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Calculating Weibull Coefficients Using the Maximum Likelihood Method and Comparing Performance Across Sites

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Highlights

- The coefficients of the WDF were calculated using the MLM.
- The performance of MLM investigated at different regions.
- The obtained results were evaluated using statistical tests.

Article Info

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Keywords

Weibull distribution function,
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Abstract

In this study, the compliance of the Weibull Distribution Function (WDF) and actual wind data (WD) from three different locations were investigated. The coefficients of the WDF were calculated using the Maximum Likelihood Method (MLM) in the Adana, Osmaniye, and Hatay regions. The main purpose of this study is to observe the performances of the MLM in determining the coefficients of the WDF in different regions in different years and to examine the success of this method in estimating the mean wind power and speed of the determined regions. The performance of the indicated approach in all three selected locations was evaluated using the Root Mean Square Error (RMSE), Coefficient of Determination (R^2), and Mean Percentage Error (MPE). Also wind power densities were estimated for all three regions, which are one of the most essential metrics for estimating a region's wind energy (WE) potential. WDF power densities were estimated and compared to real wind power densities generated from measured WD for three different places. The performance of the method described in this paper was investigated in depth in various places with varying geographic characteristics. In addition, in the same years, the performance of the chosen method was evaluated in detail in three distinct places, and it was seen how geographical factors affected the method's performance.

1. INTRODUCTION

Recently, meeting the energy needs is one of the biggest problems of countries. Increasing energy need and depleting fossil fuels are increasing this problem. Therefore, focusing on renewable energy (RE) in the world and in our country is the main element of the strategy of revitalizing and greening the global economy. Today, the need for energy is increasing due to the rapid increase in the population and the continuous advancement of technology throughout the world, and the importance of RE sources is increasing to meet this need. Due to the fact that fossil fuels will be depleted day by day and reducing the effect of harmful greenhouse gases emitted into the atmosphere, RE sources are given importance around the world [1]. The uneven distribution of fossil fuels on the earth's surface requires some countries to be dependent on foreign energy. Turkey and developed countries show a similar increase in this trend and are in an effort to increase their orientation to RE sources and to reduce their dependence on oil. Among the RE sources, WE is one of the most suitable ones in terms of applicability. WE is an environmentally friendly energy source that does not consume natural resources and does not cause global warming. Historically, wind has played an important role in human life, from sailing ships to mills, from irrigation of agricultural fields to wind turbines. Countries are developing maps and charts showing their energy potentials because of the applicability of WE, its direction in determining the potential for establishing WE systems and similar reasons. Although the cost of generating energy from wind power in the past was not preferred because it was much more than producing energy with fossil fuels, the depletion of fossil fuel resources, the damage

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it caused to the environment and the recent developments in wind power generation technologies reduced this cost, and an increase in the energy to be obtained from the wind. has created. In order to learn the WE situation in our country, wind measurements and other meteorological measurements are made by the General Directorate of Meteorology (MGM). There are many distribution functions that can be used to determine the distribution of WS. The most widely used of these are the two-parameter WDF and Rayleigh distribution functions (RDF). Although it is easier to calculate the parameters in the RDF, it is less reliable than the WDF because it is a single parameter. For this reason, the two-parameter WDF is used more frequently in WE potential analysis [2]. When it is desired to analyze with the Two-Parameter WDF function, long-term WD of a certain region that have been previously measured are needed. In order to determine the average WS and power using these WD, it is necessary to find the WDF parameters k and c first. WDF was used as a statistical approach in the evaluation of WD. In order to calculate the shape and scale factors that determine the WDF, the maximum probability method was applied using six years of data from three different regions. The results of the MLM method were compared according to the ability of the obtained distribution to represent the real data. The standard deviation, average speed, the most probable speed, the speed that makes the greatest contribution to the energy flow, the power flow and the energy flow are calculated using the maximum probability method on an annual basis for each station where WD is collected. Many research are being conducted around the world to estimate the WE potential. The WDF, which is consistent with WS data, is utilized in the majority of these research to assess the WE potential for a given region. The parameters of the WDF are the most important indicator in determining the WE potential. In the literature, a variety of numerical approaches were utilized to calculate these parameters [3, 4].

