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Evaluation and comparison of metaheuristic methods for Markowitz's mean-variance portfolio optimization model

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Abstract

Portfolio selection is the process of selecting a combination of assets among portfolios containing multiple assets to achieve a satisfactory return on investment. Mean-variance model proposed by Markowitz (1952) has been extensively used for portfolio selection problem. It is a quadratic programming model based on the minimum risk and maximum return by choosing assets in the portfolio. Generally, classical optimization algorithms have been used for solving the quadratic programming problem. Recently, metaheuristic optimization algorithms have been used in addition to classical optimization techniques for solving portfolio selection problems. Metaheuristic methods are designed to solve complex optimization problems that cannot be solved in a reasonable time with the definitive solution methods. Various metaheuristic methods have been developed for different areas. In this study, BIST30 index data set obtained from daily closing prices of 30 stocks between December 2016 - December 2017 was used. Markowitz's mean-variance model is considered to constitute an optimal portfolio. , Particle Swarm Optimization, Differential Evolution, and Artificial Bee Colony which are mostly used metaheuristic methods, are applied to determine an optimal portfolio. Performances of these methods are compared by considering risk values, i.e. portfolio variances.

Keywords: Artificial Bee Colony, Differential Evolution, Metaheuristics, Particle Swarm Optimization, Portfolio Optimization, Quadratic programming.

Öz

Markowitz'in ortalama-varyans portföy optimizasyon modeli için metasezgisel yöntemlerin değerlendirilmesi ve karşılaştırılması

Portföy seçimi, tatmin edici bir yatırım getirisi elde etmek için birden fazla varlık içeren portföyler arasından bir varlık kombinasyonu seçme sürecidir. Markowitz (1952) tarafından önerilen ortalama varyans modeli, portföy seçim problemi için yaygın olarak kullanılmaktadır. Portföydeki varlıkları minimum risk ve maksimum getiriye dayalı olarak seçen bir karesel programlama modelidir. Karesel programlama probleminin çözümü için genellikle klasik optimizasyon algoritmaları kullanılmaktadır. Son yıllarda, portföy *seçim problemlerinin çözümü için klasik optimizasyon tekniklerine ek olarak metasezgisel optimizasyon algoritmaları kullanılmaktadır. Metasezgisel yöntemler, kesin çözüm yöntemleri ile makul bir sürede çözülemeyen karmaşık optimizasyon problemlerini çözmek için tasarlanmış algoritmalardır. Farklı alanlar için çeşitli metasezgisel algoritmalar geliştirilmektedir. Bu çalışmada Aralık 2016 - Aralık 2017 tarihleri arasında BİST30 endeksinde işlem gören 30 hisse senedinin günlük kapanış fiyatlarından elde edilen veri seti kullanılmıştır. Optimal portföy oluşturmak için Markowitz'in ortalama varyans modeli ele alınmıştır. Optimal portföyü belirlemek için çoğunlukla metasezgisel yöntemlerden, Parçacık Sürüsü Optimizasyonu ve Diferansiyel Evrim ve Yapay Arı Kolonisi uygulanmıştır. Bu yöntemlerin performansları, risk değerleri yani portföy varyansları dikkate alınarak karşılaştırılmıştır.*

Anahtar sözcükler: *Yapay arı kolonisi, Diferansiyel evrim, Metasezgiseller, Parçacık Sürü Optimizasyonu, Portföy optimizasyonu, Karesel programlama.*

1. Giriş

Portfolio is the general name of all financial assets such as cash, currency, gold, stocks, etc. Portfolio selection is the process of selecting a combination of assets among portfolios containing multiple assets to achieve a satisfactory return on investment. Portfolio management is a process of creating a portfolio and deciding which investment instrument to remove from that portfolio and when to replace it. Portfolio optimization, which is the allocation of wealth among several assets, is an essential problem in modern risk management. Expected returns and risks are the most important parameters in portfolio optimization problems. Investors generally prefer to maximize returns and minimize risk. However, high returns generally involve increased risk. It is very important to find ratios of the assets in a portfolio for decision makers. Mean-variance model proposed by Markowitz (1952) that can be described in terms of the mean return of the assets and the variance of return of these assets, is extensively used for portfolio selection problem. This model is a quadratic programming model based on the minimum risk and maximum return by choosing assets in the portfolio [1]. In general, classical optimization algorithms have been used to obtain optimal ratio of the assets. Recently, in addition to classical optimization algorithms, metaheuristic optimization algorithms have been suggested for solving portfolio selection problems.
Metaheuristic optimization methods are algorithms designed to solve complex optimization problems that cannot be solved in a reasonable time with the definitive solution methods. In practice, we are usually satisfied with "good" solutions, which are obtained by heuristic/metaheuristic methods. Metaheuristic methods provide "acceptable" solutions in a reasonable time for solving hard and complex problems in science and engineering. Unlike exact (classical) optimization algorithms, metaheuristic methods do not guarantee the optimality of the obtained solutions. The metaheuristic methods have beenused for NP(Non-deterministic polynomial time)-hard problems, non-linear optimization problems, large-scale examples of P (Polynomial time) problems, real time limited problems, problems where calculation of the objective function or constraint takes a long time.

Metaheuristic methods allow much faster and more efficient solutions than classical optimization methods. The metaheuristic methods inspired by nature aim to bring appropriate solutions to combinatorial optimization problems by modeling an event that takes place in nature. Just like in nature, metaheuristic methods aim for the best performance, not the best solution. Metaheuristic methods have been applied in various fields in recent years. There are so many metaheuristic optimization algorithms in the literature. For example, Simulated Annealing (SA), Tabu Search (TS), Genetic Algorithm (GA), Ant Colony Optimization (ACO), Differential Evolution (DE), Particle Swarm Optimization (PSO), Artificle Bee Colony (ABC), etc.

In this study, different metaheuristic methods, Particle Swarm Optimization, Differential Evolution, and Artificial Bee Colony Algorithm are proposed to obtain an optimal portfolio using Markowitz's mean-variance model.

