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Determination of Relationships Between Stand Variables and Parameters of Weibull Function for *Fagus orientalis* Libsky Stands

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Abstract:

Aim of study: Information about the diameter distribution of a stand is a key resource to determine planning strategies, silvicultural treatment options and product variety. In this study, the diameter distributions of *Fagus orientalis* Libsky stands located in Karabük region were researched, and relationship between parameters of Weibull function and stand variables was investigated.

Area of study: Data used in this study were obtained from pure Oriental beech (*Fagus orientalis* Libsky) stands located in the Karabük region, north-central Turkey.

Material and methods: For this study, sixty-two sample plots from pure *Fagus orientalis* Lipsky stands located in Karabük region were taken. Maximum likelihood method was used to estimate parameters of the two-parameter Weibull probability density function. The parameters estimated were then expressed as linear functions of stand variables such as mean diameter, basal area, minimum and maximum diameters etc.

Main results: The regression model using arithmetic mean diameter as an independent variable and the model using maximum diameter of the stand as an independent variable were found superior for estimation of scale and shape parameters, respectively.

Highlights: While the Weibull distributions determined by both methods give close results, the method of determining the distribution parameters with the developed regression models seems to be superior in terms of examining the diameter distribution changes according to different stand structure simulations.

Keywords: Beech, Diameter Distribution, Karabük, Probability Density, Two Parameter Weibull

Fagus orientalis Libsky Meşcere Özellikleri ile Weibull Parametreleri Arasındaki İlişkilerin Belirlenmesi

Öz

Çalışmanın amacı: Bir meşcerenin çap dağılımına ilişkin bilgiler, planlama stratejilerini, silvikültürel müdahale seçeneklerini ve ürün çeşitliliğini belirlemek için önemli bir kaynaktır. Bu çalışmada Karabük bölgesindeki *Fagus orientalis* Libsky meşcerelerinin çap dağılımları incelenmiş ve Weibull fonksiyonunun parametre değerleri ile meşcere özellikleri arasındaki ilişkiler araştırılmıştır.

Çalışma alanı: Bu çalışmada kullanılan veriler, Türkiye'nin kuzey-orta kesiminde Karabük bölgesinde yer alan saf Doğu kayını (*Fagus orientalis* Lipsky) meşcerelerinden elde edilmiştir.

Materyal ve yöntem: Bu çalışma için Karabük bölgesinde yer alan saf *Fagus orientalis* Lipsky meşcerelerinden altmış iki adet örnek alan alınmıştır. Maksimum olabilirlik yöntemi ile tahmin edilen Weibull olasılık yoğunluk fonksiyonu parametreleri daha sonra orta çap, göğüs yüzeyi, minimum ve maksimum çaplar gibi meşcere özelliklerinin doğrusal fonksiyonları olarak modellenmiştir.

Temel sonuçlar: Bağımsız değişken olarak aritmetik orta çapı kullanan regresyon modeli ve bağımsız değişken olarak meşcerenin maksimum çap değerini kullanan model sırasıyla ölçek ve şekil parametrelerinin tahmininde üstün bulunmuştur.

Araştırma vurguları: Her iki yöntemle belirlenen Weibull dağılımları birbirine yakın sonuçlar verirken, geliştirilen regresyon modelleri ile dağılım parametrelerinin belirlenmesi, farklı meşcere yapısı simülasyonlarına göre çap dağılım değişimlerinin incelenmesi açısından daha üstün görünmektedir.

Anahtar Kelimeler: Kayın, Çap Dağılımı, Karabük, Olasılık Yoğunluk, İki Parametreli Weibull

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Introduction

Growth and yield models are systems of functions that predict growth and increment values for forests under different conditions. These models, which serve as a guide for researchers and forest managers in many ways, are used in the formation of planning strategies and silvicultural treatment options by making predictions about the future status of the forests (Vanclay, 1994). The growth and yield models are divided into three groups as individual-tree, whole-stand and diameter distribution models in terms of the modeling unit they are based on. Diameter distribution models which use statistical probability functions to define stand structure ensure detailed information compared to the whole-stand models (Zhang et al., 2010; Diamantopoulou et al., 2015; Sakici and Dal, 2021).

