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# Finding Powerful Solutions for the Generalized Hyperelastic-Rod Wave Equation

**Şeyma Tülüce Demiray[1] and Uğur Bayrakçı[1*]**

[1] *Department of Mathematics, Faculty of Science and Letters, Osmaniye Korkut Ata University, 80000, Osmaniye, Türkiye*

[*] *Corresponding author*

**Abstract**

In this paper, the generalized hyperelastic rod wave equation has been studied. The generalized exponential rational function method (GERFM) has been applied to the generalized hyperelastic rod wave equation. Thus, some new and abundant soliton solutions of the generalized hyperelastic rod wave equation have been obtained. Also, in Wolfram Mathematica 12, both 2D and 3D shapes of these built-in results have been plotted.

***Keywords:*** *GERFM, Generalized hyperelastic-rod wave equation, soliton solutions.*

***2010 Mathematics Subject Classification:*** *35C07, 35A20, 35A25.*

## 1. Introduction

In this study, GERFM have been used, the solution methods of nonlinear evolution equations (NLEEs), and these methods have been applied to the generalized hyperelastic-rod wave equation, which is a variant of NLEEs. NLEEs have very important applications in areas such as mathematical physics, optical fibres, mathematical chemistry, hydrodynamics, fluid dynamics, geochemistry, control theory, meteorology, optics, mechanics, chemical kinematics, biophysics, biogenetics, and so on. A number of methods have been developed by various researchers in order to obtain solutions of NLEEs, which have such important areas of use in the scientific World [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15]. Generalized hyperelastic-rod wave equation is given as [16, 17]:

$$u_t - u_{xxt} - \alpha u_x + 2\beta u u_x + 3\theta u^2 u_x - \gamma u_x u_{xx} - u u_{xxx} = 0, \tag{1.1}$$

where $\alpha, \beta, \theta$ and $\gamma$ are constants and we accept that $\theta$ is nonzero. This equation is used to describe finite length, small amplitude radial deformation waves in cylindrical compressible hyperelastic rods. This equation also includes important physical models. For $\beta = \frac{3}{2}, \theta = 0$ and $\gamma = 2$, the equation can be reduced to Camassa-Holm equation. For $\alpha = 1, \beta = \frac{1}{2}, \theta = 0$ and $\gamma = 3$, the equation can be reduced to Fornberg-Whitham equation. Also, for $\beta = 2, \theta = 0$ and $\gamma = 3$, the equation can be reduced to Degasperis-Procesi equation [16, 17, 18, 19, 20].

Generalized hyperelastic-rod wave equation has been studied by some researchers recently. Akcagil et al. got the travelling wave solutions of the equation with the help of expansion method [16]. Gözükızıl and Akçagıl obtained the exact solutions of the equation using the tanh-coth method [17].

This study, which was prepared to specified the soliton solutions of the generalized hyperelastic-rod wave equation using GERFM [21, 22, 23, 24, 25], was designed as follows: In Section 2, GERFM's basic principles are presented. In Section 3, some soliton solutions of generalized hyperelastic-rod wave equation have been obtained by applying methods.

## 2. Definition of GERFM

Step1: We consider NLPDE given below:

$$P(u, u_x, u_t, u_{xx}, ...) = 0. \tag{2.1}$$

We first apply the wave transform given as below to Eq. (2.1);

$$u(x,t) = u(\eta), \eta = k(x - \lambda)t. \tag{2.2}$$

***Email addresses***: **ubayrakci42@gmail.com (U. Bayrakçı), seymatuluce@gmail.com (Ş Tülüce Demiray)**

Eq. (2.1) is transformed into ordinary differential equation by using Eq. (2.2):

$$R(u,u^{'},u^{''},\cdots)=0, \tag{2.3}$$

where $k$ and $\lambda$ values that are not taken into account will be calculated later.
Step2: Assume that we think the solutions of Eq. (2.3) as:

$$u(\eta)=a_0+\sum_{i=1}^{M}a_i\Phi(\eta)^i+\sum_{i=1}^{M}\frac{b_i}{\Phi(\eta)^i}, \tag{2.4}$$

where

$$\Phi(\eta)=\frac{p_1e^{q_1\eta}+p_2e^{q_2\eta}}{p_3e^{q_3\eta}+p_4e^{q_4\eta}}. \tag{2.5}$$

Here value of $M$ is determined through the homogeneous balance principle. $p_1,p_2,p_3,p_4,q_1,q_2,q_3,q_4,a_1,a_2,\cdots a_M,b_1,b_2,\cdots b_M$ constants are determined to fit the solution.
Step3: If the Eq. (2.4) is taken into account in the Eq. (2.3), $P(e^{q_1\eta},e^{q_2\eta},e^{q_3\eta},e^{q_4\eta})=0$ equation system is obtained. A system of equations is obtained by equating all coefficients of $P$ to zero.
Step4: If we solve the obtained system of equations and the found values consider in Eq. (2.4), the solutions of the discussed NLPDE are obtained.

## 3. Application of GERFM

To find the exact solutions of Eq. (1.1) we consider the following transformation:

$$u(x,t)=u(\eta),\eta=x-vt. \tag{3.1}$$

Replace Eq. (3.1) into Eq. (1.1) and we get,

$$vu'+vu'''+\alpha u'+2\beta uu'+3\theta u^2u'-\gamma u'u''-uu'''=0. \tag{3.2}$$

Integrating Eq. (3.2) and if the integration constant is set to zero, we have

$$(\alpha-v)u+vu''+\beta u^2+\theta u^3-\frac{\gamma-1}{2}(u')^2-uu''=0. \tag{3.3}$$

By using balance principle in Eq. (3.3), we obtain

$$M=2. \tag{3.4}$$

If $M=2$ is taken into account in Eq. (2.4).

$$\mu(\eta)=a_0+a_1\Phi(\eta)+a_2\Phi^2(\eta)+\frac{b_1}{\Phi(\eta)}+\frac{b_2}{\Phi^2(\eta)}, \tag{3.5}$$

equality is achieved. So the obtained different states of the considered equation via GERFM are as follows:
Family 1: For $p=[-1,0,1,1]$ and $q=[0,0,1,0]$, Eq. (2.5) turns into the form,

$$\Phi(\eta)=\frac{-1}{1+e^{\eta}}. \tag{3.6}$$

Case1:

$$\begin{gathered}a_0=\frac{-1+2\beta+A}{6\theta},a_1=\frac{6-3A+\sqrt{3}\sqrt{7+12\alpha\theta-4A+4\beta(2+\beta-A)}}{3\theta},b_1=0,b_2=0,\\ a_2=\frac{6-3A+\sqrt{3}\sqrt{7+12\alpha\theta-4A+4\beta(2+\beta-A)}}{3\theta},v=\frac{-1-12\alpha\theta+A+\beta(1+2\beta+A)}{18\theta},\\ \gamma=\frac{1}{6}\left(-6-3A+\sqrt{3}\sqrt{7+12\alpha\theta-4A+4\beta(2+\beta-A)}\right)\end{gathered} \tag{3.7}$$

where $A=\sqrt{(1-2\beta)^2-12\alpha\theta}$.
Embedding the these values in Eqs. (3.5) and (3.6), we acquire the soliton solution of Eq. (1.1)

$$u_1(x,t)=-\frac{e^{(x-vt)}\left(5+2\beta-2A\sqrt{3}\sqrt{7+12\alpha\theta-4A+4\beta(2+\beta-A)}\right)+(-1+2\beta+A)\cosh[x-vt]}{3\left(1+e^{(x-vt)}\right)^2\theta}, \tag{3.8}$$

where $v=\frac{-1-12\alpha\theta+A+\beta(1+2\beta+A)}{18\theta}$.

