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# P.P. PROPERTIES OF GROUP RINGS

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Abstract. A ring is called left p.p. if the left annihilator of each element of $R$ is generated by an idempotent. We prove that for a ring $R$ and a group $G$, if the group ring $RG$ is left p.p. then so is $RH$ for every subgroup $H$ of $G$; if in addition $G$ is finite then $|G|^{-1} \in R$. Counterexamples are given to answer the question whether the group ring $RG$ is left p.p. if $R$ is left p.p. and $G$ is a finite group with $|G|^{-1} \in R$. Let $G$ be a group acting on $R$ as automorphisms. Some sufficient conditions are given for the fixed ring $R^G$ to be left p.p.

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## Introduction

Throughout this paper all rings are associative with identity. A ring $R$ is called Baer if the left annihilator of every nonempty subset of $R$ is generated by an idempotent. The concept of a Baer ring was introduced by Kaplansky to abstract properties of rings of operators on a Hilbert space in his 1965 book [9]. The definition of Baer is indeed left-right symmetric by [9].

Closely related to Baer rings are p.p. rings. A ring $R$ is called a left p.p. ring if each principal left ideal of $R$ is projective, or equivalently, if the left annihilator of each element of $R$ is generated by an idempotent. Similarly, right p.p. rings can be defined. A ring is called a p.p. ring if it is both a left and a right p.p. ring. The concept of a p.p. ring is not left-right symmetric by Chase [2]. A left p.p. ring $R$ is Baer (so p.p.) when $R$ is orthogonally finite by Small [11] and a left p.p. ring is p.p. when $R$ is Abelian by Endo [5]. For more details on left p.p. rings, see [3,7,8]. Baer rings are clearly p.p. rings, and von Neumann regular rings are p.p. rings by Goodearl [6].

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Given a ring $R$ and a group $G$, we will denote the group ring of $G$ over $R$ by $RG$. Write $\Delta_R(G)$ for the augmentation ideal of $RG$ generated by $\{1-g : g \in G\}$. If $H$ is a finite subgroup of $G$, we let $\hat{H} = \sum_{h\in H} h$. If $g \in G$ has finite order, we define $\hat{g} = \hat{H}$ where $H = \langle g\rangle$. We write $C_n$ for the cyclic group of order $n$, $\mathbb{Z}$ for the ring of integers and $\mathbb{Z}_n$ for the ring of integers modulo $n$. As usual, $\mathbb{Q}$ is the field of rationals and $\mathbb{C}$ is the field of complex numbers. The imaginary unit is denoted by $\mathbf{i}$. For a subset $X$ of $R$, $\mathbf{l}_R(X)$ denotes the left annihilator of $X$ in $R$.

In [13], Z. Yi and Q. Y. Zhou studied Baer properties of group rings. Motivated by them, we discuss the p.p. properties of group rings. Some methods and proofs are similar to those in [13].

## 1. Necessary Conditions

**Theorem 1.1.** *Let $R$ be a subring of a ring $S$ both with the same identity. Suppose that $S$ is a free left $R$-module with a basis $G$ such that $1 \in G$ and $ag = ga$ for all $a \in R$ and all $g \in G$. If $S$ is left p.p., then so is $R$.*

**Proof.** For $a \in R$, since $S$ is left p.p., $\mathbf{l}_S(a) = Se$ where $e^2 = e \in S$. Write $e = e_0g_0 + \cdots + e_ng_n$ where $g_0 = 1, g_i \in G$ are distinct and $e_i \in R$. Then $0 = ea = (e_0g_0 + \cdots + e_ng_n)a = e_0ag_0 + \cdots + e_nag_n$, and so $e_ia = 0$ for $i = 0,\ldots,n$. Thus $e_i \in \mathbf{l}_S(a) = Se$, implying that $e_i = e_ie$. Then $e_0g_0 = e_0 = e_0e = e_0(e_0g_0 + \cdots + e_ng_n) = e_0^2g_0 + e_0e_1g_1 + \cdots + e_0e_ng_n$, whence $e_0 = e_0^2 \in R$. Because $e_0a = 0$, we have $Re_0 \subseteq \mathbf{l}_R(a)$. For $r \in \mathbf{l}_R(a) \subseteq \mathbf{l}_S(a) = Se$, we have $r = re = r(e_0g_0 + \cdots + e_ng_n) = re_0g_0 + \cdots + re_ng_n$. So $r = re_0 \in Re_0$. Hence $\mathbf{l}_R(a) = Re_0$ and $R$ is left p.p. □

