**Iğdır Üniversitesi Fen Bilimleri Enstitüsü Dergisi, 12(3): 1770 - 1775, 2022**
**Journal of the Institute of Science and Technology, 12(3): 1770 - 1775, 2022**
**ISSN: 2146-0574, eISSN: 2536-4618**
**Mathematics** **DOI: 10.21597/jist.1095933**
**Research Article**
**Received: 30.03.2022** **Accepted: 01.07.2022**
**To Cite:** Korkmaz M, 2022. A Study on the Conversion of Wood Model Widely Used in Lactation to the Model with the Biologically Meaningful Parameters. Journal of the Institute of Science and Technology, 12(3): 1770 - 1775.

# A Study on the Conversion of Wood Model Widely Used in Lactation to the Model with the Biologically Meaningful Parameters

Mehmet KORKMAZ[1*]

**ABSTRACT:** A mathematical model is an important instrument used to get information on the attitude of a system. The mathematical models can be used to have a basic knowledge about the working of a system, lowering product costs and improving performance. In this paper, it is stated that the behavior of the system can be better understood by using biologically meaningful parameters in mathematical models. Mathematical models can be divided into two classes as empirical and mechanical models. Since the parameters not biologically menaningful in empirical models, the importance of converting these models to mechanical models containing biologically meaningful parameters has been expressed. The purpose of this manuscript is related to how Wood model widely used in lactation is converted into the model with the biologically meaningful parameters, time to maximum milk production, maximum milk production reached at time to maximum milk production and time to inflection point. For this aim, all the steps of the conversions were given stepwise.

**Keywords:** Wood model, meaningful parameters, lactation

[1] Mehmet KORKMAZ (**Orcid ID:** 0000-0002-7488-0552), Ordu University, Department of Mathematics, Ordu, Türkiye

***Corresponding Author:** Mehmet KORKMAZ, e-mail: mkorkmaz52@yahoo.com

## INTRODUCTION

Mathematical model is a forceful instrument for getting valuable information about the attitude of a system that uses a mathematical method. But the fundamental aim of using mathematical models is to get preliminary information about the operation of a system, to reduce product costs and to increase performance (Tedeschi et al., 2005; Karkach, 2006). Mathematical models fall into two categories: empirical and mechanical models (Lopez et al., 2004; Uckardes,2010). Empirical models include the parameters a, b, c, etc. Thus, these models do not directly give an idea about the attitude of a system, however they are only used to fit the data points. In this way, because the parameters in the empirical models do not include any biological meaning, it is hard to predict their starting values. In another saying, it is hard to predict the initial values of the parameters if they have no biological meaning. Since it is simple to predict the starting values for the parameters in the mechanical model, it is possible to calculate within known ranges of similar studies. So, the researchers prefer mechanical models in their studies (Lopez et al., 2004; Zwietering et al., 1990). Mechanical models defines the attitude of a system containing the parameters with biological meaning such as tmax , time to maximum milk production, A, maximum milk production reached at tmax  and ti , time to inflection point.

As a model of the lactation curve, Wood (Wood, 1967) proposed to Wood model in equation 1. Since Wood model is probably the most popular parametric model of the lactation curve, this model is used in this study. The purpose of this research is to show how Wood model widely used in lactation are transformed into the mechanical model which has the biologically meaningful parameters as above.

## MATERIALS AND METHODS

### Material

The first lactation records of Black and White Cattle in Balikesir during the 1994-1995 period from (Soysal et al., 2004) are given in Table 1. For presenting, observed average daily milk values of lactation days in Table 2 in their study was used in Table 1 in this manuscript.

**Table 1.** The observed average daily milk values

| Balikesir Population | |
|---|---|
| **Lactation Days** | **Observed average Daily milk values (kg)** |
| 30 | 22 |
| 60 | 23.6 |
| 90 | 22.1 |
| 120 | 21 |
| 150 | 19.63 |
| 180 | 17.80 |
| 210 | 16.75 |
| 240 | 15.97 |
| 270 | 15.25 |
| 300 | 12.96 |

### Methods

Wood model widely used in lactation is the following equation:

$$y=a.t^{b}e^{-ct} \quad (1)$$

where t is time, y is the milk production value depending on time, a, b and c are the parameters of Wood model, e is the base of natural logarithm.