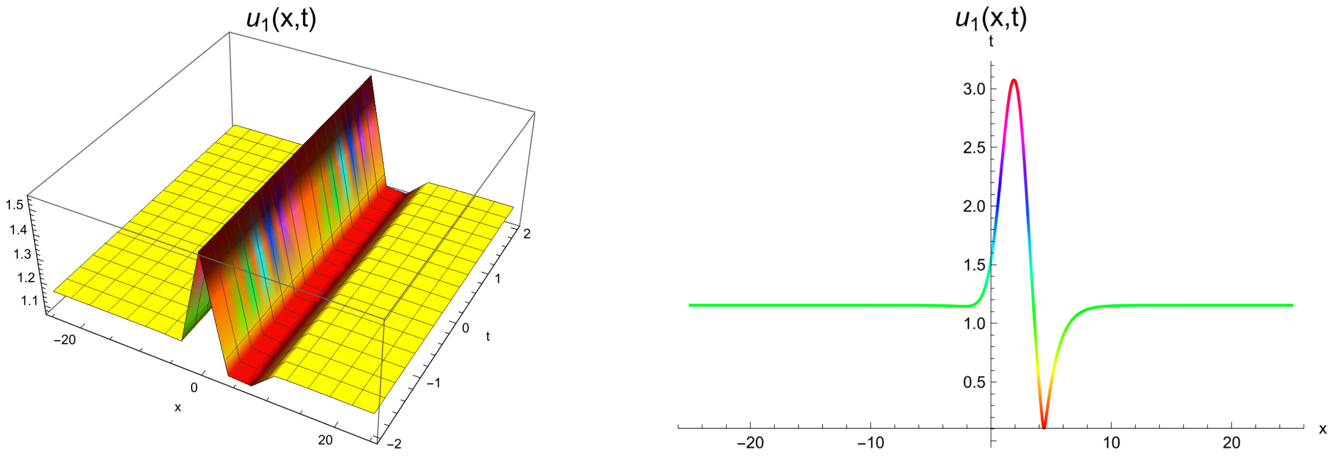


**Figure 3.1:** 3D plot of solution (3.8) for $\alpha = 2, \beta = 0.3, \theta = 0.5$ values with $-25 \leq x \leq 25, -2 \leq t \leq 2$ range and 2D plot of solution for $t = 1.5$ with these values.

Case2:

$$a_0 = -\frac{2\alpha}{-2+\beta}, a_1 = -\frac{8\alpha(-1+\beta)}{(-2+\beta)\beta}, b_1 = 0, b_2 = 0,$$
$$a_2 = -\frac{8\alpha(-1+\beta)}{(-2+\beta)\beta}, \theta = \frac{4\alpha}{(-2+\beta)\beta}, v = -\frac{2\alpha}{-2+\beta}, \gamma = -1-\beta. \tag{3.9}$$

Embedding the these values in Eqs. (3.5) and (3.6), we acquire the soliton solution of Eq. (1.1)

$$u_2(x,t) = -\frac{\alpha\left(-2+\beta-\beta\cosh\left[x+\frac{2\alpha t}{-2+\beta}\right]\right)\operatorname{sech}^2\left[\frac{x}{2}+\frac{\alpha t}{-2+\beta}\right]}{(-2+\beta)\beta}. \tag{3.10}$$

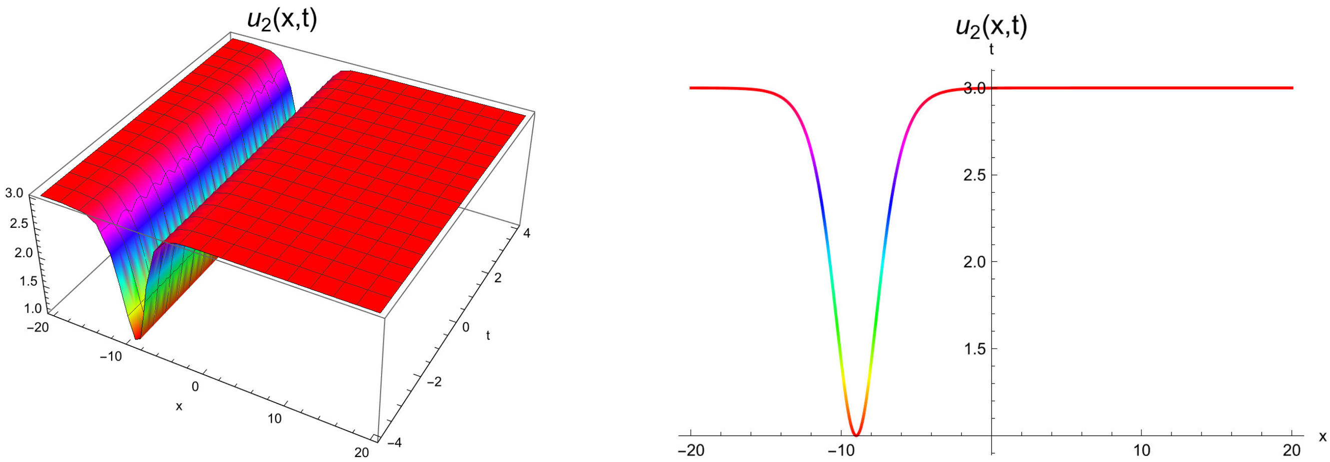


**Figure 3.2:** 3D plot of solution (3.10) for $\alpha = 1.5, \beta = 3$ values with $-20 \leq x \leq 20, -4 \leq t \leq 4$ range and 2D plot of solution for $t = 3$ with these values.

Case3:

$$a_0 = \frac{3}{5\theta}, a_1 = \frac{1}{5\theta}, a_2 = \frac{1}{5\theta}, b_1 = 0, b_2 = 0,$$
$$\alpha = \frac{753}{850\theta}, \beta = -\frac{387}{340}, \theta = \frac{4\alpha}{(-2+\beta)\beta}, v = \frac{957}{1700\theta}, \gamma = -\frac{19}{10}. \tag{3.11}$$

Embedding the these values in Eqs. (3.5) and (3.6), we acquire the soliton solution of Eq. (1.1)

$$u_3(x,t) = \frac{5+6\cosh\left[x-\frac{957t}{1700\theta}\right]}{10\theta+10\theta\cosh\left[x-\frac{957t}{1700\theta}\right]}. \tag{3.12}$$

