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Two New Ridge Parameters and A Guide for Selecting an Appropriate Ridge Parameter in Linear Regression

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ABSTRACT

The ridge regression estimator was first introduced by Hoerl and Kennard [6] as an alternative method to the ordinary least squares (OLS) estimator when multicollinearity exists among regressors. Ridge regression depends on the estimation of the ridge parameter presented in this study as k. On the other hand there is not a standard way of determining k. In the literature, there are a lot of proposed ridge parameters. The aim of this paper is to introduce two new ridge parameters and make comparison of 37 different ridge parameters including the proposed ones. A simulation study has been conducted to make comparisons in terms of the mean square error criterion. It is found that the proposed ridge parameters produce better results than most of the other parameters. The parameters proposed by Asar et. al. [2] very recently did not perform as well as their results. In fact, the parameters we have proposed did perform much better than theirs in every single case. However, there is no explicit ridge parameter that performs well in every situation. The ridge estimators act differently in various sample sizes, dimensions and collinearity degrees. We think that this study is helpful for researchers employing ridge regression as they may use the comparative results provided in the study to make a decision of choosing the best ridge parameter for their case.

Keywords: *Ridge regression, ridge parameters, collinear data generation.*

1. INTRODUCTION

In multiple linear regression, in order to get least square estimates, independent variables need to be independent from each other. However, when there is a strong linear relationship among the independent variables, a problem

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arises. This problem is called multicollinearity. When multicollinearity exists regression parameters cannot be correctly estimated and their variances become very large. One of the solutions to overcome this problem is to use ridge regression. Ridge regression is a method used for analyzing multiple regression data that has multicollinearity problem. When multicollinearity exists, ordinary least squares estimates are unbiased, but their variances become much larger so those estimates may be far from the real value. To explain how ridge regression works, consider the following linear regression model below,

$$\mathbf{y} = \mathbf{X}\alpha + \mathbf{e} \tag{1.1}$$

Where $\mathbf{y}$ is a $n \times 1$ vector of observations α is a $p \times 1$ vector of unknown regression coefficients, $\mathbf{X}$ is a $n \times p$ known design matrix of rank p and $\mathbf{e}$ is a $n \times 1$ vector random variable having multivariate normal distribution with mean vector $\mathbf{0}$ and variance-covariance matrix $\sigma^2 \mathbf{I}_n$ where $\mathbf{I}_n$ is an identity matrix of order n . The usual least squares estimate (LSE) or the maximum likelihood estimate (MLE) of α is given by

$$\hat{\alpha} = \mathbf{C}^{-1}\mathbf{X}'\mathbf{y} \tag{1.2}$$

This estimate will be depending on the characteristics of the matrix. If there are dependencies among the columns of the matrix $\mathbf{C}$, the LSE have some errors. Hoerl and Kennard [6] suggested a modified $\mathbf{C}$ to overcome this problem. They used $\mathbf{C}(k) = \mathbf{C} + kI_p$, $k \geq \mathbf{0}$. Then the resulting estimators become

$$\hat{\alpha}(k) = \left(\mathbf{C} + k\mathbf{I}_p\right)^{-1}\mathbf{X}'\mathbf{y} \tag{1.3}$$

which are known as ridge regression estimators. The constant $k > 0$ is called biasing or ridge parameter. As k increases from zero to infinity, the regression estimates tend to approach zero. Ridge regression estimators explicitly depend on the value of. However, there is not an explicit way of estimating. So, there are plenty of ridge parameter estimates proposed in the literature. Such as Hoerl and Kennard [7], [8], Hoerl et al. [6], Lawless and Wang [13], Schaeffer et al. [20], Nomura [16], Kibria [11], Khalaf and Shukur [10], Norliza et al. [17], Alkhamisi and Shukur [1], Batah et al. [3], Muniz and Kibria [15], Kibria et al. [12], Dorugade [4], Asar et al. [2]. In search of the best ridge estimator some papers have made comparisons among some ridge parameters, for example: Asar et al. [2], Dorugade [4], Kibria [11], Kibria et al. [12]., Karaibrahimoğlu et al. [9], Khalaf and Shukur [10], Saleh and Kibria [19], Sakallioglu and Kaciranlar [18]. In the meantime Gökpınar and Ebegil [21] compared popular k values according to their MSE criteria and their accept rate of the testing procedure given in Liski [22] and [23].

It is aimed in this paper to introduce two new ridge parameters and make a comparison of the proposed and other ridge parameters widely used in the literature. In order to achieve that a simulation study has been conducted to make comparisons in terms of the mean square error criterion.

