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Chebyshev polynomial approximation for solving the second kind linear fredholm integral equation

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Keywords
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Chebyshev series
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approximation method

ABSTRACT

The purpose of this study is to give a Chebyshev polynomial approximation for the solution of the second kind of Linear Fredholm integral equation. For this purpose, a new Chebyshev matrix method is introduced. This method is based on taking the truncated Chebyshev expansion of the function in the integral equations. Hence, the result matrix equation can be solved and the unknown Chebyshev coefficients can be found approximately. In addition, examples that illustrate the pertinent features of the method are presented, and the results of study are discussed.

İkinci tip lineer fredholm integral denklemlerinin chebyshev polinom yaklaşımları

Anahtar Kelimeler
Chebyshev polinomları
Chebyshev serileri, Chebyshev polinom çözümleri
Fredholm integral denklemi
Yaklaşım metodları

ÖZET

Bu çalışmada ikinci tip linear Fredholm integral denklemlerinin Chebyshev polinomları ile çözümlerinin bulunması amaçlanmıştır. Bu amaçla yeni Chebyshev matris yöntemi geliştirilmiştir. Belirtilen yöntem denklemdeki bilinmeyen fonksiyonlarının kesilmiş Chebyshev polinomlarının matris formlarının alınması esasına dayanır. Böylece elde edilen denklem sistemi çözülerek Chebyshev polinomlarının katsayıları bulunur. Yöntemin hassasiyeti çeşitli örneklerle açıklanmış ve sonuçlar tartışılmıştır.

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1.INTRODUCTION

Integral equation has been one of the principal tools in various areas of applied mathematics, physics and engineering. A computational approach to the solution of integral equation is an essential branch of scientific inquiry. It is well-known that integral equations are usually difficult to solve analytically and exact solutions are very scarce. Therefore, integral equations have been a subject of great interest of many researchers. The computational approach of solution of integral equations is an essential branch of the scientific inquiry. Indeed, in order to resolve integral equations, there were developed many methods: such as Wavelet-like bases method, Parallel iterative methods, collocation method, decomposition method, hybrid Taylor series method, Petrov-Galerkin method, Adomian decomposition method [1-8].

In principle, analytical solution is the most desired result in theory and it is almost unobtainable for most practical problems. Although numerical methods can cope with a majority of complicated problems related to a system of integral equations, the obtained results cannot be expressed in simple form. In comparison with numerical methods, one of the advantages of approximate methods lies in that it can give a solution in an analytic form with an allowable error. As a result, up-to-date approximate methods remain of much interest in spite of advanced numerical methods accompanied with the help of modern computers. Since the beginning of 1994,Taylor and Chebyshev matrix methods have also been used by Sezer et al. to solve linear differential and integro- differential equations[9-15].

In this paper, we present a novel approximate technique, based on Chebyshev series expansion, for the solution of second kind Fredholm integral equations of the form

$$y(x)-\lambda\int_a^b k(x,t)y(t)dt=f(x),\quad a\le x\le b \quad (1)$$

where the parameter λ and the functions $k(x,t)$ and $f(x)$ are given, and $y(x)$ is the unknown function[16,17]. Suppose that the solution $y(x)$ of (1) is approximated by its Chebyshev expansion and the solution is expressed in the form

$$y(x)=\sum_{n=0}^{N} a_n T_n(x) \quad (2)$$

where $T_n(x)$ denotes the Chebyshev polynomials of the first kind, a_n $(0\le n\le N)$ are unknown Chebyshev coefficients, and N is chosen any positive integer.

The rest of this paper is organized as follows. Linear Fredholm integral equation and fundamental relations are presented in Section 2. The new scheme are based on Chebyshev matrix method. The method of finding approximate solution is described in Section 3. To support our findings, we present result of numerical experiments in Section 4. Section 5 concludes this article with a brief summary.

