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On Vertex-Edge Degree Based Properties of Sierpinski Graphs

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ABSTRACT

Network science and graph theory are two important branches of mathematics and computer science. Many problems in engineering and physics are modeled with networks and graphs. Topological analysis of networks enable researchers to analyse networks in relation to some physical and engineering properties without conducting expensive experimental studies. Topological indices are numerical descriptors which are defined by using degree, distance and eigen value notions in any graph. Most of the topological indices are defined by using classical degree concept in graph theory, network and computer science. Recently two novel degree parameters have been defined in graph theory: Vertex-edge degree and Edge-vertex degree. Vertex-edge degree and edge-vertex degree based topological indices have been defined parallel to their corresponding classical degree counterparts. Sierpinski networks have important applications engineering science especially in computer science. Classical degree based topological properties of Sierpinski graphs have been investigated by many studies. In this article, vertex-edge degree based topological indices values of Sierpinski graphs have been computed.

Sierpinski Graflarının Tepe-Ayrıt Temelli Derece Özellikleri Üzerine

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ÖZ

Ağ bilimi ve çizge teorisi, matematik ve bilgisayar biliminin iki önemli dalıdır. Mühendislik ve fizikle ilgili birçok problem, ağlar ve çizgelerle modellenir. Ağların topolojik analizi, araştırmacıların pahalı deneysel çalışmalar yürütmeden, ağları bazı fiziksel ve mühendislik özellikleriyle ilgili olarak analiz etmelerini sağlar. Topolojik indeksler, herhangi bir çizgede derece, uzaklık ve öz değer kavramları kullanılarak tanımlanan sayısal tanımlayıcılardır. Topolojik indekslerin çoğu, çizge teorisi, ağ ve bilgisayar bilimlerinde klasik derece kavramı kullanılarak tanımlanır. Yakın zamanda çizge teorisinde iki yeni derece parametresi tanımlanmıştır: Tepe-ayrıt derecesi ve ayrıt-tepe derecesi. Tepe-ayrıt ve ayrıt-tepe derece temelli topolojik indeksler, klasik derece karşılıklarına parallel olarak tanımlanmıştır. Sierpinski ağları mühendislik bilimi açısından özellikle bilgisayar bilimleri açısından önemli bir uygulama alanına sahiptir. Sierpinski çizgelerinin klasik derece tabanlı topolojik özellikleri birçok çalışmada incelenmiştir. Bu makalede, Sierpinski çizgelerinin tepe-ayrıt derece temelli topolojik indeks değerleri hesaplandı.

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1. Introduction

Sierpinski graphs are fractal like graphs. Sierpinski networks have an important place in view of engineering science especially in computer science. Sierpinski networks have been used in to develop novel electronic and communication networks to facilitate information analysis in view of network security systems. There are close relationships between Sierpinski graphs and Hanoi graphs (Hinz et al., 2013).

Topological indices are used to investigate network topology for physical properties of these networks. Topological indices are numerical descriptors which are defined by using degree, distance and eigen-value notions in any graph. Most topological indices are defined by using classical degree concepts in graph theory, network and computer science. Computing topological indices for Sierpinski graphs has been intensively rising recently. Until now, many classical degree based topological indices such as Randic, Zagreb, atom-bond connectivity, geometric-arithmetic, sum-connectivity indices have been computed as indicated below. Degree-based topological indices and structural properties of the Sierpinski networks have been investigated in (Liu et al., 2019). Polymeric graphs and networks modelled by using Sierpinski graphs were examined by means of the general Randic index in (Estrada-Moreno and Rodriguez-Velazquez, 2019). Classical degree based topological features of Sierpinski networks were calculated in (Imran et al., 2017). The Graovac-Pisanski index of Sierpinski networks was reckoned in (Fathalikhani et al., 2020). Computation of Zagreb polynomials and Zagreb indices of Sierpinski networks was investigated in (Siddiqui, 2020). Certain topological features of uniform subdivision of Sierpinski networks were calculated in (Liu et al., 2021). General classical valency based topological properties and polynomials of Sierpinski graphs have been determined in (Fan et al., 2021). Two novel degree concepts; edge vertex degree and vertex edge degree notions systematically were defined in (Chellali et al., 2017). Vertex-edge degree and vertex-edge degree based topological indices have been defined by the present author (Ediz, 2017a), (Ediz, 2017b) and (Ediz, 2018). It has been shown that vertex-edge degree and edge-vertex degree based topological indices are possible tools for QSPR researches. After that many researches have been started to study mathematical and chemical properties of vertex-edge degree and edge-vertex degree based topological indices. Some related references could be seen the following articles. Edge-vertex and vertex-edge irregularities of graphs were investigated in (Horoldagva et al., 2019). Neural networks with cellular structures have been analysed in terms of vertex-edge irregularity points of view in (Husain et al., 2022). Vertex-edge degree based topological properties of metal trihalides network were computed in (Abolaban et al., 2021). Vertex-edge domination parameters of networks were calculated in (Żyliński, 2019). Vertex-edge degree and edge-vertex degree based topological features of single walled titanium dioxide nanotubes were analysed in (Zhang et al., 2021). Vertex-edge degree and edge-vertex degree related entropies firstly defined and investigated in terms of first Zagreb index entropies of networks in (Şahin and Şahin, 2021). Hexagon star graphs have been analysed with regard to vertex-edge degree topological properties in (Refaee and Ahmad, 2021). Edge-vertex based molecular topological features