Information about the diameter distribution of the standing trees in a forest is an important source, especially for forest managers. Besides, for a logical planning approach, it is essential to determine the distribution of forest assets (basal area, volume, biomass etc.) by diameter classes and therefore the product diversity in the stand (Burkhart and Tomé, 2012). Diameter distribution models, in which the number of trees in diameter classes are modeled, serve as an important base for these purposes (Vanclay, 1994; Gadow and Hui, 1999). The aforementioned models are used to estimate the distribution of trees in the stand to diameter classes with the help of various distribution functions and to obtain more detailed information about stand dynamics (Loetsch et al., 1973; Gorgoso et al., 2007).

One of the most common approaches used to reveal the distributions of diameters at breast height (*DBH*) within various size classes is the use of a probability density function (PDF). Among the probability density functions such as normal, lognormal, Johnson's SB, gamma, beta, exponential and Weibull, one of the most frequently used is the Weibull function (Liu et al., 2009; Diamantopoulou et al., 2015). The success of the two-parameter Weibull function has been underlined in many studies carried out for different tree species, such as *Pseudotsuga menziesii* (Eng, 1986; Özdemir, 2016), *Pinus sylvestris* (Maltamo et al., 1995), *Picea abies* (Maltamo et al., 1995), *Betula alba* L. (Gorgoso et al., 2007), *Pinus tabulaeformis* (Lei, 2008), *Pinus nigra* (Stankova and Zlatanov, 2010), *Quercus suber* L. (Carretero and Álvarez, 2013), *Juniperus excels* Bieb (Diamantopoulou et al., 2015), *Tetraclinis articulate* (Sghaier et al., 2016), *Eucalyptus grandis* and *Eucalyptus urophylla* (Schmidt et al., 2020), *Pinus taeda* L. (Cao, 2004; Araújo et al., 2021).

Estimation of the parameters of the distribution functions, which is directly effective for success of the diameter distribution models, could be obtained by parameter estimation methods such as maximum likelihood, regression techniques or parameter recovery methods such as moments and percentiles methods (Burkhart and Tomé, 2012; Poudel and Cao, 2013; Sakici, 2021). Among them, maximum likelihood estimation (MLE) is assumed to be the best and the most widely used method (Gorgoso et al., 2007; Diamantopoulou et al., 2015).

Fagus orientalis Libsky (Oriental beech), one of the most common and valuable tree species in Turkey, spreads on an area of 1.9 million hectares which corresponds to approximately 8.2% of Turkey's forest areas. Besides, most of these beech stands (approximately 86%) are productive forests (GDF, 2020). However, studies examining the stand structures of this species are very limited in the country and in the study area.

While, the success of different PDFs has been compared in many of the studies conducted in Turkey on diameter distribution (Ercanli and Yavuz, 2010; Sakici and Gulsunar, 2012; Sakici et al., 2016), parameter prediction methods have been examined in some studies (Bolat and Ercanli, 2017; Sivrikaya and Karakaş, 2020). In addition, in some studies, relationship between stand variables and diameter distribution of some species has been investigated (Carus, 1996; Yavuz et al., 2002; Ercanli et al., 2013). However, studies on the subject discussed in this study are very limited in Turkey. Besides, the fact that the diameter distributions of the beech stands spreading in the Karabük region have not been examined before shows the need for this study. Once the PDF parameters are estimated and related to stand variables, development of a stand for any given stand variable can be determined. Moreover, by using this linear relationship between the parameters and the stand variables, silvicultural treatments can be determined more accurately (Nokoe and Okojie, 1984).

The objectives of this study were *(i)* to investigate diameter distributions in *Fagus orientalis* Libsky stands located in Karabük using Weibull distribution function, and *(ii)* to quantify relationships between parameters of Weibull distribution and stand variables. Even though there are many different PDFs and some of them are more flexible than the Weibull function, the main purpose of this study was not to compare the success of different PDFs, but to develop simple regression equations that will enable to have basic knowledge about the diameter distributions of the stands and to create simulations for different alternatives. For this reason, two-parameter Weibull function, which is the most known and frequently used PDF, was chosen to be used in this study.