2.FUNDAMENTAL RELATIONS

To obtain the solution of equation(1) in the form(2) we first find the matrix form of each term in the equation and then differentiate it n times with respect to x to analyse it as matrix representation. First we can convert the

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solution $y(x)$ defined by a truncated Chebyshev series (2) and its derivative $y^{(k)}(x)$ to matrix forms

$$y(x) = \mathbf{T}(x)\mathbf{A} \quad (3)$$

$$y^{(k)}(x) = \mathbf{T}^{(k)}(x)\mathbf{A} \quad (4)$$

where

$$\mathbf{T}(x) = [T_0(x)\ T_1(x) \dots T_N(x)]$$

$$\mathbf{A} = [a_0\ a_1 \dots a_N]^T$$

On the other hand, it is well known [18] that the relation between the powers x^n and the Chebyshev polynomials $T_n(x)$ is

$$x^{2n} = 2^{-2n+1}\sum_{j=0}^{n}{}'\binom{2n}{n-j}T_j(x), -1 \le x \le 1 \quad (5)$$

$$x^{2n+1} = 2^{-2n}\sum_{j=0}^{n}\binom{2n}{n-j}T_{2j+1}(x), -1 \le x \le 1 \quad (6)$$

By using the expression (5) and taking n=0,1,…,N we find the corresponding matrix relation as follows

$$\mathbf{X}(x) = \mathbf{T}(x)\mathbf{D}^T \quad (7)$$

where $\mathbf{X}(x) = [1\ x \dots x^N]$ for odd N,

$$\mathbf{D} = \begin{bmatrix} 2^0\binom{0}{0} & 0 & 0 & 0 & \dots & 0 \\ 0 & 2^{-1}\binom{1}{0} & 0 & 0 & \dots & 0 \\ 2^{-2}\binom{2}{1} & 0 & 2^{-1}\binom{2}{0} & 0 & \dots & 0 \\ 0 & 2^{-3}\binom{3}{1} & 0 & 2^{-1}\binom{3}{0} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 2^{1-N}\binom{N}{\frac{N-1}{2}} & 0 & 2^{--N}\binom{N}{1} & \dots & 2^{1-N}\binom{N}{0} \end{bmatrix} \quad (8)$$

and for even N,

$$\mathbf{D} = \begin{bmatrix} 2^0\binom{0}{0} & 0 & 0 & 0 & \dots & 0 \\ 0 & 2^{-1}\binom{1}{0} & 0 & 0 & \dots & 0 \\ 2^{-2}\binom{2}{1} & 0 & 2^{-1}\binom{2}{0} & 0 & \dots & 0 \\ 0 & 2^{-3}\binom{3}{1} & 0 & 2^{-1}\binom{3}{0} & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 2^{1-N}\binom{N}{\frac{N}{2}} & 0 & 2^{1-N}\binom{N}{\frac{N-2}{2}} & 0 & \dots & 2^{1-N}\binom{N}{0} \end{bmatrix} \quad (9)$$

Then, by taking into account(7) we obtain

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$$\mathbf{T}(x) = \mathbf{X}(x)(\mathbf{D}^T)^{-1} \qquad (10)$$

and

$$\mathbf{T}^{(k)}(x) = \mathbf{X}^{(k)}(x)(\mathbf{D}^T)^{-1}, \quad k=0,1,2,... \quad (11)$$

To obtain the matrix $\mathbf{X}^{(k)}(x)$ in terms of the matrix $\mathbf{X}(x)$, we can use the following relation:

$$\mathbf{X}^{(1)}(x) = \mathbf{X}(x)\mathbf{B}^1$$

$$\mathbf{X}^{(2)}(x) = \mathbf{X}^{(1)}(x)\mathbf{B} = \mathbf{X}(x)\mathbf{B}^2$$

$$\mathbf{X}^{(3)}(x) = \mathbf{X}^{(2)}(x)\mathbf{B} = \mathbf{X}(x)\mathbf{B}^3$$

$$\mathbf{X}^{(k)}(x) = \mathbf{X}^{(k-1)}(x)\mathbf{B} = \mathbf{X}(x)\mathbf{B}^k \qquad (12)$$

where

$$\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 & \dots & 0 \\ 1 & 0 & 0 & \dots & 0 \\ 0 & 2 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & N & 0 \end{bmatrix} \qquad (13)$$

Consequently, by substituting the matrix forms (11) and (12) into (4) we have the matrix relation

$$y^{(k)}(x) = \mathbf{X}(x)\mathbf{B}^k(\mathbf{D}^T)^{-1}\mathbf{A} \quad (14)$$