2. RIDGE ESTIMATORS

Ridge regression estimators explicitly depend on the value of the ridge parameter. However, there is not an explicit way of estimating. So, there are plenty of ridge parameter estimators proposed in the literature. Such as

Hoerl and Kennard [7],[8]

$$K1 = \frac{\hat{\sigma}^2}{\hat{\alpha}^2_{\max}} \tag{2.1}$$

where $\hat{\sigma}^2$ is the estimated error variance from ordinary least square (OLS) regression and $\hat{\alpha}_{\max}^{2}$ is the square of the maximum of unknown regression coefficient estimate.

$$K2 = \frac{\hat{\sigma}^2}{\max(\hat{\alpha}_i)} \quad i = 0, 1, 2, \cdots, \square \tag{2.2}$$

Where $\hat{\alpha}_i$ is the ith unknown regression coefficient OLS estimate.

$$K3 = \frac{\hat{\sigma}^2}{\sum_{i=0}^{p} \hat{\alpha}_i^2} \tag{2.3}$$

Where $\hat{\alpha}$ is the unknown regression coefficient

Hoerl et al. [6]

$$K4 = \frac{p\hat{\sigma}^2}{\sum_{i=0}^{p} \hat{\alpha}_i^2} \tag{2.4}$$

whereis the number of regressors.

Lawless and Wang [13]

$$K5 = \frac{p\hat{\sigma}^2}{\sum_{i=0}^{p} \lambda_i \hat{\alpha}_i^2} \tag{2.5}$$

where λ_i is the ith eigenvalue of the covariance matrix $\mathbf{C}$ given in (1.2).

Schaeffer et al. [20]

$$K6 = \frac{1}{\hat{\alpha}^2_{\max}} \tag{2.6}$$

Nomura [16]

$$K7 = (p\hat{\sigma}^{2})/\left(\sum_{(i=0)}^{p} \left\{\hat{\alpha}_i/\left[1 + \left(1 + \lambda_i \left(\hat{\alpha}_i^{2}/\hat{\sigma}^{2}\right)^{(1/2)}\right)\right]\right\}\right) \tag{2.7}$$

Kibria [11]

$$K8 = \frac{\hat{\sigma}^2}{\left(\prod_{i=0}^{p} \hat{\alpha}_i^2\right)^{\frac{1}{p}}} \tag{2.8}$$

given that,

$$m_i = \sqrt{\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2}} \tag{2.9}$$

$$K9 = median\{m_i^2\} \tag{2.10}$$

$$K10 = 1/p \ \sum_{i=0}^{p} \hat{\sigma}^2/(\hat{\alpha}_i^2) \tag{2.11}$$

Khalaf and Shukur [10]

$$K11 = \frac{(\lambda_{\max})\hat{\sigma}^2}{\left((n-p-1)\hat{\sigma}^2 + \lambda_{\max}\hat{\alpha}^2_{\max}\right)} \tag{2.12}$$

Where n is the number of observations and $\lambda_{\max}$ is the maximum eigenvalue of the matrix $\mathbf{C}$ given in (1.2).

Norliza et al. [17]

$$K12 = \frac{\left\{\hat{\sigma}^2 \lambda_{\max} \sum_{i=0}^{p} (\lambda_i \hat{\alpha}_i^2) + \left[\sum_{i=0}^{p} (\lambda_i \hat{\alpha}_i^2)\right]^2\right\}}{\lambda_{\max} \sum_{i=0}^{p} (\lambda_i \hat{\alpha}_i^2)} \tag{2.13}$$

Alkhamisi and Shukur [1]

$$K13 = \max\left(\frac{\hat{\sigma}^2}{\hat{\alpha}_i^2} + \frac{1}{\lambda_i}\right) \quad i = 0, 1, 2, \cdots, p \tag{2.14}$$

Batah et al. [3]

$$K14 = \frac{p\hat{\sigma}^2}{\sum_{i=0}^{p} \left\{\frac{\hat{\alpha}_i^2}{\left[\left[\left(\frac{\hat{\alpha}_i^4 \lambda_i^2}{4\hat{\sigma}^2}\right) + \left(\frac{6\hat{\alpha}_i^4 \lambda_i}{\hat{\sigma}^2}\right)\right]^{\frac{1}{2}} - \left(\frac{\hat{\alpha}_i^2 \lambda_i}{2\hat{\sigma}^2}\right)\right]}\right\}} \tag{2.15}$$

