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# SOME RESULTS IN SOFT COMPACT FUZZY METRIC SPACES AND ITS APPLICATIONS

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**Abstract**

In this paper we introduced basic notions of soft sets and examined some important properties of soft metric and soft fuzzy metric spaces. The main object of this research article to establish some coincidence point theorems in soft compact fuzzy metric spaces and its applications.

**Keywords:** Soft fuzzy metric space, soft element, soft set, soft mappings, soft continuous mapping, soft contractive mapping, fixed point theorem.

## 1. Introduction

In the year 1999, Molodtsov [20] initiated a novel concept of soft set theory as a new mathematical tool for dealing with uncertainties. A soft set is a collection of approximate descriptions of an object. Soft systems provide a very general framework with the involvement of parameters. Since soft set theory has a rich potential, applications of soft set theory in other disciplines and real-life problems are progressing rapidly.

Maji et al. [17, 18] worked on soft set theory and presented an application of soft sets in decision making problems. Chen [6] introduced a new definition of soft set parametrization reduction and a comparison of it with attribute reduction in rough set theory, Ali et al. [2] gave some new operations in soft set theory. Shabir and Naz [23] presented soft topological spaces and investigated some properties of soft topological spaces. Later, many researches about soft topological spaces were studied in [12, 14, 19, 27]. In these studies, the concept of soft point is expressed by different approaches. In the study we use the concept of soft point which was given in [8].

It is known that there are many generalizations of metric spaces like Menger spaces, fuzzy metric spaces, generalized metric spaces, abstract (cone) metric spaces or K-metric and K-normed spaces etc. Recently Das and Samanta [8, 9] introduced a different notion of soft metric space by using a different concept of soft point and investigated some important properties of these spaces. A number of authors have defined contractive type mapping on a complete metric space which are generalizations of the well-known Banach contraction, and which have the property that each such mapping has a unique fixed point [22, 16]. The fixed point can always be found by using Picard iteration, beginning with some initial choice.

In the present study, we first give, as preliminaries, some well-known results in soft set theory. Firstly, we examine some important properties of soft metric spaces defined in [4, 5, 11, 13, 21]. Secondly, we investigate properties of soft continuous mappings on soft metric spaces. Finally, we introduced soft contractive mappings on soft metric spaces and prove some common fixed-point theorems of soft compact fuzzy metric spaces.

**Definition 1.1[17]** Let X be an initial universe set and E be a set of parameters. A pair (F, E) is called a soft set over X, if and only if F is a mapping from E into the set of all subsets of the set X, i.e., $F: E \to P(X)$, where P(X) is the power set of X.

**Definition 1.2[18]** A soft set (F, E) over X is said to be an absolute soft set denoted by $\tilde{X}$ if for all $e \in E$, $F(e) = X$.

**Definition 1.3[7]** Let R be the set of real numbers and B(R) be the collection of all non-empty bounded subsets of R and E be taken as a set of parameters. Then a mapping F: E → B(R) is called a soft real set. If a real soft set is a singleton soft set, it will be called a soft real number and denoted by $\tilde{r}$, $\tilde{t}$, $\tilde{s}$ etc. $\tilde{0}$ and $\tilde{1}$ are the soft real numbers where $\tilde{0}(e) = 0$, $\tilde{1}(e) = 1$ for all $e \in E$, respectively.

**Definition 1.4[17]** Let X be a non-empty set and E be a non-empty parameter set. Then a function ε: E → X is said to be a soft element of X. A soft element ε of X is said to belong to a soft set A of X, denoted by $\varepsilon \in A$, if $\varepsilon(e) \in A(e)$, $e \in E$. Thus, a soft set A of X with respect to the index set E can be expressed as $A(e) = \{\varepsilon(e), \varepsilon \in A\}$, $e \in E$.

Note. It is to be noted that every singleton soft set (a soft set (F; A) for which F(e) is a singleton set, $\forall \lambda \in A$) can be identified with a soft element by simply identifying the singleton set with the element that it contains $\forall \lambda \in A$.

**Definition 1.5[7]** Let $\tilde{r}$, $\tilde{s}$ be two soft real numbers. Then the following statements hold:

(i) $\tilde{r} \leq \tilde{s}$ iff $\tilde{r}(e) \leq \tilde{s}(e)$ for all $e \in E$;

(ii) $\tilde{r} \geq \tilde{s}$ if $\tilde{r}(e) \geq \tilde{s}(e)$ for all $e \in E$;

(iii) $\tilde{r} < \tilde{s}$ if $\tilde{r}(e) < \tilde{s}(e)$ for all $e \in E$;

(iv) $\tilde{r} > \tilde{s}$ if $\tilde{r}(e > \tilde{s}(e)$ for all $e \in E$

Let SE $(\tilde{X})$ be the collection of all soft points of $\tilde{X}$ and R(E)* denote the set of all non-negative soft real numbers.

**Definition 1.6[9]** A mapping d: SE $(\tilde{X})$ × SE $(\tilde{X})$ → R(E)*, is said to be a soft metric on the soft set $\tilde{X}$ if d satisfies the following conditions:

1. $\tilde{0} \leq d(\tilde{x}, \tilde{y})$ for all $\tilde{x}, \tilde{y} \in \tilde{X}$;
2. $d(\tilde{x},\tilde{y}) = 0$ if and only if $\tilde{x} = \tilde{y}$;
3. $d(\tilde{x}, \tilde{y}) = d(\tilde{y}, \tilde{x})$ for all $\tilde{x}, \tilde{y} \in \tilde{X}$;
4. $d(\tilde{x}, \tilde{y}) \leq d(\tilde{x}, \tilde{z}) + d(\tilde{z}, \tilde{y})$ for all $\tilde{x}, \tilde{y}, \tilde{z} \in \tilde{X}$

The soft set $\tilde{X}$ with a soft metric d on $\tilde{X}$ is called a soft metric space and denoted by $(\tilde{X}, d, E)$.

**Definition 1.7** Let Ψ be the family of functions ψ: R(E)*→ R(E)*satisfying the following conditions.

(i) ψ is non-decreasing,

(ii) $\sum \psi_n(\tilde{t}) < \infty$ for all $\tilde{t} > \tilde{0}$, where $\psi_n$ is the nth iterative of $\psi$.

**Remark:** For every function $\psi$: R(E)*→ R(E)* the following holds: if $\psi$ is non decreasing, then for each $\tilde{t} > \tilde{0}, \lim_{n\to\infty}\psi_n(\tilde{t}) = \tilde{0} \Rightarrow \psi(\tilde{t}) < \tilde{t} \Rightarrow \psi(\tilde{0}) = \tilde{0}$.

Therefore, if $\psi \in \Psi$ then for each $\tilde{t} > \tilde{0}$, $\psi(\tilde{t}) < \tilde{t} \Rightarrow \psi(\tilde{0}) = \tilde{0}$. The notations F(f, T) and C(f, T) stand for the set of all common fixed point and the set of all coincidence points of f and T, respectively.

**Definition 1.8.** [28] A binary operation $\tilde{*}$: $[\tilde{0}, \tilde{1}] \times [\tilde{0}, \tilde{1}] \rightarrow [0, \tilde{1}]$ is called a continuous triangular soft norm (soft $\tilde{t}$-norm) if it satisfies the following conditions:

(i) $\tilde{*}$ is associative and commutative

(ii) $\tilde{*}$ is continuous

(iii) $\tilde{a} \,\tilde{*}\, 1 = \tilde{a}$ for all $\tilde{a} \in [\tilde{0}, \tilde{1}]$

(iv) $\tilde{a} \,\tilde{*}\, \tilde{b} \leq \tilde{c} \,\tilde{*}\, \tilde{d}$, whenever $\tilde{a} \leq \tilde{c}$ and $\tilde{b} \leq \tilde{d}$, for all $\tilde{a}, \tilde{b}, \tilde{c}, \tilde{d} \in [\tilde{0}, \tilde{1}]$.

Three basic examples of continuous soft $\tilde{t}$-norms are $\tilde{a} \,\tilde{*}_1\, \tilde{b} = \min\{\tilde{a}, \tilde{b}\}$, $\tilde{a} \,\tilde{*}_2\, \tilde{b} = \tilde{a}\tilde{b}$ and $\tilde{a} \,\tilde{*}_3\, \tilde{b} = \max\{\tilde{a} + \tilde{b} - \tilde{1}, \tilde{0}\}$.

**Definition 1.9.** [15] A Soft fuzzy metric space is a triple $(\tilde{X}, \tilde{M}, *)$, where $\tilde{X}$ is a non-empty soft set, $\tilde{*}$ is a continuous $\tilde{t}$-norm and $\tilde{M}$ is a fuzzy set on $\tilde{X} \times \tilde{X} \times [\tilde{0}, +\tilde{\infty})$, satisfying the following properties:

$(K_1)$ $\tilde{M}(\tilde{x}, \tilde{y}, \tilde{0}) = \tilde{0}$ for all $\tilde{x}, \tilde{y} \in \tilde{X}$

$(K_2)$ $\tilde{M}(\tilde{x}, \tilde{y}, \tilde{t}) = \tilde{1}$ for all $\tilde{t} > \tilde{0}$ iff $\tilde{x} = \tilde{y}$

$(K_3)$ $\tilde{M}(\tilde{x}, \tilde{y}, \tilde{t}) = \tilde{M}(\tilde{y}, \tilde{x}, \tilde{t})$ for all $\tilde{x}, \tilde{y} \in \tilde{X}$ and for all $\tilde{t} > \tilde{0}$

$(K_4)$ $\tilde{M}(\tilde{x}, \tilde{y}, \tilde{\cdot})$: $[\tilde{0}, +\tilde{\infty}) \rightarrow [\tilde{0}, \tilde{1}]$ is left continuous for all $\tilde{x}, \tilde{y} \in \tilde{X}$

$(K_5)$ $\tilde{M}(\tilde{x}, \tilde{z}, \tilde{t} + \tilde{s}) \geq \tilde{M}(\tilde{x}, \tilde{y}, \tilde{t}) \,\tilde{*}\, \tilde{M}(\tilde{y}, \tilde{z}, \tilde{s})$, for all $\tilde{x}, \tilde{y}, \tilde{z} \in \tilde{X}$ and for all $\tilde{t}, \tilde{s} > \tilde{0}$.

