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QUASI-ARMENDARIZ PROPERTY ON POWERS OF COEFFICIENTS

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Dedicated to the memory of Professor Efraim P. Armendariz

Abstract. The study of Armendariz rings was initiated by Rege and Chhawchharia, based on a result of Armendariz related to the structure of reduced rings. Armendariz rings were generalized to quasi-Armendariz rings by Hirano. We introduce the concept of *power-quasi-Armendariz* (simply, *p.q.-Armendariz*) ring as a generalization of quasi-Armendariz, applying the role of quasi-Armendariz on the powers of coefficients of zero-dividing polynomials. In the process we investigate the power-quasi-Armendariz property of several ring extensions, e.g., matrix rings and polynomial rings, which have roles in ring theory.

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1. Introduction

Throughout this note every ring is associative with identity unless otherwise specified. Given a ring R, $J(R)$, $N^*(R)$ and $N(R)$ denote the Jacobson radical, the upper nilradical (i.e., sum of all nil ideals) and the set of all nilpotent elements in R, respectively. It is well-known that $N^*(R) \subseteq J(R)$ and $N^*(R) \subseteq N(R)$. We use $R[x]$ to denote the polynomial ring with an indeterminate x over given a ring R. For $f(x) \in R[x]$, let $C_{f(x)}$ denote the set of all coefficients of $f(x)$. $\mathbb{Z}$ (resp., $\mathbb{Z}_n$) denotes the ring of integers (resp., the ring of integers modulo n). Denote the n by n full (resp., upper triangular) matrix ring over a ring R by $Mat_n(R)$ (resp.,

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$U_n(R))$ for $n \geq 2$. Next let

$D_n(R)$ be the subring $\{m \in U_n(R) \mid$ the diagonal entries of m are all equal$\}$

of $U_n(R)$,

$$N_n(R) = \{(a_{ij}) \in D_n(R) \mid a_{ii} = 0 \text{ for all } i\}, \text{and}$$

$$V_n(R) = \{(a_{ij}) \in D_n(R) \mid a_{ij} = a_{(i+1)(j+1)} \text{ for } i = 1, \ldots, n-2 \text{ and } j = 2, \ldots, n-1\}.$$

Note that $V_n(R) \cong R[x]/(x^n)$, where (x^n) is the ideal of $R[x]$ generated by x^n. Use E_{ij} for the matrix with (i, j)-entry 1 and other entries 0.

For a ring R and an (R, R)-bimodule M, the *trivial extension* of R by M is the ring $T(R, M) = R \oplus M$ with the usual addition and the following multiplication: $(r_1, m_1)(r_2, m_2) = (r_1r_2, r_1m_2+m_1r_2)$. This is isomorphic to the ring of all matrices $\begin{pmatrix} r & m \\ 0 & r \end{pmatrix}$, where $r \in R$ and $m \in M$ and the usual matrix operations are used.

A ring is called *reduced* if it has no nonzero nilpotent elements. Rege and Chhawchharia [15] called a ring R (not necessarily with identity) *Armendariz* if $ab = 0$ for all $a \in C_{f(x)}$ and $b \in C_{g(x)}$ whenever $f(x)g(x) = 0$ for $f(x), g(x) \in R[x]$ based on [2, Lemma 1]. Reduced rings are clearly Armendariz. A ring is usually called *Abelian* if every idempotent is central. Armendariz rings are Abelian by [10, Lemma 7]. The concept of Armendariz ring was generalized to the quasi-Armendariz ring property by Hirano. A ring R (not necessarily with identity) is called *quasi-Armendariz* [7] provided that

$$aRb = 0 \text{ for all } a \in C_{f(x)} \text{ and } b \in C_{g(x)} \text{ whenever} f(x)Rg(x) = 0$$

for $f(x), g(x) \in R[x]$.

Semiprime rings are quasi-Armendariz rings by [7, Corollary 3.8], but not conversely in general.