The rest of the paper is organized as follows. Section 2 provides a brief review of various studies conducted on the use of metaheuristic optimization algorithms for portfolio optimization models. In Section 3, the Markowitz's mean-variance model is presented. In Section 4, algorithms of three metaheuristic methods, Particle Swarm Optimization, Differential Evolution, and Artificial Bee Colony, are explained. In Section 5, Markowitz's mean-variance model is constituted for BIST30 index data set obtained from daily closing prices of 30 stocks between December 2016 - December 2017. Then, preferredmetaheuristics methods are applied to obtain an optimal portfolio and performances of these methods are compared by considering risk value. Finally, some concluding remarks are made in Section6.

2. Literature review

This section provides a brief review of various studies conducted on the use of metaheuristic optimization algorithms for portfolio optimization models.

Crama and Schyns [2] applied Simulated Annealing approach to solve a complex portfolio selection model which is Markowitz's classical mean-variance model enriched with additional constraints. Doerner et al. [3] proposed Pareto Ant Colony Optimization for solving portfolio selection problems and compared it with other metaheuristic approaches, Simulated Annealing and Genetic Algorithm. Ehrgott and Gandibleux [4] applied Simulated Annealing, Tabu Search and Genetic Algorithm to a portfolio optimization model that extends Markowitz's mean-variance model. Cura [5] presented the Particle Swarm Optimization algorithm to the cardinality constrained mean-variance portfolio optimization problem, taking into account the data set of weekly prices from the Hang Seng index, DAX 100 index, FTSE 100 index, S&P 100 index and Nikkei index. Golmakani and Fazel [6] introduced Particle Swarm Optimization based metaheuristic method for solving the extended Markowitz mean-variance portfolio selection model and compared it with the Genetic Algorithm. Zhu et al. [7] proposed a metaheuristic approach to the portfolio optimization problem using the Particle Swarm Optimization method and tested it on various constrained and unconstrained risk investment portfolios and compared it with Genetic Algorithms. Deng et al. [8] proposed Particle Swarm Optimization method to solve cardinality constrained Markowitz portfolio optimization problem. Test results showed that the proposed PSO is much more powerful and effective than existing PSO algorithms, especially for low-risk investment portfolios Lwin and Qu [9] proposed a hybrid algorithm combining Population Based Incremental Learning and Differential Evolution algorithms for portfolio selection problems. Çelenli et al. [10] applied classical and guaranteed convergence Particle Swarm Optimization methods for portfolio optimization created from stocks included in the BIST 30 index. The results were compared with the results obtained from mathematical programming. Akyer et al. [11] developed the Particle Swarm Optimization algorithm to solve a portfolio optimization problem with cardinality constraints applied to the Istanbul Stock Exchange data. Doering et al. [12] systematically reviewed the scientific literature on the use of metaheuristic optimization algorithms to solve portfolio optimization and risk management problems. Kalaycı et al. [13] analyzed publications based on deterministic models and applications in the mean-variance portfolio optimization literature. The publications for solving this type of problems by exact solution techniques which are nonlinear programming, and inexact solution techniques which are metaheuristic methods and machine learning algorithms, were reviewed in a systematic way. Kalaycı et al. [14] introduced a hybrid metaheuristic algorithm combining continuous ant colony optimization, artificial bee colony optimization, and genetic algorithms to obtain optimal solutions for cardinality constrained portfolio optimization problem. Corazza et al. [15] proposed a hybrid metaheuristic based on Particle Swarm Optimization for a portfolio selection problem formulated as an unconstrained problem by means of penalty functions.

3. Markowitz's mean-variance portfolio optimization model

The mean -variance model proposed by Markowitz (1952) who is the founder of modern portfolio theory, has been widely used for the portfolio selection problem. Markowitz's mean variance model is a quadratic programming model based on selecting assets in the portfolio with minimum risk and maximum return [1]. In general, classical optimization algorithms have been applied to solve the quadratic programming problem. In addition to classical optimization techniques, metaheuristic optimization algorithms have been frequently used in the solution of portfolio selection problems in recent years.

Mathematical model of the mean-variance model is given as follows:

$$\begin{aligned} &Min\, Z = \sum_{i=1}^{n}\sum_{j=1}^{n} x_i x_j \sigma_{ij} \\ &\sum_{i=1}^{n} x_i E(r_i) \geq R \\ &\sum_{i=1}^{n} x_i = 1 \\ &0 \leq x_i \leq 1,\ i = 1,2,....,n \end{aligned} \tag{1}$$

In Equation (1); n is the number of available assets, σ_{ij} is the covariance value between asset i and asset j, x_i is the ratio of the asset i in the portfolio, R is the expected return level, and $E(r_i)$ is the expected return of the asset i.

4. Metaheuristic Methods

Heuristics is a solution strategy by trial-and-error to produce acceptable solutions to a complex problem within an acceptable time. Metaheuristic methods are defined as further development over the heuristic algorithms. Most metaheuristic methods are inspired by nature. Some naure-inspired algorithms have been proposed based on Darwin's evolution theory. On account of this, the methods are also called biology-inspired or bio-inspired. Unlike classical optimization algorithms, metaheuristic methods ensure "acceptable" solutions within a reasonable time to solve hard and complex optimization problems Any metaheuristic methods have two major components: selection of the best solutions and randomization. The selection of the best solutions allows that convergence to the optimality, whereas randomization prevents solutions from jamming in the local optimum as well as increase the variety of solutions. The good combination of these two components usually makes it possible to achieve the global optimality. There are various classification criteria used for metaheuristic methods: nature inspired vsnonnature inspired, memory usage versus memoryless methods, deterministic versus stochastic, population-based search versus single-solution based search, and iterative versus greedy. For instance, metaheuristic methods are divided into two groups in accordance with the solution numbers: single-solution based (eg, SA) that uses a single solution and population-based (e.g., GA, PSO, DE, and ABC) that uses a whole population of solutions, during the search [16, 17].