Muniz and Kibria [15] and Kibria et al. [12]

$$K15 = \max\left(\frac{1}{m_i}\right) \tag{2.16}$$

$$K16 = \max(m_i) \tag{2.17}$$

$$K17 = \prod_{i=0}^{p} \left(\frac{1}{m_i}\right)^{\frac{1}{p}} \tag{2.18}$$

$$K18 = \prod_{i=0}^{p} (m_i)^{\frac{1}{p}} \tag{2.19}$$

$$K19 = median\left(\frac{1}{m_i}\right) \tag{2.20}$$

$$K20 = median(m_i) \tag{2.21}$$

given that,

$$q_i = \frac{\lambda_{\max}}{(n-p)\hat{\sigma}^2 + \lambda_{\max}\hat{\alpha}_i^2} \tag{2.22}$$

$$K21 = \max\left(\frac{1}{q_i}\right) \tag{2.23}$$

$$K22 = \max(q_i) \tag{2.24}$$

$$K23 = \prod_{i=0}^{p} \left(\frac{1}{q_i}\right)^{\frac{1}{p}} \tag{2.25}$$

$$K24 = \prod_{i=0}^{p} (q_i)^{\frac{1}{p}} \tag{2.26}$$

$$K25 = median\left(\frac{1}{q_i}\right) \tag{2.27}$$

$$K26 = median(q_i) \tag{2.28}$$

Dorugade [4]

Given that,

$$Ki(AD) = \frac{2\hat{\sigma}^2}{\lambda_{\max}\hat{\alpha}_i^2} \quad i = 0, 1, 2, \cdots, p \tag{2.29}$$

$$K27 = Arithmeticmean[ki(AD)] \tag{2.30}$$

$$K28 = Median[ki(AD)] \tag{2.31}$$

$$K29 = Geometricmean[ki(AD)] \tag{2.32}$$

$$K30 = Harmonicmean[ki(AD)] \tag{2.33}$$

Asar et al. [2]

$$K31 = \frac{p^2}{\lambda_{\max}^2} \frac{\hat{\sigma}^2}{\sum_{i=0}^{p} \hat{\alpha}_i^2} \tag{2.34}$$

$$K32 = \frac{p^3}{\lambda_{\max}^3} \frac{\hat{\sigma}^2}{\sum_{i=0}^{p} \hat{\alpha}_i^2} \tag{2.35}$$

$$K33 = \frac{p}{\lambda_{\max}^{\frac{1}{3}}} \frac{\hat{\sigma}^2}{\sum_{i=0}^{p} \hat{\alpha}_i^2} \tag{2.35}$$

$$K34 = \frac{p}{\left(\sum_{i=0}^{p}\sqrt{\lambda_i}\right)^{\frac{1}{3}}}\frac{\hat{\sigma}^2}{\sum_{i=0}^{p}\hat{\alpha}_i^2} \quad (2.36)$$

$$K35 = \frac{2p}{\sqrt{\lambda_{max}}}\frac{\hat{\sigma}^2}{\sum_{i=0}^{p}\hat{\alpha}_i^2} \quad (2.37)$$

We propose two new estimators,

$$K36 = \sqrt{median(m_i)} \quad (2.38)$$

$$K37 = \frac{\hat{\sigma}^2}{(median(\hat{\alpha}_i))^2} \quad (2.39)$$

3. SIMULATION AND COMPARISON

We consider the true model as in (1.1), where e follows a normal distribution $N(0, \sigma^2 I_n)$ and the independent variables are generated [3] from

where e_{ij} is generated from an independent standart normal distribution and ρ is the correlation between x_{ij} and $x_{ij'}$ for $j, j' = 1, 2, \cdots, p$ and $j \neq j'$.

The correlation coefficient ρ may be treated as the degree of collinearity for the regressors x_{ij}

The response variable is generated as a function of both regressors and the error term as follows;

$$x_{ij} = (1-\rho^2)^{\frac{1}{2}} e_{ij} + \rho e_{ip} \quad i = 1, 2, \cdots, n, \quad j = 1, 2, \cdots, p \quad (3.1)$$

$$Y = \alpha_0 + \alpha_1 X_1 + \alpha_2 X_2 + \cdots + \alpha_p X_p + e \quad (3.2)$$

The generation of the response variable Y depends mainly on the α coefficients. We have selected two different number of regressors with three different degree of collinearity. When the number of regressors is three, the actual alfa coefficient vector used in the generation process of the response variable is considered to be $(1 \quad 1 \quad 1 \quad 1)'$. When p is 7, then the real coefficient vector is considered to be $(1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1)'$. We have considered six different sample sizes which are n=30, 50, 80, 100, 250 and 500 for any number of regressors. For any design of the generation, the simulation study has been replicated for 10 000 times. The comparisons have been made according to the value of the mean square errors obtained as follows;

$$\mathbf{MSE}_{\hat{\alpha}} = \left[\frac{\sum_{i=0}^{p}\left(\alpha - \hat{\alpha}_{ridge}\right)^2}{\mathbf{10000}}\right] \quad (3.3)$$

where α is the real coefficient parameter vector considered as $(1 \quad 1 \quad 1 \quad 1)'$ for p=3 and $(1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1)'$ for p=7 and $\hat{\alpha}_{ridge}$ is the bias estimation coefficient vector

using a ridge parameter.