DOX, RTOX and DSL Networks were investigated in (Cancan, 2019). Anti-cancer drug conjugates which contains hyaluronic acid have been investigated with regard to vertex-edge degree based topological features in (Kirmani et al., 2021).

This topic is a currently popular research area in view of computer science, network science, graph theory, physics and chemistry (Nacaroğlu and Maden, 2017(a); Nacaroğlu and Maden, 2017(b); Akgüneş and Nacaroğlu, 2019; Hong et al., 2020; Havare, 2021; Çolakoğlu, 2022; Estrada, 2022).

In this paper, we firstly computed vertex-edge degree based topological properties of Sierpinski graphs.

2. Materials and Method

In this section, necessary theoretical structures have been given. Let $G=(V(G),E(G))$ be simple connected graphs with the edge set $E(G)$ and the vertex set $V(G)$ and $u,v\in V(G)$ and $uv\in E(G)$ throughout in paper.

Definition 1. The vertex-edge degree of the vertex v, $deg_{ve}v$, is the number of all different edges between the vertex v and the other vertices with distance at most two from the vertex v.

Definition 2. The formula of the first vertex edge degree-based Zagreb alpha index of the graph G is;

$$M_1^{\alpha ve}(G)=\sum_{v\in V(G)} deg_{ve}v^2 \quad (1)$$

Definition 3. The formula of the first vertex edge degree-based Zagreb beta index of the graph G is;

$$M_1^{\beta ve}(G)=\sum_{uv\in E(G)}(deg_{ve}u+deg_{ve}v) \quad (2)$$

Definition 4. The formula of the second vertex edge degree-based Zagreb index of the graph G is;

$$M_2^{ve}(G)=\sum_{uv\in E(G)} deg_{ve}u deg_{ve}v \quad (3)$$

Definition 5. The vertex edge degree based Randic index of the graph G defined as;

$$R^{ve}(G)=\sum_{uv\in E(G)}(deg_{ve}u deg_{ve}v)^{-1/2} \quad (4)$$

Definition 6. A Sierpinski graph, where n$\geq 1, k\geq 1$, $S(n,k)$, has vertex set $\{1, 2, \cdots, k\}^n$, and there is an edge between two vertices $u=(u_1, u_2 \cdots, u_n)$ and $v=(v_1, v_2, \cdots, v_n)$ iff there is an $s\in\{1, 2, \cdots, n\}$ such that:

- $u_{l=} v_l$ for $l=1,2,\ldots,s-1$
- $u_s\neq v_s$
- $u_{l=} v_j$ and $u_{j=} v_l$ for $l=s+1,s+2,\ldots,n.$

Sierpinski graphs, $S(n,k)$, were defined originally in 1997 (Klavzar et al., 1997).

Some examples of Sierpinski graphs, see Figures 1, 2 and 3.

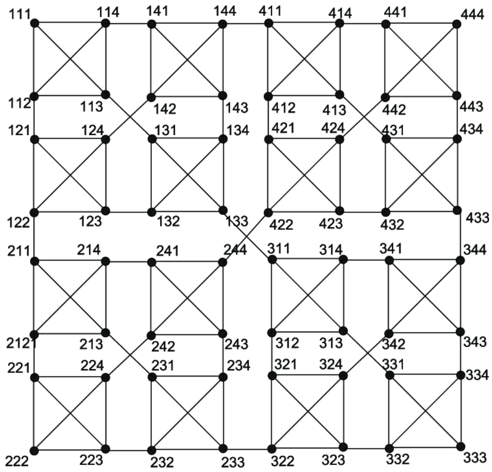

Figure 1. The Sierpinski graph S(3,4) (Klavzar et al., 1997).