We denote such space as soft KM-fuzzy metric space.

## 2. Lemma and some definitions on soft compact fuzzy metric space

**Lemma 2.1.** [15] In a soft KM-fuzzy metric space $(\tilde{X}, \tilde{M}, \tilde{*})$, $\tilde{M}(\tilde{x}, \tilde{y}, \tilde{\cdot})$ is non-decreasing for all $\tilde{x}, \tilde{y} \in \tilde{X}$. If, in the definition of Kramosil and Michalek [19], $\tilde{M}$ is a fuzzy soft set on $\tilde{X} \times \tilde{X} \times (\tilde{0}, +\tilde{\infty})$ and $(K_1)$, $(K_2)$, $(K_4)$ are replaced, respectively, with $(G_1)$, $(G_2)$, $(G_4)$ below, then $(\tilde{X}, \tilde{M}, \tilde{*})$ is called a soft fuzzy metric space in the sense of George and Veeramani [10].

$(G_1)$ $\tilde{M}(\tilde{x}, \tilde{y}, \tilde{t}) > \tilde{0}$ for all $\tilde{t} > \tilde{0}$

$(G_2)$ $\tilde{M}(\tilde{x}, \tilde{x}, \tilde{t}) = \tilde{1}$ for all $\tilde{t} > 0$ and if $\tilde{M}(\tilde{x}, \tilde{y}, \tilde{t}) \,\tilde{=}\, \tilde{1}$ for some $\tilde{t} > \tilde{0}$, then $\tilde{x} = \tilde{y}$

$(G_4)$ $\tilde{M}(\tilde{x}, \tilde{y}, \tilde{\cdot})$: $(\tilde{0}, +\tilde{\infty}) \rightarrow [\tilde{0}, \tilde{1}]$ is continuous for all $\tilde{x}, \tilde{y} \in \tilde{X}$.

We denote such space as GV –fuzzy soft metric space.

**Definition 2.2** A class {Gi} of open subset of X is said to be an open cover of X, if each point in X belongs to one $G_i$ that is U'i Gi = X. A subclass of an open cover which is at least an open cover is called a sub cover. A compact space is that space in which every open cover has finite sub cover.

**Definition 2.3.**[19]$A$ sequence $\{\tilde{x}_n\}$in a soft compact fuzzy metric space $(\tilde{X}, \tilde{M}, \tilde{*})$ is said to be Cauchy sequence if and only if for each $\epsilon > 0, \tilde{t} > 0$, there exist $n_0 \in N$ such that $\tilde{M}(\tilde{x}_n, \tilde{x}_m, \tilde{t}) > 1 - \epsilon$ for all $n, m \geqslant n_0$.

**Definition 2.4.**[19] A sequence $\{\tilde{x}_n\}$ in a soft compact fuzzy metric space $(\tilde{X}, \tilde{M}, \tilde{*})$ is said to be convergent sequence to a point $\tilde{x}$ in $\tilde{X}$if and only if each $\epsilon > 0, \tilde{t} > 0$, there exist $n_0 \in N$ such that $\tilde{M}(\tilde{x}_n, \tilde{x}, \tilde{t}) > 1 - \epsilon$ for all $n \geq n_0$.

**Definition 2.5.** [19] A soft compact fuzzy metric space $(\tilde{X}, \tilde{M}, \tilde{*})$ is said to be complete if every Cauchy sequence in it converges to a point in it.

**Definition 2.6.** [24] Two self-mappings A and S of a soft compact fuzzy metric space $(\tilde{X}, \tilde{M}, \tilde{*})$ are said to be compatible if and only if $\tilde{M}(AS\tilde{x}_n, SA\tilde{x}_n, \tilde{t}) \to 1$ for all $\tilde{t} > 0$, whenever $\{\tilde{x}_n\}$ is a sequence in $\tilde{X}$ such that $S\tilde{x}_n, A\tilde{x}_n \to \tilde{p}$ for some $\tilde{p}$ in $\tilde{X}$, as $n \to \infty$.

**Definition 2.7.** [25] Two self-mappings$A$ and $S$ of a soft compact fuzzy metric space $(\tilde{X}, \tilde{M}, \tilde{*})$ are said to be semi-compatible if and only if $\tilde{M}(AS\tilde{x}_n, S\tilde{p}, \tilde{t}) \to 1$ for all $\tilde{t} > 0$, whenever $\{\tilde{x}_n\}$ is a sequence in $\tilde{X}$such that $S\tilde{x}_n, A\tilde{x}_n \to \tilde{p}$ for Some $\tilde{p}$ in $\tilde{X}$, as $n \to \infty$.

**Definition 2.8.** [26] Two self-mappings$A$ and $S$ of a soft compact fuzzy metric space $(\tilde{X}, \tilde{M}, \tilde{*})$ are said to be weakly-compatible if they commute at their coincidence points, i.e.$A\tilde{x} = S\tilde{x}$ implies $AS\tilde{x} = SA\tilde{x}$.

**Definition 2.9.** [3] Two self-mappings$A$ and $S$ of a soft compact fuzzy metric space $(\tilde{X}, \tilde{M}, \tilde{*})$ are said to be occasionally - weakly compatible if and only if there is a point $\tilde{x}$ in $\tilde{X}$ which is coincidence point of $A$ and $S$ at which $A$ and $S$ commute.

**Definition 2.10.**[1] Two self mappings $A$ and $S$ of a soft compact fuzzy metric space $(\tilde{X}, \tilde{M}, \tilde{*})$ are said to satisfy the property E. A. if there exist sequence $\{\tilde{x}_n\}$ in $\tilde{X}$such that $\lim_{n\to\infty} A\tilde{x}_n = \lim_{n\to\infty} S\tilde{x}_n = \tilde{z}$ for some $\tilde{z} \in \tilde{X}$.

## 3. Main results

**Theorem 3.1.** Let $A, B, S$ and $T$ be self mappings on soft compact fuzzy metric space $(\tilde{x}, \tilde{M}, \tilde{*})$ satisfying the following condition:

$$\begin{aligned}\tilde{M}(A\tilde{x}, B\tilde{y}, \tilde{t}) \quad & \geqslant r[\{\tilde{M}(S\tilde{x}, T\tilde{y}, \tilde{t}) \tilde{*} \\ & \tilde{M}(S\tilde{x}, A\tilde{x}, \tilde{t}) \tilde{*} \tilde{M}(S\tilde{x}, B\tilde{y}, \tilde{t}) \tilde{*} \tilde{M}(T\tilde{y}, A\tilde{x}, \tilde{t})\}\end{aligned}$$

$$\tilde{*}\{\frac{\tilde{M}(S\tilde{x},T\tilde{y},\tilde{t})\tilde{*}\tilde{M}(S\tilde{x},B\tilde{y},\tilde{t})}{\tilde{M}(T\tilde{y},A\tilde{x},\tilde{t})}\}\tilde{*}\{\frac{(\tilde{M}(S\tilde{x},T\tilde{y},\tilde{t})\tilde{*}\tilde{M}(S\tilde{x},A\tilde{x},\tilde{t})}{\tilde{M}(T\tilde{y},A\tilde{x},\tilde{t})}\}] \qquad \ldots (3.1)$$

for all $\tilde{x}, \tilde{y} \in \tilde{X}$, where $r: [0,1] \to [0,1]$ is continuous function such that

$r(\tilde{t}) > \tilde{t}$ for each $\tilde{t} < 1$... (3.2)

and $r(\tilde{t}) = 1$ for $\tilde{t} = 1$

Also suppose the pair $(A, S)$ and $(B, T)$ share the common property (E.A.), and $S(\tilde{X})$ and $T(\tilde{X})$ are closed subsets of $\tilde{X}$, then the pair $(A, S)$ as well as $(B, T)$ have a coincidence point.

Further $A, B, S, T$ have a unique common fixed point provided the pair $(A, S)$ is semi compatible and $(B, T)$ is occasionally weakly compatible.

**Proof.** Since the pair $(A, S)$ and $(B, T)$ share the common property (E.A) then there exist two sequences $\{\tilde{x}_n\}$ and $\{\tilde{y}_n\}$ in $\tilde{X}$such that $\lim_{n\to\infty} A\tilde{x}_n = \lim_{n\to\infty} S\tilde{x}_n = \lim_{n\to\infty} B\tilde{y}_n = \lim_{n\to\infty} T\tilde{y}_n = \tilde{z}$, for some $\tilde{z} \in \tilde{X}$. Also, $S(\tilde{X})$ is closed subset of $\tilde{X}$, therefore $\lim_{n\to\infty} S\tilde{x}_n = \tilde{z} \in S(\tilde{X})$ and there is a point $u$ in $\tilde{X}$ such that

$S\tilde{u} = \tilde{z}$.