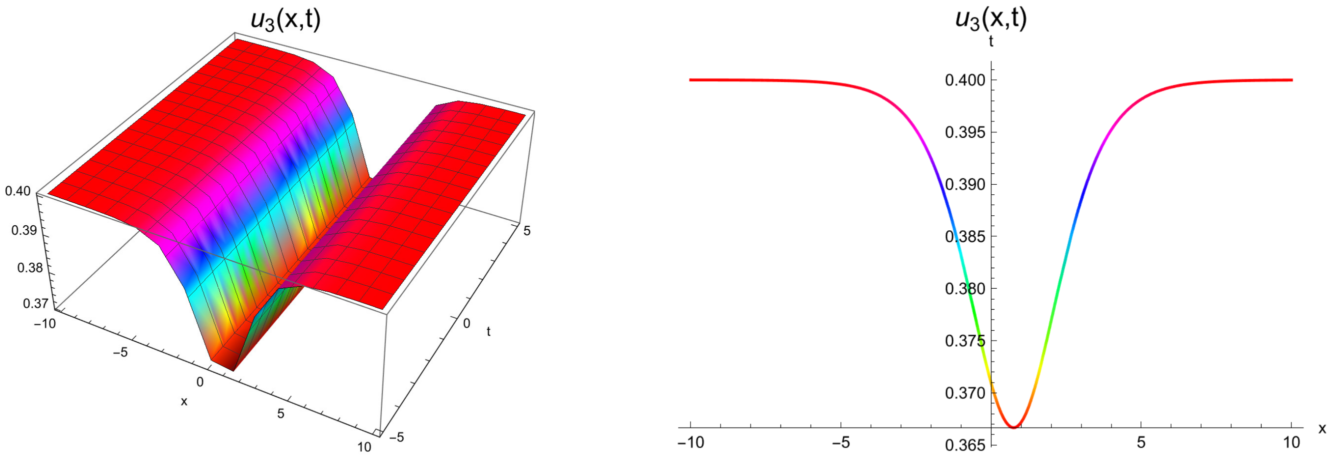


**Figure 3.3:** 3D plot of solution (3.12) for $\theta = 1.5$ values with $-10 \le x \le 10, -5 \le t \le 5$ range and 2D plot of solution for $t = 2$ with these values.

Case4:

$$a_0 = \frac{1}{3\theta}, a_1 = -\frac{1}{3\theta}, a_2 = -\frac{1}{3\theta}, b_1 = 0, b_2 = 0,$$
$$\alpha = \frac{67}{126\theta}, \beta = -\frac{67}{84}, v = \frac{95}{252\theta}, \gamma = -\frac{13}{6}. \tag{3.13}$$

Embedding the these values in Eqs. (3.5) and (3.6), we acquire the soliton solution of Eq. (1.1)

$$u_4(x,t) = \frac{4 + \operatorname{sech}^2\left[\frac{1}{2}\left(x - \frac{95t}{252\theta}\right)\right]}{12\theta}. \tag{3.14}$$

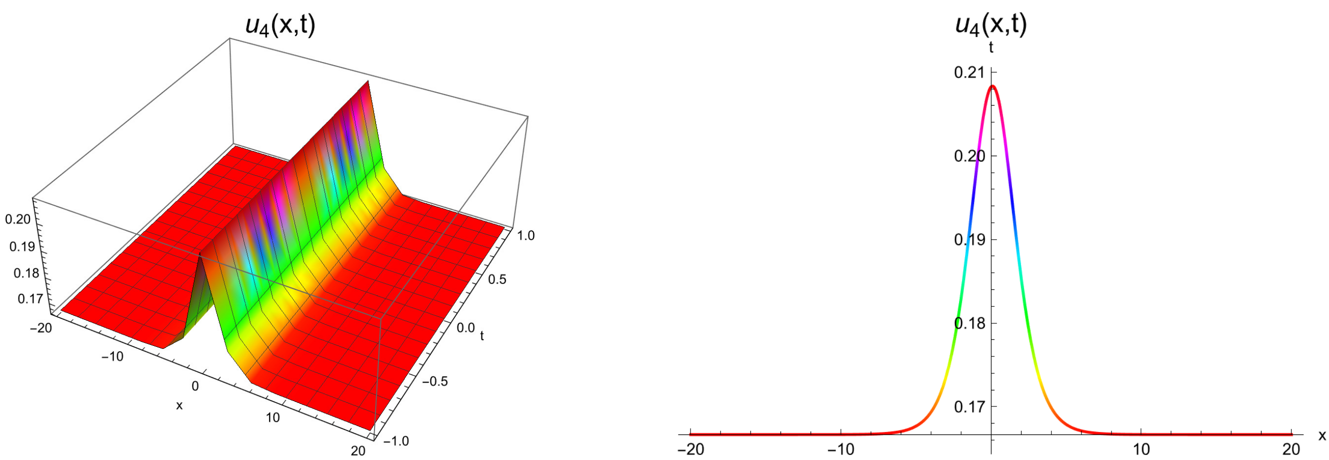


**Figure 3.4:** 3D plot of solution (3.14) for $\theta = 2$ values with $-20 \le x \le 20, -1 \le t \le 1$ range and 2D plot of solution for $t = 0.5$ with these values.

Case5:

$$a_0 = \frac{3 - i\sqrt{3}}{6\theta}, a_1 = \frac{-i}{\sqrt{3}\theta}, a_2 = \frac{-i}{\sqrt{3}\theta}, b_1 = 0, b_2 = 0,$$
$$\alpha = \frac{271 - 81i\sqrt{3}}{372\theta}, \beta = \frac{1}{372}\left(-357 + 86i\sqrt{3}\right), v = \frac{395 - 81i\sqrt{3}}{744\theta}, \gamma = -2 - \frac{i}{2\sqrt{3}}. \tag{3.15}$$

Embedding the these values in Eqs. (3.5) and (3.6), we acquire the soliton solution of Eq. (1.1)

$$u_5(x,t) = \frac{3 + 3\cosh\left[x - \frac{(395-81i\sqrt{3})t}{744\theta}\right] - i\sqrt{3}\cosh\left[x - \frac{(395-81i\sqrt{3})t}{744\theta}\right]}{6\theta + 6\theta\cosh\left[x - \frac{(395-81i\sqrt{3})t}{744\theta}\right]}. \tag{3.16}$$

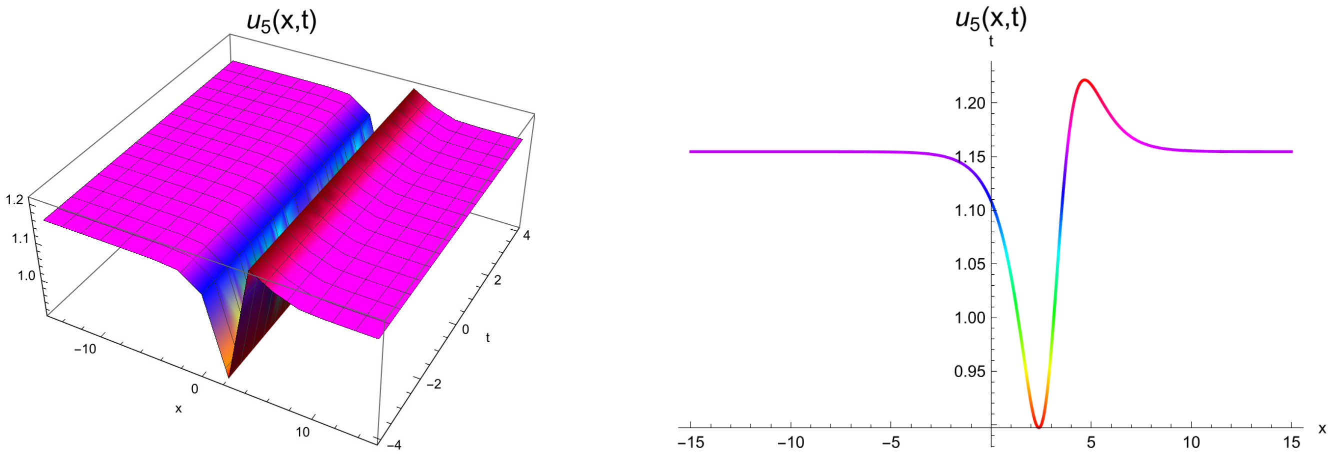


**Figure 3.5:** 3D plot of solution (3.16) for $\theta = 0.5$ values with $-15 \leq x \leq 15, -4 \leq t \leq 4$ range and 2D plot of solution for $t = 3$ with these values.