Empirical results have been given in the following tables starting from Table 3.1 and ending in Table 3.6. The results presented are obtained from the equality given in (3.3).

Table 3.1. Mean square errors of ridge parameters for different number of regressors and different degree of multicollinearity for the sample size of 30

Ridge Parameter	**n=30**					
	P=3			**P=7**		
	ρ=0.9	**ρ =0.5**	**ρ =0.3**	**ρ =0.9**	**ρ =0.5**	**ρ =0.3**
k1	0.543800	0.171740	0.150930	1.71440	0.46868	0.38371
k2	0.479650	0.168370	0.148530	1.41240	0.45451	0.37587
k3	0.596860	0.176420	0.153390	2.02490	0.49602	0.39981
k4	0.486900	0.170300	0.150290	1.31730	0.45145	0.37586
k5	0.665640	0.180180	0.155620	2.17140	0.50426	0.40462
k6	0.541890	0.171620	0.150730	1.70500	0.46779	0.38334
k7	**0.213500**	0.171300	0.171570	0.51218	0.43484	0.43987
k8	0.337890	0.166550	0.149030	0.90150	0.43602	0.37035
k9	0.379960	0.167160	0.149220	1.04850	0.44242	0.37246
k10	0.305470	0.162430	0.148770	0.85410	0.42589	0.36601
k11	0.553180	0.172490	0.151030	1.72650	0.47103	0.38532
k12	0.254160	0.152660	0.145990	0.65398	**0.40340**	0.40222

k13	0.341260	0.160050	0.149250	1.14880	0.43390	0.38274
k14	0.698890	0.181340	0.156380	2.50300	0.51212	0.40933
k15	0.302230	0.158170	0.141410	0.71818	0.41285	**0.35491**
k16	0.253150	0.160280	0.145310	0.58858	0.40793	0.35814
k17	0.420600	0.162370	0.143450	1.12320	0.43371	0.36337
k18	0.362000	0.165120	0.147000	0.96098	0.43528	0.36760
k19	0.387860	0.161690	0.143160	1.03430	0.42996	0.36204
k20	0.387300	0.165530	0.147140	1.04190	0.43873	0.36877
k21	0.225160	**0.151370**	**0.138080**	**0.40383**	0.46975	0.56906
k22	0.258260	0.162340	0.146580	1.48610	0.48314	0.39608
k23	0.362350	0.157900	0.140480	0.52601	0.45334	0.53789
k24	0.421380	0.168610	0.149150	1.66070	0.48705	0.39721
k25	0.347250	0.157040	0.140150	0.54104	0.45287	0.53675
k26	0.420590	0.168760	0.149190	1.63660	0.48692	0.39717
k27	0.661540	0.179750	0.155110	2.20170	0.50436	0.40407
k28	0.660600	0.180010	0.155450	2.21180	0.50491	0.40463
k29	0.657650	0.179990	0.155440	2.19970	0.50467	0.40451
k30	0.435730	0.173510	0.151370	0.97740	0.43287	0.36888
k31	0.665740	0.181980	0.156210	2.24890	0.50213	0.40589
k32	0.665820	0.181990	0.156210	2.24930	0.50218	0.40594
k33	0.614600	0.179060	0.154240	1.98840	0.48716	0.39648
k34	0.581730	0.177670	0.153560	1.81940	0.48169	0.39447
k35	0.618070	0.179140	0.154230	2.02730	0.48843	0.39680
k36	0.389490	0.164650	0.146150	1.03840	0.43668	0.36701
k37	0.398590	0.167680	0.149390	1.06280	0.44316	0.37266
Min MSE	**0.213500**	**0.151370**	**0.138080**	**0.40383**	**0.40340**	**0.35491**

Table 3.2.Mean square errors of ridge parameters for different number of regressors and different degree of multicollinearity for the sample size of 50

Ridge Parameter	**n=50**					
	P=3			**P=7**		
	ρ=0.9	**ρ =0.5**	**ρ =0.3**	**ρ =0.9**	**ρ =0.5**	**ρ =0.3**
k1	0.324930	0.096817	0.085210	0.96240	0.24	0.20270
k2	0.303130	0.095846	0.084532	0.86682	0.24347	0.20104
k3	0.349650	0.098518	0.086066	1.08680	0.25541	0.20720
k4	0.305950	0.096576	0.085123	0.82411	0.24339	0.20141
k5	0.375370	0.099707	0.086755	1.14180	0.25772	0.20844
k6	0.324650	0.096812	0.085206	0.96127	0.24668	0.20260
k7	**0.160150**	0.102670	0.101660	0.40703	0.25759	0.25091
k8	0.252200	0.095608	0.084816	0.67875	0.24080	0.20055
k9	0.268180	0.095713	0.084891	0.73550	0.24169	0.20076
k10	0.205950	0.094540	0.084691	0.56812	0.23739	0.19972
k11	0.329810	0.097140	0.085281	0.96928	0.24769	0.20328
k12	0.199140	**0.090049**	0.083688	0.53043	**0.22823**	0.21148
k13	0.186920	0.093548	0.084349	0.56285	0.22942	0.19869
k14	0.380890	0.099906	0.086881	1.17860	0.25880	0.20904