Now, we claim that $A\tilde{u} = \tilde{z}$. If not, then by using (3.1), we have

$$\tilde{M}(A\tilde{u}, B\tilde{y}_n, \tilde{t}) \geqslant r\Big[\{\tilde{M}(S\tilde{u}, T\tilde{y}_n, \tilde{t}) \tilde{*} \tilde{M}(S\tilde{u}, A\tilde{u}, \tilde{t}) \tilde{*} \tilde{M}(S\tilde{u}, B\tilde{y}_n, \tilde{t}) \tilde{*} \tilde{M}(T\tilde{y}_n, A\tilde{u}, \tilde{t})\} \tilde{*} \left\{\frac{\tilde{M}(S\tilde{u}, T\tilde{y}_n, \tilde{t}) \tilde{*} \tilde{M}(S\tilde{u}, B\tilde{y}_n, \tilde{t})}{\tilde{M}(T\tilde{y}_n, A\tilde{u}, \tilde{t})}\right\} \tilde{*} \{\frac{\tilde{M}(S\tilde{u}, T\tilde{y}_n, \tilde{t}) \tilde{*} \tilde{M}(S\tilde{u}, A\tilde{u}, \tilde{t})}{\tilde{M}(T\tilde{y}_n, A\tilde{u}, \tilde{t})}\}\Big]$$

Taking $n \to \infty$, we get

$$\begin{aligned}\tilde{M}(A\tilde{u}, \tilde{z}, \tilde{t}) &\geqslant r[\ \{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \tilde{*} \tilde{M}(\tilde{z}, A\tilde{u}, \tilde{t}) \tilde{*} \\ &\quad \tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \tilde{*} \tilde{M}(\tilde{z}, A\tilde{u}, \tilde{t}) \tilde{*} \left\{\frac{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \tilde{*} \tilde{M}(\tilde{z}, \tilde{z}, \tilde{t})}{\tilde{M}(\tilde{z}, A\tilde{u}, \tilde{t})}\right\} \tilde{*} \{\frac{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \tilde{*} \tilde{M}(\tilde{z}, A\tilde{u}, \tilde{t})}{\tilde{M}(\tilde{z}, A\tilde{u}, \tilde{t})}\}] \\ &= r\big[\tilde{M}(A\tilde{u}, \tilde{z}, \tilde{t})\big]\end{aligned}$$

$$\tilde{M}(A\tilde{u}, \tilde{z}, \tilde{t}) > \tilde{M}(A\tilde{u}, \tilde{z}, \tilde{t})$$

this is a contradiction. Hence $A\tilde{u} = \tilde{z}$. Thus we have $A\tilde{u} = S\tilde{u}$ or $\tilde{u}$ is a coincidence point of the pair $(A, S)$. Since $T(\tilde{X})$ is closed subset of $\tilde{X}$, therefore $\lim_{n\to\infty} T\tilde{y}_n = \tilde{z} \epsilon T(\tilde{X})$ and there exists $\tilde{w} \in \tilde{X}$ such that $T\tilde{w} = \tilde{z}$. Again using (3.1), we obtain

$$\begin{aligned}\tilde{M}(A\tilde{x}_n, B\tilde{w}, \tilde{t}) \geqslant r\Big[&\tilde{M}(S\tilde{x}_n, T\tilde{w}, \tilde{t}) \tilde{*} \tilde{M}(S\tilde{x}_n, A\tilde{x}_n, \tilde{t}) \tilde{*} \\ &\tilde{M}(S\tilde{x}_n, B\tilde{w}, \tilde{t}) \tilde{*} \tilde{M}(T\tilde{w}, A\tilde{x}_n, \tilde{t})\} \tilde{*} \\ &\left\{\frac{\tilde{M}(S\tilde{x}_n, T\tilde{w}, \tilde{t}) \tilde{*} \tilde{M}(S\tilde{x}_n, B\tilde{w}, \tilde{t})}{\tilde{M}(T\tilde{w}, A\tilde{x}_n, \tilde{t})}\right\} \tilde{*} \{\frac{\tilde{M}(S\tilde{x}_n, T\tilde{w}, \tilde{t}) \tilde{*} \tilde{M}(S\tilde{x}_n, A\tilde{x}_n, \tilde{t})}{\tilde{M}(T\tilde{w}, A\tilde{x}_n, \tilde{t})}\}\Big]\end{aligned}$$

Taking $n \to \infty$, we get

$$\begin{aligned}\tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t}) &\geq r[\{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \tilde{*} \tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \tilde{*} \tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t}) \tilde{*} \\ &\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \tilde{*} \left\{\frac{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \tilde{*} \tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t})}{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t})}\right\} \tilde{*} \{\frac{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \tilde{*} \tilde{M}(\tilde{z}, \tilde{z}, \tilde{t})}{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t})}\}] \\ &= r\big[\tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t})\big] \\ &> \tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t}).\end{aligned}$$

This implies $B\widetilde{w} = \tilde{z}$. Hence, we get $T\widetilde{w} = B\widetilde{w} = \tilde{z}$. Thus, $\widetilde{w}$ is a coincidence point of the pair $(B, T)$.

Also, $(A, S)$ is semi-compatible pair, so $\lim_n \to \infty \, S\tilde{x}_n = S\tilde{z}$ and $\lim_{n\to\infty} AS\tilde{x}_n = A\tilde{z}$. Since the limit in soft compact fuzzy metric space is unique so $S\tilde{z} = A\tilde{z}$. Now, we claim that $\tilde{z}$ is a common fixed point of the pair $(A, S)$.

Again from (3.1), we obtain

$$\tilde{M}(A\tilde{z}, B\tilde{w}, \tilde{t}) \geq r[\ \{\tilde{M}(S\tilde{z}, T\tilde{w}, \tilde{t}) \,\tilde{*}\, \tilde{M}(S\tilde{z}, A\tilde{z}, \tilde{t}) \,\tilde{*}\, \tilde{M}(S\tilde{z}, B\tilde{w}, \tilde{t}) \,\tilde{*}\, \tilde{M}(T\tilde{w}, A\tilde{z}, \tilde{t})\} \,\tilde{*} \left\{\frac{\tilde{M}(S\tilde{z}, T\tilde{w}, \tilde{t}) \,\tilde{*}\, \tilde{M}(S\tilde{z}, B\tilde{w}, \tilde{t})}{\tilde{M}(T\tilde{w}, A\tilde{z}, \tilde{t})}\right\} \tilde{*} \{\frac{S\tilde{z}, T\tilde{w}, \tilde{t}) \,\tilde{*}\, \tilde{M}(S\tilde{z}, A\tilde{z}, \tilde{t})}{\tilde{M}(T\tilde{w}, A\tilde{z}, \tilde{t})}\}]$$

Taking $n \to \infty$, we get

$$\begin{aligned} \tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) \geq\ & r[\ \{\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) \,\tilde{*}\, \tilde{M}(A\tilde{z}, A\tilde{z}, \tilde{t}) \,\tilde{*}\, \tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) \,\tilde{*} \\ & \tilde{M}(\tilde{z}, A\tilde{z}, \tilde{t})\} \,\tilde{*} \left\{\frac{\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) \,\tilde{*}\, \tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t})}{\tilde{M}(\tilde{z}, A\tilde{z}, \tilde{t})}\right\} \tilde{*} \{\frac{\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) \,\tilde{*}\, \tilde{M}(A\tilde{z}, A\tilde{z}, \tilde{t})}{\tilde{M}(\tilde{z}, A\tilde{z}, \tilde{t})}\}] \\ =\ & r\left[\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t})\right] \\ >\ & \tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}), \end{aligned}$$

implies $A\tilde{z} = \tilde{z}$. Thus $A\tilde{z} = \tilde{z} = S\tilde{z}$.

Since $\tilde{w}$ is a coincidence point of $B$ and $T$ and the pair $(B, \mathrm{T})$ is occasionally weakly compatible, so we have,

$$BT\tilde{w} = TB\tilde{w} \Rightarrow B\tilde{z} = T\tilde{z} = \tilde{z}.$$

Hence, $\tilde{z}$ is the common fixed point of $A, B, S$ and $T$. For Uniqueness, Let $\tilde{v}$ be another common fixed point of $A, B, S$ and $T$. Take $\tilde{x} = \tilde{z}$ and $\tilde{y} = \tilde{v}$ in $(3 \cdot 1)$, we get

$$\begin{gathered} \tilde{M}(A\tilde{z}, B\tilde{v}, \tilde{t}) \geq r[\{\tilde{M}(S\tilde{z}, T\tilde{v}, \tilde{t}) \,\tilde{*}\, \tilde{M}(S\tilde{z}, A\tilde{z}, \tilde{t}) \,\tilde{*}\, \tilde{M}(S\tilde{z}, B\tilde{v}, \tilde{t}) \,\tilde{*} \\ \tilde{M}(T\tilde{v}, A\tilde{z}, \tilde{t}) \,\tilde{*} \left\{\frac{\tilde{M}(S\tilde{z}, T\tilde{v}, \tilde{t}) \,\tilde{*}\, \tilde{M}(S\tilde{z}, B\tilde{v}, \tilde{t})}{\tilde{M}(T\tilde{v}, A\tilde{z}, \tilde{t})}\right\} \tilde{*} \{\frac{\tilde{M}(S\tilde{z}, T\tilde{v}, \tilde{t}) \,\tilde{*}\, \tilde{M}(S\tilde{z}, A\tilde{z}, \tilde{t})}{\tilde{M}(T\tilde{v}, A\tilde{z}, \tilde{t})}\}] \end{gathered}$$

Taking $n \to \infty$, we get

$$\begin{aligned}\tilde{M}(\tilde{z},\tilde{v},\tilde{t}) &\geq r[[\tilde{M}(\tilde{z},\tilde{v},\tilde{t}) \circledast \tilde{M}(\tilde{z},\tilde{z},\tilde{t}) \circledast \tilde{M}(\tilde{z},\tilde{v},\tilde{t}) \circledast \\ &\tilde{M}(\tilde{v},\tilde{z},\tilde{t}) \circledast \left\{\frac{\tilde{M}(\tilde{z},\tilde{v},\tilde{t}) \circledast \tilde{M}(\tilde{z},\tilde{v},\tilde{t})}{\tilde{M}(\tilde{v},\tilde{z},\tilde{t})}\right\} \circledast \{\frac{\tilde{M}(\tilde{z},\tilde{v},\tilde{t}) \circledast \tilde{M}(\tilde{z},\tilde{z},\tilde{t})}{\tilde{M}(\tilde{v},\tilde{z},\tilde{t})}\}] \\ &= r\left[\tilde{M}(A\tilde{z},\tilde{z},\tilde{t})\right] \\ &> \tilde{M}(A\tilde{z},\tilde{z},\tilde{t}),\end{aligned}$$

this implies $\tilde{z} = \tilde{v}$, Thus $\tilde{z}$ is the unique common firced point of the mappings $A, B, S$ and $T$.