Family 2: For $p = [-2-i, 2-i, -1, 1]$ and $q = [i, -i, i, -i]$, Eq. (2.5) turns into the form,

$$\Phi(\eta) = \frac{\cos(\eta) + 2\sin(\eta)}{\sin(\eta)}. \tag{3.17}$$

Case1:

$$\begin{aligned} a_0 = \frac{10(2+\gamma)}{\theta}, a_1 = 0, a_2 = 0, b_1 = -\frac{40(2+\gamma)}{\theta}, b_2 = -\frac{50(2+\gamma)}{\theta},\\ \alpha = -\frac{5(2+\gamma)(2+\beta+2\gamma)}{3\theta}, v = -\frac{(2+\gamma)(2+\beta+2\gamma)}{3\theta}. \end{aligned} \tag{3.18}$$

Embedding the these values in Eqs. (3.5) and (3.17), we acquire the soliton solution of Eq. (1.1)

$$u_6(x,t) = \frac{10(2+\gamma)}{\theta\left(\cos\left[x + \frac{t(2+\gamma)(2+\beta+2\gamma)}{3\theta}\right] + 2\sin\left[x + \frac{t(2+\gamma)(2+\beta+2\gamma)}{3\theta}\right]\right)^2}. \tag{3.19}$$

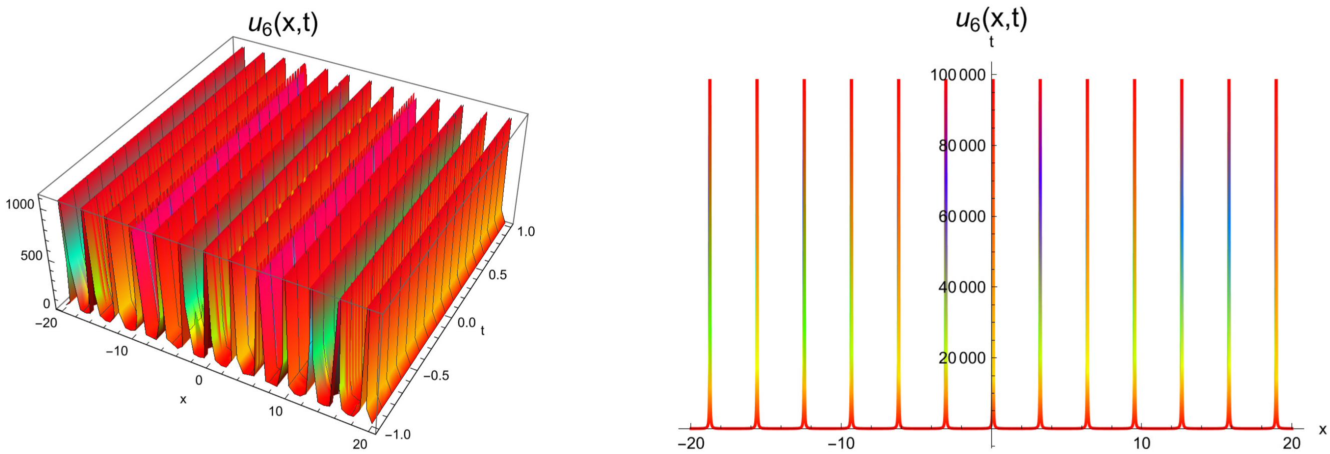


**Figure 3.6:** 3D plot of solution (3.19) for $\gamma = 2, \beta = 3, \theta = 0.5$ values with $-20 \leq x \leq 20, -1 \leq t \leq 1$ range and 2D plot of solution for $t = 0.5$ with these values.

Case2:

$$\begin{aligned} a_0 = \frac{10(2+\gamma)}{\theta}, a_1 = -\frac{8(2+\gamma)}{\theta}, a_2 = -\frac{2(2+\gamma)}{\theta}, b_1 = 0, b_2 = 0,\\ \alpha = -\frac{5(2+\gamma)(2+\beta+2\gamma)}{3\theta}, v = -\frac{(2+\gamma)(2+\beta+2\gamma)}{3\theta}. \end{aligned} \tag{3.20}$$

Embedding the these values in Eqs. (3.5) and (3.17), we acquire the soliton solution of Eq. (1.1)

$$u_7(x,t) = \frac{2(2+\gamma)\csc^2\left[x + \frac{t(2+\gamma)(2+\beta+2\gamma)}{3\theta}\right]}{3\theta}. \tag{3.21}$$

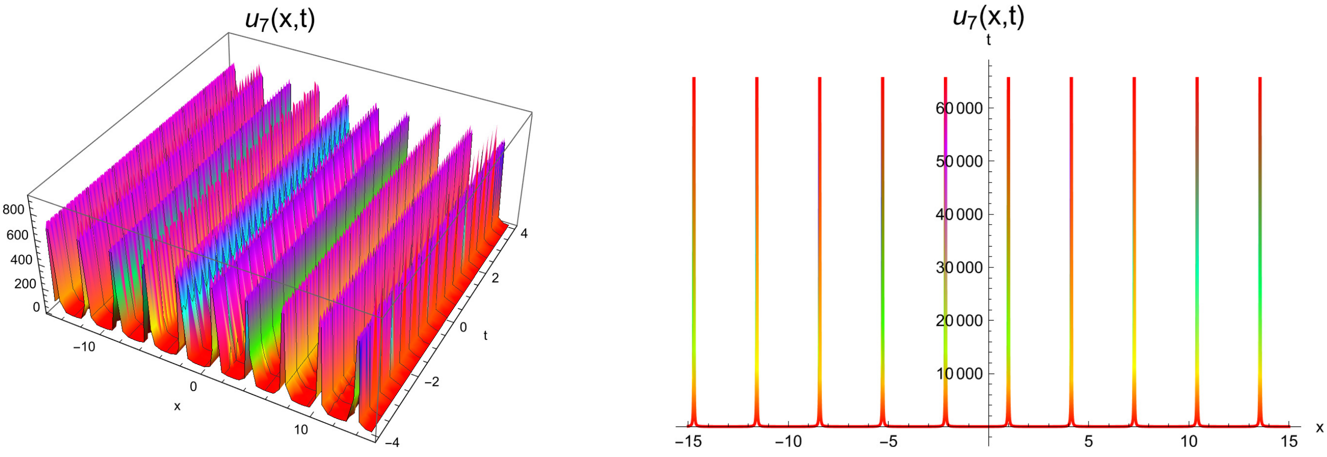


**Figure 3.7:** 3D plot of solution (3.21) for $\gamma=1,\beta=3,\theta=1$ values with $-15\le x\le 15,-4\le t\le 4$ range and 2D plot of solution for $t=3$ with these values.