k15	0.236650	0.093104	0.082694	0.59947	0.23483	**0.19655**
k16	0.202680	0.094055	0.083838	0.48494	0.23396	0.19796
k17	0.281620	0.094116	0.083156	0.76766	0.23882	0.19812
k18	0.262820	0.095113	0.084232	0.70877	0.24015	0.19969
k19	0.271120	0.093996	0.083093	0.73720	0.23828	0.19797
k20	0.271760	0.095184	0.084274	0.73848	0.24063	0.19981
k21	0.191550	0.090692	**0.081500**	**0.39557**	0.23547	0.22999
k22	0.214100	0.094933	0.084372	0.84218	0.25050	0.20556
k23	0.257880	0.092478	0.082119	0.51470	0.23299	0.22308
k24	0.286920	0.096269	0.084916	0.93426	0.25175	0.20590
k25	0.250590	0.092301	0.082030	0.52076	0.23283	0.22288
k26	0.288050	0.096304	0.084941	0.92923	0.25176	0.20589
k27	0.374990	0.099637	0.086672	1.14760	0.25775	0.20837
k28	0.374770	0.099680	0.086726	1.14850	0.25782	0.20843
k29	0.374520	0.099678	0.086725	1.14720	0.25780	0.20843
k30	0.321420	0.098640	0.085994	0.76710	0.24491	0.20084
k31	0.376230	0.101760	0.087903	1.15220	0.25808	0.20746
k32	0.376240	0.101760	0.087903	1.15220	0.25808	0.20746
k33	0.361140	0.101010	0.087419	1.08480	0.25502	0.20555
k34	0.348680	0.100560	0.087204	1.02640	0.25356	0.20499
k35	0.363330	0.101090	0.087450	1.10000	0.25550	0.20574
k36	0.272600	0.094905	0.083977	0.73890	0.24007	0.19934
k37	0.274240	0.095793	0.084910	0.74038	0.24176	0.20078
Min MSE	**0.160150**	**0.090049**	**0.081500**	**0.39557**	**0.22823**	**0.19655**

Table 3.3. Mean square errors of ridge parameters for different number of regressors and different degree of multicollinearity for the sample size of 80

Ridge Parameter	n=80					
	P=3			P=7		
	ρ=0.9	ρ =0.5	ρ =0.3	ρ =0.9	ρ =0.5	ρ =0.3
k1	0.204530	0.059204	0.052599	0.583420	0.144630	0.119060
k2	0.196630	0.058876	0.052352	0.549890	0.143750	0.118550
k3	0.216490	0.059887	0.052905	0.640830	0.147660	0.120690
k4	0.198330	0.059184	0.052581	0.534340	0.143910	0.118730
k5	0.226700	0.060317	0.053147	0.662200	0.148400	0.121120
k6	0.204500	0.059190	0.052599	0.583150	0.144610	0.119040
k7	**0.119490**	0.065153	0.064364	**0.308400**	0.163150	0.152420
k8	0.181390	0.058864	0.052488	0.486250	0.143260	0.118480
k9	0.184960	0.058901	0.052499	0.505570	0.143470	0.118530
k10	0.156710	0.058521	0.052461	0.425140	0.142520	0.118230
k11	0.207070	0.059338	0.052615	0.587340	0.145020	0.119300
k12	0.149400	**0.056539**	0.051935	0.398590	**0.139010**	0.122140
k13	0.137100	0.058277	0.052359	0.362650	0.140340	0.117710
k14	0.227880	0.060359	0.053173	0.669150	0.148600	0.121230
k15	0.172430	0.057960	0.051711	0.449320	0.141430	**0.117210**
k16	0.161120	0.058365	0.052168	0.394310	0.141560	0.117720
k17	0.187740	0.058253	0.051850	0.512610	0.142380	0.117640