**Theorem 3.2.** Let $A, B, S$ and $T$ be four self mappings on a soft compact fuzzy metric space $(\tilde{X}, \tilde{M}, \circledast)$ satisfying the following conditions:

i) The pairs $(A, S)$ and $(B, T)$share the common property $(E \cdot A)$

ii) $S(\tilde{X})$ and $T(\tilde{X})$ are closed subsets of $\tilde{X}$

iii) $q(\tilde{M}(A\tilde{x}, B\tilde{y}, \tilde{t}) \geq a\tilde{M}(T\tilde{y}, S\tilde{x}, \tilde{t}) + b\tilde{M}(S\tilde{x}, B\tilde{y}, \tilde{t}) + c\tilde{M}(A\tilde{x}, B\tilde{y}, \tilde{t}) + \{\tilde{M}(A\tilde{x}, S\tilde{x}, \tilde{t}) \circledast \tilde{M}(B\tilde{y}, T\tilde{y}, \tilde{t})\}$

for $all\ \tilde{x}, \tilde{y} \in \tilde{X}, a, b, c \geq 0, q > 0$ and $q < a + b + c + 1$, then each pair $(A, S)$ and $(B, T)$ have a point of coincidence. further, if the pair $(A, S)$ is semi-compatible and $(B, T)$ is occasionally weakly compatible, then $A, B, S$ and $T$ have a unique fixed point.

**Proof.** As the pair $(A, S)$ and $(B, T)$share the common property $(E.A)$, then there exist two sequences $\{\tilde{x}_n\}$ and $\{\tilde{y}_n\}$ in $\tilde{X}$ such that

$$\lim_{n\to\infty} A\tilde{x}_n = \lim_{n\to\infty} S\tilde{x}_n = \lim_{n\to\infty} B\tilde{y}_n = \lim_{n\to\infty} T\tilde{y}_n = \tilde{z},$$

for some $\tilde{z} \in \tilde{X}$.

Since $S(\tilde{X})$ is a closed subset of $\tilde{X}$; therefore, there exists a point $\tilde{u} \in \tilde{X}$such that Su $= \tilde{z}$. Using the above condition (iii), we have.

$$\begin{aligned}q\tilde{M}(A\tilde{u}, B\tilde{y}_n, \tilde{t}) &\geq a\tilde{M}(T\tilde{y}_n, S\tilde{u}, \tilde{t}) + b\tilde{M}(S\tilde{u}, B\tilde{y}_n, \tilde{t}) \\ &+c\tilde{M}(A\tilde{u}, B\tilde{y}_n, \tilde{t}) + \{\tilde{M}(A\tilde{u}, S\tilde{u}, \tilde{t}) \circledast \tilde{M}(B\tilde{y}_n, T\tilde{y}_n, \tilde{t})\}\end{aligned}$$

Taking $n \to \infty$, we obtain

$$\begin{aligned} q\tilde{M}(A\tilde{u},\tilde{z},\tilde{t}) &\geq a\tilde{M}(\tilde{z},\tilde{z},\tilde{t}) + b\tilde{M}(\tilde{z},\tilde{z},\tilde{t}) \\ &+c\tilde{M}(A\tilde{u},B\tilde{y}_n,\tilde{t}) + \{\tilde{M}(A\tilde{u},\tilde{z},\tilde{t}) \circledast \tilde{M}(\tilde{z},\tilde{z},\tilde{t})\} \end{aligned}$$

this gives,

$$\begin{aligned} q\tilde{M}(A\tilde{u},\tilde{z},\tilde{t}) &\geq a\tilde{M}(\tilde{z},\tilde{z},\tilde{t}) + b\tilde{M}(\tilde{z},\tilde{z},\tilde{t}) \\ &+c\tilde{M}(A\tilde{u},\tilde{z},\tilde{t}) \\ + \quad &\tilde{M}(A\tilde{u},\tilde{z},\tilde{t}), \end{aligned}$$

$$\begin{aligned} (q-c-1)\tilde{M}(A\tilde{u},\tilde{z},\tilde{t}) &\geq (a+b)\tilde{M}(\tilde{z},\tilde{z},\tilde{t}) + \\ \text{this implies,} \quad \tilde{M}(A\tilde{u},\tilde{z},\tilde{t}) &> \frac{a+b}{q-c-1} \\ &> 1. \end{aligned}$$

for all $\tilde{t} > 0$, this implies, $A\tilde{u} = \tilde{z}$. Hence $A\tilde{u} =$ S$\tilde{u}$, which shows that $\tilde{u}$ is the coincidence point of $(A,S)$.

Again, $T(\tilde{X})$ is closed subset of $\tilde{X}$, therefore there is a point $\tilde{w}$ in $\tilde{X}$ such that $T\tilde{w} = \tilde{z}$. Now take $\tilde{x} = \tilde{x}_n$ and $\tilde{y} = \tilde{w}$ in condition $(ii)$, we get

$$\begin{aligned} &q\tilde{M}(A\tilde{x}_n,B\tilde{w},\tilde{t}) \geq a\tilde{M}(T\tilde{w},S\tilde{x}_n,\tilde{t}) + b\tilde{M}(S\tilde{x}_n,B\tilde{w},\tilde{t}) \\ &+c\tilde{M}(A\tilde{x}_n,B\tilde{w},\tilde{t}) \\ &+\{\tilde{M}(A\tilde{x}_n,S\tilde{x}_n,\tilde{t}) \circledast \tilde{M}(B\tilde{w},T\tilde{w},\tilde{t})\} \end{aligned}$$

Taking $n \to \infty$, we obtain

$$\begin{aligned} q\tilde{M}(\tilde{z},B\tilde{w},\tilde{t}) \geq a\tilde{M}(\tilde{z},\tilde{z},\tilde{t}) + b\tilde{M}(\tilde{z},B\tilde{w},\tilde{t}) + c\tilde{M}(\tilde{z},B\tilde{w},\tilde{t}) \\ + \{\tilde{M}(\tilde{z},\tilde{z},\tilde{t}) \circledast \tilde{M}(B\tilde{w},\tilde{z},\tilde{t})\} \end{aligned}$$

$$\begin{aligned} q\tilde{M}(\tilde{z},B\tilde{w},\tilde{t}) \geq a\tilde{M}(\tilde{z},\tilde{z},\tilde{t}) + b\tilde{M}(\tilde{z},B\tilde{w},\tilde{t}) + c\tilde{M}(\tilde{z},B\tilde{w},\tilde{t}) \\ + \tilde{M}(B\tilde{w},\tilde{z},\tilde{t}) \end{aligned}$$

$$\text{(q-b-c-1)}\tilde{M}(\tilde{z},B\tilde{w},\tilde{t}) > a\tilde{M}(\tilde{z},\tilde{z},\tilde{t})$$

this gives $\tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t}) > \frac{a}{q-b-c-1} > 1$ for all $\tilde{t} > 0$.

This implies $B\tilde{w} = \tilde{z}$. Hence $T\tilde{w} = B\tilde{w} = \tilde{z}$ and thus $\tilde{w}$ is the coincidence point of $(B, T)$.

Further, we assume that $(A, S)$ is a semi-compatible pair, so $\lim_{n\to\infty} AS\tilde{x}_n = S\tilde{z}$ and $\lim_{n\to\infty} AS\tilde{x}_n = A\tilde{z}$. Since the limit in soft compact fuzzy metric cspace is unique, so $S\tilde{z} = A\tilde{z}$.

Now, we claim that $\tilde{z}$ is a common fixed point of the pair $A$ and $S$. using condition (iii), we get

$$q\tilde{M}(A\tilde{z}, B\tilde{w}, \tilde{t}) \geq a\tilde{M}(T\tilde{w}, S\tilde{z}, \tilde{t}) + b\tilde{M}(S\tilde{z}, B\tilde{w}, \tilde{t}) + c\tilde{M}(A\tilde{z}, B\tilde{w}, \tilde{t})$$
$$+\{\tilde{M}(A\tilde{z}, S\tilde{z}, \tilde{t}) * \tilde{M}(B\tilde{w}, T\tilde{w}, \tilde{t})\}$$

Taking $n \to \infty$, we obtain

$$q\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) \geq a\tilde{M}(\tilde{z}, A\tilde{z}, \tilde{t}) + bM(A\tilde{z}, \tilde{z}, \tilde{t})$$
$$+c\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) + \left\{\tilde{M}(A\tilde{z}, A\tilde{z}, \tilde{t}) * \tilde{M}(\tilde{z}, \tilde{z}, \tilde{t})\right\}$$

$$q\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) \geq a\tilde{M}(\tilde{z}, A\tilde{z}, \tilde{t}) + bM(A\tilde{z}, \tilde{z}, \tilde{t})$$
$$+c\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) + \tilde{M}(A\tilde{z}, A\tilde{z}, \tilde{t})$$

$$(q - a - b - c - 1)\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) \geq 1$$

This implies

$$M(A\tilde{z}, \tilde{z}, \tilde{t}) \geq \frac{1}{q - a - b - c - 1}$$
$$> 1, \text{for all } \tilde{t} > 0.$$

Thus $A\tilde{z} = \tilde{z}$. Hence $A\tilde{z} = \tilde{z} =$ S $\tilde{z}$. since $\tilde{w}$ is a coincidence point $B$ and $T$, and the pair $(B, T)$ is occasionally weak compatible. So, $BT\tilde{w} = TB\tilde{w}$ this implies $B\tilde{z} = T\tilde{z} = \tilde{z}$. Hence,$\tilde{z}$ is the common fixed point of mappings $A, S, B$ and $T$.

The uniqueness of fined point follows from taking $\tilde{x} = \tilde{z}$ and $\tilde{y} = \tilde{v}$ in condition (iii).

