International Electronic Journal of Algebra
Volume 18 (2015) 57-71

ON S-SEMISTAR-NOETHERIAN DOMAINS

Afsaneh Esmaeelnezhad and Parviz Sahandi

Received: 16 December 2014; Revised: 9 April 2015
Communicated by Burcu Üngör

Abstract. Let D be an integral domain and S be a multiplicative subset of D. Then given a semistar operation $\star$ on D, we introduced the S-$\tilde{\star}$-Noetherian domains, where $\tilde{\star}$ is the stable semistar operation of finite type associated to $\star$. Among other things, we provide many different characterization for S-$\tilde{\star}$-Noetherian domains by focusing on primary decomposition, *weak Bourbaki* associated primes and *Zariski-Samuel* associated primes of the S-saturation of a given quasi-$\tilde{\star}$-ideal I of D.

Mathematics Subject Classification (2010): 13A15, 13F05, 13G05
Keywords: Semistar operation, Noetherian domain, S-Noetherian domain, associated primes

1. Introduction

In [2], the authors introduced the concept of "almost finitely generated" to study Querré's characterization of divisorial ideals in integrally closed polynomial rings. Then Anderson and Dumitrescu [1], introduced a general concept of S-Noetherian rings. Let R denote a commutative ring with identity and S a multiplicative subset of R. Then R is called an *S-Noetherian* ring if every ideal of R is S-finite, in the sense that there exist an $s \in S$ and some finitely generated subideal J of I such that $sI \subseteq J$. Hence the authors of [1], proved several different results which also generalize well-known results on Noetherian rings. Recently in [13], the authors extended the concept of S-Noetherian domains to that of S-strong Mori domain, which is at the same time a generalization of strong Mori domains. An integral domain D is said to be *S-strong Mori domain*, if for each nonzero subideal I of D, there exist an $s \in S$ and a finitely generated ideal J of D such that $sI \subseteq J^w \subseteq I^w$, where w is the w-operation on D.

Semistar operations over an integral domain were introduced in 1994 by Okabe and Matsuda [14], to generalize the concept of star operation (in the sense of [10, Section 32]). Many classical properties of integral domains were generalized to the

case of a general semistar operation. El Baghdadi, Fontana and Picozza [5], considered the *Semistar-Noetherian domains*, i.e. those domains in which the Ascending Chain Condition holds for quasi-semistar-ideals, in order to introduce the notion of semistar Dedekind domains. Picozza [15], investigated semistar-Noetherian domains in the case of stable semistar operation, that is, semistar operations distributing over finite intersections, and proved many results which generalize classical theorems of the theory of Noetherian domains. A more accurate review has been done by Fusacchia [9], to extend the work of Heinzer and Ohm [11], on locally Noetherian rings to semistar-Noetherian domains, with respect to a stable semistar operation of finite type.

Throughout D is an integral domain with quotient field K and $S \subseteq D$ is a multiplicative set.

The goal of this note is to introduce S-semistar-Noetherian (S-$\star$-Noetherian) domains. We focus on the case where the semistar operation $\star$ is stable and of finite type. We study some basic property of S-$\tilde{\star}$-Noetherian domains and state the relation between an S-$\tilde{\star}$-Noetherian domain and an S-Noetherian domain. Moreover, we prove the S-$\tilde{\star}$-Noetherian version of Cohen's Theorem, that states that a domain is Noetherian if and only if each prime ideal is finitely generated [12, Theorem 8], and provide many different characterizations of S-$\tilde{\star}$-Noetherian domains that use weak Bourbaki associated primes and Zariski-Samuel associated primes, which is the main goal of this work. Also we prove a Hilbert basis Theorem for S-$\tilde{\star}$-Noetherian domains and then we show that if S is anti-archimedean, then D is S-$\tilde{\star}$-Noetherian if and only if the $\star$-Nagata domain of D is S-Noetherian.

We first review some basic background on semistar operations. Let $\overline{\mathcal{F}}(D)$ be the set of all nonzero D-submodule of K, $\mathcal{F}(D)$ be the set of all nonzero fractional ideals of D, and $f(D)$ be the set of all nonzero finitely generated fractional ideals of D. Obviously, $f(D) \subseteq \mathcal{F}(D) \subseteq \overline{\mathcal{F}}(D)$. As in [14], a semistar operation on D is a map $\star : \overline{\mathcal{F}}(D) \to \overline{\mathcal{F}}(D)$, $E \mapsto E^\star$, such that, for all $0 \neq x \in K$ and $E, F \in \overline{\mathcal{F}}(D)$, the following properties are satisfied

$\star_1$: $(xE)^\star = xE^\star$;
$\star_2$: $E \subseteq F$ implies that $E^\star \subseteq F^\star$;
$\star_3$: $E \subseteq E^\star$ and $E^{\star\star} := (E^\star)^\star = E^\star$.

Given a semistar operation $\star$ and an integral ideal I of D, we call the integral ideal $I^\star \cap D$ the quasi-$\star$-closure of I, and say that I is a quasi-$\star$-ideal, if $I = I^\star \cap D$. A quasi-$\star$-prime ideal of D is a prime ideal of D which is quasi-$\star$-ideal and a quasi-$\star$-maximal ideal of D is an ideal of D which is maximal in the set of all proper

quasi-$\star$-ideals of D. We denote by $\mathrm{QMax}^{\star}(D)$ (resp., $\mathrm{QSpec}^{\star}(D)$) the set of all quasi-$\star$-maximal ideals of D (resp., quasi-$\star$-prime ideals).

If $\star_1$ and $\star_2$ are two semistar operations on D, one says that $\star_1 \leq \star_2$ if $E^{\star_1} \subseteq E^{\star_2}$ for each $E \in \overline{\mathcal{F}}(D)$. This is equivalent to say that $(E^{\star_1})^{\star_2} = (E^{\star_2})^{\star_1} = E^{\star_2}$ for each $E \in \overline{\mathcal{F}}(D)$. Let d_D denote the identity (semi)star operation on D. Clearly, $d_D \leq \star$ for all semistar operation $\star$ on D. Let $\star$ be a semistar operation on D. For every $E \in \overline{\mathcal{F}}(D)$, set

$$E^{\star_f} = \bigcup \{F^{\star} | F \subseteq E, F \in f(D)\}.$$

It is easy to see that $\star_f$ is a semistar operation on D, and $\star_f$ is called the *semistar operation of finite type associated to* $\star$. Note that $(\star_f)_f = \star_f$. A semistar operation $\star$ is said to be of finite type if $\star = \star_f$; in particular $\star_f$ is of finite type.