Materials and Methods

This study investigated the diameter distribution of the *Fagus orientalis* Libsky stands located in Karabük, northern central Turkey (Figure 1). In Karabük, the winters are cold, snowy and partly cloudy while the summers are warm and clear. The annual average temperature of Karabük is 20 ℃, the annual precipitation is 522.1 mm, and the annual number of rainy days is 136 days (GDM, 2020). The main tree species of the study area are *Pinus nigra J.F.* Arnold., *Pinus brutia* Ten., *Pinus sylvestris* L., *Abies nordmanniana* (Stev.), *Juniperus* sp., *Carpinus betulus* L., *Quercus* sp., and *Fagus orientalis* Libsky.

Figure 1. Study area

For this study, sixty-two sample plots from pure *Fagus orientalis* Lipsky stands were taken. Sample plot sizes were 800, 600 and 400 m^2 for crown closures of 11–40%, 41–70 or >71, respectively.

The DBHs of all living trees thicker than 7.9 cm were measured using a caliper, and total of 1702 diameter measurements were obtained. By using the *DBH* values of the trees, number of trees (*N*, pieces ha^{-1}), quadratic mean diameter (*QMD*, cm), arithmetic mean diameter (*AMD*, cm), basal area (*G*, m^2 ha^{-1}) and relative density (*RD*, according to Curtis et al. 1981) were calculated for each sample plot. Besides, minimum diameter (D_{min}, cm) and maximum diameter (D_{max}, cm) of each sample plot were determined.

The diameter distribution of the *Fagus orientalis* stands was modeled using the two-parameter Weibull function. The form of the two-parameter Weibull function (Bailey and Dell, 1973; Schreuder and Swank, 1974) is as follows:

$$f(x) = \frac{c}{b}\left(\frac{x}{b}\right)^{c-1} exp\left[-\left(\frac{x}{b}\right)^{c}\right]$$

$x \geq 0$; b > 0; c > 0

where x is a random variable, b and c are scale and shape parameters of the distribution, respectively.

Scale and shape parameters of the Weibull distribution were estimated with MLE, using the *fitdist()* function of the "*fitdistrplus*" package in R Studio, version 4.0.2 (R Studio Team, 2020).

The relationships between estimated Weibull parameters and stand variables such as *N*, *QMD*, *AMD*, *G*, *RD*, D_{min} and D_{max} were investigated using correlation and regression analysis. Firstly, correlation coefficients between the stand variables and estimated b and c parameters were calculated. Then, these estimated b and c parameters were modelled with stepwise multiple regression analysis, using the stand variables and variables derived from the stand variables (such as $1/D_{max}$, G^2, *lnAMD*) as independent variables. This regression technique was used to determine the best predictive variables that were significant ($p<0.05$), with the highest determination of coefficient value (R^2). The model structure assumed as:

$$Y = \beta_0 + \beta_1.X_1 + \beta_2.X_2 + \ldots + \beta_n.X_n + \varepsilon$$

where; Y is Weibull parameters (b and c), $X_1, \ldots, X_n$ are the independent variables, $\beta_0, \ldots, \beta_n$ are regression coefficients.

Results and Discussion

Descriptive statistics of the estimated Weibull parameters and calculated stand variables are presented in Table 1. As seen from the table, the sample plots examined in the study represent wide ranges in terms of stand characteristics such as *N*, *QMD*, *AMD*, *G*, and *RD*. Estimated scale parameters (b) varied between 10.5 and 46.7, while shape parameters (c) varied between 1.4 and 8.5. The scale parameter value decreased with increasing number of trees, while showed positive correlation with the others. As is evident in the Table 2, the stand characteristics with the highest correlation with the shape parameter were determined as D_{max}, *QMD*, *G* and *AMD*.