Taking $A = B$ in the above theorem, we get the following corollary:

**Corollary 3.3.** Let $A, S$ and $T$ be three self mappings of a soft compact fuzzy metric space $(\tilde{X}, \tilde{M}, \tilde{*})$, satisfying the following conditions:

i) The pairs $(A, S)$ and $(A, T)$ share the common property (E.A.)

ii) $S(\tilde{X})$ and $T(\tilde{X})$ are closed subsets of $\tilde{X}$

iii) $q\tilde{M}(A\tilde{x}, A\tilde{y}, \tilde{t}) \geq a\tilde{M}(T\tilde{y}, S\tilde{x}, \tilde{t}) + b\tilde{M}(S\tilde{x}, A\tilde{y}, \tilde{t}) + c\tilde{M}(A\tilde{x}, A\tilde{y}, \tilde{t})$
$+\{\tilde{M}(A\tilde{x}, S\tilde{x}, \tilde{t}) \tilde{*} \tilde{M}(A\tilde{y}, T\tilde{y}, \tilde{t})\}$
for all $\tilde{x}, \tilde{y} \in \tilde{X}, a, b, c \geq 0, q > 0$ and $q < a + b + c + 1$, then the pairs $(A, S)$ and $(A, T)$ have a point of coincidence.
Further, if the pair $(A, S)$ is semi-compaitible and $(A, T)$ is occasionally weakly compatible then $A, S$ and $T$ have a unique common fixed point

$$\tilde{M}(A\tilde{u}, B\tilde{y}_n, \tilde{t}) \geqslant r[\{\tilde{M}(S\tilde{u}, T\tilde{y}_n, \tilde{t}) \tilde{*} \tilde{M}(S\tilde{u}, A\tilde{u}, \tilde{t}) \tilde{*} \tilde{M}(S\tilde{u}, B\tilde{y}_n, \tilde{t}) \tilde{*} \tilde{M}(T\tilde{y}_n, A\tilde{u}, \tilde{t})\} \tilde{*} \left\{\frac{\tilde{M}(S\tilde{u}, T\tilde{y}_n, \tilde{t}) \tilde{*} \tilde{M}(S\tilde{u}, B\tilde{y}_n, \tilde{t})}{\tilde{M}(T\tilde{y}_n, A\tilde{u}, \tilde{t})}\right\} \tilde{*} \{\frac{\tilde{M}(S\tilde{u}, T\tilde{y}_n, \tilde{t}) \tilde{*} \tilde{M}(S\tilde{u}, A\tilde{u}, \tilde{t})}{\tilde{M}(T\tilde{y}_n, A\tilde{u}, \tilde{t})}\}]$$

Taking $n \to \infty$, we get

$$\begin{aligned}\tilde{M}(A\tilde{u}, \tilde{z}, \tilde{t}) &\geqslant r[\ \{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \tilde{*} \tilde{M}(\tilde{z}, A\tilde{u}, \tilde{t}) \tilde{*} \\ &\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \tilde{*} \tilde{M}(\tilde{z}, A\tilde{u}, \tilde{t}) \tilde{*} \left\{\frac{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \tilde{*}\ \tilde{M}(\tilde{z}, \tilde{z}, \tilde{t})}{\tilde{M}(\tilde{z}, A\tilde{u}, \tilde{t})}\right\} \tilde{*} \{\frac{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \tilde{*} \tilde{M}(\tilde{z}, A\tilde{u}, \tilde{t})}{\tilde{M}(\tilde{z}, A\tilde{u}, \tilde{t})}\}] \\ &= r[\tilde{M}(A\tilde{u}, \tilde{z}, \tilde{t})]\end{aligned}$$

$$\tilde{M}(A\tilde{u}, \tilde{z}, \tilde{t}) > \tilde{M}(A\tilde{u}, \tilde{z}, \tilde{t})$$

this is a contradiction. Hence $A\tilde{u} = \tilde{z}$. Thus we have $A\tilde{u} = S\tilde{u}$ or $\tilde{u}$ is a coincidence point of the pair $(A, S)$. Since $T(\tilde{X})$ is closed subset of $\tilde{X}$, therefore $\lim_{n\to\infty} T\tilde{y}_n = \tilde{z} \epsilon T(\tilde{X})$ and there exists $\tilde{w} \in \tilde{X}$ such that $T\tilde{w} = \tilde{z}$.
Again using (3.1), we obtain

$$\tilde{M}(A\tilde{x}_n, B\tilde{w}, \tilde{t}) \geqslant r\Big[\tilde{M}(S\tilde{x}_n, T\tilde{w}, \tilde{t}) \circledast \tilde{M}(S\tilde{x}_n, A\tilde{x}_n, \tilde{t}) \circledast \tilde{M}(S\tilde{x}_n, B\tilde{w}, \tilde{t}) \circledast \tilde{M}(T\tilde{w}, A\tilde{x}_n, \tilde{t})\Big\} \circledast \left\{\frac{\tilde{M}(S\tilde{x}_n, T\tilde{w}, \tilde{t}) \circledast \tilde{M}(S\tilde{x}_n, B\tilde{w}, \tilde{t})}{\tilde{M}(T\tilde{w}, A\tilde{x}_n, \tilde{t})}\right\} \circledast \left\{\frac{\tilde{M}(S\tilde{x}_n, T\tilde{w}, \tilde{t}) \circledast \tilde{M}(S\tilde{x}_n, A\tilde{x}_n, \tilde{t})}{\tilde{M}(T\tilde{w}, A\tilde{x}_n, \tilde{t})}\right\}\Big]$$

Taking $n \to \infty$, we get

$$\begin{aligned}\tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t}) &\geq r[\{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \circledast \tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \circledast \tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t}) \circledast \tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \circledast \left\{\frac{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \circledast \tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t})}{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t})}\right\} \circledast \{\frac{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \circledast \tilde{M}(\tilde{z}, \tilde{z}, \tilde{t})}{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t})}\}] \\ &= r\big[\tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t})\big] \\ &> \tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t}).\end{aligned}$$

This implies $B\widetilde{w} = \tilde{z}$. Hence, we get $T\widetilde{w} = B\widetilde{w} = \tilde{z}$. Thus,$\widetilde{w}$ is a coincidence point of the pair $(B, T)$.

Also, $(A, S)$ is semi-compatible pair, so $\lim_n \to \infty$ $S\tilde{x}_n = S\tilde{z}$ and $\lim_{n\to\infty} AS\tilde{x}_n = A\tilde{z}$. Since the limit in soft compact fuzzy metric space is unique so $S\tilde{z} = A\tilde{z}$. Now, we claim that $\tilde{z}$ is a common fixed point of the pair $(A, S)$.

Again from (3.1), we obtain

$$\tilde{M}(A\tilde{z}, B\tilde{w}, \tilde{t}) \geq r[\ \{\tilde{M}(S\tilde{z}, T\tilde{w}, \tilde{t}) \circledast \tilde{M}(S\tilde{z}, A\tilde{z}, \tilde{t}) \circledast \tilde{M}(S\tilde{z}, B\tilde{w}, \tilde{t}) \circledast \tilde{M}(T\tilde{w}, A\tilde{z}, \tilde{t})\} \circledast \left\{\frac{\tilde{M}(S\tilde{z}, T\tilde{w}, \tilde{t}) \circledast \tilde{M}(S\tilde{z}, B\tilde{w}, \tilde{t})}{\tilde{M}(T\tilde{w}, A\tilde{z}, \tilde{t})}\right\} \circledast \{\frac{S\tilde{z}, T\tilde{w}, \tilde{t}) \circledast \tilde{M}(S\tilde{z}, A\tilde{z}, \tilde{t})}{\tilde{M}(T\tilde{w}, A\tilde{z}, \tilde{t})}\}]$$

Taking $n \to \infty$, we get

$$\begin{aligned}\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) \geq\ & r[\ \{\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) \circledast \tilde{M}(A\tilde{z}, A\tilde{z}, \tilde{t}) \circledast \tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) \circledast \tilde{M}(\tilde{z}, A\tilde{z}, \tilde{t})\} \circledast \left\{\frac{\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) \circledast \tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t})}{\tilde{M}(\tilde{z}, A\tilde{z}, \tilde{t})}\right\} \circledast \{\frac{\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) \circledast \tilde{M}(A\tilde{z}, A\tilde{z}, \tilde{t})}{\tilde{M}(\tilde{z}, A\tilde{z}, \tilde{t})}\}] \\ =\ & r\big[\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t})\big] \\ >\ & \tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}),\end{aligned}$$

implies $A\tilde{z} = \tilde{z}$. Thus $A\tilde{z} = \tilde{z} = S\tilde{z}$.

Since $\tilde{w}$ is a coincidence point of $B$ and $T$ and the pair $(B, \mathrm{T})$ is occasionally weakly compatible, so we have,

$$BT\tilde{w} = TB\tilde{w} \Rightarrow B\tilde{z} = T\tilde{z} = \tilde{z}.$$

Hence, $\tilde{z}$ is the common fixed point of $A, B, S$ and $T$.
For Uniqueness,
Let $\tilde{v}$ be another common fixed point of $A, B, S$ and $T$.
Take $\tilde{x} = \tilde{z}$ and $\tilde{y} = \tilde{v}$ in $(3 \cdot 1)$, we get

$$\tilde{M}(A\tilde{z}, B\tilde{v}, \tilde{t}) \geq r[\{\tilde{M}(S\tilde{z}, T\tilde{v}, \tilde{t}) \circledast \tilde{M}(S\tilde{z}, A\tilde{z}, \tilde{t}) \circledast \tilde{M}(S\tilde{z}, B\tilde{v}, \tilde{t}) \circledast$$
$$\tilde{M}(T\tilde{v}, A\tilde{z}, \tilde{t}) \circledast \left\{\frac{\tilde{M}(S\tilde{z}, T\tilde{v}, \tilde{t}) \circledast \tilde{M}(S\tilde{z}, B\tilde{v}, \tilde{t})}{\tilde{M}(T\tilde{v}, A\tilde{z}, \tilde{t})}\right\} \circledast \{\frac{\tilde{M}(S\tilde{z}, T\tilde{v}, \tilde{t}) \circledast \tilde{M}(S\tilde{z}, A\tilde{z}, \tilde{t})}{\tilde{M}(T\tilde{v}, A\tilde{z}, \tilde{t})}\}]$$

Taking $n \to \infty$, we get

$$\begin{aligned}
\tilde{M}(\tilde{z}, \tilde{v}, \tilde{t}) \quad &\geq r[[\tilde{M}(\tilde{z}, \tilde{v}, \tilde{t}) \circledast \tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \circledast \tilde{M}(\tilde{z}, \tilde{v}, \tilde{t}) \circledast \\
&\tilde{M}(\tilde{v}, \tilde{z}, \tilde{t}) \circledast \left\{\frac{\tilde{M}(\tilde{z}, \tilde{v}, \tilde{t}) \circledast \tilde{M}(\tilde{z}, \tilde{v}, \tilde{t})}{\tilde{M}(\tilde{v}, \tilde{z}, \tilde{t})}\right\} \circledast \{\frac{\tilde{M}(\tilde{z}, \tilde{v}, \tilde{t}) \circledast \tilde{M}(\tilde{z}, \tilde{z}, \tilde{t})}{\tilde{M}(\tilde{v}, \tilde{z}, \tilde{t})}\}] \\
&= r\left[\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t})\right] \\
&> \tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}),
\end{aligned}$$

this implies $\tilde{z} = \tilde{v}$, Thus $\tilde{z}$ is the unique common firced point of the mappings $A, B, S$ and $T$.