In Wood model which expressed by the equation (1), the a,b and c parameters do not make any biological sense. This model was transformed into

$$y=\boldsymbol{A t_{max}^{\frac{-t_{max}^2}{(t_i-t_{max})^2}} e^{\frac{-(t-t_{max})t_{max}}{(t_i-t_{max})^2}} t^{\frac{t_{max}^2}{(t_i-t_{max})^2}}} \quad (2)$$

form using the biologically meaningful parameters ti, tmax and A where
ti= inflection point time
tmax= the maximum milk production time
A= the maximum milk production value at tmax
Lactation growth curve with the biologically meaningful parameters is given in Figure 1.

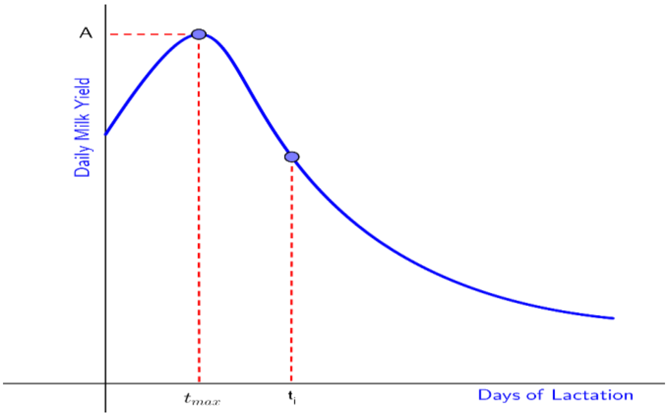


**Figure 1.** Lactation Growth Curve with biologically meaningful parameters, tmax , A and ti

All of the steps of conversions of Wood model transformed into mechanical model containing the biologically meaningful parameters are given here.

The production rate is shown as the following equation.

$$\mu = \frac{dy}{dt} \tag{3}$$

We need to change Wood model including the parameters with biological meaning, tmax , A and ti .

Wood model with empirical or mechanical form draws the same curve. That's the graph of the model does not change. What has changed is the meaningfulness of the parameters of Wood model. In this manuscript, the meaningful parameters of Wood model are the inflection point time, ti , the maximum milk production time, tmax and the maximum milk production value, A, at tmax.

The following steps of the modification of Wood model are presented:

**Step 1.** To get the maximum milk production time, tmax , the first derivative of the function with respect to t is obtained in equation 4.

$$\frac{dy}{dt} = \frac{at^b be^{-ct}}{t} - at^b ce^{-ct} \tag{4}$$

By equating this first derivative to zero, tmax value is found as following.

$$t_{max} = \frac{b}{c} \tag{5}$$

**Step 2.** To obtain the inflection point time, ti , the second derivative of the function with respect to t is obtained in equation 6.

$$\frac{d^2y}{dt^2} = \frac{at^b b^2 e^{-ct}}{t^2} - \frac{at^b be^{-ct}}{t^2} - \frac{2at^b bce^{-ct}}{t} + at^b c^2 e^{-ct} \tag{6}$$

By equating this second derivative to zero, ti value is found as following.

$$t_i = \frac{b \mp \sqrt{b}}{c} \tag{7}$$

**Step 3.** The production rate, μ, of Wood model at ti can be found in equation 8.

$$\mu = \frac{a\left(\frac{b\mp\sqrt{b}}{c}\right)^b ce^{-b\pm\sqrt{b}}\sqrt{b}}{b\mp\sqrt{b}} \tag{8}$$

**Step 4.** The parameters, b and c of Wood model used in the study are rewritten with respect to tmax and ti as following, respectively.

$$\boldsymbol{b} = \frac{t_{max}^2}{(t_i - t_{max})^2} \tag{9}$$

$$\boldsymbol{c} = \frac{t_{max}}{(t_i - t_{max})^2} \tag{10}$$

**Step 5.** How the parameter a of Wood model can be written with respect to tmax , ti and A will be given.

To get the maximum milk production value, A, putting t_max=b/c in the equation (1) instead of t time we get the following equation.