On the other hand, Han et al. [6] called a ring R (not necessarily with identity) *power-Armendariz* if whenever $f(x)g(x) = 0$ for $f(x), g(x) \in R[x]$, there exist $m, n \geq 1$ such that

$$a^m b^n = 0 \text{ for all } a \in C_{f(x)}, b \in C_{g(x)}.$$

The class of quasi-Armendariz rings and the class of power-Armendariz rings do not imply each other by Example 2.1 to follow.

2. Power-quasi-Armendariz rings

We first consider the following condition (†): There exist $m, n \geq 1$ such that

$$a^m R b^n = 0 \text{ for any } a \in C_{f(x)} \text{ and } b \in C_{g(x)}, \text{ whenever } f(x)Rg(x) = 0$$

for $f(x), g(x) \in R[x]$, where R is a ring, not necessarily with identity.

It is obvious that $a^m R b^n = 0$ for some $m, n \geq 1$ if and only if $a^\ell R b^\ell = 0$ for some $\ell \geq 1$, in the condition (†) above. Quasi-Armendariz rings clearly satisfy the condition (†), but each part of the following example shows that the class of rings satisfying the condition (†) need not be quasi-Armendariz or power-Armendariz.

Example 2.1. *(1) Consider a ring $R = D_n(T)$ where $T = T(W, W)$ for a division ring W and $n \geq 2$. Let $f(x) = \sum_{i=0}^{s} A_i x^i, g(x) = \sum_{j=0}^{t} B_j x^j \in R[x]$ with $f(x)Rg(x) = 0$. Since $J(R) = N_n(T)$ and $\frac{R}{N_n(T)} \cong T$, $f(x)Rg(x) = 0$ implies that $A_i, B_j \in N_n(T)$ for all i, j. Then $A_i^n = 0 = B_j^n$ and so $A_i^n R B_j^n = 0$, showing that R satisfies the condition (†). However, R is not quasi-Armendariz by help of* [3, Example 2.5]*. Note that R is power-Armendariz.*

(2) Consider a ring $R = Mat_n(A)$ where A is a quasi-Armendariz ring and $n \geq 2$. Then R is quasi-Armendariz by [7, Theorem 3.12] *and so it satisfies the condition (†), but not power-Armendariz by* [6, Example 1.5(1)].

Based on the above, we will call a ring R (not necessarily with identity) *power-quasi-Armendariz* (shortly, *p.q.-Armendariz*) if it satisfies the condition (†). Hence, the concept of p.q.-Armendariz ring is a generalization of a quasi-Armendariz ring.

Due to Lambek [13], an ideal I of a ring R is called *symmetric* if $abc \in I$ implies $acb \in I$ for all $a, b, c \in R$. If the zero ideal of a ring R is symmetric then R is called *symmetric*. Following Bell [4], a ring R is called to satisfy the *Insertion-of-Factors-Property* (simply, an *IFP* ring) if $ab = 0$ implies $aRb = 0$ for $a, b \in R$. Note that $N(R) = N^*(R)$ for an IFP ring R by [16, Theorem 1.5]. Reduced rings are symmetric and symmetric rings are IFP, and a simple computation yields that IFP rings are Abelian. We see that $D_3(R)$ is an IFP ring and $D_n(R)$ is not IFP for $n \geq 4$ in [11], where R is a reduced ring.

Recall that a ring R is called *almost symmetric* [16] if R is IFP and satisfies the following condition:

$$\text{(S II)} \quad ab^m c^m = 0 \text{ for some positive integer } m \text{ whenever } a(bc)^n = 0$$

for given $n \geq 1$ and $a, b, c \in R$.

Symmetric rings are almost symmetric, but not conversely by [16, Proposition 1.4 and Example 5.1], and almost symmetric rings are obviously IFP, however the class of IFP rings and the class of rings satisfying the condition (S II) are independent of each other by [16, Example 5.1(c) and Example 5.2(b)]. Symmetric rings are power-Armendariz by [6, Proposition 1.1(4)].