A semistar operation on D is said to be stable if $(E \cap F)^{\star} = E^{\star} \cap F^{\star}$ for every $E, F \in \overline{\mathcal{F}}(D)$. Starting from a semistar operation $\star$, it is possible to define the following associated semistar operation:

$$E^{\tilde{\star}} = \bigcup \{(E : J) | J \in \mathcal{F}^{\star} \cap f(D)\},$$

where $\mathcal{F}^{\star}$ denotes the set of ideals J of D such that $J^{\star} = D^{\star}$. It is known that $\tilde{\star}$ is a stable semistar operation, with $\tilde{\star} \leq \star$. A semistar operation $\star$ is stable and of finite type if and only if $\star = \tilde{\star}$ (see [6, Theorem 2.10 and Corollary 3.9]).

If Δ is a nonempty subset of the prime spectrum $\mathrm{Spec}(D)$ of D, we can define a semistar operation $\star_\Delta$ on D, defined by $E^{\star_\Delta} = \bigcap_{P \in \Delta} ED_P$ for each $E \in \overline{\mathcal{F}}(D)$. A semistar operation $\star$ on D is said to be spectral if there exists $\Delta \subseteq \mathrm{Spec}(D)$ such that $\star = \star_\Delta$. Then $\star$ is stable. Finally, we recall that given a semistar operation $\star$, the induced operation $\tilde{\star}$ is always spectral, defined by the family $\Delta := \mathrm{QMax}^{\star_f}(D)$; i.e.,

$$E^{\tilde{\star}} := \bigcap \{ED_P | P \in \mathrm{QMax}^{\star_f}(D)\}, \text{ for each } E \in \overline{\mathcal{F}}(D).$$

Let X be an indeterminante over K. For each $f \in K[X]$, we let $c_D(f)$ denote the content ideal of the polynomial f, i.e., the (fractional) ideal of D generated by the coefficients of f. Let $\star$ be a semistar operation on D. If $N_\star := \{g \in D[X] | g \neq 0 \text{ and } c_D(g)^{\star} = D^{\star}\}$, then $N_\star = D[X] \setminus \bigcup \{P[X] | P \in \mathrm{QMax}^{\tilde{\star}}(D)\}$ is a saturated multiplicative subset of $D[X]$. The ring of fractions

$$\mathrm{Na}(D, \star) := D[X]_{N_\star}$$

is called the $\star$-*Nagata domain* (of D with respect to the semistar operation $\star$).

The most widely studied semistar operations on D have been the identity d, v, $t := v_f$, and $w := \widetilde{v}$ operations, where $E^v := (E^{-1})^{-1}$, with $E^{-1} := (D : E) := \{x \in K | xE \subseteq R\}$, for $E \in \overline{\mathcal{F}}(D)$.

2. S-$\tilde{\star}$-Noetherian domains

Throughout this section $\star$ is a semistar operation on D.

Definition 2.1. An integral domain D is said to be *S-$\star$-Noetherian*, in the case every nonzero ideal I of D is S-$\star$-finite, in the sense that there exist an $s \in S$ and a finitely generated ideal J of D such that $sI \subseteq J^\star \subseteq I^\star$.

It is clear that if $\star = d_D$, then an S-$\star$-Noetherian domain is just an S-Noetherian domain and if $\star = w$, then an S-$\star$-Noetherian domain is an S-strong Mori domain. Assume that $\star_1 \leq \star_2$ are two semistar operations on D. Then it is easy to see that if D is an S-$\star_1$-Noetherian, then D is an S-$\star_2$-Noetherian. In particular every S-Noetherian domain is an S-$\star$-Noetherian domain for every semistar operation $\star$ on D. If $\star$ is a (semi)star operation (that is if $D^\star = D$) and if D is an S-$\tilde{\star}$-Noetherian domain, then D is an S-strong Mori domain.

Remark 2.2. (1) *Let I be an S-$\star_f$-finite ideal of D. In the same way as in the proof of* [8, Lemma 2.3]*, we can choose a finitely generated subideal J of I such that $sI \subseteq J^\star \subseteq I^{\star_f}$ for some $s \in S$.*

(2) *Let $\overline{S}$ be the saturation of S. Then by the same argument of* [13, Proposition 1.3(2)]*, D is an S-$\star$-Noetherian domain if and only if D is an $\overline{S}$-$\star$-Noetherian domain.*

The following proposition shows the relation between an S-$\tilde{\star}$-Noetherian domain and an S-Noetherian domain. Recall that D has $\star$-finite character if every element of D is contained in finitely many element of $\mathrm{QMax}^{\tilde{\star}}(D)$.

Proposition 2.3. *The following assertions hold.*

(1) *If D is an S-$\tilde{\star}$-Noetherian domain, then D_P is S-Noetherian for each $P \in \mathrm{QSpec}^{\tilde{\star}}(D)$.*

(2) *Suppose that D has $\star$-finite character and D_M is an S-Noetherian domain for each $M \in \mathrm{QMax}^{\tilde{\star}}(D)$. Then D is S-$\tilde{\star}$-Noetherian.*

Proof. (1) Assume that D is an S-$\tilde{\star}$-Noetherian domain, and let $P \in \mathrm{QSpec}^{\tilde{\star}}(D)$. Let I be a non-zero ideal of D. If $ID_P = D_P$, then for every $s \in S$ we have $sID_P \subseteq sD_P \subseteq ID_P$. So we may assume that ID_P is a proper ideal of D_P. Since D is an S-$\tilde{\star}$-Noetherian domain, there exist an $s \in S$ and a finitely generated subideal

J of I such that $sI \subseteq J^{\tilde{\star}} \subseteq I^{\tilde{\star}}$. By [6, Lemma 4.1(2)] we have $J^{\tilde{\star}}D_P = JD_P$ and $I^{\tilde{\star}}D_P = ID_P$. Thus we have $sID_P \subseteq J^{\tilde{\star}}D_P = JD_P \subseteq I^{\tilde{\star}}D_P = ID_P$, which shows that ID_P is S-finite. So D_P is an S-Noetherian domain.

(2) Let I be a nonzero ideal of D and $0 \neq a \in I$. Then a is contained in only finitely many quasi-$\tilde{\star}$-maximal ideal of D, say $M_1, \cdots, M_n$. Fix an $i \in \{1, \cdots, n\}$. Since D_{M_i} is S-Noetherian, there exist $s_i \in S$ and a finitely generated subideal J_i of I such that $s_i ID_{M_i} \subseteq J_i D_{M_i}$. Let $s = s_1 \cdots s_n$ and set $J = (a) + J_1 + \cdots + J_n$. Then $sID_{M_i} \subseteq JD_{M_i}$ for every $1 \leqslant i \leqslant n$. Now suppose that $M' \in \mathrm{QMax}^{\tilde{\star}}(D)$ and $M' \neq M_i$ for each $1 \leqslant i \leqslant n$. Thus a is a unit element of $D_{M'}$ so that $ID_{M'} = D_{M'} = JD_{M'}$. Therefore for each $M \in \mathrm{QMax}^{\tilde{\star}}(D)$, $sID_M \subseteq JD_M$. Hence we have

$$sI^{\tilde{\star}} = s(\bigcap_{M \in \mathrm{QMax}^{\tilde{\star}}(D)} ID_M) \subseteq \bigcap_{M \in \mathrm{QMax}^{\tilde{\star}}(D)} sID_M \subseteq \bigcap_{M \in \mathrm{QMax}^{\tilde{\star}}(D)} JD_M = J^{\tilde{\star}}.$$

Note that J is a finitely generated subideal of I. Hence $sI \subseteq J^{\tilde{\star}} \subseteq I^{\tilde{\star}}$. Thus D is an S-$\tilde{\star}$-Noetherian domain. □

In the next theorem we prove the S-$\tilde{\star}$-Noetherian analogue of Cohen's Theorem. We need the following lemma.