**Theorem 3.2.** Let $A, B, S$ and $T$ be four self mappings on a soft compact fuzzy metric space $(\tilde{X}, \tilde{M}, \circledast)$ satisfying the following conditions:

iv) The pairs $(A, S)$ and $(B, T)$share the common property $(E \cdot A)$

v) $S(\tilde{X})$ and $T(\tilde{X})$ are closed subsets of $\tilde{X}$

vi) $q(\tilde{M}(A\tilde{x}, B\tilde{y}, \tilde{t}) \geq a\tilde{M}(T\tilde{y}, S\tilde{x}, \tilde{t}) + b\tilde{M}(S\tilde{x}, B\tilde{y}, \tilde{t}) + c\tilde{M}(A\tilde{x}, B\tilde{y}, \tilde{t}) + \{\tilde{M}(A\tilde{x}, S\tilde{x}, \tilde{t}) \circledast \tilde{M}(B\tilde{y}, T\tilde{y}, \tilde{t})\}$

for $all\ \tilde{x}, \tilde{y} \in \tilde{X}, a, b, c \geq 0, q > 0$ and $q < a + b + c + 1$, then each pair $(A, S)$ and $(B, T)$ have a point of coincidence. further, if the pair $(A, S)$ is semi-compatible and $(B, T)$ is occasionally weakly compatible, then $A, B, S$ and $T$ have a unique fixed point.

**Proof.** As the pair $(A, S)$ and $(B, T)$share the common property $(E.A)$, then there exist two sequences $\{\tilde{x}_n\}$ and $\{\tilde{y}_n\}$ in $\tilde{X}$ such that

$$\lim_{n\to\infty} A\tilde{x}_n = \lim_{n\to\infty} S\tilde{x}_n = \lim_{n\to\infty} B\tilde{y}_n = \lim_{n\to\infty} T\tilde{y}_n = \tilde{z},$$

for some $\tilde{z} \in \tilde{X}$.

Since $S(\tilde{X})$ is a closed subset of $\tilde{X}$; therefore, there exists a point $\tilde{u} \in \tilde{X}$such that Su $= \tilde{z}$. Using the above condition (iii), we have.

$$\begin{aligned} q\tilde{M}(A\tilde{u}, B\tilde{y}_n, \tilde{t}) &\geq a\tilde{M}(T\tilde{y}_n, S\tilde{u}, \tilde{t}) + b\tilde{M}(S\tilde{u}, B\tilde{y}_n, \tilde{t}) \\ &+c\tilde{M}(A\tilde{u}, B\tilde{y}_n, \tilde{t}) + \{\tilde{M}(A\tilde{u}, S\tilde{u}, \tilde{t}) * \tilde{M}(B\tilde{y}_n, T\tilde{y}_n, \tilde{t})\} \end{aligned}$$

Taking $n \to \infty$, we obtain

$$\begin{aligned} q\tilde{M}(A\tilde{u}, \tilde{z}, \tilde{t}) &\geq a\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) + b\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \\ &+c\tilde{M}(A\tilde{u}, B\tilde{y}_n, \tilde{t}) + \{\tilde{M}(A\tilde{u}, \tilde{z}, \tilde{t}) * \tilde{M}(\tilde{z}, \tilde{z}, \tilde{t})\} \end{aligned}$$

this gives,

$$\begin{aligned} q\tilde{M}(A\tilde{u}, \tilde{z}, \tilde{t}) &\geq a\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) + b\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) \\ &+c\tilde{M}(A\tilde{u}, \tilde{z}, \tilde{t}) \\ + &\ \tilde{M}(A\tilde{u}, \tilde{z}, \tilde{t}), \end{aligned}$$

$$(q - c - 1)\tilde{M}(A\tilde{u}, \tilde{z}, \tilde{t}) \geq (a + b)\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) +$$

this implies, $\tilde{M}(A\tilde{u}, \tilde{z}, \tilde{t}) > \dfrac{a+b}{q-c-1} > 1.$

for all $\tilde{t} > 0$, this implies, $A\tilde{u} = \tilde{z}$. Hence $A\tilde{u} =$ S$\tilde{u}$, which shows that $\tilde{u}$ is the coincidence point of $(A, S)$.

Again, $T(\tilde{X})$ is closed subset of $\tilde{X}$, therefore there is a point $\tilde{w}$ in $\tilde{X}$ such that $T\tilde{w} = \tilde{z}$. Now take $\tilde{x} = \tilde{x}_n$ and $\tilde{y} = \tilde{w}$ in condition $(ii)$, we get

$$\begin{aligned} &q\tilde{M}(A\tilde{x}_n, B\tilde{w}, \tilde{t}) \geq a\tilde{M}(T\tilde{w}, S\tilde{x}_n, \tilde{t}) + b\tilde{M}(S\tilde{x}_n, B\tilde{w}, \tilde{t}) \\ &+c\tilde{M}(A\tilde{x}_n, B\tilde{w}, \tilde{t}) \\ &+\{\tilde{M}(A\tilde{x}_n, S\tilde{x}_n, \tilde{t}) * \tilde{M}(B\tilde{w}, T\tilde{w}, \tilde{t})\} \end{aligned}$$

Taking $n \to \infty$, we obtain

$$\begin{aligned} q\tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t}) \geq a\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) + b\tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t}) + c\tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t}) \\ + \{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) * \tilde{M}(B\tilde{w}, \tilde{z}, \tilde{t})\} \end{aligned}$$

$$q\tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t}) \geq a\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}) + b\tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t}) + c\tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t}) + \tilde{M}(B\tilde{w}, \tilde{z}, \tilde{t})$$

$$(q\text{-}b\text{-}c\text{-}1)\tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t}) > a\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t})$$

this gives $\tilde{M}(\tilde{z}, B\tilde{w}, \tilde{t}) > \frac{a}{q-b-c-1} > 1$ for all $\tilde{t} > 0$.

This implies $B\tilde{w} = \tilde{z}$. Hence $T\tilde{w} = B\tilde{w} = \tilde{z}$ and thus $\tilde{w}$ is the coincidence point of $(B, T)$.

Further, we assume that $(A, S)$ is a semi-compatible pair, so $\lim_{n\to\infty} AS\tilde{x}_n = S\tilde{z}$ and $\lim_{n\to\infty} AS\tilde{x}_n = A\tilde{z}$. Since the limit in soft compact fuzzy metric cspace is unique, so $S\tilde{z} = A\tilde{z}$.

Now, we claim that $\tilde{z}$ is a common fixed point of the pair $A$ and $S$. using condition (iii), we get

$$q\tilde{M}(A\tilde{z}, B\tilde{w}, \tilde{t}) \geq a\tilde{M}(T\tilde{w}, S\tilde{z}, \tilde{t}) + b\tilde{M}(S\tilde{z}, B\tilde{w}, \tilde{t}) + c\tilde{M}(A\tilde{z}, B\tilde{w}, \tilde{t}) + \{\tilde{M}(A\tilde{z}, S\tilde{z}, \tilde{t}) \circledast \tilde{M}(B\tilde{w}, T\tilde{w}, \tilde{t})\}$$

Taking $n \to \infty$, we obtain

$$q\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) \geq a\tilde{M}(\tilde{z}, A\tilde{z}, \tilde{t}) + bM(A\tilde{z}, \tilde{z}, \tilde{t}) + c\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) + \left\{\tilde{M}(A\tilde{z}, A\tilde{z}, \tilde{t}) \circledast \tilde{M}(\tilde{z}, \tilde{z}, \tilde{t})\right\}$$

$$q\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) \geq a\tilde{M}(\tilde{z}, A\tilde{z}, \tilde{t}) + bM(A\tilde{z}, \tilde{z}, \tilde{t}) + c\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) + \tilde{M}(A\tilde{z}, A\tilde{z}, \tilde{t})$$

$$(q - a - b - c - 1)\tilde{M}(A\tilde{z}, \tilde{z}, \tilde{t}) \geq 1$$

This implies

$$M(A\tilde{z}, \tilde{z}, \tilde{t}) \geq \frac{1}{q - a - b - c - 1} > 1, \text{ for all } \tilde{t} > 0.$$

Thus $A\tilde{z} = \tilde{z}$. Hence $A\tilde{z} = \tilde{z} =$ S $\tilde{z}$. since $\tilde{w}$ is a coincidence point $B$ and $T$, and the pair $(B, T)$ is occasionally weak compatible. So, $BT\tilde{w} = TB\tilde{w}$ this implies $B\tilde{z} = T\tilde{z} = \tilde{z}$. Hence,$\tilde{z}$ is the common fixed point of mappings $A, S, B$ and $T$.