The three metaheuristic methods, PSO, DE, and ABC, used for obtaining optimal portfolio are briefly explained in the following subsections.

4.1. Particle swarm optimization

Particle swarm optimization (PSO), one of the stochastic population-based metaheuristic, is designed for continuous optimization problem. The first application of the PSO is carried out by Kennedy and Eberhart (1995, 2001) for optimization problems [16, 25, 26, 27]. On the basis of the behavior of a colony or swarm of insects, such as ants, termites, bees, and wasps; a flock of birds; or a school of fish, the PSO mimics the social behavior of these organisms [26, 28]. The particle means that a bee in a colony or a bird in a flock. If one particle finds out a good path to food, the rest of the swarm will also be able to follow the good path instantly even if their location is far away in the swarm. The swarm is assumed to be of specified or fixed size with each particle located initially at random locations in the multidimensional design space. It is suggested that each particle have two characteristics: a position and a velocity. Each particle circuits around in the design space and remembers the best position concerning the food source or objective function value. The particles communicate information or good positions to each other and regulate their individual positions and velocities on the basis of received information about the good positions [28].

The purpose of PSO is to find the best global among all the current best solutions until the objective function no longer improves or until a predetermined number of iterations [29]. The population is formed of a set of particles. Each particle records its personal best position ($pbest$) and swarm's best position found by all particles ($gbest$). Then, all particles update their velocity (v_i) and position (x_i) in each iteration [16, 17, 29].

Due to the ease of application and high convergence speed, the PSO is one of the most widely used metaheuristic methods [29, 30]. Also, PSO is much simpler than GA and ant algorithms as it does not use mutation/crossover operators or pheromones [17].

<u>Pseudo code of Particle Swarm Optimization (PSO) Algorithm</u>

Determine the objective function $f = f(x)$
Set particle numbers (n), max iteration number (G)
Set inertia weight (w) and acceleration coefficients (c_1, c_2)
Initialize position (x_i) and velocity (v_i) of n particles
Produce random numbers (r_1, r_2) from uniform distribution in $[0,1]$,
Find $gbest^g$ from $\min\{f(x_1), f(x_2),.., f(x_n)\}$ at $g = 0$
while $g < G$ or stopping criteria
    for particles $i = 1, 2, ..., n$
    Evaluate $f(x_i^g)$
    Determine the $pbest_i^g$ and $gbest^g$
        if $f(x_i^g) < f(pbest_i^g)$ *then* $pbest_i = x_i^g$
        if $f(x_i^g) < f(gbest^g)$ *then* $gbest^g = x_i^g$
        Generate new velocity $v_i^{g+1} = wv_i^g + c_1 r_1 \left(pbest_i^g - x_i^g \right) + c_2 r_2 \left(gbest^g - x_i^g \right)$
        Calculate new position $x_i^{g+1} = x_i^g + v_i^{g+1}$
    end for
    $g = g + 1$
end while

4.2. Differential evolution

Differential evolution (DE), one of the population-based evolutionary algorithm, is proposed by Storn (1996) and Storn and Price (1997) for continuous optimization problems. Like Genetic

Algorithm (GA), DE algorithm uses crossover, mutation, and selection operators. DE uses the mutation operator as a search mechanism and selection operator to navigate the candidate areas in the search space to obtain better solutions unlike GA uses crossover operator. The mutation operator in the DE is based on the difference between randomly selected two solutions from the population [16, 33, 34]. The notation $DE/x/y/z$ is generally used to classify the different variants of the DE. x specifies the vector to be used in the mutation, y is the number of difference vectors and z indicates the crossing stage. In this study, the standard variant, $DE/rand/1/bin$, has been used. "*rand*" indicates a randomly selected population vector for x, "*bin*" implies the cross based on independent binomial experiments for z [16, 35, 36].

<u>Pseudo code of Differential Evolution (DE) Algorithm</u>

Determine the objective function $f = f(x)$

Set max iteration number (G) and population size ($NP \geq 4$)

Set scaling factor ($F \in (0,1]$) and crossover factor (C_r).

Initialize the population $\vec{x}_i^g = [x_{1,i}^g, x_{2,i}^g, ..., x_{D,i}^g]$

Calculate the $f(x)$ of each solution of the population

while $g < G$ or stopping criteria

    for $i = 1,\ i \leq k,\ i = i + 1$

      Mutate and Recombine

      Generate a random index $j_{rand} \in [0, D]$

      for $j = 1,\ j \leq D,\ j = j + 1$ and distinct vectors $r_1, r_2, r_3; r_1 \neq r_2 \neq r_3 \neq i \in (0, NP)$

        if $rand_{i,j}(0,1) \leq C_r$ *or* $j = j_{rand}$

         then generate new donor vector by $\vec{u}_{j,i}^{g+1} = \vec{v}_{j,i}^g = \vec{x}_{r1}^g + F\left(\vec{x}_{r2}^g - \vec{x}_{r3}^g\right)$

         else $\vec{u}_{j,i}^{g+1} = \vec{x}_{j,i}^g$

        Replace

$$\vec{x}_i^{g+1} = \begin{cases} \vec{u}_i^g, & \log L(\vec{u}_i^g) \geq \log L(\vec{x}_i^g) \\ \vec{x}_i^g, & \log L(\vec{u}_i^g) < \log L(\vec{x}_i^g) \end{cases}$$

      end for $g = g + 1$

end while

4.3. Artificial Bee Colony

The Artificial Bee Colony (ABC) algorithm, population based stochastic metaheuristic algorithm, is proposed by Karaboğa (2005) for numerical optimization problem inspiring the intelligent foraging behavior of honey bees. Later, it was adapted to solve optimization problem by Akay ve Karaboğa (2012). In the ABC algorithm, artificial bees are classified into three groups: employed bees, onlooker bees, and scout bees [37, 38]. The employed bees are pioneers of the swarm discovering food sources, gathering honey, and sharing information on the nectar amount the food sources within the hive. The onlooker bees select and exploit better food sources on the basis of this information. A bee with a bad food source randomly changes to be a scout bee in search of new food sources [39]. In ABC, position of a food source indicates a possible solution to the problem and the nectar amount of a food source corresponds to the quality (fitness) of the associated solution. Due to the fact that employed bee is associated with one and only one food source, the number of employed bees is equal to the number of food sources (i.e. solutions) [38].