Case3:

$$a_0=\frac{-3\left(\beta-36(2+\gamma)\right)+\sqrt{3\left(64+3\beta^2-16\gamma^2-8\beta(2+\gamma)\right)}}{12\theta},a_1=-\frac{8(2+\gamma)}{\theta},a_2=\frac{2(2+\gamma)}{\theta},b_1=0,b_2=0,$$
$$\alpha=\frac{\left(3\beta^2-8(2+\gamma)(7+\gamma)+2\beta(11+7\gamma)\right)+(6-\beta+2\gamma)\sqrt{3\left(64+3\beta^2-16\gamma^2-8\beta(2+\gamma)\right)}}{12\theta},$$
$$v=\frac{4(1+\gamma)(2+\gamma)-\beta(5+\gamma)-(1+\gamma)\sqrt{3\left(64+3\beta^2-16\gamma^2-8\beta(2+\gamma)\right)}}{12\theta}. \tag{3.22}$$

Embedding the these values in Eqs. (3.5) and (3.17), we acquire the soliton solution of Eq. (1.1)

$$u_8(x,t)=\frac{-3\beta+\sqrt{3\left(64+3\beta^2-16\gamma^2-8\beta(2+\gamma)\right)}+12(2+\gamma)\left(1+2\cot^2[x-vt]\right)}{12\theta}, \tag{3.23}$$

where $v=\frac{4(1+\gamma)(2+\gamma)-\beta(5+\gamma)-(1+\gamma)\sqrt{3(64+3\beta^2-16\gamma^2-8\beta(2+\gamma))}}{12\theta}$.

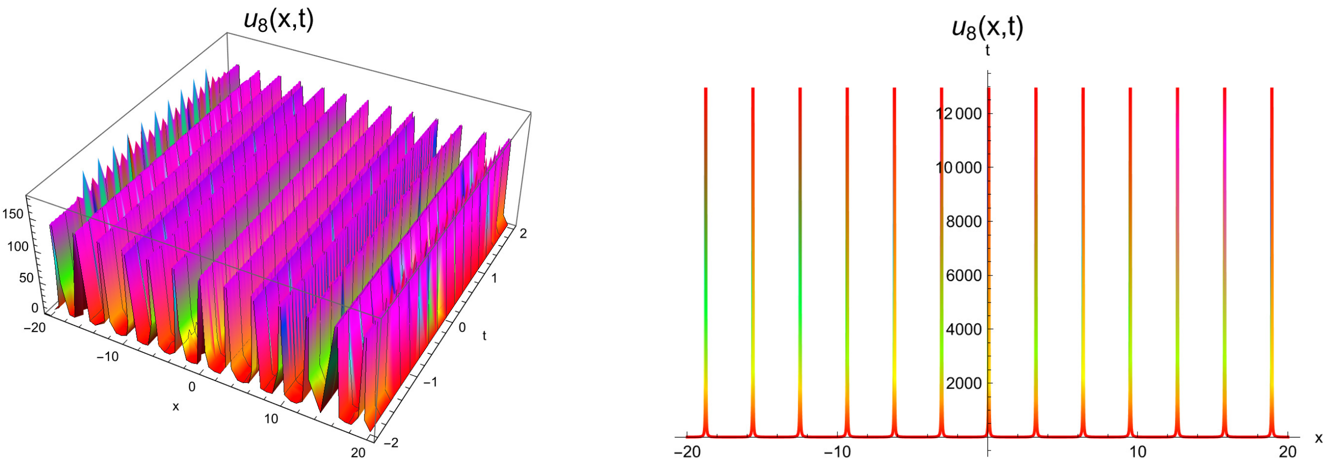


**Figure 3.8:** 3D plot of solution (3.23) for $\gamma=0.1,\beta=-3,\theta=2$ values with $-20\le x\le 20,-2\le t\le 2$ range and 2D plot of solution for $t=1$ with these values.

Case4:

$$a_0=\left(-2+8\sqrt{11}\right)a_2,a_1=-2\left(1+\sqrt{11}\right)a_2,b_1=-10\left(1+\sqrt{11}\right)a_2,b_2=25a_2,$$
$$\alpha=-18\left(-4+\sqrt{11}\right)a_2,\beta=28-6\sqrt{11},\theta=\frac{1}{a_2},\gamma=\frac{-3}{2},v=6\left(-4+\sqrt{11}\right)a_2. \tag{3.24}$$

Embedding the these values in Eqs. (3.5) and (3.17), we acquire the soliton solution of Eq. (1.1)

$$u_9(x,t)=\frac{\csc^4[x-vt]\left(1-4\sqrt{11}+4\left(1+\sqrt{11}\right)\cos[2x-2vt]-3\cos[4x-4vt]-2\left(1+\sqrt{11}\right)\sin[2x-2vt]+4\sin[4x-4vt]\right)a_2}{2\left(2+\cot[x-vt]\right)^2}, \tag{3.25}$$

where $v=6\left(-4+\sqrt{11}\right)a_2$.

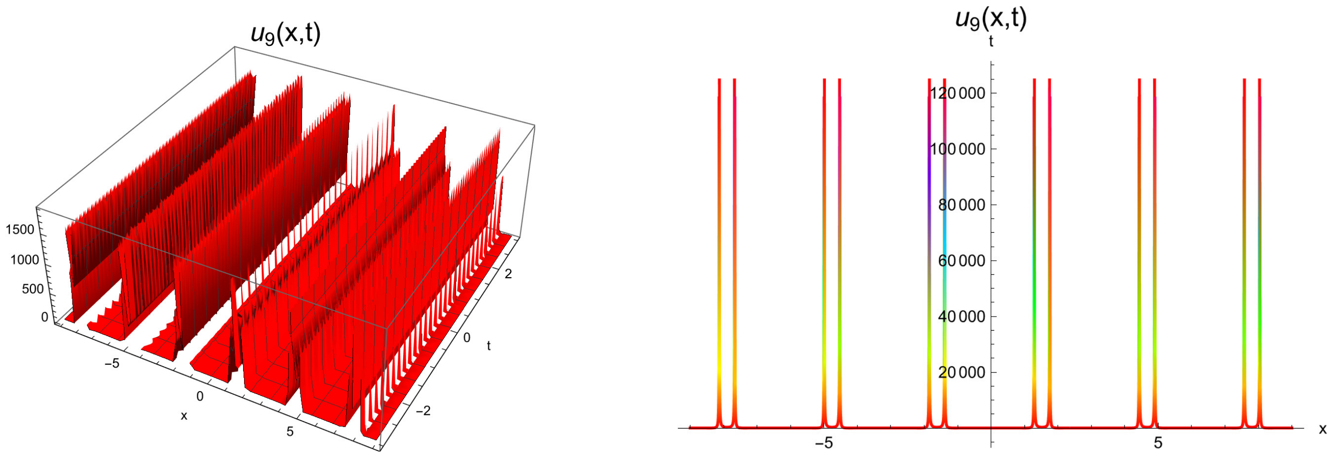


**Figure 3.9:** 3D plot of solution (3.25) for $a_2 = 4$ values with $-25 \leq x \leq 25, -3 \leq t \leq 3$ range and 2D plot of solution for $t = 2$ with these values.