Lemma 2.4. *Let I be an ideal of D which is maximal among non-S-$\tilde{\star}$-finite ideals of D. Then I is a prime ideal of D with $I \cap S = \emptyset$.*

Proof. Assume in the contrary that I is not prime. Then there exist $a, b \in D \backslash I$ such that $ab \in I$. By maximality of I, (I, a) is S-$\tilde{\star}$-finite. Hence there exists an $s \in S$ such that $s(I, a) \subseteq (r_1 + ad_1, \cdots, r_l + ad_l)^{\tilde{\star}}$ for some $r_1, \cdots, r_l \in I$ and some $d_1, \cdots, d_l \in D$. On the other hand $(I :_D a)$ is an S-$\tilde{\star}$-finite ideals of D, since it contains I and b. Hence there is $t \in S$ such that $t(I :_D a) \subseteq (q_1, \cdots, q_n)^{\tilde{\star}}$ for some $q_1, \cdots, q_n \in (I :_D a)$. Let $x \in I$. Then $sx \in (r_1 + ad_1, \cdots, r_l + ad_l)^{\tilde{\star}}$, and there exists a finitely generated ideal J_1 of D such that $J_1^{\star} = D^{\star}$ and $sxJ_1 \subseteq (r_1 + ad_1, \cdots, r_l + ad_l)$. Write $J_1 = (j_1, \cdots, j_p)$. Then for each $i = 1, \cdots, p$, there exist some $u_{i1}, \cdots, u_{il} \in D$ such that

$$sxj_i = u_{i1}(r_1 + ad_1) + \cdots + u_{il}(r_l + ad_l).$$

Hence we have,

$$\begin{aligned} sxJ_1 =& (\{u_{i1}(r_1 + ad_1) + \cdots + u_{il}(r_l + ad_l) | i = 1, \cdots, p\}) \\ =& (\{(u_{i1}r_1 + \cdots + u_{il}r_l) + a(u_{i1}d_1 + \cdots + u_{il}d_l) | i = 1, \cdots, p\}). \end{aligned}$$

Therefore $(\{u_{i1}d_1 + \cdots + u_{il}d_l | i = 1, \cdots, p\}) \subseteq (I :_D a)$. Thus

$$t(\{u_{i1}d_1 + \cdots + u_{il}d_l | i = 1, \cdots, p\}) \subseteq t(I :_D a) \subseteq (q_1, \cdots, q_n)^{\tilde{\star}}.$$

Hence there exists a finitely generated ideal J_2 of D such that $J_2^{\star} = D^{\star}$ and

$$t(\{u_{i1}d_1 + \cdots + u_{il}d_l | i = 1, \cdots, p\})J_2 \subseteq (q_1, \cdots, q_n)$$

Write $J_2 = (e_1, \cdots, e_m)$. Then for each $k = 1, \cdots, m$ and $i = 1, \cdots, p$, we have

$$te_k(u_{i1}d_1 + \cdots + u_{il}d_l) = v_{ik1}q_1 + \cdots + v_{ikn}q_n$$

for some $v_{ik1}, \cdots, v_{ikn} \in D$. Therefore

$$tJ_2(\{u_{i1}d_1 + \cdots + u_{il}d_l | i = 1, \cdots, p\})$$

$$=(\{v_{ik1}q_1 + \cdots + v_{ikn}q_n | i = 1, \cdots, p \text{ and } k = 1, \cdots, m\}).$$

Thus we obtain

$$\begin{aligned} stxJ_1J_2 =& (\{(u_{i1}r_1 + \cdots + u_{il}r_l) + a(u_{i1}d_1 + \cdots + u_{il}d_l) | i = 1, \cdots, p\})tJ_2 \\ \subseteq& (\{(u_{i1}r_1 + \cdots + u_{il}r_l) | i = 1, \cdots, p\})tJ_2 + (\{a(u_{i1}d_1 + \cdots + u_{il}d_l) | i = 1, \cdots, p\})tJ_2 \\ \subseteq& (tr_1, \cdots, tr_l, aq_1, \cdots, aq_n). \end{aligned}$$

Since x is arbitrary,

$$stIJ_1J_2 \subseteq (tr_1, \cdots, tr_l, aq_1, \cdots, aq_n) \subseteq I.$$

Hence $stI \subseteq (tr_1, \cdots, tr_l, aq_1, \cdots, aq_n)^{\tilde{\star}} \subseteq I^{\tilde{\star}}$. This indicate that I is S-$\tilde{\star}$-finite, which is a contradiction. Finally we show that $I \cap S = \emptyset$. If $s \in I \cap S$, then $sI \subseteq sD \subseteq (sD)^{\tilde{\star}} \subseteq I^{\tilde{\star}}$ so I is S-$\tilde{\star}$-finite which is absurd. □

Theorem 2.5. *An integral domain D is an S-$\tilde{\star}$-Noetherian domain if and only if every prime ideal disjoint from S is S-$\tilde{\star}$-finite.*

Proof. The only if part is clear by definition of S-$\tilde{\star}$-Noetherian domain. For the if part assume in the contrary that D is not an S-$\tilde{\star}$-Noetherian domain. Then the set $\mathcal{T} = \{I | I \text{ is an ideal of } D \text{ which is not } S\text{-}\tilde{\star}\text{-finite }\}$ is not empty. Let $\{I_\alpha\}_{\alpha \in \Lambda}$ be a chain in $\mathcal{T}$ and set $I = \bigcup_{\alpha \in \Lambda} I_\alpha$. If I is S-$\tilde{\star}$-finite, then there exist an $s \in S$ and a finitely generated subideal J of I such that $sI \subseteq J^{\tilde{\star}} \subseteq I^{\tilde{\star}}$. Since J is finitely generated, $J \subseteq I_\alpha$ for some $I_\alpha \in \mathcal{T}$, so $sI_\alpha \subseteq sI \subseteq J^{\tilde{\star}} \subseteq I_\alpha^{\tilde{\star}}$ this contradicts to choice of I_α. Hence I is not S-$\tilde{\star}$-finite. Clearly I is an upper bound of $\{I_\alpha\}_{\alpha \in \Lambda}$ so by Zorn's lemma, we can choose a maximal element P in $\mathcal{T}$. By Lemma 2.4, P is a prime ideal of D which is disjoint from S hence by assumption P is an S-$\tilde{\star}$-finite which is absurd. □