**Corollary 1.2.** *Let $R$ be a ring and $G$ be a group. If $RG$ is left p.p., then so is $R$.*

**Proof.** Note that $S = RG = \oplus_{g\in G}Rg$ is a free left $R$-module with a basis $G$ satisfying the assumptions of Theorem 1.1. □

**Corollary 1.3.** *If $R[x]$ or $R[x, x^{-1}]$ is left p.p., then so is $R$.*

**Proof.** Note that $R[x]$ and $R[x, x^{-1}]$ are free $R$-modules with bases $\{x^i : i = 0, 1, \ldots\}$ and $\{x^i : i = 0, \pm1, \ldots\}$ satisfying the assumptions of Theorem 1.1. □

**Corollary 1.4.** *If $R[x]/(x^n + a_1x^{n-1} + \cdots + a_n)$ is left p.p., where $a_1, \cdots, a_n \in R$ and $n$ is a positive integer, then $R$ is left p.p.*

**Proof.** Note that $S = R[x]/(x^n + a_1x^{n-1} + \ldots + a_n) = \oplus_{i=0}^{n-1}Rx^i$ is a free left $R$-module with a basis $\{1, x, \ldots, x^{n-1}\}$ satisfying the assumptions of Theorem 1.1. □

**Theorem 1.5.** *If $RG$ is left p.p., then so is $RH$ for every subgroup $H$ of $G$.*

**Proof.** For $x \in RH$, because $RG$ is left p.p. and $RH \subseteq RG$, we have $\mathbf{l}_{RG}(x) = RGe$, where $e^2 = e \in RG$. Write $e = \sum_{h \in H} a_h h + \sum_{g \notin H} b_g g$. Then

$$0 = ex = (\textstyle\sum_{h \in H} a_h h)x + (\sum_{g \notin H} b_g g)x.$$

Note that if $h \in H$ and $g \notin H$ then $hg \notin H$. This shows that the support of $(\sum_{g \notin H} b_g g)x$ is contained in $G \backslash H$. So by the above equality that $\alpha := \sum_{h \in H} a_h h \in \mathbf{l}_{RH}(x) \subseteq \mathbf{l}_{RG}(x) = RGe$, and hence

$$\textstyle\sum_{h \in H} a_h h = (\sum_{h \in H} a_h h)e = (\sum_{h \in H} a_h h)^2 + (\sum_{h \in H} a_h h)(\sum_{g \notin H} b_g g).$$

Therefore, $\alpha^2 = \alpha$ and $RH\alpha \subseteq \mathbf{l}_{RH}(x)$. If $y \in \mathbf{l}_{RH}(x)$, then $yx = 0$. So $y = ye = y(\sum_{h \in H} a_h h) + y(\sum_{g \notin H} b_g g)$, showing that $y = y(\sum_{h \in H} a_h h) = y\alpha$. Hence $RH\alpha = \mathbf{l}_{RH}(x)$ and $RH$ is left p.p. □

**Theorem 1.6.** *If $G$ is a finite group and $RG$ is left p.p., then $|G|^{-1} \in R$.*

**Proof.** It is well-known that $\mathbf{l}_{RG}(\hat{G}) = \Delta_R(G)$. Since $RG$ is left p.p., we have $\Delta_R(G) = \mathbf{l}_{RG}(\hat{G}) = RGe$ where $e^2 = e \in RG$. Then $\Delta_R(G)$ is a direct summand of $RG$. By [10, Lemma 3.4.6], $|G|$ is invertible in $R$. □