k18	0.183920	0.058661	0.052268	0.495710	0.142970	0.118190
k19	0.185130	0.058218	0.051839	0.502270	0.142250	0.117600
k20	0.186020	0.058683	0.052275	0.505690	0.143080	0.118220
k21	0.151160	0.057085	**0.051239**	0.350800	0.139330	0.120770
k22	0.166370	0.058709	0.052372	0.514880	0.145420	0.119820
k23	0.178120	0.057627	0.051448	0.423530	0.139350	0.119680
k24	0.192860	0.059080	0.052511	0.567910	0.145950	0.119980
k25	0.175520	0.057571	0.051433	0.423440	0.139250	0.119650
k26	0.193360	0.059096	0.052513	0.567710	0.145960	0.119980
k27	0.226650	0.060303	0.053128	0.663440	0.148410	0.121100
k28	0.226690	0.060312	0.053140	0.663580	0.148420	0.121110
k29	0.226640	0.060312	0.053140	0.663410	0.148420	0.121110
k30	0.218070	0.060095	0.052983	0.568550	0.145980	0.119300
k31	0.222900	0.060592	0.052456	0.668810	0.148440	0.120950
k32	0.222900	0.060592	0.052456	0.668820	0.148440	0.120950
k33	0.218030	0.060369	0.052313	0.647910	0.147590	0.120380
k34	0.213150	0.060210	0.052234	0.625570	0.147080	0.120160
k35	0.219060	0.060407	0.052332	0.653970	0.147780	0.120470
k36	0.186090	0.058571	0.052165	0.505240	0.142880	0.118070
k37	0.186730	0.058919	0.052503	0.506770	0.143490	0.118530
Min MSE	**0.119490**	**0.056539**	**0.051239**	**0.308400**	**0.139010**	**0.117210**

Table 3.4. Mean square errors of ridge parameters for different number of regressors and different degree of multicollinearity for the sample size of 100

Ridge Parameter	**n=100**					
	P=3			**P=7**		
	ρ=0.9	**ρ =0.5**	**ρ =0.3**	**ρ =0.9**	**ρ =0.5**	**ρ =0.3**
k1	0.165070	0.046887	0.040583	0.465550	0.113890	0.092941
k2	0.160230	0.046688	0.040443	0.445190	0.113400	0.092672
k3	0.173390	0.047328	0.040762	0.504350	0.115780	0.093892
k4	0.161560	0.046889	0.040583	0.435900	0.113540	0.092811
k5	0.179920	0.047597	0.040901	0.517760	0.116230	0.094137
k6	0.165030	0.046884	0.040578	0.465380	0.113880	0.092939
k7	**0.102960**	0.052434	0.050748	**0.269060**	0.131650	0.121610
k8	0.151840	0.046697	0.040541	0.407360	0.113180	0.092687
k9	0.153520	0.046720	0.040548	0.418780	0.113300	0.092717
k10	0.136620	0.046493	0.040543	0.375190	0.112780	0.092575
k11	0.166880	0.046974	0.040587	0.468410	0.114150	0.093087
k12	0.128220	**0.045213**	0.040306	0.343920	**0.110470**	0.095353
k13	0.122560	0.046372	0.040490	0.322620	0.111650	0.092361
k14	0.180500	0.047618	0.040913	0.521040	0.116320	0.094190
k15	0.145380	0.046158	0.040074	0.383480	0.112100	**0.091999**
k16	0.139790	0.046409	0.040360	0.350370	0.112250	0.092309
k17	0.154420	0.046319	0.040139	0.422250	0.112590	0.092195
k18	0.152900	0.046571	0.040404	0.412780	0.112990	0.092517
k19	0.153110	0.046300	0.040133	0.416160	0.112520	0.092177
k20	0.153900	0.046585	0.040408	0.418690	0.113050	0.092533

k21	0.131310	0.045618	**0.039831**	0.318750	0.110520	0.093190
k22	0.142690	0.046635	0.040467	0.414680	0.114240	0.093319
k23	0.148190	0.045919	0.039922	0.371220	0.110690	0.092722
k24	0.158430	0.046836	0.040534	0.453780	0.114590	0.093415
k25	0.146700	0.045888	0.039912	0.370280	0.110620	0.092701
k26	0.158770	0.046846	0.040536	0.454100	0.114610	0.093416
k27	0.179910	0.047590	0.040893	0.518370	0.116230	0.094130
k28	0.179930	0.047595	0.040898	0.518430	0.116240	0.094136
k29	0.179910	0.047594	0.040898	0.518360	0.116240	0.094136
k30	0.176620	0.047489	0.040826	0.476310	0.115070	0.093291
k31	0.175570	0.047504	0.041940	0.522370	0.116170	0.094487
k32	0.175570	0.047504	0.041940	0.522380	0.116170	0.094487
k33	0.172710	0.047369	0.041853	0.510570	0.115700	0.094182
k34	0.169610	0.047262	0.041801	0.496840	0.115400	0.094050
k35	0.173400	0.047396	0.041867	0.514310	0.115820	0.094241
k36	0.153860	0.046516	0.040339	0.418320	0.112920	0.092443
k37	0.154340	0.046728	0.040551	0.419440	0.113310	0.092718
Min MSE	**0.102960**	**0.045213**	**0.039831**	**0.269060**	**0.110470**	**0.091999**