ABC is a very simple and robust technique that perform better than PSO and DE in most studies [17, 40].

Pseudo code of Artficial Bee Colony (ABC) Algorithm

Determine the objective function $f = f(x)$

Set the population size (*SN*) and number of variables to be optimized (*D*)

Generate the initial population $x_i = \left[x_{i,1}, x_{i,2}, ..., x_{i,n}\right]^T$ by $x_{ij} = x_j^{\min} + rand[0,1](x_j^{\max} - x_j^{\min}),\ i = 1,2,....,SN;\ j = 1,2,..,D$

while $g < G$ or stopping criteria

for each employed bee

Produce new solution $v_{ij} = x_{ij} + \phi_{ij}\left(x_{ij} - x_{mj}\right)$ with a random number ϕ_{ij} in $[-1,1]$

Calculate the fitness value of the solution by

$$fit_i(x_i) = \begin{cases} \dfrac{1}{1+f_i(x_i)}, & if \quad f_i(x_i) \geq 0 \\ 1 + abs\left(f_i(x_i)\right), & if \quad f_i(x_i) < 0 \end{cases}$$

Apply greedy selection process between x_i and v_i, then select the better one

Calculate the probability values $p_i = \dfrac{fit_i(x_i)}{\sum_{i=1}^{SN} fit_i(x_i)}$ for the solution x_i

for each onlooker bee

Select a solution x_i depending on p_i

Produce new solution v_i

Calculate the fit_i value

Apply greedy selection process between x_i and v_i, then select the better one

if there is an abandoned solution x_i

then

Replace it using a scout bee with a new solution randomly produced by x_{ij}

Memorise the best solution so far

$g = g + 1$

end while

5. Numerical evaluation and comparison of metaheuristic methods

Borsa İstanbul (BIST), formerly known as the Istanbul Stock Exchange, was established in Turkey in 1985. BIST is the institution that provides custody and clearing services to Turkish banks, foreign banks and intermediary institutions operating in the capital market. It brings together all buyers, sellers and intermediary institutions by ensuring that the transactions are carried out within legal frameworks. The BIST 30 index measures the joint performance of the stocks of 30 companies with the highest market value and transaction volume, traded on the Borsa Istanbul. In order to determine the stocks to be included in the BIST 30 index, the market values and daily average trading volumes of the stocks in circulation are ranked in decreasing order. Those who are at the top of both rankings are included in the index.

In this study, daily closing prices of 30 stocks traded on the BIST30 index between December 1, 2016 and December 29, 2017 are considered. The stocks between the dates discussed are GARAN, AKBNK, EREGL, TCELL, TUPRS, BIMAS, THYAO, KCHOL, SAHOL, ISCTR, HALKB, EKGYO, PETKM, ASELS, VAKBN, TOASO, SISE, ARCLK, YKBNK, TAVHL, TKFEN, TTKOM, KRDMD, KOZAL, PGSUS, KOZAA, DOHOL, OTKAR, ECILC, SKBNK. At first, average monthly closing prices of the stocks are calculated using the daily closing prices for the periods between December 2016 and December 2017, as shown in Table 5.1. Then, returns and expected returns (means) of all stocks are calculated for the 13 periods discussed, and are given in Table 5.2. For instance, SKBNK's return for the period December 2017 is computed by using the average closing prices of that period and the previous period, with the following formula:

$$r_{\text{SKBNK\_Dec2017}} = \left(\frac{P_{Dec2017} - P_{Nov2017}}{P_{Nov2017}} \right) = \left(\frac{1,5542 - 1,3645}{1,3645} \right) = 0,1391$$

By using the returns given in Table 5.2, deviation values related to all stocks are calculated by taking the difference between the stocks' return in that period and its average of all periods. For example, deviation value of SKBNK for the period December 2017 is computed as follows:

$$\text{SKBNK\_Dec2017} = \left(r_{Dec2017} - \overline{r}_{2017} \right) = \left(0,1391 - 0,0280 \right) = 0,1110$$

In case of two or more stocks, the risk can be measured with the covariance matrix. Covariance matrix of all stocks is given in Table5.3. After calculating the returns and covariance matrix for the stocks using the discussed one-year data, Markowitz's mean variance model is created for portfolio selection. The value of expected return level R is considered as 0 in the constraint $\sum_{i=1}^{n} x_i E(r_i) \geq R$. The created model is solved with the classical optimization technique as well as metaheuristic optimization methods, Particle Swarm Optimization, Differential Evolution, and Artificial Bee Colony.

Table 5.1. Average monthly closing prices of all stocks for the periods between December 2016 and December 2017