**Example 1.7.** *$\mathbb{Z}G$ is not left p.p. for any nontrivial finite group $G$.*

**Example 1.8.** *Let $G$ be a finite group and $n$ be an integer with $n > 1$. Then the following are equivalent:*

(i) *$\mathbb{Z}_n G$ is Baer;*

(ii) *$\mathbb{Z}_n G$ is (left) p.p.;*

(iii) $\gcd(n, |G|) = 1$ *and $n$ is square-free.*

**Proof.** (i) clearly implies (ii).

Suppose that (ii) holds. Write $n = p_1^{s_1} \cdots p_k^{s_k}$ where all $p_i$ are prime numbers and $s_i > 0$. Then $\mathbb{Z}_n \cong \mathbb{Z}_{p_1^{s_1}} \times \cdots \times \mathbb{Z}_{p_k^{s_k}}$, and $\mathbb{Z}_n G \cong \mathbb{Z}_{p_1^{s_1}} G \times \cdots \times \mathbb{Z}_{p_k^{s_k}} G$. It follows from (ii) that each $\mathbb{Z}_{p_i^{s_i}} G$ is p.p. So $\mathbb{Z}_{p_i^{s_i}}$ is (left) p.p. and $p_i^{s_i} \nmid |G|$ by Theorem 1.6.

Claim. If $\mathbb{Z}_{p_i^{s_i}}$ is left p.p. then $s_i = 1$.

Proof. Assume that $s_i > 1$. Since $\mathbb{Z}_{p_i^{s_i}}$ is left p.p., $\mathbf{l}_{\mathbb{Z}_{p_i^{s_i}}}(p_i) = \mathbb{Z}_{p_i^{s_i}} e$, where $e^2 = e \in \mathbb{Z}_{p_i^{s_i}}$. Because $\mathbb{Z}_{p_i^{s_i}}$ is local, either $e = 0$ or $e = 1$. Then $\mathbf{l}_{\mathbb{Z}_{p_i^{s_i}}}(p_i) = 0$ or $\mathbf{l}_{\mathbb{Z}_{p_i^{s_i}}}(p_i) = \mathbb{Z}_{p_i^{s_i}}$, a contradiction. Thus $s_i = 1$ and $p_i \nmid |G|$. Hence (iii) holds.

If (iii) holds, then $\mathbb{Z}_n G$ is a semisimple ring by Maschke's Theorem, hence (i) holds. □

**Proposition 1.9.** *Let $R$ be a von Neumann regular ring and $G$ be a locally finite group. Then the following are equivalent:*

(i) *$RG$ is (left) p.p.;*

(ii) *the order of every finite subgroup of $G$ is a unit in $R$.*

**Proof.** Suppose that (i) holds. Since $RG$ is left p.p., by Theorem 1.5 we have $RH$ is left p.p for every finite subgroup $H$ of $G$. So we have $|H|^{-1} \in R$ by Theorem 1.6. Hence (ii) holds.

Suppose (ii) holds. By [1], $RG$ is von Neumann regular, so $RG$ is left p.p. □

In the following, $S_3$ denotes the symmetric group of order 6.

**Lemma 1.10.** [4, Lemma 4.7 ] *If $6^{-1} \in R$, then $RS_3 \cong R \times R \times \mathbb{M}_2(R)$.*

By [8, Proposition 9(i)], if $R$ is a left p.p. ring then so is $eRe$ for $e^2 = e \in R$. Thus if $\mathbb{M}_2(R)$ is left p.p. then $R$ is left p.p. So we have

**Corollary 1.11.** *If $6^{-1} \in R$, then $RS_3$ is left p.p. if and only if $\mathbb{M}_2(R)$ is left p.p.*

## 2. Group Rings of Finite Cyclic Groups

Let $R$ be a ring and $G$ be a finite group. If the group ring $RG$ is left p.p. then $R$ is left p.p. and $|G|^{-1} \in R$ by Corollary 1.2 and Theorem 1.6. Thus it is natural to ask whether the converse holds. In this section, counterexamples to this question are given.