Family 3: For $p = [i, -i, 1, 1]$ and $q = [i, -i, i, -i]$, Eq. (2.5) turns into the form,

$$\Phi(\eta) = \frac{-\sin(\eta)}{\cos(\eta)}. \tag{3.26}$$

Case1:

$$a_0 = \frac{2(2+\gamma)}{\theta}, a_1 = 0, a_2 = \frac{2(2+\gamma)}{\theta}, b_1 = 0, b_2 = 0,$$
$$\alpha = -\frac{5(2+\gamma)(2+\beta+2\gamma)}{3\theta}, v = -\frac{(2+\gamma)(2+\beta+2\gamma)}{3\theta}. \tag{3.27}$$

Embedding the these values in Eqs. (3.5) and (3.26), we acquire the soliton solution of Eq. (1.1)

$$u_{10}(x,t) = \frac{2(2+\gamma)\sec^2\left[x + \frac{t(2+\gamma)(2+\beta+2\gamma)}{3\theta}\right]}{\theta}. \tag{3.28}$$

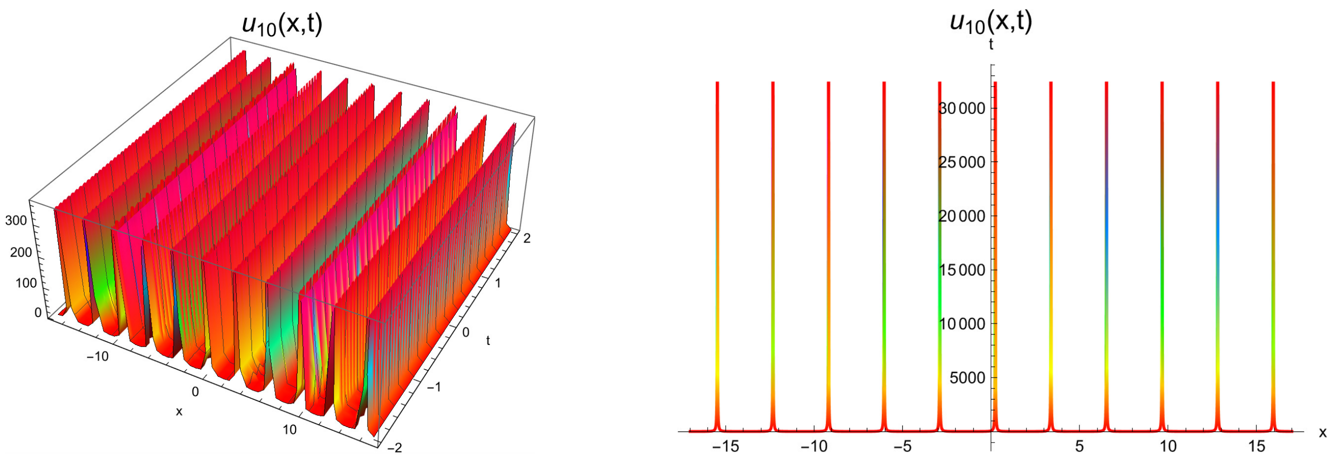


**Figure 3.10:** 3D plot of solution (3.28) for $\gamma = -0.1, \beta = 0.3, \theta = 1$ values with $-17 \leq x \leq 17, -2 \leq t \leq 2$ range and 2D plot of solution for $t = 1$ with these values.

Case2:

$$a_0 = \frac{-3\beta\theta + \sqrt{3\left(3\beta^2 - 32\left(\beta + 8(-2+\gamma)\right)(2+\gamma)\right)}}{12\theta}, a_1 = 0, a_2 = \frac{2(2+\gamma)}{\theta}, b_1 = 0, b_2 = \frac{2(2+\gamma)}{\theta},$$
$$\alpha = \frac{\left(3\beta^2 - 32(2+\gamma)(31+7\gamma) + 2\beta(59+31\gamma)\right) + (30-\beta+14\gamma)\sqrt{3\left(3\beta^2 - 32\beta(2+\gamma) - 256(-4+\gamma^2)\right)}}{12\theta},$$
$$v = \frac{16(1+\gamma)(2+\gamma) - \beta(5+\gamma) - (1+\gamma)\sqrt{3\left(3\beta^2 - 32\beta(2+\gamma) - 256(-4+\gamma^2)\right)}}{12\theta}. \tag{3.29}$$

Embedding the these values in Eqs. (3.5) and (3.26), we acquire the soliton solution of Eq. (1.1)

$$u_{11}(x,t) = \frac{-3\beta + \sqrt{3\left(3\beta^2 - 32\beta(2+\gamma) - 256(-4+\gamma^2)\right)} + 24(2+\gamma)\left(1 + \cot^4[x - vt]\right)\tan^2[x - vt]}{12\theta}, \tag{3.30}$$

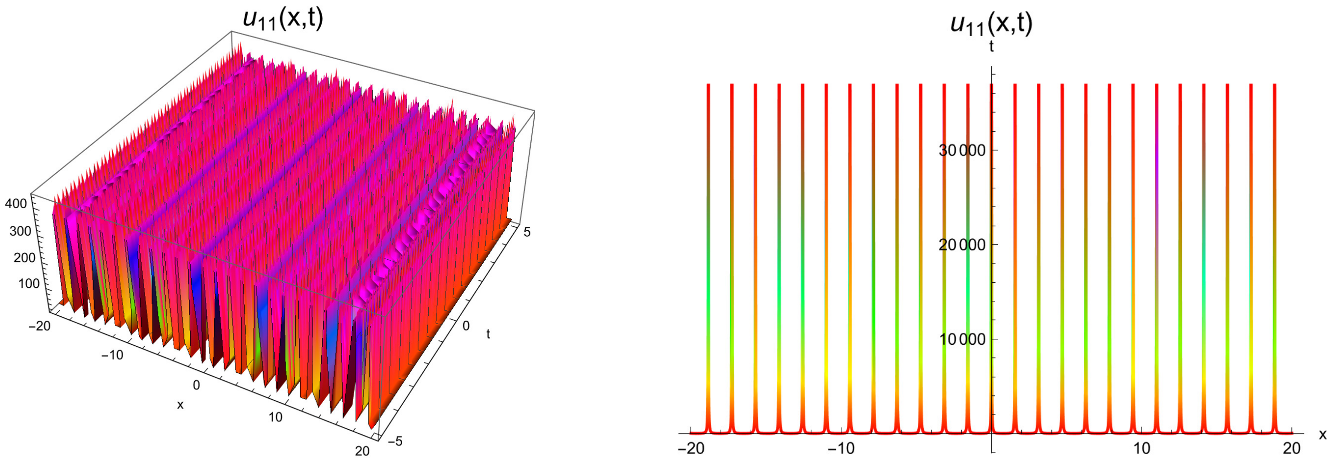


**Figure 3.11:** 3D plot of solution (3.30) for $\gamma=-1,\beta=3,\theta=1$ values with $-20\leq x\leq 20,-5\leq t\leq 5$ range and 2D plot of solution for $t=2$ with these values.

where $v=\frac{16(1+\gamma)(2+\gamma)-\beta(5+\gamma)-(1+\gamma)\sqrt{3(3\beta^2-32\beta(2+\gamma)-256(-4+\gamma^2))}}{12\theta}$.
Family 4: For $p=[1,1,-1,1]$ and $q=[1,-1,1,-1]$, Eq. (2.5) turns into the form,

$$\Phi(\eta)=\frac{-\cosh(\eta)}{\sinh(\eta)}. \tag{3.31}$$

Case1:

$$\begin{aligned}a_0&=-\frac{2(2+\gamma)}{\theta},a_1=0,a_2=\frac{2(2+\gamma)}{\theta},b_1=0,b_2=0,\\ \alpha&=\frac{5(2+\gamma)(\beta-2(1+\gamma))}{3\theta},v=-\frac{(2+\gamma)(2-\beta+2\gamma)}{3\theta}.\end{aligned} \tag{3.32}$$

Embedding the these values in Eqs. (3.5) and (3.31), we acquire the soliton solution of Eq. (1.1)

$$u_{12}(x,t)=\frac{2(2+\gamma)\operatorname{csch}^2\left[x-\frac{t(2+\gamma)(2-\beta+2\gamma)}{3\theta}\right]}{\theta}. \tag{3.33}$$