Proposition 2.2. (1) *If R is a p.q.-Armendariz ring, then so is eRe for $0 \neq e^2 = e \in R$.*

(2) *The class of p.q.-Armendariz rings is closed under direct sum.*

(3) *Almost symmetric rings are p.q.-Armendariz.*

(4) *Power-Armendariz IFP rings are p.q.-Armendariz.*

Proof. (1) Let $f(x), g(x) \in eRe[x]$ such that $f(x)(eRe)g(x) = 0$. Since $f(x)e = f(x)$ and $eg(x) = g(x)$, we have $f(x)Rg(x) = 0$. Assume that R is p.q.-Armendariz. Then there exist $m, n \geq 1$ such $a^m Rb^n = 0$ for any $a \in C_{f(x)}, b \in C_{g(x)}$. Since $a = ae$ and $b = eb$, $0 = a^m Rb^n = \underbrace{a \cdots a}_{m-1} aeReb \underbrace{b \cdots b}_{n-1} = a^m(eRe)b^n$ and thus eRe is p.q.-Armendariz.

(2) Let R_u be p.q.-Armendariz rings for all $u \in U$ and $E = \oplus_{u \in U} R_u$, the direct sum of R_u's. Suppose that $f(x)Eg(x) = 0$ for $0 \neq f(x) = \sum_{i=0}^{s}(a(i)_u)x^i, 0 \neq g(x) = \sum_{j=0}^{t}(b(j)_u)x^j \in E[x]$. We apply the proof of [6, Proposition 1.1(1)]. Note that $f(x)$ and $g(x)$ can be rewritten by

$$f(x) = (\sum_{i=0}^{s} a(i)_u x^i),\ g(x) = (\sum_{j=0}^{t} b(j)_u x^j) \in \oplus_{u \in U} R_u[x].$$

$f(x)Eg(x) = 0$ yields $(\sum_{i=0}^{s} a(i)_u x^i)E(\sum_{j=0}^{t} b(j)_u x^j) = 0$ for all $u \in U$. Note that finitely many polynomials in $\{(\sum_{i=0}^{s} a(i)_u x^i), (\sum_{j=0}^{t} b(j)_u x^j) \mid u \in U\}$ are nonzero. Since R_u is p.q.-Armendariz for all $u \in U$. Then there exists $h \geq 1$ such that $[a(i)_u]^h[b(j)_u]^h = 0$ for all i, j, u. This implies that $(a(i)_u)^h E(b(j)_u)^h = 0$ for all i, j, showing that E is p.q.-Armendariz.

(3) Let R be an almost symmetric ring. Then $N(R) = N^*(R)$. Suppose that $f(x)Rg(x) = 0$ for $f(x) = \sum a_i x^i, g(x) = \sum b_j x^j \in R[x]$. We use $\bar{R}$ and $\bar{r}$ to denote $R/N(R)$ and $r + N(R)$, respectively. Since $R/N(R)$ is reduced (hence quasi-Armendariz) and $(\sum \bar{a}_i x^i)\bar{R}(\sum \bar{b}_j x^j) = 0$, we have $aRb \subseteq N(R)$ for any $a \in C_{f(x)}$ and $b \in C_{g(x)}$. Then $(aRb)^n = 0$ and so $(ab)^n = 0$ for some $n \geq 1$. Since R is almost symmetric, $a^l b^l = 0$ and so $a^l Rb^l = 0$ for some $l \geq 1$. Thus R is p.q.-Armendariz.