The uniqueness of fined point follows from taking $\tilde{x} = \tilde{z}$ and $\tilde{y} = \tilde{v}$ in condition (iii).

Taking $A = B$ in the above theorem, we get the following corollary:

**Corollary 3.3.** Let $A, S$ and $T$ be three self mappings of a soft compact fuzzy metric space $(\tilde{X}, \tilde{M}, \tilde{*})$, satisfying the following conditions:

iv) The pairs $(A, S)$ and $(A, T)$ share the common property (E.A.)

v) $S(\tilde{X})$ and $T(\tilde{X})$ are closed subsets of $\tilde{X}$

vi) $q\tilde{M}(A\tilde{x}, A\tilde{y}, \tilde{t}) \geq a\tilde{M}(T\tilde{y}, S\tilde{x}, \tilde{t}) + b\tilde{M}(S\tilde{x}, A\tilde{y}, \tilde{t}) + c\tilde{M}(A\tilde{x}, A\tilde{y}, \tilde{t})$
$+\{\tilde{M}(A\tilde{x}, S\tilde{x}, \tilde{t}) \tilde{*} \tilde{M}(A\tilde{y}, T\tilde{y}, \tilde{t})\}$
for all $\tilde{x}, \tilde{y} \in \tilde{X}, a, b, c \geq 0, q > 0$ and $q < a + b + c + 1$, then the pairs $(A, S)$ and $(A, T)$ have a point of coincidence.
Further, if the pair $(A, S)$ is semi-compaitible and $(A, T)$ is occasionally weakly compatible then $A, S$ and $T$ have a unique common fixed point.

## 4. Applications

**Theorem 4.1.** Let $A, B, S$ and $T$ be self-mappings on a soft compact fuzzy metric space $(\tilde{X}, \tilde{M}, \tilde{*})$ satisfying the condition

$$\int_0^{\tilde{M}(A\tilde{x}, B\tilde{y}, \tilde{t})} \phi(t)dt \geqslant \int_0^{r[\tilde{m}(\tilde{x}, \tilde{y}, \tilde{t})]} \phi(t)dt \qquad \text{.... (4.1)}$$

where $\phi: R^+ \longrightarrow R^+$is a Lebesgue-integrable mapping which is summable, non-negative such that $\int_0^\varepsilon \phi(t)dt > 0$ for each $\varepsilon > 0$, and

$$\tilde{m}(\tilde{x}, \tilde{y}, \tilde{t}) = \min\{(1 + \tilde{M}(S\tilde{x}, T\tilde{y}, \tilde{t}))^2, (1 + \tilde{M}(S\tilde{x}, A\tilde{x}, \tilde{t}))^2, (1 + \tilde{M}(S\tilde{x}, B\tilde{y}, \tilde{t}))^2, (1 + \tilde{M}(T\tilde{y}, A\tilde{x}, \tilde{t}))^2\}$$

for all $\tilde{x}, \tilde{y} \in \tilde{X}$ and $r: [0,1] \rightarrow [01]$ is continuous function such that $r(\tilde{t}) > \tilde{t}$ for all $\tilde{t} < 1$ and $r(\tilde{t}) = 1$ for $\tilde{t} = 1$, also suppose that the pairs $(A, S)$ and $(B, T)$ share the common property $(E \cdot A)$ and $S(\tilde{X})$ and $T(\tilde{X})$ are closed subset of $\tilde{X}$. Then the pair $(A, S)$ as well as $(B, T)$ have a coincidence point. Further, if $A, B, S$ and $T$ have a unique common fixed point provided the pair $(A, S)$ is semi-compatible and $(B, T)$ is occasionally weakly compatible.

**Proof.** Since the pair $(A, S)$ and $(B, T)$ share The common property (E. A) then there exist two sequences $\{\tilde{x}_n\}$ and $\{\tilde{y}_n\}$ in $\tilde{X}$such that $\lim_{n\to\infty} A\tilde{x}_n = \lim_{n\to\infty} S\tilde{x}_n = \lim_{n\to\infty} B\tilde{y}_n = \lim_{n\to\infty} T\tilde{y}_n = \tilde{z}$ for some $\tilde{z} \in \tilde{X}$. Since $S(\tilde{X})$ is closed subset of $\tilde{X}$, then $\lim_{n\to\infty} S\tilde{x}_n = \tilde{z} \in S(\tilde{X})$, therefore there is a point $\tilde{u}$ in $\tilde{X}$ such that $S\tilde{u} = \tilde{z}$. we claim that $A\tilde{u} = \tilde{z}$. If not then by using (4.1), we nave

$$\int_0^{\tilde{M}(A\tilde{u}, B\tilde{y}_n, \tilde{t})} \phi(t)dt \geq \int_0^{r[\tilde{m}(\tilde{u}, \tilde{y}_n, \tilde{t})]} \phi(t)dt$$

Now,

$$r[\tilde{m}(\tilde{u}, \tilde{y}_n, \tilde{t})] = r\left[\min\left\{\left(1 + \tilde{M}(S\tilde{u}, T\tilde{y}_n, \tilde{t})\right)^2, (1 + \tilde{M}(S\tilde{u}, A\tilde{u}, \tilde{t}))^2 \quad \left(1 + \tilde{M}(S\tilde{u}, B\tilde{y}_n, \tilde{t})\right)^2, \left(1 + \tilde{M}(T\tilde{y}_n, A\tilde{u}, \tilde{t})\right)^2\right\}\right]$$

$$\geqslant r\left[\min\{\tilde{M}^2(S\tilde{u}, T\tilde{y}_n, \tilde{t}),\ \tilde{M}^2(S\tilde{u}, A\tilde{u}, \tilde{t}),\ M^2(S\tilde{u}, B\tilde{y}_n, \tilde{t}),\ \tilde{M}^2(T\tilde{y}_n, A\tilde{u}, \tilde{t})\}\right]$$

$$\geqslant r\left[\min\{\tilde{M}(S\tilde{u}, T\tilde{y}_n, \tilde{t}), \tilde{M}(S\tilde{u}, A\tilde{u}, \tilde{t}), \tilde{M}(S\tilde{u}, B\tilde{y}_n, \tilde{t}), \tilde{M}(T\tilde{y}_n, A\tilde{u}, \tilde{t})\}\right]$$

Taking $n \to \infty$, we obtain

$$\begin{aligned} r[\tilde{m}(\tilde{u}, \tilde{z}, \tilde{t})] &\geq r[\min\{\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}), \tilde{M}(\tilde{z}, A\tilde{u}, \tilde{t}), \\ &\tilde{M}(\tilde{z}, \tilde{z}, \tilde{t}), \tilde{M}(\tilde{z}, A\tilde{u}, \tilde{t})\}] \\ &= r\left[\tilde{M}(A\tilde{u}, \tilde{z}, \tilde{t})\right] \\ &> \tilde{M}(A\tilde{u}, \tilde{z}, \tilde{t}) \end{aligned}$$

Now from

$$\int_0^{\tilde{M}(A\tilde{u},B\tilde{y}_n,\tilde{t})} \phi(t)dt \geqslant \int_0^{r[\tilde{m}(\tilde{u},\tilde{y}_n,\tilde{t})]} \phi(t)dt$$

Taking

$$\lim_{n\to\infty} \int_0^{\tilde{M}((A\tilde{u},\tilde{z},\tilde{t})} \phi(t)dt \geqslant \int_0^{r[\tilde{m}(\tilde{u},\tilde{z},\tilde{t})]} \phi(t)dt$$

or $\int_0^{\tilde{M}(A\tilde{u},\tilde{z},\tilde{t})} \phi(t)dt \geqslant \int_0^{\tilde{M}(A\tilde{u},\tilde{z},\tilde{t})} \phi(t)dt$

this is contradiction, this implies $A\tilde{u} = \tilde{z}$

Hence $A\tilde{u} = S\tilde{u}$ or $\tilde{u}$ is a coincidence point of the pair $(A, S)$. But $T(\tilde{X}\ )$ is closed subset of $\tilde{X}$ , then $\lim_{n\to\infty} T\tilde{y}_n = \tilde{z} \in T(\tilde{X}\ )$, therefore there exists $\tilde{w} \in \tilde{X}$ such that $T\tilde{w} = \tilde{z}$.

using $(4 \cdot 1)$, we obtain

$$\int_0^{\tilde{M}(A\tilde{x}_n,B\tilde{w},\tilde{t})} \phi(t)dt \geqslant \int_0^{r[\tilde{m}(\tilde{x}_n,\tilde{w},\tilde{t})]} \phi(t)dt$$

$$\int_0^{\tilde{M}(\tilde{z},zB\tilde{w},\tilde{t})} \phi(t)dt \geqslant \int_0^{r[\tilde{m}(\tilde{z},\tilde{w},\tilde{t})]} \phi(t)dt$$

or

Now,

$$r[\tilde{m}(\tilde{x}_n,\tilde{w},\tilde{t})] = r \quad \Big[\min\Big\{\Big(1 + \tilde{M}(S\tilde{x}_n, T\tilde{w}, \tilde{t})\Big)^2, \Big(1 + \tilde{M}(S\tilde{x}_n, A\tilde{x}_n, \tilde{x})\Big)^2, (1 + \tilde{M}(S\tilde{x}_n, B\tilde{w}, \tilde{t})]^2, \Big(1 + \tilde{M}(T\tilde{w}, A\tilde{x}_n, \tilde{x})\Big)^2\Big\}\Big]$$

