Gamma$ so $a\gamma b \in P$. If $a\gamma\, b = 0 \in A$, then

$$(a + A)\gamma\, (b + A) = 0,$$

a contradiction. So if *P* is weakly left primary, then either $a \in \sqrt{P}$ or $b \in P$, hence either $a + A \in P/A$ or $b + A \in P/A$, as required.

(ii) Let $0 \neq a\gamma\, b \in P$, where $a,b \in R$, so $(a + A)\gamma\, (b + A) \in P/A$. For $a\gamma b \in A$, if A is weakly left primary, then either

$$a \in A \subseteq \sqrt{P} \text{ or } b \in A \subseteq P.$$

So we may assume that $a\gamma\, b \notin A$. Then either $a + A \in \sqrt{P / P}$ or $b + A \in P/A$. It follows that either $a \in \sqrt{P}$ or $b \in P$ as needed.

Theorem 3.17. Let *P* and *Q* be weakly left primary ideals of a Γ-LA –ring *R* that are not left primary. Then $P + Q$ is a weakly left primary ideal of *R*.

Proof. Since $(P + Q) / Q \approx Q / (P \cap Q)$, we get that $(P+Q) / Q$ is weakly left primary by Proposition 3.16 (i). Now the assertion follows from Proposition 3.16 (ii).

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