One of the main goals of this research is to see how the MLM's performance varies in different places with different mean WSs. For the purpose of justifying the MLM's performance, the statistical fits of the computed and measured hourly WS data were analyzed. MLM was utilized in this paper, and its performance was examined in three separate regions with varying average wind speed (WS) and characteristics. As a result, it has been noted how the WDF's performance varies depending on geographical factors and mean WS. Using various error analysis methodologies, the performance of the MLM approach used to find the coefficients of WDF has been thoroughly investigated.

In the literature; Wang et al. [5] examined the distributions of low-order statistics, peak factors, and low-occurring strong WSs. The goodness of fit of the WDF to the wind speed data was analyzed. Muhammad et al. [6] analyzed the different monsoon seasons and used WDF to evaluate the average wind strength in their chosen region. In this article they did, they used hourly WS data. According to the results they obtained, they suggested that the performances of the methods used while determining the WDF coefficients varied. Hussain et al. [7] stated that Pakistan is currently facing serious energy crises and global warming effects, and therefore they argued that there is an urgent need to use RE generation. This article explores and numerically analyzes the WE density potential of coastal regions. They calculated the coefficients of the WD function in different regions using eight different methods. Kaplan [8] found the coefficients of the WD function in a region she chose in her study with the moving least squares approach and suggested that this method she used showed a very successful performance

2. WIND MEASUREMENT SITES MATERIAL METHOD

Figure 1 shows the regions that were chosen. In this study, three different points with different wind characteristics were determined. These locations are shown in detail on the map below. Although the regions selected in this study seem close on the map, they are different in terms of wind character and potential. Thus, it will be possible to observe how the performance of the method to be used to find the coefficients of WDF changes in different wind characteristics. Three distinct regions with varied average WSs were chosen in order to examine the performance of the suggested approach at various WSs.