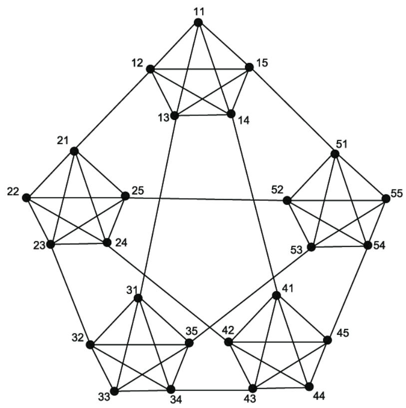

Figure 2. The Sierpinski graph S(2,5)

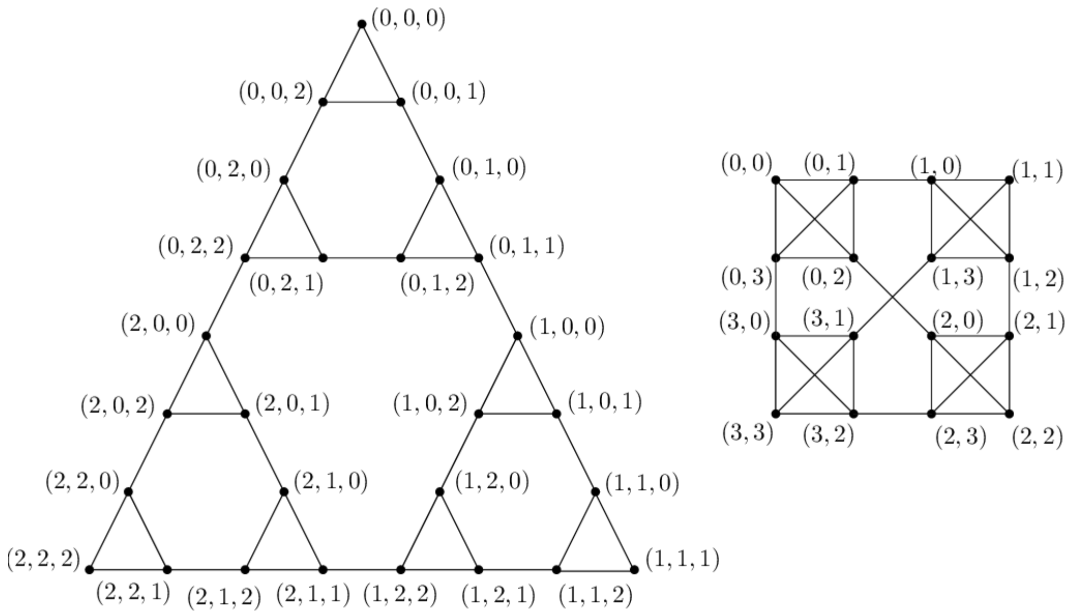

Figure 3. The Sierpinski graphs S(3,3) and S(2,4)

A vertex of $S(n,k)$ of the form $i,i,\dots,i \in \{1,2,\dots,k\}$ will be called an extreme vertex and the other vertices are called inner vertices. The degree of extreme vertices is $k-1$, while the degree of the inner vertices is k. The edges of $S(n,k)$ that lie in no induced K_k are called bridge edges. Note that bridge edges consist of the vertices $(i,j,\dots,j)$ and $(j,i,\dots,i)$ for $i \neq j$.

3. Results and Discussion

In this section we begin to compute vertex-edge numerical descriptors for the Sierpinski graphs. Following lemmas are indispensable tools for our calculations.

Lemma 1. (Daniele, 2009) $S(n,k)$ has k extreme vertices with degree $\frac{k}{2}(k^n-1)-1$ and k^n-k inner vertices with degree k.

Lemma 2. (Daniele, 2009) $S(n,k)$ has $\frac{k}{2}(k^n-1)$ edges.

From the definition of the Sierpinski graphs and above lemmas, we can directly state the following observation and lemmas.

Observation 3. Let $S(n,k)$ be a Sierpinski graph. $S(n,k)$ consists of two kinds of edges according to with endpoints. The first kind of edges are with endpoints inner-extreme vertices and second kind of edges with endpoints extreme-extreme vertices.

Lemma 4. Let $S(n,k)$ be a Sierpinski graph then the number of the first kind of edges (inner vertex-extreme vertex) is $k(k-1)$.

Proof. There are k extreme vertices each of them adjacent to $k-1$ vertices. That is there are k extreme vertices each of them incident to $k-1$ edges. From this, we get that $S(n,k)$ has k^2-k edges with end points are between inner and extreme vertices.