3. METHOD OF SOLUTION

We now ready to construct the fundamental matrix equation corresponding to Eq. (1), for this purpose, we first substitute (2) into Eq. (1) and then simplify. Thus we have the fundamental matrix equation

$$\mathbf{X}(x)(\mathbf{D}^T)^{-1}\mathbf{A} = f(x) + \lambda\int_a^b k(x,t)\mathbf{X}(t)(\mathbf{D}^T)^{-1}\mathbf{A}dt \qquad (15)$$

or clearly

$$\left[\mathbf{X}(x)(\mathbf{D}^T)^{-1} - (\lambda\int_a^b k(x,t)\mathbf{X}(t)dt)(\mathbf{D}^T)^{-1}\right]\mathbf{A} = f(x) \qquad (16)$$

Suppose that the solution $y(x)$ of (1) is approximated by its Chebyshev expansion(2). We first differentiate both sides of (1) with respect to x to get (N+1) equation and substitute (14) in the expression to get that for $a \le x \le b$,

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$$\left[\mathbf{X}^{(1)}(x)(\mathbf{D}^T)^{-1} - (\lambda\int_a^b \frac{\partial}{\partial x}(k(x,t))\mathbf{X}(t)dt)(\mathbf{D}^T)^{-1}\right]\mathbf{A} = f'(x)$$

$$\left[\mathbf{X}^{(2)}(x)(\mathbf{D}^T)^{-1} - (\lambda\int_a^b \frac{\partial^2}{\partial x^2}(k(x,t))\mathbf{X}(t)dt)(\mathbf{D}^T)^{-1}\right]\mathbf{A} = f''(x)$$

$$\left[\mathbf{X}^{(N)}(x)(\mathbf{D}^T)^{-1} - (\lambda\int_a^b \frac{\partial^N}{\partial x^N}(k(x,t))\mathbf{X}(t)dt)(\mathbf{D}^T)^{-1}\right]\mathbf{A} = f^{(N)}(x) \qquad (17)$$

Substitute $\mathbf{X}^{(N)}(x)$ in the integrals in Eqs.(17) to obtain that for $a \le x \le b$,

$$\left[\mathbf{X}(x)\mathbf{B}(\mathbf{D}^T)^{-1} - (\lambda\int_a^b \frac{\partial}{\partial x}(k(x,t))\mathbf{X}(t)dt)(\mathbf{D}^T)^{-1}\right]\mathbf{A} = f'(x).$$

$$\left[\mathbf{X}(x)\mathbf{B}^2(\mathbf{D}^T)^{-1} - (\lambda\int_a^b \frac{\partial^2}{\partial x^2}(k(x,t))\mathbf{X}(t)dt)(\mathbf{D}^T)^{-1}\right]\mathbf{A} = f''(x)$$

...

$$\left[\mathbf{X}(x)\mathbf{B}^N(\mathbf{D}^T)^{-1} - (\lambda\int_a^b \frac{\partial^N}{\partial x^N}(k(x,t))\mathbf{X}(t)dt)(\mathbf{D}^T)^{-1}\right]\mathbf{A} = f^{(N)}(x) \qquad (18)$$

$$\left[\mathbf{X}(x)\mathbf{B}^k(\mathbf{D}^T)^{-1} - \mathbf{Z}_k(\mathbf{D}^T)^{-1}\right]\mathbf{A} = f^{(k)}(x), k = 0,1,2,...,N \qquad (19)$$

where

$$\mathbf{Z}_k = \lambda\int_a^b \frac{\partial^k}{\partial x^k}(k(x,t))\mathbf{X}(t)dt), \quad k = 0,1,2,...,N$$