Note that $\overline{\mathcal{F}}(D_S) = \{ED_S | E \in \overline{\mathcal{F}}(D)\}$. Thus given a semistar operation $\star$ on D, we can define a semistar operation $\star_S$ on D_S as follow; $\star_S : \overline{\mathcal{F}}(D_S) \to \overline{\mathcal{F}}(D_S)$ such that $(ED_S)^{\star_S} := E^{\star}D_S$ for each $ED_S \in \overline{\mathcal{F}}(D_S)$. It is easy to check that if $\star$ is stable and of finite type then $\star_S$ is also stable and of finite type; i.e. $\tilde{\star}_S = \widetilde{(\star_S)}$.

Remark 2.6. *If I is a quasi-$\star$-ideal of D, then it is easy to see that ID_S is a quasi-$\star_S$-ideal of D. The converse hold when I is a prime ideal of D, indeed if PD_S is a quasi-$\star_S$-prime ideal of D_S, then we have*

$$(P^{\star} \cap D)D_S = P^{\star}D_S \cap D_S = (PD_S)^{\star_S} \cap D_S = PD_S;$$

so

$$P^{\star} \cap D \subseteq (P^{\star} \cap D)D_S \cap D = PD_S \cap D = P.$$

Thus P is a quasi-$\star$-prime ideal of D.

A prime ideal P is said to be a *weak Bourbaki associated prime* ideal for an ideal I of D (or simply a B_w-prime of I), if P is minimal over $(I :_D x)$ for some $x \in D \backslash I$. Also P is said to be a *Zariski-Samuel associated prime ideal for I* (or simply a ZS-prime of I), if it is the radical of some $(I :_D x)$. Obviously a ZS-prime ideal of I is also a B_w-prime.

Our aim is to characterize the S-$\tilde{\star}$-Noetherian domains using the notions of primary decomposition, weak Bourbaki associated primes and Zariski-Samuel associated primes. On the other hand G. Fusacchia [9], has characterized the $\tilde{\star}$-Noetherian domains with those notions, so we wish to link the notion of S-$\tilde{\star}$-Noetherianity, with that of $\tilde{\star}_S$-Noetherianity. For an ideal I of D, $\mathrm{Sat}_S(I)$ denotes the *S-saturation* of I, that is $\mathrm{Sat}_S(I) := ID_S \cap D$.

Theorem 2.7. *The integral domain D is S-$\tilde{\star}$-Noetherian if and only if D_S is $\tilde{\star}_S$-Noetherian, and for every ideal I of D, $\mathrm{Sat}_S(I)^{\tilde{\star}} = (I^{\tilde{\star}} :_{D^{\tilde{\star}}} t)$ for some $t \in S$.*

Proof. Assume that D is S-$\tilde{\star}$-Noetherian and let ID_S be an ideal of D_S. Then there exist an $s \in S$ and a finitely generated subideal J of I such that $sI \subseteq J^{\tilde{\star}} \subseteq I^{\tilde{\star}}$. Therefore $ID_S \subseteq J^{\tilde{\star}}D_S$; so $(ID_S)^{\tilde{\star}_S} \subseteq (J^{\tilde{\star}}D_S)^{\tilde{\star}_S} = (JD_S)^{\tilde{\star}_S}$. Thus $(ID_S)^{\tilde{\star}_S} = (JD_S)^{\tilde{\star}_S}$. Since JD_S is a finitely generated subideal of ID_S, D_S is $\tilde{\star}_S$-Noetherian by [5, Lemma 3.3]. Now let I be an ideal of D. Then there exists $u \in S$ such that $u\,\mathrm{Sat}_S(I) \subseteq J^{\tilde{\star}} \subseteq \mathrm{Sat}_S(I)^{\tilde{\star}}$ for some finitely generated subideal J of $\mathrm{Sat}_S(I)$. Since J is finitely generated, there exists $v \in S$ such that $vJ \subseteq I$. So $uv\,\mathrm{Sat}_S(I) \subseteq vJ^{\tilde{\star}} \subseteq I^{\tilde{\star}}$. Hence $\mathrm{Sat}_S(I)^{\tilde{\star}} \subseteq (I^{\tilde{\star}} :_{D^{\tilde{\star}}} uv)$. Now let $x \in (I^{\tilde{\star}} :_{D^{\tilde{\star}}} uv)$. Then $xuv \in I^{\tilde{\star}}$ and there exists a finitely generated ideal $J' \in \mathcal{F}^{\star}$ such that $xuvJ' \subseteq I$. Hence $xJ'D_S \subseteq ID_S$ which implies that $xJ' \subseteq x\,\mathrm{Sat}_S\,J' \subseteq xJ'D_S \cap xD \subseteq ID_S \cap D^{\tilde{\star}} \subseteq$

$(ID_S)^{\tilde{\star}} \cap D^{\tilde{\star}} = (\mathrm{Sat}_S I)^{\tilde{\star}}$. Thus $x \in \mathrm{Sat}_S(I)^{\tilde{\star}}$. Therefore $\mathrm{Sat}_S(I)^{\tilde{\star}} = (I^{\tilde{\star}} :_{D^{\tilde{\star}}} uv)$. Thus the only if part is complete.

For the if part assume that D_S is $\tilde{\star}_S$-Noetherian, and for every ideal I of D, $\mathrm{Sat}_S(I)^{\tilde{\star}} = (I^{\tilde{\star}} :_{D^{\tilde{\star}}} t)$ for some $t \in S$. Let I be an ideal of D. Then $(ID_S)^{\tilde{\star}_S} = (JD_S)^{\tilde{\star}_S}$ for some finitely generated subideal J of I. Let $a \in I$. Then $\frac{a}{1} \in I^{\tilde{\star}}D_S = J^{\tilde{\star}}D_S$ hence there exists $u \in S$ such that $au \in J^{\tilde{\star}}$. Thus $auJ' \subseteq J$ for some finitely generated ideal J' of D such that $J'^{\tilde{\star}} = D^{\tilde{\star}}$. Hence $aJ'D_S \subseteq JD_S$ and we have,

$$aJ' \subseteq a(J'D_S \cap D) \subseteq aJ'D_S \cap aD \subseteq JD_S \cap D = \mathrm{Sat}_S(J).$$

Therefore $a \in \mathrm{Sat}_S(J)^{\tilde{\star}} = (J^{\tilde{\star}} :_{D^{\tilde{\star}}} t)$. Since a is an arbitrary element of I, we get $I \subseteq (J^{\tilde{\star}} :_{D^{\tilde{\star}}} t)$ that is $tI \subseteq J^{\tilde{\star}} \subseteq I^{\tilde{\star}}$. Thus I is S-$\tilde{\star}$-finite as required. □

Recall that a topological space Δ is said to be *Noetherian* if Δ satisfies the descending chain condition on closed sets. Consider $\Delta \subseteq \mathrm{Spec}(D)$ with the relative topology induced by Zariski topology on $\mathrm{Spec}(D)$. We will call an intersection of members of Δ a Δ-*radical ideal*. Recall that an ideal I of D is Δ-radically finite if there exists a finitely generated ideal $J \subseteq I$ such that I and J are contained in the same members of Δ. Also Δ is a Noetherian topological space if and only if every prime ideal of D is Δ-radically finite (see [16, Section 1]).