Table 3.5. Mean square errors of ridge parameters for different number of regressors and different degree of multicollinearity for the sample size of 250

Ridge Parameter	**n=250**					
	P=3			**P=7**		
	ρ=0.9	**ρ =0.5**	**ρ =0.3**	**ρ =0.9**	**ρ =0.5**	**ρ =0.3**
k1	0.066265	0.018294	0.016053	0.186560	0.043313	0.035283
k2	0.065660	0.018268	0.016032	0.183910	0.043257	0.035253
k3	0.068005	0.018366	0.016087	0.194730	0.043623	0.035420
k4	0.066069	0.018303	0.016056	0.18319	0.043297	0.035280
k5	0.069031	0.018406	0.016110	0.196850	0.043690	0.035454
k6	0.066259	0.018294	0.016053	0.186560	0.043311	0.035283
k7	**0.050888**	0.021574	0.020546	**0.137880**	0.053702	0.048253
k8	0.064900	0.018278	0.016048	0.180170	0.043251	0.035266
k9	0.065014	0.018280	0.016049	0.181130	0.043264	0.035269
k10	0.063266	0.018252	0.016046	0.174990	0.043203	0.035254
k11	0.066678	0.018309	0.016056	0.187320	0.043360	0.035306
k12	0.060076	**0.018048**	0.015983	0.165040	**0.042816**	0.035712
k13	0.062097	0.018252	0.016040	0.161090	0.043108	0.035239
k14	0.069064	0.018407	0.016111	0.197030	0.043695	0.035456
k15	0.063975	0.018205	0.015980	0.176390	0.043120	0.035181
k16	0.063752	0.018245	0.016023	0.171850	0.043153	0.035226
k17	0.064895	0.018219	0.015987	0.180730	0.043163	0.035196
k18	0.064907	0.018258	0.016028	0.180390	0.043222	0.035242
k19	0.064797	0.018217	0.015987	0.180180	0.043156	0.035194
k20	0.064978	0.018260	0.016028	0.180890	0.043228	0.035244
k21	0.061927	0.018131	**0.015936**	0.168140	0.042915	0.035127
k22	0.064322	0.018281	0.016044	0.174440	0.043292	0.035297
k23	0.063806	0.018157	0.015947	0.176580	0.042975	0.035127

k24	0.065788	0.018297	0.016051	0.183890	0.043360	0.035314
k25	0.063656	0.018154	0.015946	0.175950	0.042964	**0.035124**
k26	0.065832	0.018299	0.016051	0.184230	0.043365	0.035315
k27	0.069031	0.018405	0.016109	0.196890	0.043690	0.035453
k28	0.069032	0.018406	0.016110	0.196890	0.043690	0.035454
k29	0.069032	0.018406	0.016110	0.196890	0.043690	0.035454
k30	0.068906	0.018400	0.016105	0.195210	0.043624	0.035405
k31	0.069096	0.018538	0.015951	0.194410	0.044203	0.035583
k32	0.069096	0.018538	0.015951	0.194410	0.044203	0.035583
k33	0.068762	0.018524	0.015942	0.193140	0.044158	0.035553
k34	0.068276	0.018509	0.015934	0.191150	0.044118	0.035533
k35	0.068880	0.018529	0.015945	0.193670	0.044174	0.035562
k36	0.064944	0.018249	0.016018	0.180740	0.043210	0.035232
k37	0.065064	0.018281	0.016049	0.181170	0.043264	0.035270
Min MSE	**0.050888**	**0.018048**	**0.015936**	**0.137880**	**0.042816**	**0.035124**

Table 3.6. Mean square errors of ridge parameters for different number of regressors and different degree of multicollinearity for the sample size of 500