Now these equations given by (19) for $k = 0,1,2,...,N$ become a linear system of N+1 algebraic equations for N+1 unknowns $a_0 a_1,...,a_N$. Hence, the fundamental matrix equation (19) corresponding to Eq. (1) can be written in the form

$$\mathbf{WA} = \mathbf{F} \text{ or } [\mathbf{W};\mathbf{F}], \quad \mathbf{W} = [w_{i,j}], \; i, j = 0,1,...,N \qquad (20)$$

where

$$\mathbf{W} = [\mathbf{W}_0 \; \mathbf{W}_1 ... \; \mathbf{W}_N]^T, \quad \mathbf{W}_k = \mathbf{X}(x)\mathbf{B}^k(\mathbf{D}^T)^{-1} - \mathbf{Z}_k(\mathbf{D}^T)^{-1}, \; \text{k=0,1,…N} \qquad (21)$$

$$\mathbf{F} = [f^{(0)}(x) \; f^{(1)}(x) \; ... \; f^{(N)}(x)]^T$$

The fundamental matrix $\mathbf{W}$ is depend on x variable. In application we choose any point $x = x_q \in [a,b], \; q = 0,1,2,...$ and we solve the problem in any point in the given interval.

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Here, Eq. (20) corresponds to a system of $(N+1)$ linear algebraic equations with unknown Chebyshev coefficients $a_0, a_1, ..., a_N$. If $rank\mathbf{W} = rank[\mathbf{W};\mathbf{F}] = N+1$, then we can write

$$\mathbf{A} = (\mathbf{W})^{-1}\mathbf{F} \tag{22}$$

Thus the matrix $\mathbf{A}$ (thereby the coefficients $a_0, a_1, \ldots, a_N$) is uniquely determined. Also the Eq.(1) has a unique solution. This solution is given by truncated Chebyshev series $y(x) = \sum_{n=0}^{N} a_n T_n(x)$.

We can easily check the accuracy of the suggested method. Since the truncated Chebyshev series (2) is an approximate solution of Eq.(1), when the solution $y_N(x)$ is substituted in Eq.(1), the resulting equation must be satisfied approximately; that is, for $x = x_q \in [a,b]$, $q = 0,1,2,...$

$$E(x_q) = \left| y_N(x) - \lambda \int_a^b k(x,t) y_N(t) dt - f(x) \right| \cong 0 \tag{23}$$

and $E(x_q) \le 10^{-k_q}$ (k_q positive integer). If $\max 10^{-k_q} = 10^{-k}$ (k positive integer) is prescribed, then the truncation limit N is increased until the difference $E(x_q)$ at each of the points becomes smaller than the prescribed 10^{-k}. On the other hand, the error can be estimated by the function

$$E_N(x) = y_N(x) - \lambda \int_a^b k(x,t) y_N(t) dt - f(x) \tag{24}$$

If $E_N(x) \to 0$, when N is sufficiently large enough, then the error decreases.

4.NUMERICAL RESULTS

In this section, several numerical examples are given to illustrate the accuracy and effectiveness properties of the method. The absolute errors in Tables are the values of $|y(x) - y_N(x)|$ at selected points. All computations were carried out using Maple10 on Personal Computer, and it took only several minutes to get all the computed results.

Example 1.

Let us first consider the second kind Fredholm integral equation

$$y(x) = e^x - 1 + \int_0^1 e^{-t} y(t) dt$$

and seek the solution $y(x)$ as a truncated Chebyshev series

$$y(x) = \sum_{n=0}^{N} a_n T_n(x)$$

Here $k(x,t) = e^{-t}$, $f(x) = e^x - 1$, $\lambda = 1$, $a = 0, b = 1$ and exact solution $y(x) = e^x$.

Then, fundamental matrix equation of the problem is defined by

$$\mathbf{WA} = \mathbf{F} \tag{25}$$

where the matrices are defined by

$$\mathbf{X}(x) = [1 \; x \; x^2 \; x^3 \; x^4 \; x^5 \; x^6]$$

$$\mathbf{Z}_0 = [0.632120 \quad 0.264241 \quad -0.310914 \quad -0.337007 \quad 0.049988 \quad 0.183461 \quad -0.039540]$$