**Proposition 2.1.** *$RC_2$ is left p.p. if and only if $R$ is left p.p. and $2^{-1} \in R$.*

**Proof.** By [13, Lemma 2.1], if $2^{-1} \in R$ then $RC_2 \cong R \times R$. Thus the result follows from Corollary 1.2 and Theorem 1.6. □

**Proposition 2.2.** *$RC_4$ is left p.p. if and only if $R[x]/(x^2+1)$ is left p.p. and $2^{-1} \in R$.*

**Proof.** By [13, Lemma 2.3], if $2^{-1} \in R$ then $RC_4 \cong R \times R \times R[x]/(x^2+1)$. Thus the result follows from Corollary 1.4 and Theorem 1.6. □

**Proposition 2.3.** *If $R \subseteq \mathbb{C}$, then $RC_3$ is left p.p. if and only if $R[x]/(x^2+x+1)$ is left p.p. and $3^{-1} \in R$.*

**Proof.** By [13, Lemma 2.5], if $R \subseteq \mathbb{C}$ and $3^{-1} \in R$ then

$$RC_3 \cong R \times R[x]/(x^2+x+1).$$

Thus the result follows from Corollary 1.4 and Theorem 1.6. □

The proof of the next theorem is similar to that of [13, Theorem 2.6].

**Theorem 2.4.** *Let $R$ be a subring of $\mathbb{C}$ and let $Q(R)$ denote the quotient field of $R$. Consider the polynomial $x^2 + a_1x + a_2 \in R[x]$ with $a_1^2 - 4a_2 \neq 0$. Let $\alpha$ be a solution of $x^2 + a_1x + a_2 = 0$ in $\mathbb{C}$. Then $R[x]/(x^2 + a_1x + a_2)$ is left p.p. if and only if either $\alpha \in R$ or $R\alpha \cap R = 0$ (i.e., $\alpha \notin Q(R)$).*

**Proof.** Let $T$ denote the ring $R[x]/(x^2+a_1x+a_2)$ and $x^2+a_1x+a_2 = (x-\alpha)(x-\beta)$ where $\alpha, \beta \in\mathbb{C}$. By hypothesis, $\alpha \neq \beta$. First suppose $\alpha \notin Q(R)$. Then $T$ is a domain. In particular $T$ is p.p.

Next suppose $\alpha \in Q(R)$. Then $\beta \in Q(R)$. Define the map $\varphi : R[x] \to Q(R) \times Q(R)$ by $\varphi(f(x)) = (f(\alpha), f(\beta))$. Then the kernel of $\varphi$ is $(x^2 + a_1x + a_2)$. Hence $T$ can be regarded as a subring of $Q(R) \times Q(R)$. It is clear that $T$ is not a domain.

Claim. $T$ is (left) p.p. if and only if $T$ contains the idempotent $(0, 1) \in Q(R) \times Q(R)$.

Proof. " $\Rightarrow$ " Since $T$ is not a domain, if $T$ is (left) p.p. then $T$ contains the nontrivial idempotents of $Q(R)\times Q(R)$. The nontrivial idempotents of $Q(R)\times Q(R)$ are exactly $(1, 0)$ and $(0, 1)$. So $(0, 1) \in T$.