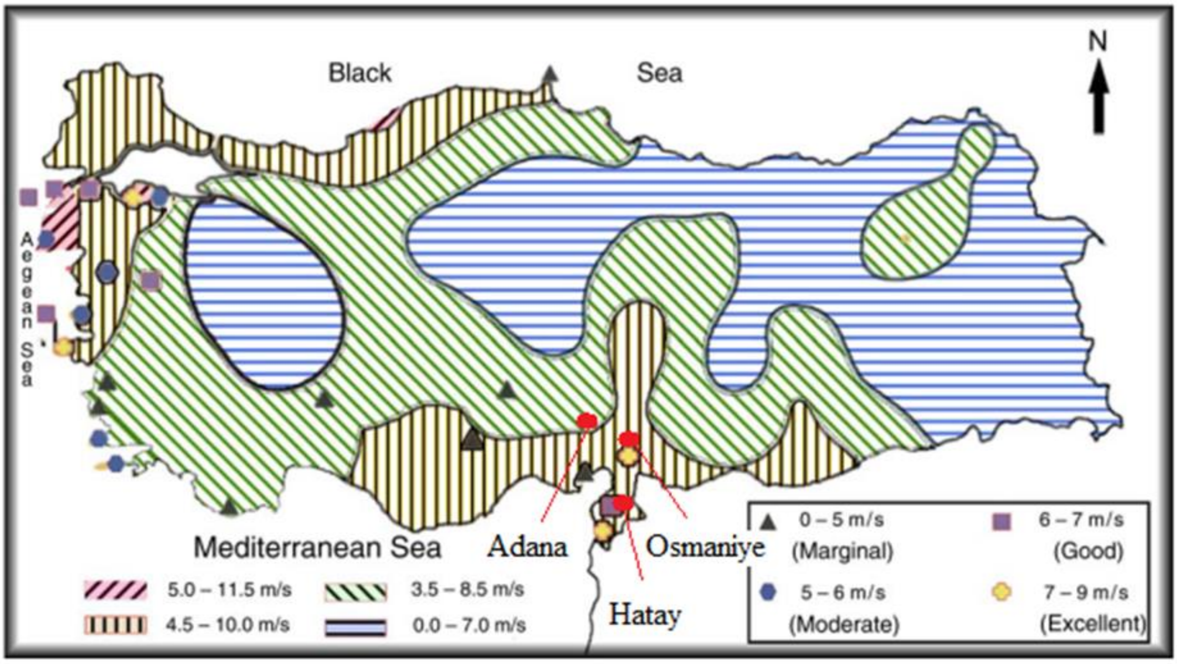

Figure 1. *Locations of WS observations used in this study*

The WE Association in Turkey provides statistical data on the provinces' wind potential in the form of tables and graphs based on data that is regularly updated. Another goal of this study is to compare data collected at a height of 10 meters in various areas and provinces of Turkey with findings estimated using the WDF.

In order to satisfy rising energy demand, Turkey should expand its usage of RE and enhance installed capacity of WE. The new WE targets must be established, as well as the associated investments. It is vital to build more efficient systems in order to minimize WE production costs and raise the share of WE in total energy production. The goal of wind turbine technology advancements is to increase production capacity while maintaining physical strength and operating quality.

3. OUTLINE OF THE METHODOLOGY

There are many distribution functions used for the analysis of wind speed data. The most commonly used distribution functions are the two-parameter WDF and RDF. The most used and best performing distribution function for the analysis of wind speed data is WDF. Because the RDF has just one parameter, it is less flexible than the WDF; nonetheless, estimating its parameters is significantly easier. Thanks to the detailed study for these selected regions, the WE potential for each region will be determined by MLM, and it will be examined whether the performance of MLM has changed in different regions [9, 10].

Nomenclature

A:	swept area	fi:	wind blowing frequency
v:	wind speed	xi:	i-th WDF data
k:	shape paremeter	yi:	i-th measured data
c:	scale parameter	zi:	i-th mean real data
ρ:	air density	Pm:	actual wind power
σ:	standard deviation	Pw:	predicted wind power
Γ:	Gama function		

3.1. Determination of WDF Parameters

The WDF is now widely used in life-time data analysis and statistical models in engineering. In recent studies, WDF has been widely used for short-term prediction of WS. In the study, firstly, the WS data in the hourly WS raw data obtained from the Meteorology Directorate were examined in detail. Then, six years of processed hourly WS data from the Meteorology Directorate for three different regions were estimated using MLM for various input sizes with the best accuracy of WSs at certain time intervals. Wind potential is calculated using probability density functions in a specific region. The probability density function is a mathematical function that describes the WS distribution. The parameters of the WDF are calculated using a variety of numerical methods. The following Equation 1 and Equation 2 offer the general expression for the WDF. The dimensionless form 'k' coefficient and the scale 'c' coefficient, which has the same unit as WS, are the two parameters of the WDF [11, 12]:

$$p(v) = \left(\frac{k}{c}\right)\left(\frac{v}{c}\right)^{k-1} exp\left[-\left(\frac{v}{c}\right)^{k}\right] \quad (1)$$

$$P(v) = 1 - exp\left[-\left(\frac{v}{c}\right)^{k}\right] \quad . \quad (2)$$

3.2. Maximum Likelihood Method

Actual values obtained from meteorological stations or wind farms are used in predictions for wind (speed, direction, potential, etc.). In particular, the WD measured in the past plays an important role for the estimation phase of the MLM model to be applied. In this study, hourly WS data covering six years obtained from the General Directorate of Meteorology were used. These data were statistically analyzed and adapted to the MLM model. The MLM is one of the approaches for calculating the shape and scale parameters of the WDF. Many iterations may be required in the MLM to compute the WDF's 'k' parameter, as seen in Equation (3) and the 'c' value can be simply obtained using Equation (4) [13, 14, 15]

$$k = \left(\frac{\sum_1^n {v_i}^k ln(v_i)}{\sum_1^n {v_i}^k} - \frac{\sum_1^n ln(v_i)}{n}\right)^{-1} \quad (3)$$

$$c = \left(\frac{1}{n}\sum_1^n {V_i}^k\right)^{1/k} . \quad (4)$$

The dimensionless form 'k' parameter and scale 'c' parameter were found using hourly average WS data in this study. Between 2010 and 2015, WS values were measured hourly in all three zones.