Lemma 5. Let $S(n,k)$ be a Sierpinski graph then the number of the second kind of edges (extreme vertex-extreme vertex) is $\frac{k}{2}(k^n-2k+1)$.

Proof. By Observation 3, we know that there are k^2-k edges with end points are between inner and extreme vertices. If we subtract this value from the total value of edge number $\frac{k}{2}(k^n-1)$, then we get the desired result.

Lemma 6. Let $v \in S(n,k)$ and $k \geq 3$,then v lies on exactly $\frac{(k-1)(k-2)}{2}$ different triangles.

Proof. Any triangle of $S(n,k)$ lies in only the components of $S(n,k)$ that is complete graphs K_k. And we know that a vertex of any complete graph K_k and remaining two vertices constitute a triangle graph. Therefore let $v \in S(n,k)$ and $k \geq 3$,then v lies on exactly $\binom{k-1}{2} = \frac{(k-1)(k-2)}{2}$ different triangles.

Lemma 7. (Şahin and Ediz, 2018) Let G be a connected graph and $v \in V(G)$ then $deg_{ve}v = \sum_{u\in N(v)} \deg(u) - n_v$ where $n_v$is the number of triangles in which contains the vertex v.

Lemma 8. Let $S(n,k)$ be a Sierpinski graph, $v \in S(n,k)$, v is an extreme vertex and $k \geq 4$,then $deg_{ve}v = \frac{(k-1)(k+2)}{2}$.

Proof. Every neighbour of an extreme vertex has the degree k. And an extreme vertex has $k-1$ neighbours. Therefore $\sum_{u \in N(v)} \deg(u) = k(k-1)$. We know that every extreme vertex lies in exactly $\frac{(k-1)(k-2)}{2}$ different triangles. Therefore, we get that $deg_{ve}v = \sum_{u \in N(v)} \deg(u) - k + 1 = k(k-1) - \frac{(k-1)(k-2)}{2} = \frac{(k-1)(k+2)}{2}$.

Lemma 9. Let $S(n,k)$ be a Sierpinski graph, $v \in S(n,k)$, v is an inner vertex and $k \geq 4$,then $deg_{ve}v = \frac{k(k-1)}{2}$.

Proof. Every inner vertex has a neighbour of extreme vertex with degree $k-1$ and has $k-1$ neighbours of inner vertex with degree k. Therefore, for any inner vertex v, we can write that $\sum_{u \in N(v)} \deg(u) = (k+1)(k-1)$. Also, we know that every inner vertex lies in exactly $\frac{(k-1)(k+2)}{2}$ triangles. Thus, we get that $deg_{ve}v = \sum_{u \in N(v)} \deg(u) - k + 1 = (k+1)(k-1) - \frac{(k-1)(k+2)}{2} = \frac{k(k-1)}{2}$.

And now, we can start to compute vertex-edge degree based topological indices of Sierpinski networks with the help of above observations and lemmas.

Proposition 10. The first ve-degree Zagreb alpha index of the graph $S(n,k)$ is $M_1^{\alpha ve}(S(n,k)) = k\frac{(k-1)^2(k+2)^2}{4} + (k^n - k)\frac{(k-1)^2k^2}{4}$.

Proof. The first ve-degree Zagreb alpha index of the graph $S(n,k)$ is defined as; $M_1^{\alpha ve}(S(n,k)) = \sum_{v \in V(S(n,k))} deg_{ve}v^2$. We know that $S(n,k)$ has k extreme vertices with vertex-edge degree $\frac{(k-1)(k+2)}{2}$ and $k^n - k$ inner vertices with vertex-edge degree $\frac{k(k-1)}{2}$ from the above observations and lemmas. By using these facts in the the first ve-degree Zagreb alpha index formula we get:

$$M_1^{\alpha ve}(S(n,k)) = k\frac{(k-1)^2(k+2)^2}{4} + (k^n - k)\frac{(k-1)^2k^2}{4}$$

Proposition 11. The first ve-degree Zagreb beta index of the graph $S(n,k)$ is $M_1^{\beta ve}\big(S(n,k)\big) = k(k-1)^2(k+1) + \frac{k^2}{2}(k^n - 2k + 1)(k-1)$.