" $\Leftarrow$ " Assume $(0, 1) \in T$. Then $(1, 0) \in T$. Consider any $(0, 0) \neq (a, b) \in T$, where $a, b \in Q(R)$. If $a \neq 0, b \neq 0$, $\mathbf{l}_T((a, b)) = 0$; if $a = 0, b \neq 0$, $\mathbf{l}_T((a, b)) = T(1, 0)$; if $a \neq 0, b = 0$, $\mathbf{l}_T((a, b)) = T(0, 1)$. So $T$ is (left) p.p.

Moreover, $(0, 1) \in T$ if and only if there exists $ax+b \in R[x]$ such that $a\alpha+b = 0$ and $a\beta + b = 1$. Since $x^2 + a_1x + a_2 = (x - \alpha)(x - \beta)$, we have that $(a_1a - 1)b = [-(\alpha + \beta)a - 1]b = [-(1 - 2b) - 1]b = 2b(b - 1) = 2(-a\alpha)(-a\beta) = 2a^2a_2$. Hence $b = a(a_1b - 2aa_2)$. So $\alpha = -\frac{b}{a} \in R$. □

**Example 2.5.** Let $R_0 = \{n/2^k : n, k \in \mathbb{Z}, k \geq 0\}$. Then $R_0$ is a subring of $\mathbb{Q}$. Set

$$R = \{a + pb\mathbf{i} : a, b \in R_0\}$$

where $p > 2,\ p$ is a prime. Then $R$ is a subring of $\mathbb{C}$ with $\frac{1}{2} \in R$. Because $R$ is a domain, it is certainly p.p. Clearly $\mathbf{i} \notin R$. Moreover, for $r = p$ and $s = p\mathbf{i}$, we have $s = p\mathbf{i} \in R \cap R\mathbf{i}$. So, by Theorem 2.4, $R[x]/(x^2 + 1)$ is not (left) p.p. Hence $RC_4$ is not (left) p.p. by Proposition 2.2.

**Example 2.6.** [13, Example 2.8] Let $R_0 = \{n/3^k : n, k \in \mathbb{Z},\ k \geq 0\}$. Then $R_0$ is a subring of $\mathbb{Q}$. Set

$$R = \{a + \sqrt{3}b\mathbf{i} : a, b \in R_0\}.$$

Then $R$ is a subring of $\mathbb{C}$ with $\frac{1}{3} \in R$. Because $R$ is a domain, it is certainly p.p. Clearly $\alpha = \frac{-1+\sqrt{3}\mathbf{i}}{2} \notin R$. Let $r = 2\sqrt{3}\mathbf{i}, s = -(3+\sqrt{3})\mathbf{i}$. Then $s = r\alpha \in R\alpha \cap R$. Hence $RC_3$ is not (left) p.p. by Proposition 2.3 and Theorem 2.4.

## 3. Fixed Rings

Let $G$ be a group acting on $R$ as automorphisms and let $R^G$ be the fixed ring of $G$ acting on $R$. Here we study the conditions under which $R^G$ becomes left p.p.

**Theorem 3.1.** *Let $R$ be a ring and $G$ be a group acting on $R$ as automorphisms such that either (i) $ee^g = e^g e$ for all $e^2 = e \in R$ and all $g \in G$ or (ii) $G$ is finite with $|G|^{-1} \in R$. If $R$ is left p.p., so is $R^G$.*

**Proof.** For any $a \in R^G$, since $R$ is left p.p., we have $\mathbf{l}_R(a) = Re$ where $e^2 = e \in R$. For $g \in G$,

$$Re^g = R^g e^g = (Re)^g = (\mathbf{l}_R(a))^g = \mathbf{l}_{R^g}(a^g) = \mathbf{l}_R(a) = Re.$$

It follows that

$$e^g = e^g e \text{ and } e = ee^g \text{ for all } g \in G. \qquad (3.1)$$

Suppose that (i) holds. It follows that $e = e^g$ for all $g \in G$, so $e \in R^G$. Since $ea = 0$, we have that $R^G e \subseteq \mathbf{l}_{R^G}(a)$. For $r \in \mathbf{l}_{R^G}(a)$, we have $ra = 0$, so $r \in \mathbf{l}_R(a) = Re$. Thus $r = re \in R^G e$. Hence $\mathbf{l}_{R^G}(a) = R^G e$.