Period	GARAN	AKBNK	EREGL	TCELL	TUPRS	BIMAS	THYAO	KCHOL	SAHOL	ISCTR	HALKB	EKGYO	PETKM	ASELS	VAKBN
Dec 2016	7,3377	7,5118	4,7018	8,3459	65,5686	47,6273	4,8991	12,7836	8,8764	4,8686	9,0432	2,9595	3,3255	11,5905	4,3009
Jan 2017	7,5132	7,6064	5,0255	9,4282	70,0577	50,4023	5,1332	13,7232	9,2286	5,2123	10,0014	3,0091	3,6355	13,1500	4,5009
Feb 2017	8,4805	8,5565	5,4830	10,3085	78,6760	51,5310	5,6640	14,9915	10,0345	5,9840	11,3630	3,1255	3,9615	13,8550	5,1935
Mar 2017	8,6300	8,6633	5,5792	10,9679	82,6379	54,4454	5,6513	15,0679	9,9167	6,3575	11,0267	3,0729	4,4029	16,8167	5,3717
Apr 2017	9,2180	9,0960	5,7815	10,9280	85,3740	55,2950	5,6400	15,9180	10,2705	6,7845	11,0160	2,8895	4,7660	17,7305	5,8150
May 2017	9,5171	9,4014	6,4538	10,5757	93,5357	60,3914	6,7257	16,5638	10,7129	7,0019	11,9757	2,9190	5,4300	21,1229	6,0890
Jun 2017	9,6780	9,7850	6,5150	10,8225	98,9650	63,7675	7,4880	16,4480	10,8715	7,1815	12,8530	2,9595	5,8835	21,9170	6,4780
Jul 2017	10,3005	10,2662	7,5600	11,3738	104,8786	66,3471	8,6543	16,5186	10,9743	7,6843	14,1381	3,0876	6,2414	22,6686	6,9452
Aug 2017	10,6932	10,3505	7,9945	12,0332	114,9045	71,1345	9,2436	17,2668	10,5682	7,5509	14,8077	3,0350	6,2600	26,0591	7,0823
Sep 2017	10,3021	9,9521	7,8168	12,0263	121,2684	74,8332	9,4937	17,0621	10,4189	7,1595	12,8053	2,7726	6,0942	26,4689	6,6358
Oct 2017	10,0332	9,6009	8,3718	13,1255	128,0455	74,8432	9,4750	16,4095	10,1877	6,9091	11,6982	2,6595	6,3068	28,8673	6,2495
Nov 2017	10,0627	9,5245	9,1473	13,8764	128,6409	76,8223	10,7718	16,9027	10,6591	6,6145	9,9091	2,5945	6,5677	35,4795	5,8591
Dec 2017	10,0795	9,3024	9,2986	14,8857	118,4286	75,1000	14,2600	17,5490	10,6119	6,6238	10,1443	2,6624	7,1605	32,3033	6,2114
Period	**TOASO**	**SISE**	**ARCLK**	**YKBNK**	**TAVHL**	**TKFEN**	**TTKOM**	**KRDMD**	**KOZAL**	**PGSUS**	**KOZAA**	**DOHOL**	**OTKAR**	**ECILC**	**SKBNK**
Dec 2016	22,7059	3,1695	20,0664	3,3700	13,0282	6,1732	5,2077	1,1045	16,2105	12,6786	2,2900	0,6855	122,0605	2,7023	1,1468
Jan 2017	24,0914	3,4509	20,6418	3,4805	14,2532	6,6809	5,4564	1,1795	17,1395	14,5895	2,0995	0,8155	132,1659	2,7936	1,1882
Feb 2017	25,8875	3,5205	21,2830	3,8915	15,0140	7,4635	5,6795	1,2590	20,1875	15,9445	2,1130	0,8190	136,5820	3,0865	1,2340
Mar 2017	26,6025	3,6363	21,3717	3,8717	14,5663	8,5554	5,8058	1,1608	19,5263	14,7317	2,0271	0,6988	125,7688	3,1079	1,2050
Apr 2017	28,3500	3,7845	22,8670	4,1080	14,7540	9,0010	5,9830	1,2290	20,2640	15,2530	2,0695	0,7185	125,2300	3,4150	1,1845
May 2017	29,0400	4,1157	23,8876	4,3571	16,1252	9,6129	6,2910	1,4681	18,2943	17,2500	2,1124	0,7314	131,6429	3,8490	1,2190
Jun 2017	29,4570	4,2760	25,9220	4,4400	18,2050	8,8880	6,2855	1,6540	19,2850	18,8930	2,5775	0,7400	116,9350	4,4545	1,2165
Jul 2017	30,1333	4,4005	26,2676	4,5905	20,5438	9,7886	6,5929	1,9295	26,7200	21,3576	3,9829	0,7762	121,1333	4,7152	1,3690
Aug 2017	31,6373	4,3936	24,6755	4,5218	21,0209	12,1714	7,1005	2,1559	32,8436	26,0100	6,3277	0,8577	117,3682	4,6177	1,3555
Sep 2017	31,3916	4,1716	23,0579	4,4568	19,1911	11,6747	6,7605	2,3753	31,1326	27,1453	7,2505	0,9358	109,0053	4,4653	1,5374
Oct 2017	30,3709	4,2327	22,0318	4,3964	17,8559	12,4464	6,6223	2,5232	32,2027	27,8882	6,7232	0,9232	105,8955	4,3641	1,5155
Nov 2017	32,6136	4,4977	20,1645	4,3705	19,0132	14,3073	5,9118	2,3309	30,7627	27,7427	5,3968	0,8818	117,5545	4,5050	1,3645
Dec 2017	32,0257	4,3786	20,5610	4,2171	21,2871	15,9695	6,1814	2,7248	33,0771	31,3724	5,7257	0,8438	119,6952	4,3076	1,5543

Table 5.2. Returns and expected returns (means) of all stocks for the periodsbetween December 2016 and December 2017