(4) is simply checked through a simple computation. □

Corollary 2.3. *Let e be a central idempotent of a ring R. Then R is p.q.-Armendariz if and only if eR and $(1-e)R$ are both p.q.-Armendariz.*

Proof. It follows from Proposition 2.2(1,2), since $R \cong eR \oplus (1-e)R$. □

Example 2.4. *The ring $R = U_2(D)$ for a domain D is quasi-Armendariz by* [7, Corollary 3.15] *and hence R is p.q.-Armendariz, but not IFP.*

Proposition 2.5. *Let R be a ring and I be a proper ideal of R. If R/I is a p.q.-Armendariz ring and I is reduced as a ring without identity, then R is p.q.-Armendariz.*

Proof. We adapt the proof of [6, Theorem 1.11(4)]. Let $f(x)Rg(x) = 0$ for $f(x), g(x) \in R[x]$. Since R/I is p.q.-Armendariz, there exists $s \geq 1$ such that $a^sRb^s \subseteq I$ for any $a \in C_{f(x)}$ and $b \in C_{g(x)}$. By the same computation as in the proof of [6, Theorem 1.11(4)], we have $aIb = 0$ for any $a \in C_{f(x)}$ and $b \in C_{g(x)}$, and thus

$$a^{s+1}Rb^{s+1} = a(a^sRb^s)b \in aIb,$$

and hence $a^{s+1}Rb^{s+1} = 0$. Therefore R is p.q.-Armendariz. □

Proposition 2.6. *For a ring R, if $Mat_n(R)$ (resp., $U_n(R)$) is p.q-Armendariz for $n \geq 2$, then R is p.q.-Armendariz.*

Proof. If $Mat_n(R)$ is p.q.-Armendariz, then $R \cong E_{11}Mat_n(R)E_{11}$ is p.q.-Armendariz by Proposition 2.2(1). □

We actually do not know whether $Mat_n(R)$ (resp., $U_n(R)$) is p.q.-Armendariz if R is a p.q.-Armendariz ring.

Question. If R is a p.q.-Armendariz ring, then is $Mat_n(R)$ (resp., $U_n(R)$) p.q.-Armendariz?

But we find the following kinds of subrings of $Mat_n(R)$ which preserve the p.q. Armendariz property.

Theorem 2.7. *Let R be an IFP ring and $n \geq 2$. The following conditions are equivalent:*

(1) *R is p.q.-Armendariz.*
(2) *$D_n(R)$ is p.q.-Armendariz.*
(3) *$V_n(R)$ is p.q.-Armendariz.*
(4) *$T(R, R)$ is p.q.-Armendariz.*

Proof. (1)⇒(2): Let $f(x) = \sum_{i=0}^{s} A_i x^i, g(x) = \sum_{j=0}^{t} B_j x^j \in D_n(R)[x]$ satisfy $f(x)D_n(R)g(x) = 0$, where $A_i = (a(i)_{cd})$ and $B_j = (b(j)_{hk})$ for $0 \leq i \leq s$ and $0 \leq j \leq t$. The proof is similar to one of [6, Theorem 1.4(1)], but we write it here for completeness.

Note that $f(x)$ and $g(x)$ can be expressed by

$$f(x) = \begin{pmatrix} f_{11}(x) & f_{12}(x) & f_{13}(x) & \cdots & f_{1n}(x) \\ 0 & f_{22}(x) & f_{23}(x) & \cdots & f_{2n}(x) \\ 0 & 0 & f_{33}(x) & \cdots & f_{3n}(x) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & f_{nn}(x) \end{pmatrix}$$

and

$$g(x) = \begin{pmatrix} g_{11}(x) & g_{12}(x) & g_{13}(x) & \cdots & g_{1n}(x) \\ 0 & g_{22}(x) & g_{23}(x) & \cdots & g_{2n}(x) \\ 0 & 0 & g_{33}(x) & \cdots & g_{3n}(x) \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & g_{nn}(x) \end{pmatrix},$$

where

$$f_{11}(x) = \cdots = f_{nn}(x) = \sum_{i=0}^{s} a(i)_{11} x^i,\ f_{cd}(x) = \sum_{i=0}^{s} a(i)_{cd} x^i$$

and

$$g_{11}(x) = \cdots = g_{nn}(x) = \sum_{j=0}^{t} b(j)_{11} x^j,\ g_{hk}(x) = \sum_{j=0}^{t} b(j)_{hk} x^j.$$

Since $f(x)D_n(R)g(x) = 0$, $f_{11}(x)Rg_{11}(x) = 0$ and so there exist $w \geq 1$ such that $a(i)_{11}^w Rb(j)_{11}^w = 0$ for all i, j since R is p.q.-Armendariz.