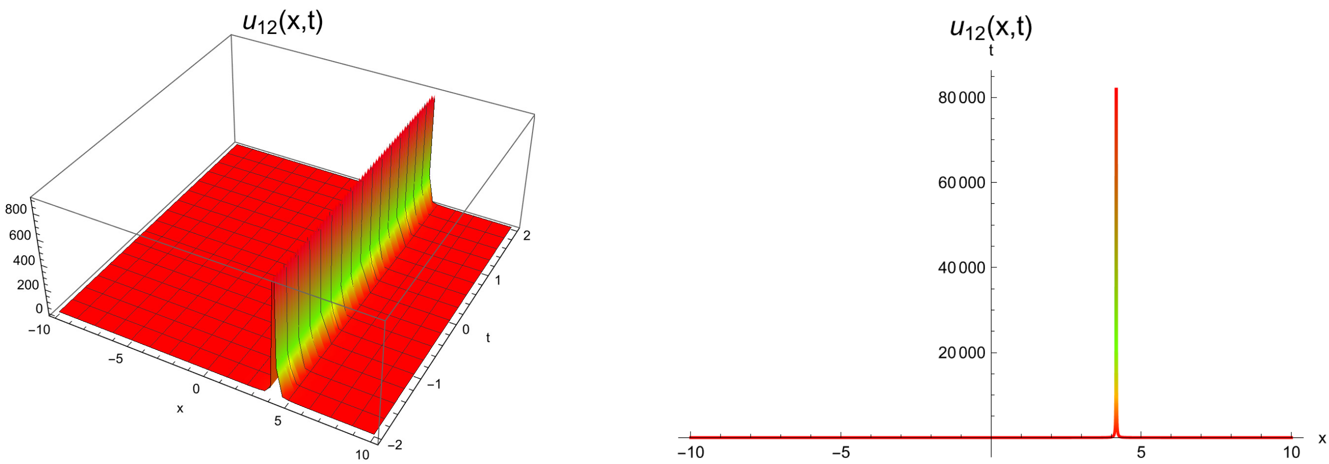


**Figure 3.12:** 3D plot of solution (3.33) for $\gamma=3,\beta=0.5,\theta=3$ values with $-10\leq x\leq 10,-2\leq t\leq 2$ range and 2D plot of solution for $t=1$ with these values.

Case2:

$$\begin{aligned}a_0&=-\frac{3\left(8+\beta+4\gamma\right)+\sqrt{3\left(64+3\beta^2-16\gamma^2-8\beta(2+\gamma)\right)}}{12\theta},a_1=0,a_2=\frac{2(2+\gamma)}{\theta},b_1=0,b_2=0,\\ \alpha&=\frac{3\left(\beta^2-8(2+\gamma)(3+\gamma)-2\beta(7+3\gamma)\right)+(10+\beta+6\gamma)\sqrt{3\left(64+3\beta^2-16\gamma^2-8\beta(2+\gamma)\right)}}{24\theta},\\ v&=\frac{-4(1+\gamma)(2+\gamma)-\beta(5+\gamma)-(1+\gamma)\sqrt{3\left(64+3\beta^2-16\gamma^2+8\beta(2+\gamma)\right)}}{12\theta}.\end{aligned} \tag{3.34}$$

Embedding the these values in Eqs. (3.5) and (3.31), we acquire the soliton solution of Eq. (1.1)

$$u_{13}(x,t) = -\frac{3(8+\beta+4\gamma)+\sqrt{3\left(64+3\beta^2-16\gamma^2+8\beta(2+\gamma)\right)}-24(2+\gamma)\coth^2[x-vt]}{12\theta}, \tag{3.35}$$

where $v = \frac{-4(1+\gamma)(2+\gamma)-\beta(5+\gamma)-(1+\gamma)\sqrt{3(64+3\beta^2-16\gamma^2+8\beta(2+\gamma))}}{12\theta}$.

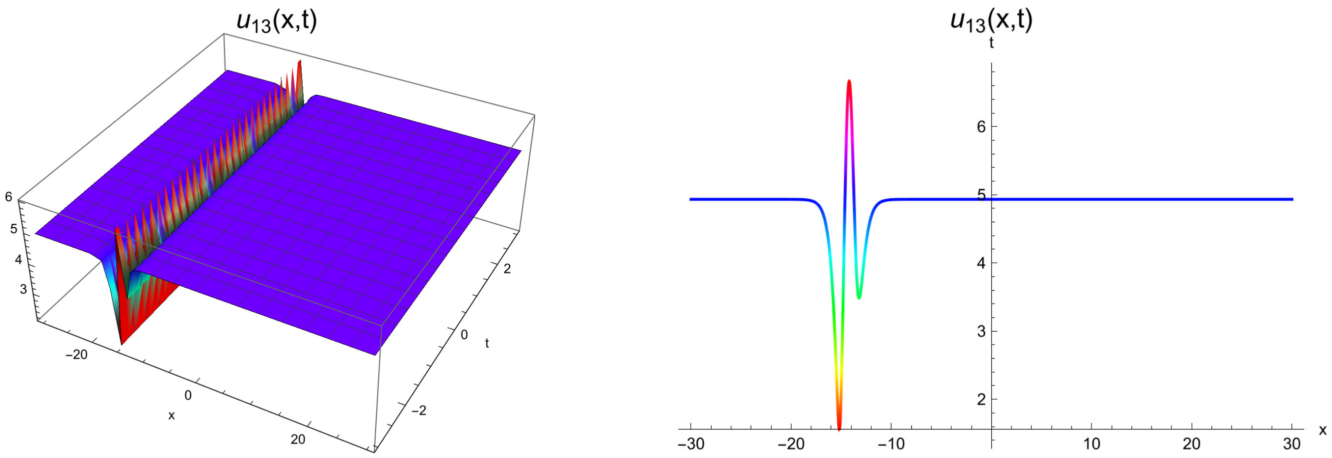


**Figure 3.13:** 3D plot of solution (3.35) for $\gamma = 3, \beta = 0.7, \theta = 1$ values with $-30 \le x \le 30, -3 \le t \le 3$ range and 2D plot of solution for $t = 2$ with these values.

Case3:

$$a_0 = -\frac{3(8+\beta+4\gamma)+\sqrt{3\left(64+3\beta^2-16\gamma^2+8\beta(2+\gamma)\right)}}{12\theta}, a_1 = 0, a_2 = 0, b_1 = 0, b_2 = \frac{2(2+\gamma)}{\theta},$$
$$\alpha = \frac{3\left(\beta^2-8(2+\gamma)(3+\gamma)-2\beta(7+3\gamma)\right)+(10+\beta+6\gamma)\sqrt{3\left(64+3\beta^2-16\gamma^2+8\beta(2+\gamma)\right)}}{24\theta},$$
$$v = \frac{-4(1+\gamma)(2+\gamma)-\beta(5+\gamma)-(1+\gamma)\sqrt{3\left(64+3\beta^2-16\gamma^2+8\beta(2+\gamma)\right)}}{12\theta}. \tag{3.36}$$

Embedding the these values in Eqs. (3.5) and (3.31), we acquire the soliton solution of Eq. (1.1)

$$u_{14}(x,t) = -\frac{3(8+\beta+4\gamma)+\sqrt{3\left(64+3\beta^2-16\gamma^2+8\beta(2+\gamma)\right)}-24(2+\gamma)\tanh^2[x-vt]}{12\theta}, \tag{3.37}$$

where $v = \frac{-4(1+\gamma)(2+\gamma)-\beta(5+\gamma)-(1+\gamma)\sqrt{3(64+3\beta^2-16\gamma^2+8\beta(2+\gamma))}}{12\theta}$.