Proof. The first ve-degree Zagreb beta index of $S(n,k)$ is defined as; $M_1^{\beta ve}(S(n,k)) = \sum_{uv \in E(S(n,k))}(deg_{ve}u + deg_{ve}v)$. From the Lemmas 4 and 5, we know that a Sierpinski graph has the number of the first kind of edges which consists of end points are inner vertex and extreme vertex is $k(k-1)$. And again a Sierpinski graph has the number of the second kind of edges which consists of end points are inner vertices is $\frac{k}{2}(k^n - 2k + 1)$. The vertex-edge degree of an extreme vertex is $\frac{(k-1)(k+2)}{2}$ and the vertex-edge degree of an inner vertex is $\frac{k(k-1)}{2}$ (see Lemmas 8 and 9). By using these facts in the first ve-degree Zagreb beta index formula we get:

$$M_1^{\beta ve}\big(S(n,k)\big) = k(k-1)(\frac{(k-1)(k+2)}{2} + \frac{k(k-1)}{2}) + \frac{k}{2}(k^n - 2k + 1)k(k-1)$$

$= k(k-1)^2(k+1) + \frac{k^2}{2}(k^n - 2k + 1)(k-1).$

Proposition 12. The second ve-degree Zagreb index of the graph $S(n,k)$ is $M_2^{ve}(S(n,k)) ==$ $\frac{k^2}{4}(k-1)^3(k+2) + \frac{k^3}{8}(k-1)^2(k^n - 2k + 1).$

Proof. The second ve-degree Zagreb index of is defines as $M_2^{ve}(S(n,k)) = \sum_{uv\in E(S(n,k))} deg_{ve}u deg_{ve}v$. From the Lemmas 4 and 5, we know that a Sierpinski graph has the number of the first kind of edges which consists of end points are inner vertex and extreme vertex is $k(k-1)$. And again a Sierpinski graph has the number of the second kind of edges which consists of end points are inner vertices is $\frac{k}{2}(k^n - 2k + 1)$. The vertex-edge degree of an extreme vertex is $\frac{(k-1)(k+2)}{2}$ and the vertex-edge degree of an inner vertex is $\frac{k(k-1)}{2}$ (see Lemmas 8 and 9). By using these facts in the second ve-degree Zagreb beta index formula we get:

$$M_2^{ve}\big(S(n,k)\big) = k(k-1)\frac{(k-1)(k+2)}{2}\frac{k(k-1)}{2} + \frac{k}{2}(k^n - 2k + 1)\frac{k^2(k-1)^2}{4}$$

$= \frac{k^2}{4}(k-1)^3(k+2) + \frac{k^3}{8}(k-1)^2(k^n - 2k + 1).$

Proposition 13. The ve-degree Randic index of the graph $S(n,k)$ is

$R^{ve}(S(n,k)) = 2k(k(k+2))^{-1/2} + \frac{2}{k-1}(k^n - 2k + 1).$

Proof. The ve-degree Randic index of the graph $S(n,k)$ defined as $R^{ve}(S(n,k)) = \sum_{uv\in E(S(n,k))}(deg_{ve}u deg_{ve}v)^{-1/2}$. From the Lemmas 4 and 5, we know that a Sierpinski graph has the number of the first kind of edges which consists of end points are inner vertex and extreme vertex is $k(k-1)$. And again a Sierpinski graph has the number of the second kind of edges which consists of end points are inner vertices is $\frac{k}{2}(k^n - 2k + 1)$. The vertex-edge degree of an extreme vertex is $\frac{(k-1)(k+2)}{2}$ and the vertex-edge degree of an inner vertex is $\frac{k(k-1)}{2}$ (see Lemmas 8 and 9). By using these facts in the ve-degree Randic index formula we get:

$$R^{ve}(S(n,k)) = k(k-1)\left(\frac{(k-1)(k+2)}{2}\frac{k(k-1)}{2}\right)^{-1/2} + \frac{k}{2}(k^n - 2k + 1)\left(\frac{k^2(k-1)^2}{4}\right)^{-1/2}$$

$= 2k(k(k+2))^{-1/2} + \frac{2}{k-1}(k^n - 2k + 1).$

4. Conclusion

Graph theory plays an important role in modelling and studying many networks in computer science. Networks are analysed by means of topological indices frequently in recent years. The numerical results obtained from these analyses are important in terms of the characteristics of networks. These calculations are used to understand and characterize the topologies hidden under these networks. Thanks to these calculations, applications can have information about the structural properties of the networks without making expensive experiments. Classical degree-based topological properties of Sierpinski graphs have been the subject of many studies over the last three years. Vertex-edge based topological indices were defined in 2018 and the calculation of the values of vertex-edge degree based indices in network science has been very popular in recent years. In this paper, we firstly computed vertex-edge degree degree based topological properties of Sierpinski graphs. These calculations are essential to understand underlying topology of Sierpinski networks.

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Conflict of interest

The author declares no conflict of interest.

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