Next note that every sum-factor of each entry of A_i^{wn} (resp., B_j^{wn}) contains $a(i)_{11}^w$ (resp., $b(j)_{11}^w$) in its product by [9, Lemma 1.2(1)]. Now since R is IFP, we get $A_i^{wn} RB_j^{wn} = 0$ because every sum-factor in each entry of $A_i^{wn} RB_j^{wn}$ is of the form

$$sa(i)_{11}^w tb(j)_{11}^w u = 0,$$

for any $s, t, u \in R$.

(2)⇒(1): Suppose that $D_n(R)$ is p.q.-Armendariz. Let $f(x)Rg(x) = 0$ for $f(x), g(x) \in R[x]$. Then

$$(f(x)\sum_{i=1}^{n} E_{ii})D_n(R)[x](g(x)\sum_{i=1}^{n} E_{ii}) = 0.$$

Since $D_n(R)$ is p.q.-Armendariz, there exist $s, t \geq 1$

$$(a\sum_{i=1}^{n} E_{ii})^s D_n(R)(b\sum_{i=1}^{n} E_{ii})^t = 0$$

for any $a \in C_{f(x)}$ and $b \in C_{g(x)}$. In particular, for any $r \in R$, we get

$$(a\sum_{i=1}^{n} E_{ii})^s (r\sum_{i=1}^{n} E_{ii})(b\sum_{i=1}^{n} E_{ii})^t = 0,$$

implying that $a^s R b^t = 0$ for any $a \in C_{f(x)}$ and $b \in C_{g(x)}$. Therefore R is p.q.-Armendariz.

(1)⇔(3) and (1)⇔(4) can be obtained by the same argument as in the proof of (1)⇔(2). □

The following result comes from Theorem 2.7 and Proposition 2.2(3).

Corollary 2.8. *If R is an almost symmetric ring, then $D_n(R)$ is p.q.-Armendariz for any $n \geq 2$.*

Recall that a ring R is called *directly finite* if $ba = 1$ whenever $ab = 1$ for $a, b \in R$. Abelian rings are directly finite and power-Armendariz rings are Abelian by [6, Proposition 1.1(5)]. However, there exists a p.q.-Armendariz ring which is not directly finite (hence non-Abelian) by the following.

Example 2.9. *There exists a domain (hence p.q.-Armendariz) D such that $R = Mat_2(D)$ is not directly finite by* [14, Theorem 1.0]*. Then R is quasi-Armendariz by* [7, Theorem 3.12]*, and so it is p.q.-Armendariz. But R is non-Abelian obviously.*

A ring R is called (*von Neumann*) *regular* if for each $a \in R$ there exists $b \in R$ such that $a = aba$. in [5]. Notice that a regular ring R is power-Armendariz if and only if R is Armendariz if and only if R is Abelian if and only if R is reduced by help of [6, Theorem 1.8]. However, there exists a von Neumann regular p.q.-Armendariz ring but not reduced, by considering $Mat_2(D)$ with D a division ring in Example 2.9.