3.3. Wind Power Calculation

Currently, research are being conducted to determine the WE potential of various places. To extract WE from a location, it is necessary to first estimate the region's WE potential. The WE is generally stated as the following Equation (5) [16, 17]. Here, P is the wind power, A = the area swept by the propeller (r is the turbine radius) and v is the wind speed. Here ρ=1.225 kg/m^3 is the density of air at sea level

$$P_A = \frac{1}{2}\rho A v^3. \quad (5)$$

Because WE is related to WS, the WS is the most essential input in calculating the WE potential, as can be seen from the preceding equation. WE can vary dramatically depending on small variations in wind intensity. The frequency and duration of the desired wind intensity are the most critical indicators when establishing WE farms [18, 19, 20].

4. RESULTS AND DISCUSSION

The "k" of the WDF is very important for investigating and finding the characteristics of WS in a location. The "c" indicates the wind potential of that area. The larger the "c" parameter value, the higher the wind potential. These two parameters of the WDF affect the distribution curve, so these parameters must be considered together to determine the wind characteristics of a region. The determined shape and scale coefficients of WDF for three different sites were illustrated from Figure 2 to Figure 7.

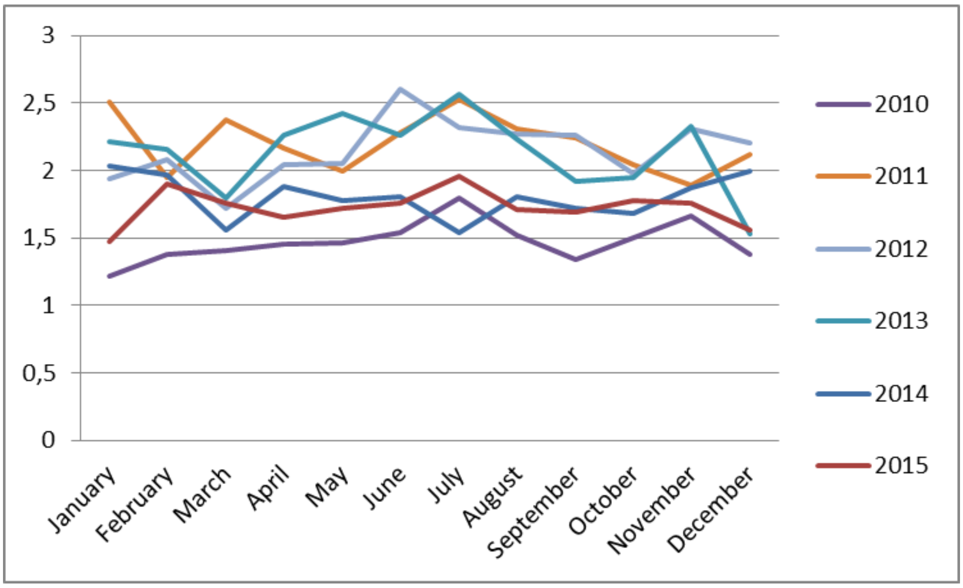

Figure 2. *The "k" parameter for Adana Region*

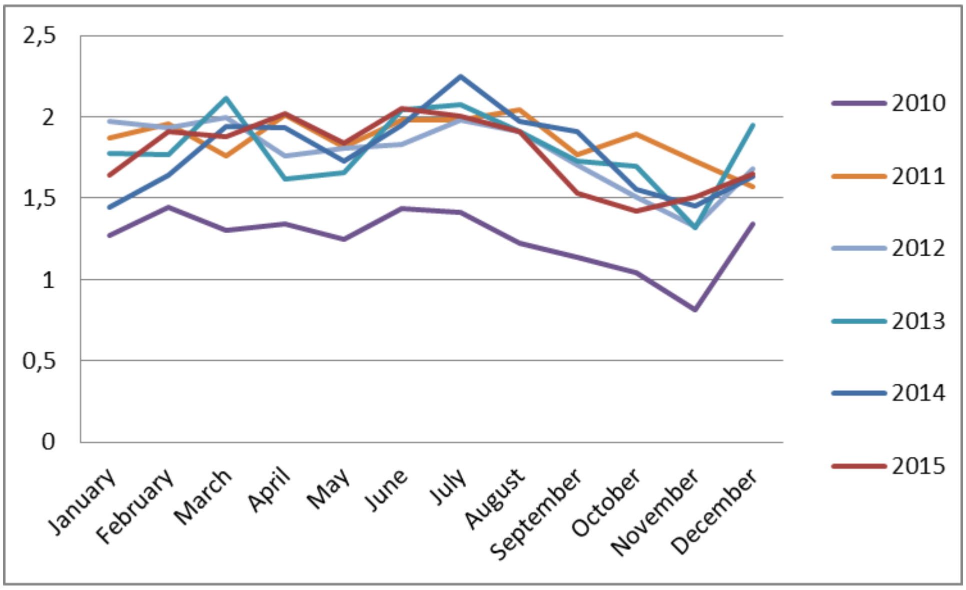

Figure 3. *The "c" parameter for Adana Region*

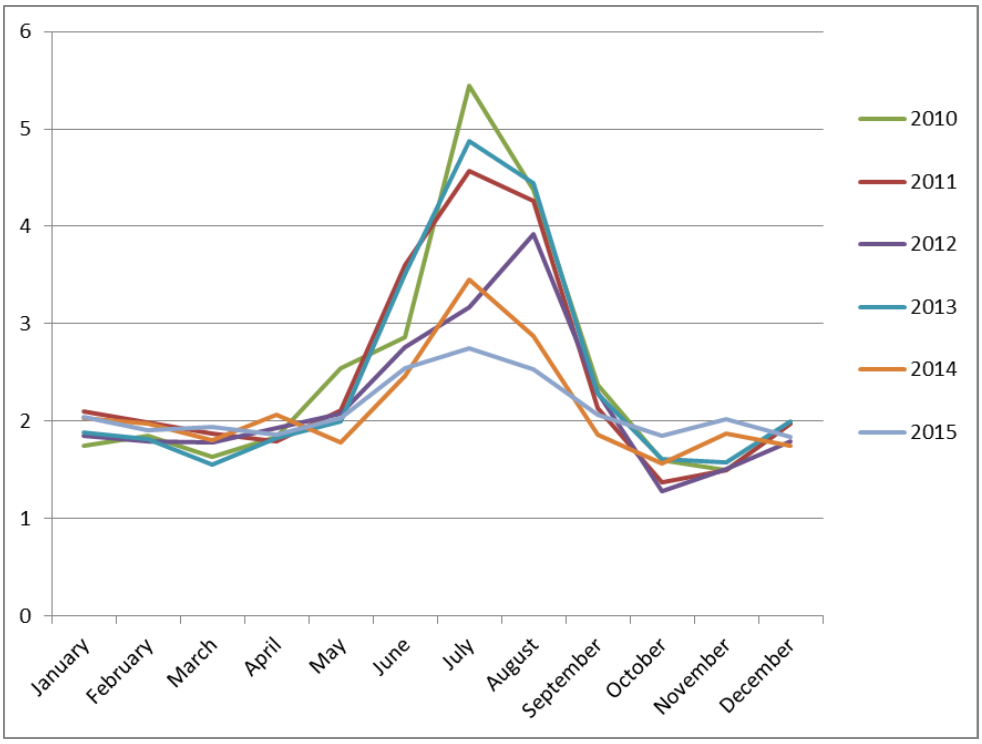

Figure 4. *The "k" parameter for Hatay Region*

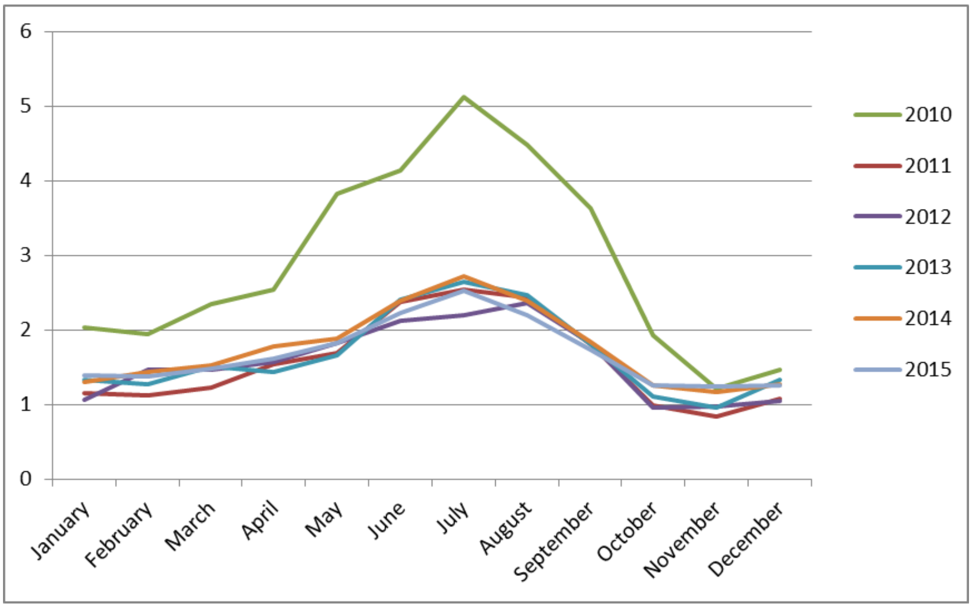

Figure 5. *The "c" parameter for Hatay Region*

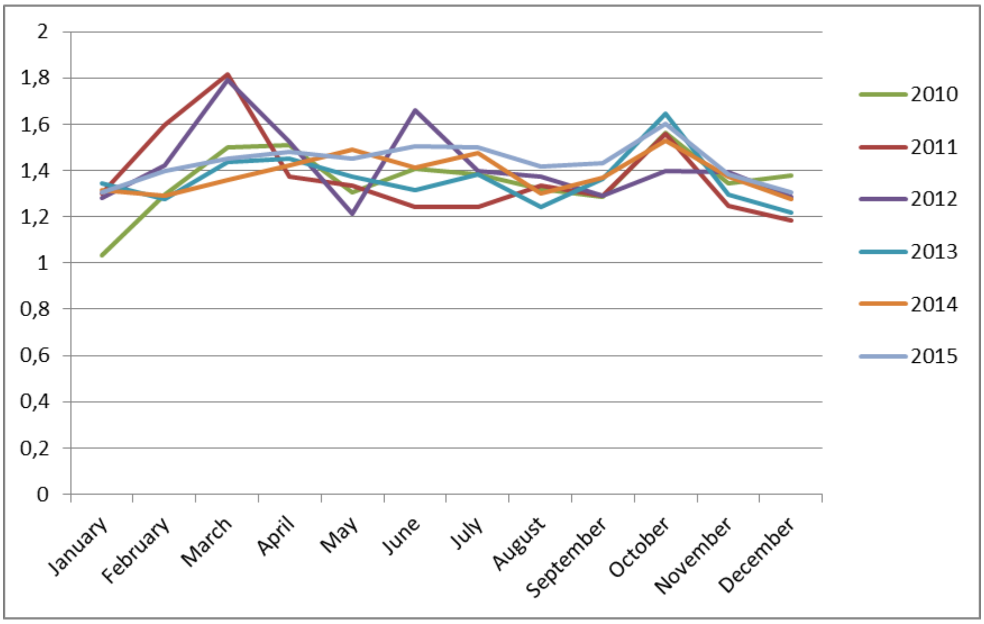

Figure 6. *The "k" parameter for Osmaniye Region*

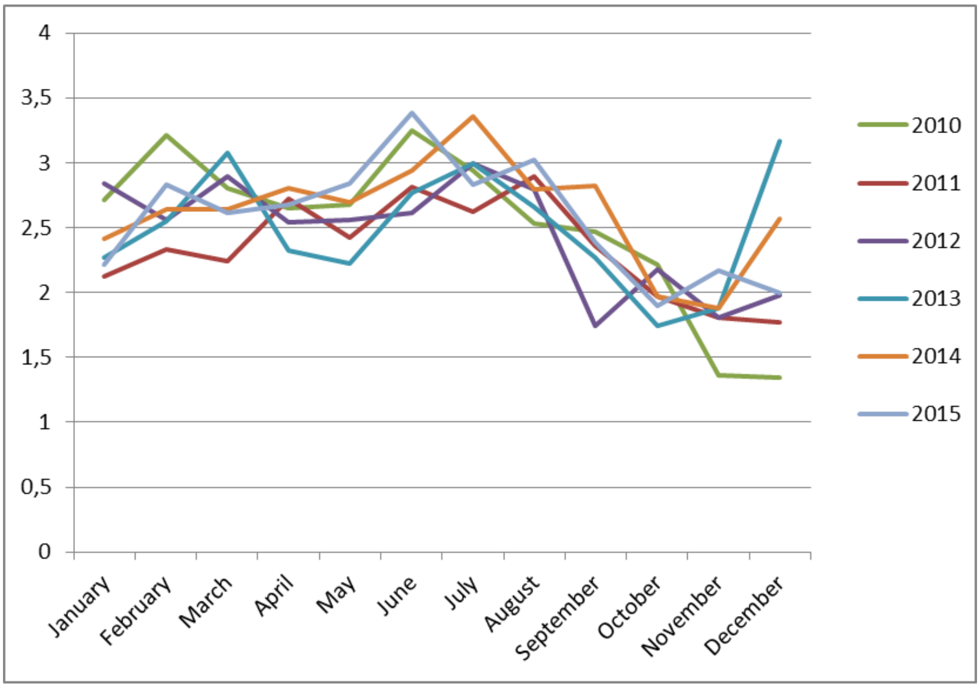

Figure 7. *The "k" parameter for Osmaniye Region*

One of the most essential aspects in determining the wind characteristics of the determined points is the dimensionless 'k' parameter. The estimated 'k' parameter is often more than '1' for three regions, hence these values show that the WSs fluctuate