Table 1. Summary statistics of the stand attributes and estimated Weibull parameters

Stand variables	Min	Max	Mean
N (trees ha^{-1})	187	1475	652
QMD (cm)	8.8	47.7	27.5
AMD (cm)	8.8	41.0	24.1
D_{min} (cm)	8.0	22.0	9.2
D_{max} (cm)	12.0	110.0	55.4
G (m^2 ha^{-1})	4.4	82.4	35.7
RD	1.4	14.6	6.6
Weibull Parameters			
b	10.5382	46.7434	27.7603
c	1.3504	8.5327	2.7905

Where *N* is number of trees, *QMD* is quadratic mean diameter, *AMD* is arithmetic mean diameter, D_{min} and D_{max} are minimum and maximum diameters, *G* is basal area, *RD* is relative density, *b* and *c* are estimated scale and shape parameters of the Weibull distribution, respectively.

When correlations between estimated Weibull parameters and stand variables were investigated (Figure 2 and Table 2), all the correlations were found statistically significant ($p<0.01$) except between D_{min} and shape parameter ($p>0.05$). Mean diameter of the sampled stands, especially *AMD*, showed the highest correlations with scale parameter.

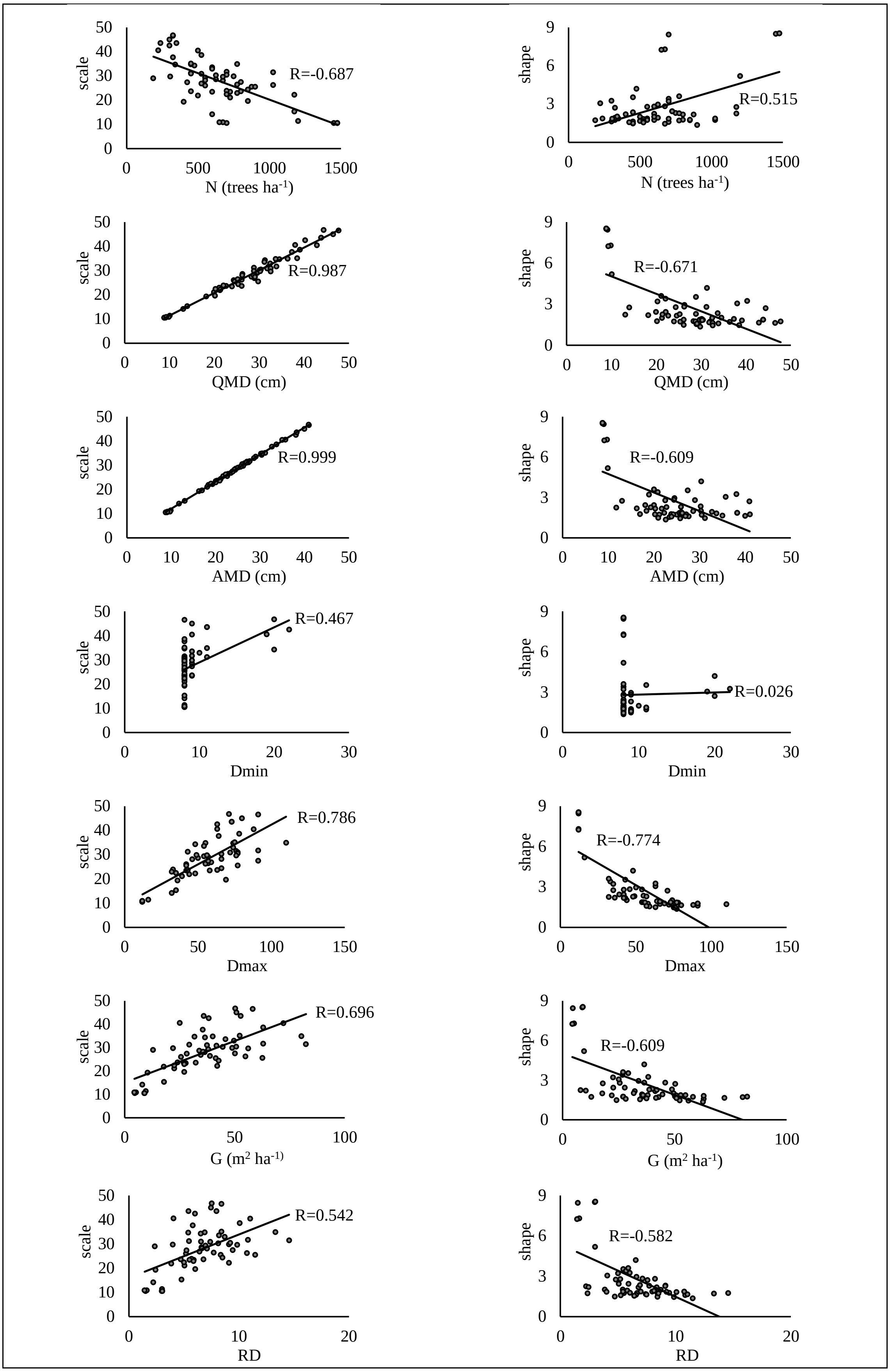

Figure 2. Correlations between stand variables and Weibull parameters

Table 2. Correlation coefficients between stand attributes and Weibull parameters

Stand variables	Weibull parameters	
	b	*c*
N (trees ha^{-1})	-0.687**	0.515**
QMD (cm)	0.987**	-0.671**
AMD (cm)	0.999**	-0.609**
D_{min} (cm)	0.467**	0.026^{ns}
D_{max} (cm)	0.786**	-0.774**
G (m^2 ha^{-1})	0.696**	-0.609**
RD	0.542**	-0.582**