Lemma 2.8. *Assume that $\mathcal{P}$ is the family of all quasi-$\tilde{\star}$-prime ideals P such that $P \cap S = \emptyset$. If D_P is Noetherian for every $P \in \mathcal{P}$, and every quasi-$\tilde{\star}$-ideal I with $I \cap S = \emptyset$ has finitely many minimal prime ideals, then every prime ideal Q of D with $Q \cap S = \emptyset$ is $\mathcal{P}$-radically finite.*

Proof. Assume that ID_S is a proper ideal of D_S. If ID_S is a quasi-$\tilde{\star}_S$-ideal, then

$$ID_S = (ID_S)^{\tilde{\star}_S} \cap D_S = (I^{\tilde{\star}} \cap D)D_S.$$

Thus $(I^{\tilde{\star}} \cap D)D_S$ is a proper ideal of D_S. Therefore by assumption $I^{\tilde{\star}} \cap D$ has finitely many minimal prime ideals. Thus ID_S has finitely many minimal prime ideals. Let PD_S be a quasi-$\tilde{\star}_S$-prime ideal of D_S. Then $(D_S)_{PD_S}$ is Noetherian, since by Remark 2.6, P is a quasi-$\tilde{\star}$-prime ideal of D, and by assumption, D_P is Noetherian. Therefore by [9, Lemma 4.4], the family $\mathrm{QSpec}^{\tilde{\star}_S}(D_S)$ of quasi-$\tilde{\star}_S$-prime ideals of D_S is a Noetherian topological space; i.e. every prime ideal of D_S is $\mathrm{QSpec}^{\tilde{\star}_S}(D_S)$-radically finite. Now let Q be a prime ideal of D such that $Q \cap S = \emptyset$. Then QD_S is a prime ideal of D_S, thus there exists a finitely generated ideal $J \subseteq Q$ such that JD_S and QD_S are contained in the same members of $\mathrm{QSpec}^{\tilde{\star}_S}(D_S)$. A simple check shows that Q and J are contained in the same members of $\mathcal{P}$; that is Q is $\mathcal{P}$-radically finite. □

Now composing Theorem 2.7 with [9, Corollary 4.6], we get the following characterization for S-$\tilde{\star}$-Noetherian domains.

Corollary 2.9. *The following are equivalent:*

(1) *D is S-$\tilde{\star}$-Noetherian.*

(2) *The following hold:*
   (i) *D_S is $\tilde{\star}_S$-Noetherian.*
   (ii) *For every ideal I of D, $\mathrm{Sat}_S(I)^{\tilde{\star}} = (I^{\tilde{\star}} :_{D^{\tilde{\star}}} t)$ for some $t \in S$.*

(3) *The following hold:*
   (i) *D_P is Noetherian for every quasi-$\tilde{\star}$-prime ideal P with $P \cap S = \emptyset$.*
   (ii) *For every quasi-$\tilde{\star}$-ideal I, with $I \cap S = \emptyset$, ID_S and $Sat_S(I)$ admit a primary decomposition.*
   (iii) *For every ideal I of D, $\mathrm{Sat}_S(I)^{\tilde{\star}} = (I^{\tilde{\star}} :_{D^{\tilde{\star}}} t)$ for some $t \in S$.*

(4) *The following hold:*
   (i) *D_P is Noetherian for every quasi-$\tilde{\star}$-prime ideal P with $P \cap S = \emptyset$.*
   (ii) *For every quasi-$\tilde{\star}$-ideal I, with $I \cap S = \emptyset$, $Sat_S(I)$ and ID_S have finitely many* ZS*-prime ideals.*
   (iii) *Every prime ideal P of D with $P \cap S$, is $\mathcal{P}$-radically finite, where $\mathcal{P}$ is the same as Lemma 2.8.*
   (iv) *For every ideal I of D, $\mathrm{Sat}_S(I)^{\tilde{\star}} = (I^{\tilde{\star}} :_{D^{\tilde{\star}}} t)$ for some $t \in S$.*

(5) *The following hold:*
   (i) *D_P is Noetherian for every quasi-$\tilde{\star}$-prime ideal P with $P \cap S = \emptyset$.*
   (ii) *Every quasi-$\tilde{\star}$-ideal I, with $I \cap S = \emptyset$, has finitely many B_w-prime ideal P such that $P \cap S = \emptyset$.*
   (iii) *For every ideal I of D, $\mathrm{Sat}_S(I)^{\tilde{\star}} = (I^{\tilde{\star}} :_{D^{\tilde{\star}}} t)$ for some $t \in S$.*

Proof. (1) $\Rightarrow$ (2) Follows by Theorem 2.7.

(2) $\Rightarrow$ (3) (i) Let P be a quasi-$\tilde{\star}$-prime ideal of D with $P \cap S = \emptyset$. Then PD_S is a quasi-$\tilde{\star}_S$-prime ideal of D_S. Since by assumption D_S is a $\tilde{\star}_S$-Noetherian domain, [9, Corollary 4.6] implies that, $(D_S)_{PD_S} = D_P$ is a Noetherian domain.

(ii) Let I be a quasi-$\tilde{\star}$-ideal of D. Then ID_S is a quasi-$\tilde{\star}_S$-ideal of D_S. Hence by [9, Corollary 4.6], ID_S admits a primary decomposition. Suppose that $ID_S = \bigcap_{i=1}^{n} Q_i D_S$ for some P_i-primary ideals Q_i of D such that $P_i \cap S = \emptyset$. Therefore $ID_S \cap D = (\bigcap_{i=1}^{n} Q_i D_S) \cap D = \bigcap_{i=1}^{n} (Q_i D_S \cap D) = \bigcap_{i=1}^{n} Q_i$. Hence $\mathrm{Sat}_S(I)$ admits a primary decomposition and so has finitely many minimal prime ideals which implies that I also has finitely many minimal prime ideals.