Period	GARAN	AKBNK	EREGL	TCELL	TUPRS	BIMAS	THYAO	KCHOL	SAHOL	ISCTR	HALKB	EKGYO	PETKM	ASELS	VAKBN
Dec 2016															
Jan 2017	0,0239	0,0126	0,0688	0,1297	0,0685	0,0583	0,0478	0,0735	0,0397	0,0706	0,1060	0,0167	0,0932	0,1346	0,0465
Feb 2017	0,1287	0,1249	0,0910	0,0934	0,1230	0,0224	0,1034	0,0924	0,0873	0,1481	0,1361	0,0387	0,0897	0,0536	0,1539
Mar 2017	0,0176	0,0125	0,0175	0,0640	0,0504	0,0566	-0,0023	0,0051	-0,0117	0,0624	-0,0296	-0,0168	0,1114	0,2138	0,0343
Apr 2017	0,0681	0,0499	0,0363	-0,0036	0,0331	0,0156	-0,0020	0,0564	0,0357	0,0672	-0,0010	-0,0597	0,0825	0,0543	0,0825
May 2017	0,0325	0,0336	0,1163	-0,0322	0,0956	0,0922	0,1925	0,0406	0,0431	0,0320	0,0871	0,0102	0,1393	0,1913	0,0471
Jun 2017	0,0169	0,0408	0,0095	0,0233	0,0580	0,0559	0,1133	-0,0070	0,0148	0,0256	0,0733	0,0139	0,0835	0,0376	0,0639
Jul 2017	0,0643	0,0492	0,1604	0,0509	0,0598	0,0405	0,1558	0,0043	0,0095	0,0700	0,1000	0,0433	0,0608	0,0343	0,0721
Aug 2017	0,0381	0,0082	0,0575	0,0580	0,0956	0,0722	0,0681	0,0453	-0,0370	-0,0174	0,0474	-0,0170	0,0030	0,1496	0,0197
Sep 2017	-0,0366	-0,0385	-0,0222	-0,0006	0,0554	0,0520	0,0271	-0,0119	-0,0141	-0,0518	-0,1352	-0,0864	-0,0265	0,0157	-0,0630
Oct 2017	-0,0261	-0,0353	0,0710	0,0914	0,0559	0,0001	-0,0020	-0,0382	-0,0222	-0,0350	-0,0865	-0,0408	0,0349	0,0906	-0,0582
Nov 2017	0,0029	-0,0080	0,0926	0,0572	0,0047	0,0264	0,1369	0,0301	0,0463	-0,0426	-0,1529	-0,0244	0,0414	0,2291	-0,0625
Dec 2017	0,0017	-0,0233	0,0165	0,0727	-0,0794	-0,0224	0,3238	0,0382	-0,0044	0,0014	0,0237	0,0261	0,0903	-0,0895	0,0601
Mean	*0,0277*	*0,0189*	*0,0596*	*0,0503*	*0,0517*	*0,0391*	*0,0969*	*0,0274*	*0,0156*	*0,0275*	*0,0140*	*-0,0080*	*0,0670*	*0,0929*	*0,0330*

Period	TOASO	SISE	ARCLK	YKBNK	TAVHL	TKFEN	TTKOM	KRDMD	KOZAL	PGSUS	KOZAA	DOHOL	OTKAR	ECILC	SKBNK
Dec 2016															
Jan 2017	0,0610	0,0888	0,0287	0,0328	0,0940	0,0822	0,0477	0,0679	0,0573	0,1507	-0,0832	0,1897	0,0828	0,0338	0,0361
Feb 2017	0,0746	0,0202	0,0311	0,1181	0,0534	0,1171	0,0409	0,0674	0,1778	0,0929	0,0064	0,0043	0,0334	0,1048	0,0386
Mar 2017	0,0276	0,0329	0,0042	-0,0051	-0,0298	0,1463	0,0222	-0,0780	-0,0328	-0,0761	-0,0407	-0,1468	-0,0792	0,0069	-0,0235
Apr 2017	0,0657	0,0408	0,0700	0,0610	0,0129	0,0521	0,0305	0,0587	0,0378	0,0354	0,0209	0,0283	-0,0043	0,0988	-0,0170
May 2017	0,0243	0,0875	0,0446	0,0606	0,0929	0,0680	0,0515	0,1945	-0,0972	0,1309	0,0207	0,0180	0,0512	0,1271	0,0292
Jun 2017	0,0144	0,0389	0,0852	0,0190	0,1290	-0,0754	-0,0009	0,1266	0,0542	0,0952	0,2202	0,0117	-0,1117	0,1573	-0,0021
Jul 2017	0,0230	0,0291	0,0133	0,0339	0,1285	0,1013	0,0489	0,1666	0,3855	0,1305	0,5452	0,0489	0,0359	0,0585	0,1254
Aug 2017	0,0499	-0,0016	-0,0606	-0,0150	0,0232	0,2434	0,0770	0,1173	0,2292	0,2178	0,5887	0,1050	-0,0311	-0,0207	-0,0099
Sep 2017	-0,0078	-0,0505	-0,0656	-0,0144	-0,0870	-0,0408	-0,0479	0,1017	-0,0521	0,0436	0,1458	0,0910	-0,0713	-0,0330	0,1342
Oct 2017	-0,0325	0,0147	-0,0445	-0,0136	-0,0696	0,0661	-0,0205	0,0623	0,0344	0,0274	-0,0727	-0,0135	-0,0285	-0,0227	-0,0143
Nov 2017	0,0738	0,0626	-0,0848	-0,0059	0,0648	0,1495	-0,1073	-0,0762	-0,0447	-0,0052	-0,1973	-0,0448	0,1101	0,0323	-0,0996
Dec 2017	-0,0180	-0,0265	0,0197	-0,0351	0,1196	0,1162	0,0456	0,1690	0,0752	0,1308	0,0609	-0,0431	0,0182	-0,0438	0,1391
Mean	*0,0297*	*0,0281*	*0,0034*	*0,0197*	*0,0443*	*0,0855*	*0,0157*	*0,0815*	*0,0687*	*0,0812*	*0,1013*	*0,0207*	*0,0005*	*0,0416*	*0,0280*

Table 5.3. Covariance matrix related to all stocks for the periods between December 2016 and December 2017