**Significant at the 0.01 level. ^{ns}non-significant at the 0.05 level.

Scale and shape parameters estimated with MLE were modeled by stepwise linear regression analysis, using the stand variables given in the Table 2 as independent variables. The best results were obtained by using *AMD* as an independent variable for estimating *b* parameter, while $1/D_{max}$ for estimating *c* parameter. Coefficient estimates and fit statistics of linear regression models are given in Table 3. As seen from the table, model fits, with high R^2 and low error values, seemed good.

Table 3. Parameter estimates and fit statistics for the best regression models

Dependent variable	Independent variables	Coefficients estimate	Coefficients Std. error	t	p	R^2	*SE*
scale (*b*)	Constant	0.709	0.127	5.590	<0.001	0.999	0.327
	AMD	1.121	0.005	225.599	<0.001		
shape (*c*)	Constant	0.593	0.106	5.609	<0.001	0.924	0.529
	$1/D_{max}$	87.838	3.261	26.937	<0.001		

Table 3 provides formulation for estimation of the Weibull parameters, and hence estimating a diameter distribution for any scenario. In order to make an estimation with this formula, it is sufficient to use the *AMD* and D_{max} values of a subject stand. Patterns of residuals for predicted Weibull parameters by regression models are presented graphically (Figure 3). As seen from the figure, the regression model ensured indiscriminate patterns of residuals around zero and no apparent trends with homogenous variance.