(3) $\Rightarrow$ (4) (ii) is clear, since if every ideal has a primary decomposition, then it must have finitely many ZS-prime ideals (see [3, Theorem 4.5]).

(iii) hold from Lemma 2.8.

(4) $\Rightarrow$ (5) (ii) Let PD_S be a quasi-$\tilde{\star}_S$-ideal of D_S. Then by Remark 2.6, P is a quasi-$\tilde{\star}$-prime ideal of D. Hence by assumption (i), $D_P = (D_S)_{PD_S}$ is Noetherian. On the other hand it is easy to see that $\mathrm{QSpec}^{\tilde{\star}_S}(D_S)$ is a Noetherian topological space if and only if every prime ideal Q of D with $Q \cap S = \emptyset$ is $\mathcal{P}$-radically finite. Let I be a quasi-$\tilde{\star}$-ideal of D with $I \cap S = \emptyset$. Thus [9, Corollary 4.6] implies that, ID_S has finitely many B_w-prime ideals. Therefore I has finitely many B_w-prime ideals P such that $P \cap S = \emptyset$, since the B_w-prime ideals of ID_S are corresponding to the B_w-prime ideals of I which don't meet S (see [11, Proposition 1.2]).

(5) $\Rightarrow$ (1) Assumptions (i) and (ii) together with [9, Corollary 4.6] imply that, D_S is $\tilde{\star}_S$-Noetherian. Thus the result follows by (iii) and Theorem 2.7. □

In the above characterization, taking $\tilde{\star} = w$, we get a new characterization for S-strong Mori domains.

Corollary 2.10. *The following are equivalent:*

(1) *D is an S-strong Mori domain.*

(2) *The following hold:*
   - (i) *D_P is Noetherian for every quasi-w-prime ideal P with $P \cap S = \emptyset$.*
   - (ii) *For every quasi-w-ideal I, with $I \cap S = \emptyset$, ID_S and $Sat_S(I)$ admit a primary decomposition.*
   - (iii) *For every ideal I of D, $\mathrm{Sat}_S(I)^w = (I^w :_D t)$ for some $t \in S$.*

(3) *The following hold:*
   - (i) *D_P is Noetherian for every quasi-w-prime ideal P with $P \cap S = \emptyset$.*
   - (ii) *For every quasi-w-ideal I, with $I \cap S = \emptyset$, $Sat_S(I)$ and ID_S have finitely many* ZS*-prime ideals.*
   - (iii) *Every prime ideal P of D with $P \cap S$, is $\mathcal{P}$-radically finite, where $\mathcal{P}$ is the family of quasi-w-prime ideals P such that $P \cap S = \emptyset$.*
   - (iv) *For every ideal I of D, $\mathrm{Sat}_S(I)^w = (I^w :_D t)$ for some $t \in S$.*

(4) *The following hold:*
   - (i) *D_P is Noetherian for every quasi-w-prime ideal P with $P \cap S = \emptyset$.*
   - (ii) *Every quasi-w-ideal I, with $I \cap S = \emptyset$, has finitely many B_w-prime ideal P such that $P \cap S = \emptyset$.*
   - (iii) *For every ideal I of D, $\mathrm{Sat}_S(I)^w = (I^w :_D t)$ for some $t \in S$.*

Now we prove a Hilbert Basis Theorem for S-$\tilde{\star}$-Noetherian domains. To this purpose we use the semistar operation $\star[X]$ on $D[X]$, induced canonically from the semistar operation $\star$ on D, introduced by the second author [17] (see also [18]).

Theorem 2.11. [17, Theorem 2.1] *Let X and Y be two indeterminate over D and let $\star$ be a semistar operation on D. Set $D_1 := D[X]$, and $K_1 := K(X)$ and take the following subset of* $\mathrm{Spec}(D_1)$*:*

$$\Theta_1^\star := \{Q_1 \in \mathrm{Spec}(D_1) | Q_1 \cap D = (0) \text{ or } (Q_1 \cap D)^{\star_f} \subsetneq D^\star\}.$$

Set $\mathfrak{S}_1^\star := S(\Theta_1^\star) := D_1[Y]\backslash(\bigcup\{Q_1[Y] | Q_1 \in \Theta_1^\star\})$ and

$$E^{\mathcal{O}_{\mathfrak{S}_1^\star}} := E[Y]_{\mathfrak{S}_1^\star} \cap K_1, \text{ for all } E \in \overline{\mathcal{F}}(D_1).$$

(a) *The mapping $\star[X] := \mathcal{O}_{\mathfrak{S}_1^\star} : \overline{\mathcal{F}}(D_1) \to \overline{\mathcal{F}}(D_1)$, $E \mapsto E^{\mathcal{O}_{\mathfrak{S}_1^\star}}$ is a stable semistar operation of finite type on $D[X]$, i.e., $\widetilde{\star[X]} = \star[X]$.*

(b) $\tilde{\star}[X] = \star_f[X] = \star[X]$.

(c) $\star[X] \leq [\star]$, *where $[\star]$ is the semistar operation canonically associated to $\star$ introduced in* [4].

(c) $d_D[X] = d_{D[X]}$.

Remark 2.12. *Assume that J is an ideal of D such that $J^{\tilde{\star}} = D^{\tilde{\star}}$. Set $J_1 := J[X]$. Thus $(J_1 \cap D)^{\tilde{\star}} = J^{\tilde{\star}} = D^{\tilde{\star}}$. Therefore $J_1[Y] \cap \mathfrak{S}_1^{\tilde{\star}} \neq \emptyset$; so that $J_1[Y]_{\mathfrak{S}_1^{\tilde{\star}}} = D_1[Y]_{\mathfrak{S}_1^{\tilde{\star}}}$. Hence using the notation of Theorem 2.11 we get;*

$$J[X]^{\star[X]} = J_1[Y]_{\mathfrak{S}_1^{\tilde{\star}}} \cap K_1 = D_1[Y]_{\mathfrak{S}_1^{\tilde{\star}}} \cap K_1 = D[X]^{\star[X]}.$$

On the other hand if $J[X]^{\star[X]} = D[X]^{\star[X]}$, then by Theorem 2.11(c), $J[X]^{[\star]} = D[X]^{[\star]}$. Thus by [4, Theorem 2.3(c)]*, $J^{\tilde{\star}} = D^{\tilde{\star}}$.*

Recall that a multiplicative subset S of an integral domain D is *anti-archimedean* if $\bigcap_{n\geq 1} s^n D \cap S \neq \emptyset$ for every $s \in S$.