Ridge Parameter	**n=500**					
	P=3			**P=7**		
	ρ=0.9	**ρ =0.5**	**ρ =0.3**	**ρ =0.9**	**ρ =0.5**	**ρ =0.3**
k1	0.03322	0.00908	0.00800	0.09546	0.02119	0.01751
k2	0.03309	0.00907	0.00799	0.09493	0.02118	0.01750
k3	0.03373	0.00909	0.00800	0.09785	0.02127	0.01755
k4	0.03323	0.00908	0.00800	0.09492	0.02120	0.01751
k5	0.03399	0.00910	0.00801	0.09838	0.02128	0.01756
k6	0.03322	0.00908	0.00800	0.09547	0.02119	0.01751
k7	**0.02811**	0.01101	0.01039	**0.08050**	0.02745	0.02426
k8	0.03296	0.00907	0.00799	0.09431	0.02119	0.01751
k9	0.03299	0.00907	0.00799	0.09448	0.02119	0.01751
k10	0.03260	0.00907	0.00799	0.09355	0.02118	0.01750
k11	0.03335	0.00908	0.00800	0.09572	0.02121	0.01752
k12	0.03163	**0.00902**	0.00798	0.09001	**0.02111**	0.01759
k13	0.03250	0.00907	0.00799	0.09097	0.02117	0.01750
k14	0.03400	0.00910	0.00801	0.09840	0.02128	0.01756
k15	0.03277	0.00906	0.00798	0.09355	0.02116	0.01749
k16	0.03275	0.00907	0.00799	0.09294	0.02117	0.01750
k17	0.03293	0.00906	0.00798	0.09429	0.02117	0.01749
k18	0.03295	0.00907	0.00799	0.09431	0.02118	0.01750
k19	0.03292	0.00906	0.00798	0.09420	0.02117	0.01749
k20	0.03297	0.00907	0.00799	0.09439	0.02118	0.01750
k21	0.03230	0.00904	**0.00797**	0.09217	0.02113	**0.01747**
k22	0.03293	0.00908	0.00799	0.09257	0.02119	0.01751
k23	0.03264	0.00905	0.00797	0.09373	0.02114	0.01747
k24	0.03318	0.00908	0.00800	0.09481	0.02120	0.01752
k25	0.03261	0.00904	0.00797	0.09357	0.02114	0.01747
k26	0.03319	0.00908	0.00800	0.09492	0.02120	0.01752

k27	0.03399	0.00910	0.00801	0.09839	0.02128	0.01756
k28	0.03399	0.00910	0.00801	0.09839	0.02128	0.01756
k29	0.03399	0.00910	0.00801	0.09839	0.02128	0.01756
k30	0.03398	0.00910	0.00801	0.09823	0.02127	0.01755
k31	0.034906	0.009159	0.008012	0.096817	0.021746	0.017572
k32	0.034906	0.009159	0.008012	0.096817	0.021746	0.017572
k33	0.034838	0.009156	0.008010	0.096569	0.021736	0.017566
k34	0.034720	0.009152	0.008008	0.096097	0.021725	0.017562
k35	0.034867	0.009157	0.008011	0.096689	0.021740	0.017568
k36	0.03296	0.00907	0.00799	0.09435	0.02118	0.01750
k37	0.03299	0.00907	0.00799	0.09448	0.02119	0.01751
Min MSE	**0.02811**	**0.00902**	**0.00797**	**0.08050**	**0.02111**	**0.01747**

4. RESULTS AND DISCUSSIONS

The remarkable conclusions and findings may be given as follows;

- One of the proposed ridge parameters k3, inspired by the parameter k20, seems to produce better results than k20 no matter what the number of regressors and sample sizes are. Hence, this modification is successful and seems to be promising.
- The next proposed ridge parameter is freely developed and seems to be better than most ridge parameters used widely in linear regression.
- The ridge parameter presented as k7 has the highest priority to produce the minimum MSE for strong degrees of multicollinearity when the number of regressors is three. In fact it is free of sample size. When the number of regressors is 7, it is still good and promising for the sample size that is greater than 50.
- When the degree of multicollinearity is fairly large as 0.5, then k12 is the best ridge parameter. The sample size and the number of regressors do not have any effect on this ridge parameter for fairly large inter-association.
- When the degree of multicollinearity is significant and as low as 0.3 for three number of regressors, k21 seems to be the best for any sample size. However as the number of regressors' increases to 7, we observe k15 to be the best for a sample size that is less than 250. Moreover, for large sample sizes, k25 and k21 produce the best results.
- The parameters given as k31, k32, k33, k34, k35 and proposed by Asar et. al. [2] very recently did not perform as well as their results. In fact the parameters we have proposed did perform much better than theirs in every single case.
- In conclusion, we suggest researchers who employ ridge regression to check and measure the degree of collinearity first, then use the ridge parameter that fits best for their case of sample size and number of regressors.

Researchers who are making similar comparisons may feel free to demand the codes we have written in Minitab for our study. In our future study we are going to develop a new and robust ridge parameter that is free of sample size, number of regressors and degree of collinearity. We intend to develop this parameter using a collection or a combination of the best ridge parameters iteratively. Since the comparison cannot be made analytically, a wide comparison is also going to be realized using generated data for more case of sample sizes and number of regressors.

CONFLICT OF INTEREST

No conflict of interest was declared by the authors.

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