Suppose that (ii) holds. Let $f = \frac{1}{|G|}\sum_{g\in G} e^g$. Note that, for all $g, h \in G$, (3.1) implies $e^h e^g = (e^h e)e^g = e^h(ee^g) = e^h e = e^h$. This shows that

$$\begin{aligned} f^2 &= \left(\tfrac{1}{|G|}\textstyle\sum_{h\in G} e^h\right)\left(\tfrac{1}{|G|}\textstyle\sum_{g\in G} e^g\right) = \tfrac{1}{|G|^2}\textstyle\sum_{h\in G}\sum_{g\in G} e^h e^g \\ &= \tfrac{1}{|G|^2}\textstyle\sum_{h\in G}\sum_{g\in G} e^h = \tfrac{1}{|G|}\textstyle\sum_{h\in G} e^h = f. \end{aligned}$$

Moreover, $f^g = f$ for all $g \in G$. So $f \in R^G$. Because $ea = 0$ and $f = \frac{1}{|G|}\sum_{g\in G} e^g = \frac{1}{|G|}\sum_{g\in G} e^g e \in Re$ (by (3.1)), we have $R^G f \subseteq \mathbf{l}_{R^G}(a)$. Note that $\mathbf{l}_{R^G}(a) \subseteq \mathbf{l}_R(a) = Re^g$ for all $g \in G$. Thus, for $r \in \mathbf{l}_{R^G}(a)$, $r = re^g$ for all $g \in G$. Hence $r = \frac{1}{|G|}(|G|r) = \frac{1}{|G|}\sum_{g\in G} re^g = rf \in R^G f$, so $\mathbf{l}_{R^G}(a) = R^G f$. Therefore, $R^G$ is left p.p. □

The assumptions (i) and (ii) in the previous theorem are necessary by the next example.

**Example 3.2.** [12, Example 6.4] Let $K$ be a field with $\operatorname{char}(K) = p > 0$. Let $R = \mathbb{M}_2(K)$ and $G = \langle g \rangle$ where $g : R \to R, r \mapsto u^{-1}ru$, with $u = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$. Then $R$ is left p.p. (simple Artinian indeed). Direct calculations show that $R^G =$

$\left\{\begin{pmatrix} a & b \\ 0 & a \end{pmatrix} : a, b \in K\right\}$. So $J(R^G) = \left\{\begin{pmatrix} 0 & b \\ 0 & 0 \end{pmatrix} : b \in K\right\}$. If $x = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$, then $\mathbf{l}_{R^G}(x) = J(R^G)$. Because $J(R^G)$ can not be generated by an idempotent, $R^G$ is not left p.p. If $e = \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix} \in R$, then $e^2 = e$ and $e^g = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$. It is clear that $ee^g = e \neq e^g = e^g e$. Moreover, $|G| = p$ is zero in $R$.

The next example shows that $R$ being left p.p. is not necessary for $R^G$ to be left p.p.

**Example 3.3.** [13, Example 3.3] Let $K$ be a field with $2^{-1} \in K$ and $R$ be the ring $\left\{\begin{pmatrix} a & b \\ 0 & a \end{pmatrix} : a, b \in K\right\}$. Let $g : R \to R$ be given by $\begin{pmatrix} a & b \\ 0 & a \end{pmatrix} \mapsto \begin{pmatrix} a & -b \\ 0 & a \end{pmatrix}$, and $G = \langle g \rangle$. Then $R^G = \left\{\begin{pmatrix} a & 0 \\ 0 & a \end{pmatrix} : a \in K\right\} \cong K$. So $R^G$ is p.p., but $R$ is not left p.p. by Example 3.2.

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