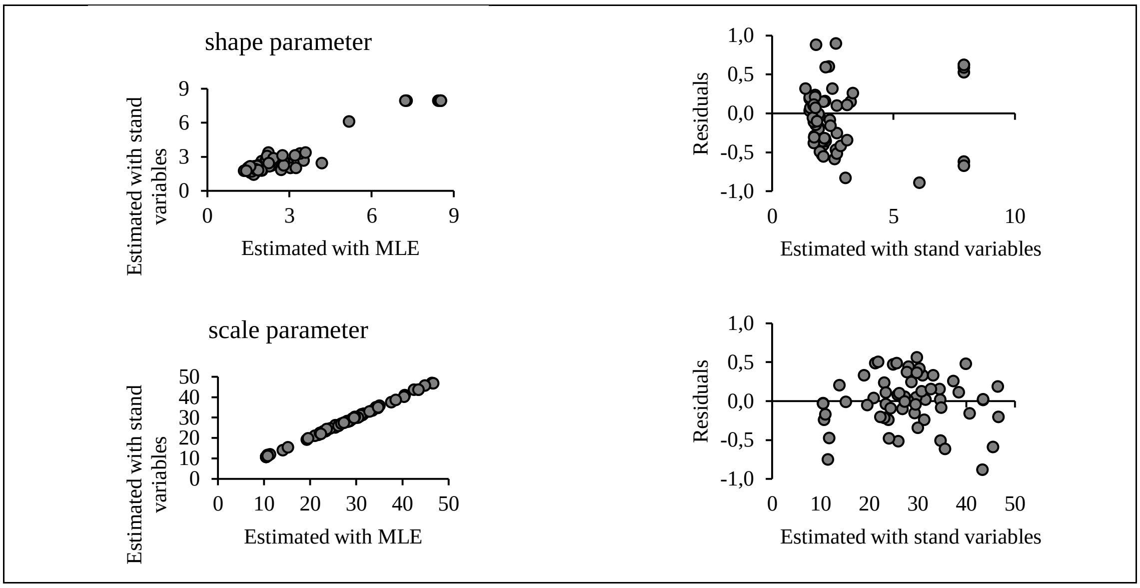

Figure 3. Plots of predicted Weibull parameters by MLE vs. regression model (left) and residuals vs. predicted parameters by regression model (right)

In order to visualize the comparison between the observed and predicted diameter distributions both by MLE and RMs, diameter distribution graphs of some sample plots were drawn in Figure 4. Although they outperform each other in some sample plots and in some diameter classes, in general, the Weibull distributions obtained by both MLE and RMs showed close trends. However, the main factor to be considered here is the required inputs for both estimation methods to be used. In order to have information about the diameter distribution of a stand, it is necessary to measure the diameters of all trees for the MLE method, while it is sufficient to have opinion about the *AMD* and D_{max} values values in the developed regression models. Moreover, there is an option to get an idea of diameter distribution scenarios for different forest types using these regression models.

In this study, significant and high correlations between stand variables and Weibull distribution parameters were determined. These results support previous studies investigating relationships between Weibull distribution parameters and stand variables. In the study, in which diameter distributions of three different species (*Lovoa trichilioides*, *Khaya ivorensis* and *Entandiophragma cylindricum*) distributed in Nigeria were examined, it was determined that there were strong correlations between Weibull parameters and stand characteristics such as *AMD*, *QMD*, *G*, D_{max} and age (Nokoe and Okojie, 1984). Besides, strong correlations between Weibull parameters and stand mean diameters were found in the studies conducted on broadleaved-Korean pine mixed forest in China (Wang et al., 2006), *Tetraelinis articulata* stands located in Tunisia (Sghaier et al., 2016), clonal *Eucalyptus* plantations in Brazil (Schmidt et al., 2020), mixed uneven-aged stands located in Romania (Ciceu et al., 2021). These strong correlations between stand characteristics and Weibull parameters and developed regression models using these relationships offers an option that is easy to use in practice and that we can easily predict for different stand simulations. For example, in the study, in which diameter distributions of *Azadirachta indica* plantations located in northern Ghana were examined, successful predictions were obtained with regression models that included stand age, mean diameter and height as independent variables (Nanang, 1998).

Conclusions

The main conclusion obtained in this and previous studies is that Weibull parameters estimated using stand variables give successful results. The regression models including stand variables as independent variables can be used to quickly estimate the Weibull parameters. However, these models mentioned, lead to a loss in precision in the predicted diameter distributions, and so they are recommended for the studies where only rough estimations of diameter distributions are needed. Besides, these regression models can be easily used to quickly determine diameter distributions for alternative stand structures.

If it is desired to have information about the diameter distribution of a stand, all diameter values of the trees in that stand are needed. However, in practical applications such as silvicultural and management decision processes, simple regression models developed in this study will be useful in order to make predictions about the dynamics of stands for different structures.

Ethics Committee Approval

N/A

Peer-review

Externally peer-reviewed.

Conflict of Interest

The author has no conflicts of interest to declare.