Theorem 2.10. (1) *If $R[x]$ is a p.q.-Armendariz ring, then so is R.*

(2) *Let R be an IFP ring. If R is p.q.-Armendariz, then so is $R[x]$.*

Proof. (1) Suppose that $R[x]$ is a p.q.-Armendariz ring. Let $f(x)Rg(x) = 0$ for $f(x), g(x) \in R[x]$. Let y be an indeterminate over $R[x]$. Then $f(y)Rg(y) = 0$ and so $f(y)R[x]g(y) = 0$ for $f(y), g(y) \in R[x][y]$, since x commutes with y. By hypothesis, there exist $s, t \geq 1$ such that $a^s R[x] b^t = 0$ for any $a \in C_{f(y)}$ and

$b \in C_{g(y)}$. This implies that $a^s R b^t = 0$ for any $a \in C_{f(x)}$ and $b \in C_{g(x)}$, and thus R is p.q.-Armendariz.

(2) We apply the proof of [6, Proposition 2.2] which was done by help of Anderson and Camillo [1, Theorem 2]. Suppose that R is a p.q.-Armendariz IFP ring. Let $p(y) = \sum_{i=0}^{m} f_i(x)y^i$ and $q(y) = \sum_{j=0}^{n} g_j(x)y^j \in (R[x])[y]$ with $p(y)R[x]q(y) = 0$. Next let $f_i(x) = a_{i_0} + a_{i_1}x + \cdots + a_{i_w}x^{i_w}$, $g_j(x) = b_{j_0} + b_{j_1}x + \cdots + b_{j_v}x^{j_v}$ for each i, j, where $a_{i_0}, \ldots, a_{i_w}$, $b_{j_0}, \ldots, b_{j_v} \in R$. Let $k = \sum_{i=0}^{m} deg(f_i(x)) + \sum_{j=0}^{n} deg(g_j(x))$, where the degree is considered as polynomials in $R[x]$ and the degree of zero polynomial is taken to be 0. Let $p(x^k) = \sum_{i=0}^{m} f_i(x)(x^k)^i$ and $q(x^k) = \sum_{j=0}^{n} g_j(x)(x^k)^j \in R[x]$. Then the set of coefficients of the f_i's (resp., g_j's) equals the set of coefficients of $p(x^k)$ (resp., $q(x^k)$). From $p(y)R[x]q(y) = 0$, we have $p(y)Rq(y) = 0$ and so $p(x^k)Rq(x^k) = 0$. Since R is p.q.-Armendariz, there exists $v \geq 1$ such that

$$a_\alpha^v R b_\beta^v = 0 \text{ for all } \alpha, \beta.$$

Since R is IFP, we also have

$$a_\alpha R_1 a_\alpha R_2 \cdots R_{v-1} a_\alpha R_v b_\beta R_{v+1} b_\beta R_{v+2} \cdots R_{2v-1} b_\beta = 0, \tag{1}$$

where $R_1 = \ldots = R_{2v-1} = R$. Note that some $a_{\alpha'}$ (resp., some $b_{\beta'}$) occurs at least v-times (resp., v-times) in the coefficient of each monomial in

$$f_i(x)^{(m+1)v} \text{ (resp., } g_j(x)^{(n+1)v}).$$

From this we have

$$f_i(x)^{(m+1)v} R g_j(x)^{(n+1)v} = 0$$

by the equality (1). This implies that $R[x]$ is p.q.-Armendariz. □

Recall that a ring R is called *strongly IFP* [12] if $R[x]$ is IFP, equivalently, whenever polynomials $f(x), g(x)$ in $R[x]$ satisfy $f(x)g(x) = 0$, $f(x)Rg(x) = 0$. Clearly strongly IFP rings are IFP, but not conversely by [8, Example 2].

Let R be a strongly IFP ring. Then the Armendariz ring property coincides with the quasi-Armendariz ring property by [12, Proposition 3.18]. This yields the following equivalent conditions by help of [1, Theorem 2] and the fact that the quasi-Armendariz property is closed under subrings:

(1) R is quasi-Armendariz;

(2) R is Armendariz;

(3) $R[x]$ is Armendariz;

(4) $R[x]$ is quasi-Armendariz.

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