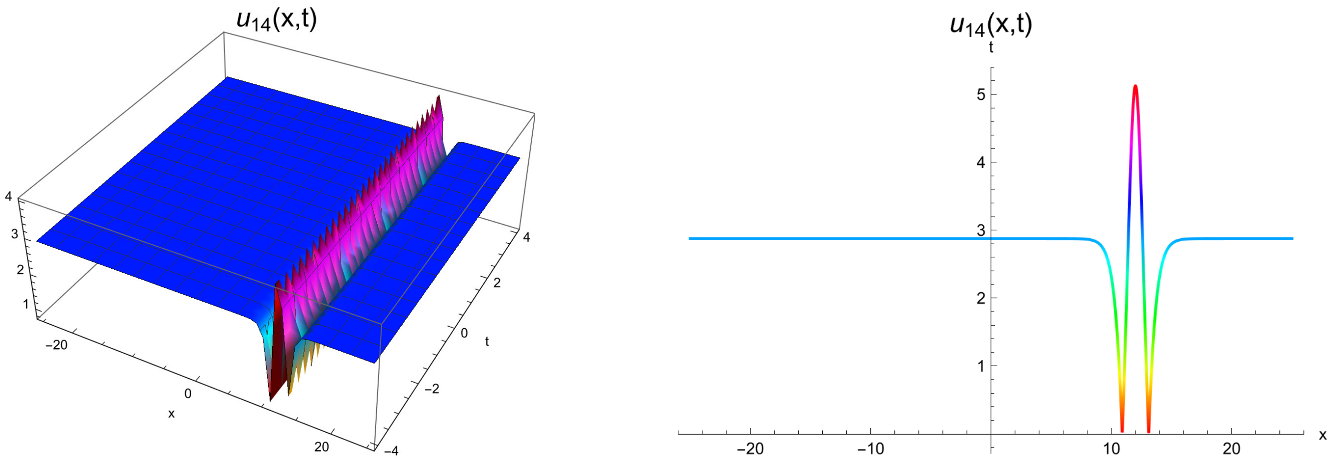


**Figure 3.14:** 3D plot of solution (3.37) for $\gamma = -1, \beta = -3, \theta = 0.25$ values with $-25 \le x \le 25, -4 \le t \le 4$ range and 2D plot of solution for $t = 3$ with these values.

Case4:

$$a_0=-\frac{3\beta\theta+\sqrt{3\left(3\beta^2+32\left(16+\beta-8\gamma\right)\left(2+\gamma\right)\right)}}{12\theta},a_1=0,a_2=\frac{2(2+\gamma)}{\theta},b_1=0,b_2=\frac{2(2+\gamma)}{\theta},$$

$$\alpha=\frac{3\left(\beta^2-32(2+\gamma)(11+3\gamma)-2\beta(23+11\gamma)\right)+(34+\beta+18\gamma)\sqrt{3\left(3\beta^2+32\beta(2+\gamma)-256(-4+\gamma^2)\right)}}{24\theta},$$

$$v=\frac{-16(1+\gamma)(2+\gamma)-\beta(5+\gamma)-(1+\gamma)\sqrt{3\left(3\beta^2+32\beta(2+\gamma)-256(-4+\gamma^2)\right)}}{12\theta}. \tag{3.38}$$

Embedding the these values in Eqs. (3.5) and (3.31), we acquire the soliton solution of Eq. (1.1)

$$u_{15}(x,t)=-\frac{3\beta+\sqrt{3\left(3\beta^2+32\beta(2+\gamma)-256(-4+\gamma^2)\right)}-24(2+\gamma)\left(1+\coth^4[x-vt]\right)\tanh^2[x-vt]}{12\theta}, \tag{3.39}$$

where $v=\frac{-16(1+\gamma)(2+\gamma)-\beta(5+\gamma)-(1+\gamma)\sqrt{3(3\beta^2+32\beta(2+\gamma)-256(-4+\gamma^2))}}{12\theta}$.

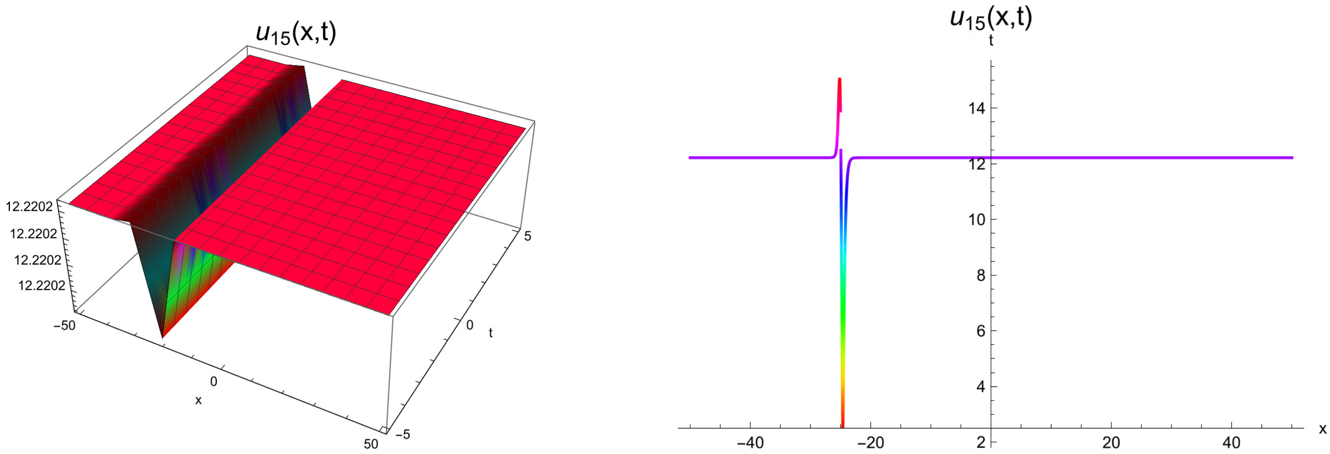


**Figure 3.15:** 3D plot of solution (3.39) for $\gamma=-4,\beta=4,\theta=1$ values with $-50\le x\le 50,-5\le t\le 5$ range and 2D plot of solution for $t=3$ with these values.

## 4. Result and Discussion

Some soliton solutions of the generalized hyperelastic rod wave equation have been obtained by applying GERFM. These results, obtained with the help of Wolfram Mathematica 12, have been graphically represented and their accuracy has been proven. When the previous results related to this equation are compared with the results obtained in this study, our (3.35) and (3.37) solutions are similar to the (42) solutions given by Gozükızıl and Akçagıl. Other solutions we have obtained according to our research have not been shown before and are new.

## 5. Conclusion

In this study, the generalized hyperelastic-rod wave equation was examined. GERFM, which is the solution method of NLEEs, was applied to this equation and thus some trigonometric function, hyperbolic function, complex hyperbolic function, combo soliton, singular soliton, dark soliton and bright soliton solutions of the equation were obtained. In addition, certain values and ranges were given to the obtained solutions and 2D and 3D graphs of these solutions were drawn with the help of Wolfram Mathematica. The most important advantage of the method used in this study is that a wide variety of solution families can be created. It is a more general method compared to other methods, as it offers a wide variety of solution families.

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