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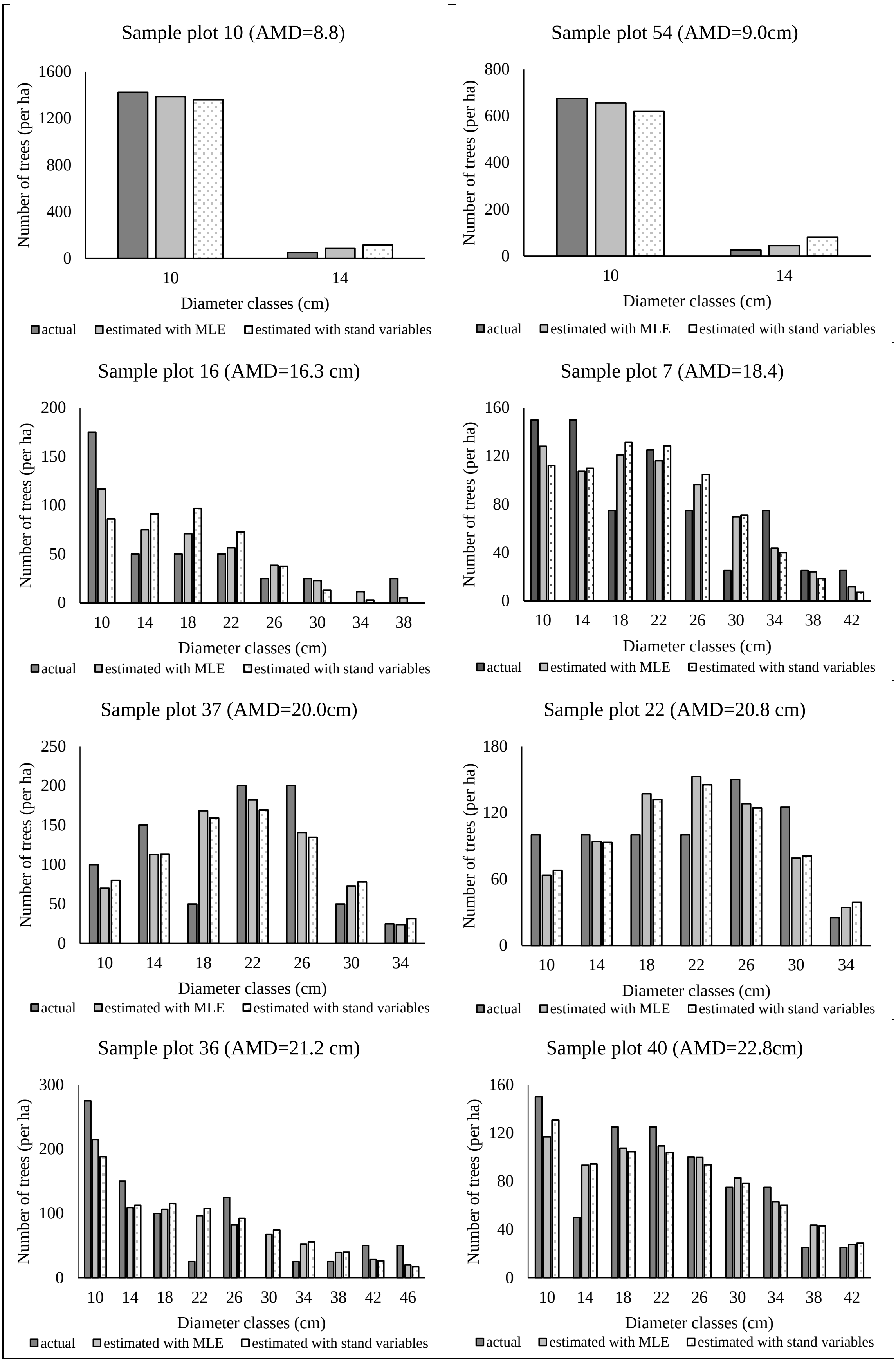

Figure 4. Observed and estimated diameter distributions of some sample plots

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