Theorem 2.13. *Let S be an anti-archimedean subset of D. Then D is S-$\tilde{\star}$-Noetherian if and only if the polynomial ring $D[X]$ is S-$\star[X]$-Noetherian.*

Proof. Assume that D is S-$\tilde{\star}$-Noetherian and A is a nonzero ideal of $D[X]$. For each $h \in \mathbb{N}$ suppose that I_h is the ideal of D generated by the set of leading coefficients of the polynomials in A of degree less than or equal to h. Since D is S-$\tilde{\star}$-Noetherian, each I_h is S-$\tilde{\star}$-finite, that is for each $h \in \mathbb{N}$ there exist $s_h \in S$ and a finitely generated ideal $J_h \subseteq I_h$ of D such that $s_h I_h \subseteq J_h^{\tilde{\star}} \subseteq I_h^{\tilde{\star}}$. Note that $I_0 = A \cap D \subseteq I_1 \subseteq \cdots$, thus $I = \bigcup_{h\geqslant 0} I_h$ is an ideal of D. Therefore there exist $s \in S$ and a finitely generated ideal J of D such that $sI \subseteq J^{\tilde{\star}} \subseteq I^{\tilde{\star}}$. Since J is finitely generated, there exists $m \in \mathbb{N}$ such that $J \subseteq I_m$. Let $J = (b_1, \cdots, b_k)$ and assume that $f_1, \cdots, f_k$ are polynomials in A having leading coefficients $b_1, \cdots, b_k$ and degrees $n_1, \cdots, n_k$ respectively. For each $h \leqslant m$ set $b_{1,h}, \cdots, b_{k_h,h}$ the generators of J_h

and let $g_{1,h}, \cdots, g_{k_h,h}$ be polynomials in A of degrees $n_{1,h}, \cdots, n_{k_h,h}$ which have $b_{1,h}, \cdots, b_{k_h,h}$ as leading coefficients respectively. Let $f \in A$ with leading coefficient a and degree n. Then $a \in I$ and $sa \in sI \subseteq J^{\tilde{\star}} = (b_1, \cdots, b_k)^{\tilde{\star}}$. Therefore there exists a finitely generated ideal Q of D such that $Q^{\tilde{\star}} = D^{\tilde{\star}}$ and $saQ \subseteq (b_1, \cdots b_k)$. Assume that $Q = (d_1, \cdots, d_z)$ so $sad_i = \sum_{j=1}^{k} r_{ij}b_j$ for each $i = 1, \cdots, z$ and some $r_{ij} \in D$. If $n > m$, then we set $g_i = d_i sf - \sum_{j=1}^{k} r_{ij} f_j X^{n-n_j}$ for each $i = 1, \cdots, z$. Then $g_i \in A$ and has degree strictly less than n. If we still have $\deg g_i > m$ for some $i \in \{1, \cdots, z\}$, we repeat the same process. After finitely many steps, we can find a finitely generated ideal Q' of D such that $(Q')^{\tilde{\star}} = D^{\tilde{\star}}$ and an integer $q \geqslant 1$ such that $Q's^q f \subseteq (f_1, \cdots, f_k)D[X] + T$, where T denotes the set of all polynomials in A of degree less than or equal to m. Now let $F \in T$ with leading coefficient b and degree $n' \leqslant m$; so that $b \in I_m$. Therefore $s_m b \in s_m I_m \subseteq J_m^{\tilde{\star}} = (b_{1,m}, \cdots, b_{k_m,m})^{\tilde{\star}}$. Thus there exists a finitely generated ideal Q_1 of D such that $Q_1^{\tilde{\star}} = D^{\tilde{\star}}$ and $s_m b Q_1 \subseteq (b_{1,m}, \cdots, b_{k_m,m})$. Let $Q_1 = (e_1, \cdots, e_y)$. Then $s_m b e_i = \sum_{j=1}^{k_m} u_{ij} b_{j,m}$ for each $i = 1, \cdots, y$ and some $u_{ij} \in D$. Now assume that $G_i = s_m F e_i - \sum_{j=1}^{k_m} u_{ij} g_{j,m} X^{n'-n_{j,m}}$ for each $i = 1, \cdots, y$. Note that G_i has degree strictly less than F so $G_i \in T$ and the leading coefficient of G_i is contained in I_{m-1} for each $i = 1, \cdots, y$. We repeat this process for G_i for all $i = 1, \cdots, y$. After finitely many steps we can find a finitely generated ideal Q_1' of D such that $(Q_1')^{\tilde{\star}} = D^{\tilde{\star}}$ and $tFQ_1' \subseteq B$, where $B := (\{g_{1,h}, \cdots, g_{k_h,h} | h = 1, \cdots, m\})D[X]$ and $t = s_1 s_2 ... s_m$. Therefore $tFQ_1'[X] \subseteq B$. Note that $Q'[X]$ and $Q_1'[X]$ are finitely generated ideals of $D[X]$ and by Remark 2.12, $Q'[X]^{\star[X]} = Q_1'[X]^{\star[X]} = D[X]^{\star[X]}$. Thus $tF \in B^{\star[X]}$ that is $tT \subseteq B^{\star[X]}$. Let $v \in \bigcap_{i \geqslant 1} s^i D \cap S$ therefore

$$\begin{aligned} Q'[X]tvf \subseteq & t(f_1, \cdots, f_m)D[X] + tTD[X] \\ \subseteq & (f_1, \cdots, f_m)D[X] + B^{\star[X]}. \end{aligned}$$

Thus $tvf \in ((f_1, \cdots, f_m)D[X] + B^{\star[X]})^{\star[X]} = ((f_1, \cdots, f_m)D[X] + B)^{\star[X]}$. Since $(f_1, \cdots, f_m)D[X] + B$ is a finitely generated subideal of A, A is S-$\star[X]$-finite. Thus $D[X]$ is an S-$\star[X]$-Noetherian domain.

Conversely assume that $D[X]$ is S-$\star[X]$-Noetherian and I is a nonzero ideal of D. By Theorem 2.11(c), $D[X]$ is an S-$[\star]$-Noetherian domain. Therefore $ID[X]$ is an S-$[\star]$-finite ideal of $D[X]$. So there exist an $s \in S$ and a finitely generated subideal J of I such that $sID[X] \subseteq JD[X]^{[\star]} \subseteq ID[X]^{[\star]}$. Note that $ED[X]^{[\star]} = E^{\tilde{\star}}D[X]$ for every $E \in \overline{\mathcal{F}}(D)$ [4, Theorem 2.3(d)]. Thus $sI \subseteq J^{\tilde{\star}} \subseteq I^{\tilde{\star}}$, that is, I is S-$\tilde{\star}$-finite ideal of D. Thus D is an S-$\tilde{\star}$-Noetherian domain. □

Remark 2.14. *Let S be an anti-archimedean subset of D. Since $\star[X] \leq [\star]$, it is easy to see that D is S-$\tilde{\star}$-Noetherian if and only if the polynomial ring $D[X]$ is S-$[\star]$-Noetherian.*