on average level. Smaller 'k' values correspond to more variable winds, while larger 'k' values correspond to less variable winds. The scale parameter 'c' shows the relative cumulative WS frequency. In simple words, the parameter c changes depending on the average speed. If the average speed is high, parameter c is also high [4].

4.1. Statistical Error Analysis

The performance of the MLM method, which is used to predict the WE potential of selected regions using WDF, has been examined with three different statistical tests given below. According to the results obtained, the MLM method performed very well in all three different regions [21, 22, 23, 24].

● The root mean square error (RMSE)

$$RMSE = \left[\frac{1}{N}\sum_{i=1}^{N}(y_i - x_i)^2\right]^{\frac{1}{2}}. \tag{6}$$

● Analysis of variance (R^2)

$$R^2 = \frac{\sum_{i=1}^{N}(y_i - z_i)^2 - \sum_{i=1}^{N}(y_i - x_i)^2}{\sum_{i=1}^{N}(y_i - z_i)^2}. \tag{7}$$

● Mean percentage error (MPE)

$$MPE = \frac{1}{N}\sum_{i=1}^{N}\left(\frac{x_i - y_i}{y_i}\right) * 100\% . \tag{8}$$

When the results given in Table 1 are evaluated in general, the MLM method performed very well in all three different regions. The fact that the results in the RMSE and MPE tests are close to 0 is an indication that the estimation performance of the method used is successful. In the R^2 test, the closer the result is to 1, the closer the estimated value by the method used is to the real value [11, 25].

Table 1. *Statistical test results for selected locations*

MLM	Adana			Hatay			Osmaniye		
Year	R^2	RMSE	MPE	R^2	RMSE	MPE	R^2	RMSE	MPE
2010	0.9996	$2.93\text{x}10^{-3}$	0.0179	0.999	$8.82\text{x}10^{-3}$	0.0919	0.9992	$4.5\text{x}10^{-3}$	0.2946
2011	0.9972	$6.3\text{x}10^{-3}$	0.3086	0.9992	0.0152	1.1482	0.9992	$2.8\text{x}10^{-3}$	0.2708
2012	0.9985	$6.81\text{x}10^{-3}$	0.3534	0.9978	0.0205	1.4810	0.9929	0.0275	1.1299
2013	0.9986	$7.23\text{x}10^{-3}$	0.4064	0.9986	0.0190	1.3071	0.9992	$3.36\text{x}10^{-3}$	0.4321
2014	0.9987	$7.63\text{x}10^{-3}$	0.4485	0.9981	0.0193	0.6135	0.9981	$4.39\text{x}10^{-3}$	0.6347
2015	0.9956	0.0122	0.2205	0.9940	0.0788	1.2748	0.9988	$3.83\text{x}10^{-3}$	0.5706