	GARAN	**AKBNK**	**EREGL**	**TCELL**	**TUPRS**	**BIMAS**	**THYAO**	**KCHOL**	**SAHOL**	**ISCTR**	**HALKB**	**EKGYO**	**PETKM**	**ASELS**	**VAKBN**
GARAN	0,0020	0,0019	0,0011	0,0002	0,0010	0,0001	0,0003	0,0012	0,0010	0,0023	0,0031	0,0010	0,0009	-0,0001	0,0025
AKBNK	0,0019	0,0021	0,0010	0,0000	0,0013	0,0002	0,0001	0,0010	0,0012	0,0024	0,0032	0,0010	0,0010	0,0000	0,0026
EREGL	0,0011	0,0010	0,0026	0,0003	0,0009	0,0002	0,0012	0,0004	0,0007	0,0010	0,0020	0,0011	0,0006	0,0014	0,0008
TCELL	0,0002	0,0000	0,0003	0,0022	-0,0001	-0,0005	-0,0003	0,0004	0,0001	0,0006	0,0007	0,0007	0,0000	0,0000	0,0002
TUPRS	0,0010	0,0013	0,0009	-0,0001	0,0026	0,0011	-0,0025	0,0004	0,0004	0,0012	0,0020	0,0001	0,0000	0,0020	0,0008
BIMAS	0,0001	0,0002	0,0002	-0,0005	0,0011	0,0010	-0,0007	0,0001	0,0000	0,0001	0,0008	0,0000	0,0001	0,0017	0,0000
THYAO	0,0003	0,0001	0,0012	-0,0003	-0,0025	-0,0007	0,0094	0,0007	0,0006	-0,0001	0,0027	0,0024	0,0014	-0,0034	0,0016
KCHOL	0,0012	0,0010	0,0004	0,0004	0,0004	0,0001	0,0007	0,0014	0,0009	0,0014	0,0020	0,0006	0,0007	0,0002	0,0016
SAHOL	0,0010	0,0012	0,0007	0,0001	0,0004	0,0000	0,0006	0,0009	0,0013	0,0013	0,0013	0,0006	0,0009	0,0004	0,0012
ISCTR	0,0023	0,0024	0,0010	0,0006	0,0012	0,0001	-0,0001	0,0014	0,0013	0,0035	0,0044	0,0014	0,0018	-0,0003	0,0035
HALKB	0,0031	0,0032	0,0020	0,0007	0,0020	0,0008	0,0027	0,0020	0,0013	0,0044	0,0094	0,0031	0,0026	-0,0016	0,0055
EKGYO	0,0010	0,0010	0,0011	0,0007	0,0001	0,0000	0,0024	0,0006	0,0006	0,0014	0,0031	0,0016	0,0011	-0,0005	0,0018
PETKM	0,0009	0,0010	0,0006	0,0000	0,0000	0,0001	0,0014	0,0007	0,0009	0,0018	0,0026	0,0011	0,0022	0,0007	0,0019
ASELS	-0,0001	0,0000	0,0014	0,0000	0,0020	0,0017	-0,0034	0,0002	0,0004	-0,0003	-0,0016	-0,0005	0,0007	0,0088	-0,0018
VAKBN	0,0025	0,0026	0,0008	0,0002	0,0008	0,0000	0,0016	0,0016	0,0012	0,0035	0,0055	0,0018	0,0019	-0,0018	0,0043
	TOASO	**SISE**	**ARCLK**	**YKBNK**	**TAVHL**	**TKFEN**	**TTKOM**	**KRDMD**	**KOZAL**	**PGSUS**	**KOZAA**	**DOHOL**	**OTKAR**	**ECILC**	**SKBNK**
TOASO	0,0013	0,0008	0,0002	0,0009	0,0007	0,0012	0,0001	-0,0012	0,0007	0,0002	-0,0008	0,0006	0,0012	0,0011	-0,0013
SISE	0,0008	0,0017	0,0009	0,0008	0,0014	0,0002	0,0002	-0,0006	-0,0010	0,0000	-0,0035	0,0003	0,0014	0,0017	-0,0016
ARCLK	0,0002	0,0009	0,0030	0,0014	0,0022	-0,0019	0,0015	0,0018	0,0005	0,0007	-0,0002	0,0000	-0,0005	0,0027	0,0006
YKBNK	0,0009	0,0008	0,0014	0,0019	0,0009	-0,0004	0,0007	0,0005	0,0011	0,0004	-0,0010	0,0006	0,0009	0,0023	-0,0002
TAVHL	0,0007	0,0014	0,0022	0,0009	0,0057	0,0002	0,0014	0,0028	0,0037	0,0031	0,0037	0,0008	0,0021	0,0026	0,0008
TKFEN	0,0012	0,0002	-0,0019	-0,0004	0,0002	0,0071	0,0013	-0,0019	0,0039	0,0012	0,0032	-0,0010	0,0024	-0,0022	-0,0017
TTKOM	0,0001	0,0002	0,0015	0,0007	0,0014	0,0013	0,0027	0,0025	0,0035	0,0026	0,0062	0,0012	-0,0001	0,0005	0,0013
KRDMD	-0,0012	-0,0006	0,0018	0,0005	0,0028	-0,0019	0,0025	0,0075	0,0041	0,0053	0,0116	0,0033	-0,0003	0,0010	0,0043
KOZAL	0,0007	-0,0010	0,0005	0,0011	0,0037	0,0039	0,0035	0,0041	0,0187	0,0058	0,0248	0,0033	0,0007	0,0000	0,0033
PGSUS	0,0002	0,0000	0,0007	0,0004	0,0031	0,0012	0,0026	0,0053	0,0058	0,0063	0,0118	0,0047	0,0012	0,0004	0,0023
KOZAA	-0,0008	-0,0035	-0,0002	-0,0010	0,0037	0,0032	0,0062	0,0116	0,0248	0,0118	0,0589	0,0071	-0,0050	-0,0006	0,0068
DOHOL	0,0006	0,0003	0,0000	0,0006	0,0008	-0,0010	0,0012	0,0033	0,0033	0,0047	0,0071	0,0072	0,0012	0,0001	0,0018
OTKAR	0,0012	0,0014	-0,0005	0,0009	0,0021	0,0024	-0,0001	-0,0003	0,0007	0,0012	-0,0050	0,0012	0,0045	0,0002	-0,0005
ECILC	0,0011	0,0017	0,0027	0,0023	0,0026	-0,0022	0,0005	0,0010	0,0000	0,0004	-0,0006	0,0001	0,0002	0,0046	-0,0012
SKBNK	-0,0013	-0,0016	0,0006	-0,0002	0,0008	-0,0017	0,0013	0,0043	0,0033	0,0023	0,0068	0,0018	-0,0005	-0,0012	0,0053

Analyses are performed for two population sizes (50, 100), five iteration numbers (100, 250, 500, 750, 1000), and three run times (100, 250, 500) for the metaheuristic optimization algorithms. The minimum and maximum values for the parameter variables $(x_{\min} = 0.2, x_{\max} = 0.8)$ and the crossover value $(CR = 0.2)$ are determined for DE algorithm. The cooling rate $(\alpha = 0.99)$, initial temperatıure value $(T = 1000)$, and final temperatıure value $(T_s = 0.1)$ are considered for SA algorithm. The number of onlooker bee (Pop) and dropping limit of the food source $(0{,}6 * nVar(30) * Pop)$ are determined for ABC algorithm. The acceleration coefficients $(c_1 = 1.5, c_2 = 2)$ and inertia weight $(w = 1)$ are considered for PSO algorithm.