$$\boldsymbol{A} = \boldsymbol{a}\left(\frac{b}{c}\right)^{b} e^{-b} \tag{11}$$

From the equation (11), the parameter, a of Wood model is the following.

$$\boldsymbol{a} = \frac{A}{(\frac{b}{c})^{b} e^{-b}} \tag{12}$$

**Step 6.** By using equations 9 and 10, the parameter, a in Wood model used in the study are rewritten with respect to ti , tmax and A as following.

$$\boldsymbol{a} = \frac{A}{t_{max}^{\frac{t_{max}^2}{(t_i - t_{max})^2}} e^{\frac{-t_{max}^2}{(t_i - t_{max})^2}}} \tag{13}$$

**Step 7.** Substituting the equations 9, 10 and 13, the new equation of milk production (y) can be rewritten with respect to ti , tmax and A as following.

$$\boldsymbol{y} = \boldsymbol{A} t_{max}^{\frac{-t_{max}^2}{(t_i - t_{max})^2}} e^{\frac{-(t - t_{max}) t_{max}}{(t_i - t_{max})^2}} t^{\frac{t_{max}^2}{(t_i - t_{max})^2}} \tag{14}$$

It is more convenient to work with the new parameters. Note that although it seems that there are two turning points in equation 7, one of them will be meaningless. Although one of turning points is pointless, both turning points did not affect the a, b and c parameter transformations in equations 9, 10, 12 and 13. Therefore, the model transformation with the biologically meaningful parameters in equation 2 did not change either.

## RESULTS AND DISCUSSION

By using Table 1, the estimates of Wood model and observed average Daily milk values for the lactation days were given in Table 2. It is shown that the calculation made by selecting certaining a, b and c parameters in Wood model and using the corresponding biologically meaningful parameters in compatible with daily milk values.

**Table 2.** The observed and estimated daily milk values for Wood model

| | **Balikesir Population** | |
|---|---|---|
| **Lactation Days** | **Observed average Daily milk values (kg)** | **Wood** |
| 30 | 22 | 22.35 |
| 60 | 23.6 | 22.85 |
| 90 | 22.1 | 22.17 |
| 120 | 21 | 21.05 |
| 150 | 19.63 | 19.74 |
| 180 | 17.80 | 18.39 |
| 210 | 16.75 | 17.03 |
| 240 | 15.97 | 15.72 |
| 270 | 15.25 | 14.46 |
| 300 | 12.96 | 13.28 |

The parameters, a, b and c of Wood model used in this manuscript for the given data set are given in Table 3.

**Table 3**. The parameters of Wood model

| The parameters of the modified wood model | | |
|---|---|---|
| a | b | c |
| 13.305 | 0.183 | 0.003 |

The new parameters, ti , tmax and A of Wood model used in this manuscript for the given data set are given in Table 4.

**Table 4.** The new parameters of the modified Wood model

| New parameters of the modified wood model | | |
|---|---|---|
| $t_i$ | $t_{max}$ | A |
| 175.13 | 52.50 | 22.89 |

As seen in Table 3 and Table 4, although the parameters of Wood model do not show a direct meaning, the new parameters of the model modified contain direct meaning. In Table 4, since the inflection point time (ti) is 175.13, we can say that the rapid decline in lactation curve will slow after this point. According to Table 4 we can say that the maximum milk production time, tmax, and its maximum milk production value, A, are 52.50 and 22.89, respectively.

Although the model parameters change, the error sum of squares (SSE) will be exactly the same, since the model itself does not change. In this manuscript the SSE of Wood model is 1.92.

The empirical and mechanical forms of Wood model are given in equations 1 and 2, respectively.

It is seen that the empirical model and the mechanical model have the same number of parameters. The equation of the mechanical form is longer than the equation of the empirical form. Since all calculating for finding the parameters are made by using the package programs, the most important factor limiting the use of the model is the number of the parameters, not the length of the model (Motulsky and Ransnas, 1987; Zwitering et al., 1990; Khamis et al., 2005). Although the modified equation of Wood model seems like a long equation, number of parameters in both equations is the same. Because of that, the operations to be performed by the computer will not cause any difficulty in terms of computation. It is said that the model can be easily entered into the data set when the meanings of the parameters are known.