Taking $n \to \infty$, we obtain

$$\begin{aligned} r[\tilde{m}(\tilde{z},\tilde{w},\tilde{t})] = r \quad & [\min\{(1+\tilde{M}(\tilde{z},\tilde{z},\tilde{t}))^2, \\ & (1+\tilde{M}(\tilde{z},\tilde{z},\tilde{t}))^2, (1+\tilde{M}(\tilde{z},B\tilde{w},\tilde{t}))^2, \\ & (1+\tilde{M}(\tilde{z},\tilde{z},\tilde{t}))^2\}] \\ \geqslant \quad & r[\min\{\tilde{M}^2(\tilde{z},\tilde{z},\tilde{t}), \tilde{M}^2(\tilde{z},\tilde{z},\tilde{t}), \tilde{M}^2(\tilde{z},B\tilde{w},\tilde{t}), \\ & \tilde{M}^2(\tilde{z},\tilde{z},\tilde{t})\}] \\ \geqslant \quad & r[\min\{\tilde{M}(\tilde{z},\tilde{z},\tilde{t}), M(\tilde{z},\tilde{z},\tilde{t}), \tilde{M}(\tilde{z},B\tilde{w},\tilde{t}), \\ & \tilde{M}(\tilde{z},\tilde{z},\tilde{t})\}] \\ = \quad & r[\tilde{M}(\tilde{z},B\tilde{w},\tilde{t})] \\ > \quad & \tilde{M}(\tilde{z},B\tilde{w},\tilde{t}) \end{aligned}$$

So, $\int_0^{\tilde{M}(\tilde{z},B\tilde{w},\tilde{t})} \phi(t)dt \geqslant \int_0^{r[\tilde{m}(\tilde{z},\tilde{w},\tilde{t})]} \phi(t)dt \geqslant \int_0^{\tilde{M}(\tilde{z},B\tilde{w},\tilde{t})} \phi(t)dt$

This gives $B\tilde{w} = \tilde{z}$. Hence $T\tilde{w} = B\tilde{w} = \tilde{z}$ or $\tilde{w}$ is a coincidence point of the pair (B, $T$ ). Also $(A,S)$ is a semi-compatible pair, So, $\lim_{n\to\infty} AS\tilde{x}_n = S\tilde{z}$ and $\lim_{n\to\infty} AS\tilde{x}_n = A\tilde{z}$. Since the limit in soft compact fuzzy metric space is unique, therefore $S\tilde{z} = A\tilde{z}$.

we claim that $\tilde{z}$ is common fixed point of the pair $(A,S)$.

from $(4 \cdot 1)$, we have $\int_0^{\tilde{M}(A\tilde{z},A\tilde{w},\tilde{t})} \phi(t)dt \geqslant \int_0^{r[\tilde{m}(\tilde{z},\tilde{w},\tilde{t})]} \phi(t)dt$ or $\int_0^{\tilde{M}(A\tilde{z},\tilde{z},\tilde{t})} \phi(t)dt \geqslant \int_0^{r[\tilde{m}(\tilde{z},\tilde{w},\tilde{t})]} \phi(t)dt$

where,

$$\begin{aligned} r[\tilde{m}(\tilde{z},\tilde{w},\tilde{t})] \quad & = r[\min\{(1+\tilde{M}(A\tilde{z},\tilde{z},\tilde{t}))^2, \\ & (1+\tilde{M}(A\tilde{z},A\tilde{z},\tilde{t}))^2, (1+\tilde{M}(A\tilde{z},\tilde{z},\tilde{t}))^2, \\ & (1+\tilde{M}(\tilde{z},A\tilde{z},\tilde{t}))^2\}] \end{aligned}$$

$$\begin{aligned} r[\tilde{m}(\tilde{z},\tilde{w},\tilde{t})] \geqslant \quad & r[\min\{\tilde{M}^2(A\tilde{z},\tilde{z},\tilde{t}), \tilde{M}^2(A\tilde{z},A\tilde{z},\tilde{t}) \\ & \tilde{M}^2(A\tilde{z},\tilde{z},\tilde{t}), M^2(\tilde{z},A\tilde{z},t)\}] \\ \geqslant \quad & r[\min\{\tilde{M}(A\tilde{z},\tilde{z},\tilde{t}), \tilde{M}(A\tilde{z},A\tilde{z},\tilde{t}), \\ & \tilde{M}(A\tilde{z},\tilde{z},\tilde{t}), \tilde{M}(\tilde{z},A\tilde{z},\tilde{t})\}] \\ = \quad & r[\tilde{M}(A\tilde{z},\tilde{z},\tilde{t})] \\ > \quad & \tilde{M}(A\tilde{z},\tilde{z},\tilde{t}) \end{aligned}$$

So,

$$\int_0^{\tilde{M}(A\tilde{z},\tilde{z},\tilde{t})} \phi(t)dt \geqslant \int_0^{r[\tilde{m}(\tilde{z},\tilde{w},\tilde{t})]} \phi(t)dt$$
$$\geqslant \int_0^{\tilde{M}(A\tilde{z},\tilde{z},\tilde{t})} \phi(t)dt$$

this implies $A\tilde{z} = \tilde{z}$ and hence $A\tilde{z} = \tilde{z} = S\tilde{z}$. Since $\tilde{w}$ is a coincidence point of $B$ and $T$, and the pair $(B, T)$ is occasionally weakly compatible, so we have $BT\tilde{w} = TB\tilde{w}$

$$\Rightarrow B\tilde{z} = T\tilde{z} = \tilde{z}$$

Hence $\tilde{z}$ is the common fixed point of $A, S, B$ and $T$.

For uniqueness, let $\tilde{v}$ be another common fixed point of $A, B, S$ and $T$. Take $\tilde{x} = \tilde{z}$ and $\tilde{y} = \tilde{v}$ in (4.1), we get

$$\int_0^{\tilde{M}(A\tilde{z},B\tilde{v},\tilde{t})} \phi(t)dt \geqslant \int_0^{r[\tilde{m}(\tilde{z},\tilde{v},\tilde{t})]} \phi(t)dt$$
$$\text{or } \int_0^{\tilde{M}(\tilde{z},\tilde{v},\tilde{t})} \phi(t)dt \geqslant \int_0^{r[\tilde{m}(\tilde{z},\tilde{v},\tilde{t})]} \phi(t)dt$$

where

$$\begin{aligned} r[\tilde{m}(\tilde{z},\tilde{v},\tilde{t})] &= r\big[\min\big\{(1+\tilde{M}(S\tilde{z},T\tilde{v},\tilde{t}))^2, (1+\tilde{M}(S\tilde{z},A\tilde{z},t))^2 \\ &\quad (1+\tilde{M}(S\tilde{z},B\tilde{v},\tilde{t}))^2, (1+\tilde{M}(T\tilde{v};A\tilde{z},\tilde{t}))^2\big\}\big] \end{aligned}$$

$$\begin{aligned} &= r\big[\min\big\{\tilde{M}^2(S\tilde{z},T\tilde{v},\tilde{t}), \tilde{M}^2(S\tilde{z},A\tilde{z},\tilde{t}), \tilde{M}^2\big(\tilde{S}z,B\tilde{v},\tilde{t}\big) \\ &\quad \tilde{M}^2(T\tilde{v},A\tilde{z},\tilde{t})\big\}\big] \\ &= r[\min\{\tilde{M}(\tilde{z},T\tilde{v},\tilde{t}), \tilde{M}(S\tilde{z},A\tilde{z},\tilde{t}), \\ &\quad \tilde{M}(S\tilde{z},B\tilde{v},\tilde{t}), \tilde{M}(T\tilde{v},A\tilde{z},\tilde{t})\}] \\ &= r[\min\{\tilde{M}(\tilde{z},\tilde{v},\tilde{t}), \tilde{M}(\tilde{z},\tilde{z},\tilde{t}), \tilde{M}(\tilde{z},\tilde{v},\tilde{t}), \\ &\quad \tilde{M}(\tilde{v},\tilde{z},\tilde{t})\}] \\ &= r[\tilde{m}(\tilde{z},\tilde{v},\tilde{t})] \\ &> \tilde{M}(\tilde{z},\tilde{v},\tilde{t}) \end{aligned}$$

So, $\int_0^{\tilde{M}(\tilde{z},\tilde{v},\tilde{t})}\phi(t)dt \geqslant \int_0^{r[\tilde{m}(\tilde{z},\tilde{v},\tilde{t})]}\phi(t)dt$

$$> \int_0^{\tilde{M}(\tilde{z},\tilde{v},\tilde{t})}\phi(t)dt$$

This implies $\tilde{z}=\tilde{v}$ and thus the unique common fixed point of the mappings $A, B, S$, and $T$.

**Theorem 4.2.** Let $A, B, S$ and $T$ be four self mappings on soft compact fuzzy metric space $(\tilde{X}, \tilde{M}, \tilde{*})$ satisfing the following conditions:

1 The pair $(A, S)$ and $(B, T)$ share the common property (E.A.);

2 $S(\tilde{X})$ and $T(\tilde{X})$ are closed subsets of $\tilde{X}$;

3 $q\int_0^{\tilde{M}(A\tilde{x},B\tilde{y},\tilde{t})}\phi(t)dt \geqslant a\int_0^{\tilde{M}(T\tilde{y},S\tilde{x},\tilde{t})}\phi(t)dt$

$$+b\int_0^{\tilde{M}(S\tilde{x},B\tilde{y},\tilde{t})}\phi(t)dt + c\int_0^{\tilde{M}(A\tilde{x},B\tilde{y},\tilde{t})}\phi(t)dt$$

$$+\int_0^{\max\{\tilde{M}(A\tilde{x},S\tilde{x},\tilde{t}),\tilde{M}(B\tilde{y},T\tilde{y},\tilde{t})\}}\phi(t)dt$$

for $all\ \tilde{x},\tilde{y} \in \tilde{X}, a, b, c \geqslant 0, q > 0$ and $q < a+b+c$,

then the pair (A, S) and (B, T) have a point of coincidence each. Further if the pair (A, S) and (B,T) is semi-compatible and (B,T) is occasionally weakly compatible, then A, B,S and T have a unique common fixed point.

**Proof.** The proof follows from theorem 4.1.

**Conclusion:** Our outcome gives a new direction to generalized the concept of coincidence point and useful to future examination of coincidence point hypothesis in soft compact fuzzy metric spaces. some applications are given in support of our result.

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