Investmentable stocks and their optimal ratios with the minimum portfolio variances obtained from the PSO, DE, ABC, and classical optimization method are shown in Table 5.4.

Table 5.4. Optimal solutions obtained from metaheuristic and classical optimization methods

	Methods			
	PSO	**DE**	**ABC**	**Classical optimization**
Pop size	100	50	50,100	-
Run time	500	100	100,250,500	-
Iteration number	100	750	1000	-
Variance	0,0002568557	0,0002568614	0,0002568557	0,0002570465
TCELL	0,2142	0,2146	0,2142	0,2144
BIMAS	0,3943	0,3935	0,3943	0,3934
THYAO	0,0162	0,0169	0,0162	0,0169
SAHOL	0,1734	0,1705	0,1735	0,1696
ASELS	0	0	0	0,0300
TOASO	0,0270	0,0305	0,0269	0,0324
ARCLK	0,0510	0,0511	0,0510	0,0505
TKFEN	0,0279	0,0267	0,0279	0,0264
SKBNK	0,0961	0,0962	0,0961	0,0963

According to Table 5.4., minimum portfolio variance (0.0002568557) is computed using the ABC and the optimal ratios of the stocks related to this variance are BIMAS (39.43%), TCELL (21.42%), SAHOL (17.35%), SKBNK (9.61%), ARCLK (5.10%), TKFEN (2.79%), TOASO (2.69%), and THYAO (1.62%). With the classical optimization method, the optimal ratios of the stocks regarding the minimum portfolio variance (0.0002570465) are computed as BIMAS (39.34%), TCELL (21.44%), SAHOL (16.96%), SKBNK (9.63%).), ARCLK (5.05%), TOASO (3.24%), ASELS (3%), TKFEN (2.64%) and THYAO (1.69%). The minimum portfolio variances with ratios of the stocks obtained from the both classical optimization method and preferred metaheuristic optimization methods are very close to each other.

6. Conclusions

The most important decision in portfolio management is the choice of asset mix that refers to the division of the portfolio investment into different asset types such as stocks, bonds, warrants, treasury bills, financing bills, asset-backed securities, repo, gold, and foreign currency. In the traditional portfolio theory, diversification is made without considering the interaction of the assets in the portfolio with each other. The key element is the number of financial assets in the portfolio. Accordingly, it is thought that the risk can be reduced by simply increasing the number of assets in the portfolio, without considering the relationships between the returns of the assets in the portfolio. However, in the modern portfolio theory, it is predicted that the assets in the portfolio move in the same or the opposite direction, and therefore the risk cannot be reduced by simply diversifying the portfolio. The modern portfolio theory is based on a number of theories that explain how the information available in the market is analyzed, how investors behave, and how these behaviors affect prices. While traditional portfolio theory is based on qualitative variables, modern portfolio theory tries to quantify the relevant variables. The mean-variance model is provided to find out which assets will be in the portfolio and the ratios of the stocks in the portfolio.

In this study, it is aimed to determine optimal portfolio with minimum risk and maximum return by using the Markowitz' mean-variance model which is a quadratic programming problem. As an application, stocks traded in the BIST30 index between December 1, 2016 and December 29, 2017 are discussed. Optimal solutions for this problem are obtained with both classical optimization and metaheuristic optimization methods, PSO, DE, and ABC. In the application of metaheuristic optimization methods, various trials are carried out for different population sizes, iteration numbers and run times. It is found that there are very small differences between the objective function values, i.e. portfolio variances, obtained from the PSO, DE, and ABC.

When the results obtained from all metaheuristic methods are compared, it is seen that the minimum portfolio variance is calculated as 0.0002568557 from the ABC. The optimal ratios of the stocks to be included in the portfolio for this variance are ordered as BIMAS (39.43%), TCELL (21.42%), SAHOL (17.35%), SKBNK (9.61%), ARCLK (5.10%), TKFEN (2.79%), TOASO (2.69%), and THYAO (1.62%). On the other hand, the optimal ratios of the stocks obtained according to the minimum portfolio variance of 0.0002570465 calculated by the classical optimization method are BIMAS (39.34%), TCELL (21.44%), SAHOL (16.96%), SKBNK (9%, 63), ARCLK (5.05%), TOASO (3.24%), ASELS (3%), TKFEN (2.64%) and THYAO (1.69%). It is seen that the portfolio variances and ratios of the stocks obtained from the all methods are very close to each other.

The differences between the metaheuristic methods discussed are due to the structure of the algorithms and the different parameters they use. Although PSO, DE, and ABC methods give optimal solutions very close to each other, they have differences in terms of fastest convergence. PSO has the fastest convergence performance, followed by ABC and DE, respectively.

In future studies, it can be suggested to obtain solutions for portfolio selection problems with mean absolute deviation models proposed by Konno and Yamazaki [41], Feinstein and Thapa [42] using the metaheuristic and classical optimization methods. Also, hybrid algorithms which are composed with a derivative-free simple local search algorithm, e.g. Nelder-Mead Simplex and population based artificial intelligence algorithms, Genetic Algorithm and Particle Swarm Optimization, proposed by Türkşen and Tez [43] can be applied to portfolio selection problems.

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