It is important to know what the parameters of both models mean biologically. Indeed, it is very important to know the meanings of the parameters in both models. Once the meanings of the parameters are known, the model can easily fit into the data set.

In some cases it is mentioned that whether the model studied is mechanical or empirical, and this difference is mentioned in this study of Korkmaz and Uckardes (2013). In addition, it is mentioned that the conversion of some important sigmoidal empirical models into mechanical models is given step-by-step in order to give an idea in the studies (Zwitering et al., 1990; Korkmaz and Uckardes, 2013; Korkmaz, 2016; Korkmaz, 2017) when there is no study on the lactation data.

## CONCLUSION

In this manuscript, it is shown stepwise how Wood model with non-biologically meaningful parameters in empirical structure, which is widely used in lactation, is transformed into Wood model with biologically meaningful parameters in mechanical structure. More about the lactation with the parameters of the new model will be obtained. Although this study has been prepared in a theoretical way, experimental data has been employed.

It is stated that although the parameters have changed, the error sum of squares remains exactly the same because the model itself has not changed. From this point of view, it is said transforming models such as Logistic, Gompertz, Brody, von Bertalanffy and etc. into meaningful models can be examined. This study provides a biological interpretation of the mathematical model used for lactation.

Since only the parameters change between Wood model with non-biologically meaningful parameters and Wood model with biologically meaningful parameters, there will be no difference in terms of other model selection criteria used in statistics. In another saying, in both models, the same values will be obtained in the same model selection criteria such as SSE, AIC (Akaike Information Criterion), R2 (coefficient of determination), R‾^2(adjusted coefficient of determination) as in SSE.

In this manuscript, although it is explained how the a, b and c parameters in the Wood model in the empirical form are transformed into the biologically meaningful parameters, tmax , A and ti, the transformation of Wood model into other meaningful parameters can also be investigated.

Moreover, from this point of view, the transformations of the parameters in the growth models such as Logistic, Gompertz, Brody, von Bertalanffy and etc. into meaningful parameters can also be investigated.

## REFERENCES

Karkach AS, 2006. Trajectories and models of individual growth. Demographic Research, 15: 347-400.

Khamis A, Ismail Z, Haron H, Mohammed TA, 2005. Nonlinear growth models for modeling oil palm yield growth. Journal of Mathematics and Statistics, 1(3): 225-233.

Korkmaz M, Uckardes F, 2013. Transformation to some growth models widely used in agriculture. The J. Anim Plan Sci., 23(3): 840-844.

Korkmaz M, 2016. Transformations of some growth models widely used in gas production Measurements. Journal of Scientific and Engineering Research, 3(2): 109-115.

Korkmaz M, 2017. An Applied Study on Converting Some Sigmoidal Models in Empirical Form to Meaningful Parameterized Mechanistic Models. Journal of Scientific and Engineering Research, 4(7):82-92.

Lopez S, Prieto M, Dijkstra J, Dhanoa MS, France J, 2004. Statistical evaluation of mathematical models for microbial growth. International Journal of Food Microbiology, 96: 289-300.

Motulsky HJ, Ransnas LA, 1987. Fitting curves to data using nonlinear regression: A Practical and Nonmathematical Review. FASEB J., 1, 365–374.

Soysal MI, Sirlar FG, Gurcan EK, 2004. An investigation on the lactation biometry of black and white dairy cattle herds raised in some public intensive farms in Turkey. Trakia Journal of Sciences, 2(3): 54-58.

Tedeschi OL, Fox GD, Saintz DR, Barioni GL, Medeiros RS, Boin C, 2005. Mathematical models in ruminant nutrition. Scielo Agricalture, Brazil, 62(1): 76-91.

Uckardes F, 2010. Kahramanmaras Sutcu Imam University Graduate School of Natural and Applied Sciences, (PhD thesis) Kahramanmaras, Turkey.

Wood PDP, 1967. Algebraic model of lactation curve in cattle. Nature, 216: 164-165.

Zwietering MH, Jongenburger I, Rombouts FM, Van’t Riet K, 1990. Modeling of the bacterial growth curve. Applied And Environmental Microbiology, 56(6): 1875-1991.