When the results are evaluated in general, it is seen that the MLM method performs very well in three different regions, although there are small differences from year to year. When the studies on similar subjects in the literature are examined, it is understood that these small differences are normal and the methods used may perform differently in different regions.

4.2. Power Density Assesment

The difference between the WDF-estimated WE and the calculated WE from actual time series data, as is well known, can be used as a metric to evaluate the method's performance. The average Ws 'Vm' and wind power density 'Pm' can be calculated using Equation (11) and Equation (12) [26, 27]:

$$v_m = \frac{1}{n}\sum_{i=1}^{n} v_i \tag{9}$$

$$P_M = \frac{1}{2}\rho\overline{V^3}. \tag{10}$$

Using the parameters of the WDF, the average WS and average WE power of the region we have chosen can be estimated with the equations given below. In this study, the actual values and estimated values for the three different regions determined are shown in Table 2 [28, 29]:

$$V_w = c\Gamma\left(1+\frac{1}{k}\right) \tag{11}$$

$$P_w = \frac{1}{2}\rho c^3 \Gamma\left(1+\frac{3}{k}\right). \tag{12}$$

Table 2. *Comparison of actual values and forecast values in terms of average WS and power*

	The predicted by MLM				Actual Values	
2009-2017	k	c (m/s)	V_w (m/s)	P_w (W/m^2)	V_m (m/s)	P_m (W/m^2)
Adana	1.8347	1.7108	1.5201	4.5031	1.4734	5.0740
Hatay	1.8534	1.8735	1.6637	5.8168	1.9111	12.4114
Osmaniye	1.1701	2.0996	1.9812	20.014	2.2735	25.2215

The real average WS and calculated power of WE for the three points were determined, as well as the average WS and predicted power of WE, using the WDF, and the results are presented in Table 2. When obtained results are compared, the Osmaniye region with the highest average WS is found to be the best suitable in terms of WE capacity. Based on computed and predicted power densities based on nine-year WS data, it was discovered that MLM gave very good performance for three separate locations.

5. CONCLUSIONS

The major goal of this paper is to assess MLM's performance in estimating WDF coefficients in three different regions with varying average WSs. The findings generated by MLM were evaluated using three different statistical error tests. At three different locations, statistical indicators revealed that the calculated values for MLM are near to true WS values. The conformance of the methods used to determine coefficients of WDF with real data may fluctuate in line with the mean WS, according to the results derived from data collected over a six-year period.

The following are some of the most important outcomes of this article:

- In this study, using long-term WS data, statistical analysis of WS distribution parameters at different selected points was performed, and when the results are evaluated in general, it is seen that the MLM method performs very well in three different regions, although there are small differences according to the selected regions.
- This study demonstrates the MLM's compliance with real WS data for the specified regions, and this research may provide a forecast for wind power investments in these areas.
- Based on the findings of the statistical error analysis, it can be concluded that the MLM for determining WDF coefficients performs admirably in some areas. The results vary depending on the year and the WS, therefore it cannot be said that this error analysis technique gives the best results for the region. Instead, it is determined that the same statistical test does not generate the best conclusions in various locations.
- When the literature is examined, many different methods and approaches have been used while determining the coefficients of WDF. The performances of these used methods are examined using different error tests. It was observed that the MLM, which was clearly used in this study, performed very well in different regions.
- This research's unique feature is that no extensive investigations of MLM have ever been done, therefore this detailed paper can make significant contributions to WE-related scientific studies.

CONFLICTS OF INTEREST

No conflict of interest was declared by the author.

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