Theorem 2.15. *If S is an anti-archimedean subset of an integral domain D, then D is S-$\tilde{\star}$-Noetherian if and only if $\mathrm{Na}(D,\star)$ is an S-Noetherian domain.*

Proof. Assume that D is S-$\tilde{\star}$-Noetherian. Let $I\,\mathrm{Na}(D,\star)$ be a non-zero ideal of $\mathrm{Na}(D,\star)$, where I is an ideal of $D[X]$. By Theorem 2.13, $D[X]$ is S-$\star[X]$-Noetherian. So there exist $s \in S$ and a finitely generated subideal J of I such that $sI \subseteq J^{\star[X]} \subseteq I^{\star[X]}$. Now let $* := \star_{\triangle}$ be the spectral semistar operation on $D[X]$ defined by the set $\Delta := \{PD[X] | P \in \mathrm{QMax}^{\tilde{\star}}(D)\}$. Then $\star[X] \leq *$ (cf. [15, Proposition 3.4(1)]). Hence $(J^{\star[X]})^* = J^*$ and $(I^{\star[X]})^* = I^*$ therefore $sI \subseteq J^* \subseteq I^*$; so $sI\,\mathrm{Na}(D,\star) \subseteq J^*\,\mathrm{Na}(D,\star) \subseteq I^*\,\mathrm{Na}(D,\star)$. Now we have

$$\begin{aligned} I^*\,\mathrm{Na}(D,\star) &= \Big(\bigcap_{P\in \mathrm{QMax}^{\tilde{\star}}(D)} ID[X]_{PD[X]}\Big)\,\mathrm{Na}(D,\star) \\ &= \Big(\bigcap_{P\in \mathrm{QMax}^{\tilde{\star}}(D)} (ID[X]_{N_\star})_{PD[X]_{N_\star}}\Big)\,\mathrm{Na}(D,\star) \\ &\overset{\dagger}{=} (I\,\mathrm{Na}(D,\star))\,\mathrm{Na}(D,\star) = I\,\mathrm{Na}(D,\star). \end{aligned}$$

(for (†) see the proof of [7, Proposition 3.4(1)]). Similarly we can show that $J^*\,\mathrm{Na}(D,\star) = J\,\mathrm{Na}(D,\star)$. Therefore we get $sI\,\mathrm{Na}(D,\star) \subseteq J\,\mathrm{Na}(D,\star) \subseteq I\,\mathrm{Na}(D,\star)$, that is, $\mathrm{Na}(D,\star)$ is an S-Noetherian domain.

Conversely assume that $\mathrm{Na}(D,\star)$ is an S-Noetherian domain. Let I be a nonzero ideal of D. Then there exist an $s \in S$ and a finitely generated subideal J of $ID[X]$ such that $sI\,\mathrm{Na}(D,\star) \subseteq J\,\mathrm{Na}(D,\star) \subseteq I\,\mathrm{Na}(D,\star)$. Let $a \in I$. Then $sag \in J$ for some $g \in N_\star$. Hence $(sa)c_D(g) \subseteq c_D(J)$. Since $c_D(g)$ is a finitely generated ideal of D with $c_D(g)^\star = D^\star$, $sa \in (c_D(J))^{\tilde{\star}}$. As a is an arbitrary element of I, we get $sI \subseteq (c_D(J))^{\tilde{\star}}$. Note that $c_D(J)$ is a finitely generated subideal of I hence I is S-$\tilde{\star}$-finite as required. □

Acknowledgment. We would like to thank the referee for helpful suggestions.

References

1. D. D. Anderson and T. Dumitrescu, *S-Noetherian rings*, Comm. Algebra, 30(9) (2002), 4407–4416.
2. D. D. Anderson, D. J. Kwak, and M. Zafrullah, *Agreeable domains*, Comm. Algebra, 23(13) (1995), 4861–4883.

3. M. F. Atiyah and I. G. Macdonald, Introduction to Commutative Algebra, Addison-Wesley Publishing Co., Reading, Mass.-London-Don Mills, Ont. 1969.
4. G. W. Chang and M. Fontana, *Uppers to zero and semistar operations in polynomial rings*, J. Algebra, 318(1) (2007), 484–493.
5. S. El Baghdadi, M. Fontana and G. Picozza, *Semistar Dedekind domains*, J. Pure Appl. Algebra, 193(1-3) (2004), 27–60.
6. M. Fontana and J. A. Huckaba, *Localizing systems and semistar operations*, Non-Noetherian commutative ring theory, 169-197, Math. Appl., 520, Kluwer Acad. Publ., Dordrecht, 2000.
7. M. Fontana and K. A. Loper, *Kronecker function rings: a general approach*, Ideal theoretic methods in commutative algebra (Columbia, MO, 1999), 189-205, Lecture Notes in Pure and Appl. Math., 220, Dekker, New York, 2001.
8. M. Fontana and G. Picozza, *Semistar invertibility on integral domains*, Algebra Colloq., 12(4) (2005), 645–664.
9. G. Fusacchia, *Strong semistar noetherian domains*, Houston J. Math., 39(1) (2013), 1–20.
10. R. Gilmer, Multiplicative Ideal Theory, Pure and Applied Mathematics, 12, Marcel Dekker, Inc., New York, 1972.
11. W. Heinzer and J. Ohm, *Locally Noetherian commutative rings*, Trans. Amer. Math. Soc., 158 (1971), 273–284.
12. I. Kaplansky, Commutative Rings, Revised edition, The University of Chicago Press, Chicago, Ill.-London, 1974.
13. H. Kim, M. O. Kim and J. W. Lim, *On S-strong Mori domains*, J. Algebra, 416 (2014), 314–332.
14. A. Okabe and R. Matsuda, *Semistar-operations on integral domains*, Math. J. Toyama Univ., 17 (1994), 1–21.
15. G. Picozza, *A note on semistar noetherian domains*, Houston. J. Math., 33(2) (2007), 415–432.
16. D. E. Rush and L. J. Wallace, *Noetherian maximal spectrum and coprimely packed localization of polynomial rings*, Houston. J. Math., 28(3) (2002), 437–448.
17. P. Sahandi, *Semistar-Krull and valuative dimension of integral domains*, Ric. Mat., 58(2) (2009), 219-242.
18. P. Sahandi, *Universally catenarian integral domains, strong S-domains and semistar operations*, Comm. Algebra, 38(2) (2010), 673–683.

Afsaneh Esmaeelnezhad
Faculty of Mathematics and Computer Science
Kharazmi University
Tehran, Iran
e-mail: esmaeilnejad@gmail.com

Parviz Sahandi
Department of Mathematics
University of Tabriz
Tabriz, Iran
e-mails: sahandi@ipm.ir
sahandi@tabrizu.ac.ir