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$$\mathbf{W}=\begin{bmatrix} 0.367879 & -0.264241+x & -0.689085+2x^2 & 0.337007-3x+4x^3 & 0.950011-8x^2+8x^4 \\ 0 & 1 & 4x & -3+12x^2 & -16x+32x^3 \\ 0 & 0 & 4 & 24x & -16+96x^2 \\ 0 & 0 & 0 & 24 & 192x \\ 0 & 0 & 0 & 0 & 192 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\begin{bmatrix} -0.183461+5x-20x^3+16x^5 & -0.960460+18x^2-48x^4+32x^6 \\ 5-60x^2+80x^4 & 36x-192x^3+192x^5 \\ -120x+320x^3 & 36-576x^2+960x^4 \\ -120+960x^2 & -1152x+3840x^3 \\ 1920x & -1152+11520x^2 \\ 1920 & 23040x \\ 0 & 23040 \end{bmatrix}$$

$$\mathbf{B}=\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}, \quad \mathbf{D}=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1/4 & 0 & 1/2 & 0 & 0 & 0 & 0 \\ 0 & 3/4 & 0 & 1/4 & 0 & 0 & 0 \\ 3/8 & 0 & 1/2 & 0 & 1/8 & 0 & 0 \\ 0 & 5/8 & 0 & 5/16 & 0 & 1/16 & 0 \\ 5/16 & 0 & 15/32 & 0 & 3/16 & 0 & 1/32 \end{bmatrix}$$

We take $x=1$ for our computations and then, the fundamental matrix **W** of the equation and the matrix **F** is defined by

$$\mathbf{W}=\begin{bmatrix} 0.367879 & 0.735758 & 1.310914 & 1.337007 & 0.950011 & 0.816539 & 1.03954 \\ 0 & 1 & 4 & 9 & 16 & 25 & 36 \\ 0 & 0 & 4 & 24 & 80 & 200 & 420 \\ 0 & 0 & 0 & 24 & 192 & 840 & 2688 \\ 0 & 0 & 0 & 0 & 192 & 1920 & 10368 \\ 0 & 0 & 0 & 0 & 0 & 1920 & 23040 \\ 0 & 0 & 0 & 0 & 0 & 0 & 23040 \end{bmatrix} \quad \mathbf{F}=\begin{bmatrix} 1.718282 \\ 2.718282 \\ 2.718282 \\ 2.718282 \\ 2.718282 \\ 2.718282 \\ 2.718282 \end{bmatrix}$$

This system has the solution

A=[1.277883 1.109965 0.2849241 0.03775392 0.007786744 0.0 0.000117981]

Therefore, we find the approximate solution

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$y(x) = 1.277883T_0(x) + 1.109965T_1(x) + 0.2849241T_2(x) + 0.03775392T_3(x) + 0.007786744\,T_4(x) + 0.000117981T_6(x)$

or the simplified form of the above result is

$$y(x) = 1.000628 + 0.9967032x + 0.5096778x^2 + 0.1510157x^3 + 0.05663088x^4 + 0.003775392x^6$$

Taking N=6,8,10 the obtained solutions are compared with the exact solution in Table1.

Table1 Error analysis of Example 1 for the x value

x	Exact Solution	Present Method					
		N=6	N_e=6	N=8	N_e=8	N=10	N_e=10
0.0	1.000000	1.000628	0.628E-3	1.000012	0.120E-4	0.999979	0.210E-5
0.1	1.105171	1.105552	0.381E-3	1.105178	0.700E-5	1.105169	0.200E-5
0.2	1.221403	1.221655	0.252E-3	1.221409	0.600E-5	1.221401	0.200E-5
0.3	1.349859	1.350049	0.190E-3	1.349864	0.500E-5	1.349856	0.300E-5
0.4	1.491825	1.491987	0.162E-3	1.491830	0.500E-5	1.491822	0.300E-5
0.5	1.648721	1.648874	0.153E-3	1.648727	0.600E-5	1.648719	0.200E-5
0.6	1.822119	1.822268	0.149E-3	1.822124	0.500E-5	1.822117	0.200E-5
0.7	2.013753	2.013901	0.148E-3	2.013758	0.500E-5	2.013751	0.200E-5
0.8	2.225541	2.225691	0.150E-3	2.225546	0.500E-5	2.225540	0.100E-5
0.9	2.459603	2.459752	0.149E-3	2.459609	0.600E-5	2.459602	0.100E-5
1.0	2.718282	2.718431	0.149E-3	2.718288	0.600E-5	2.718282	0.00000

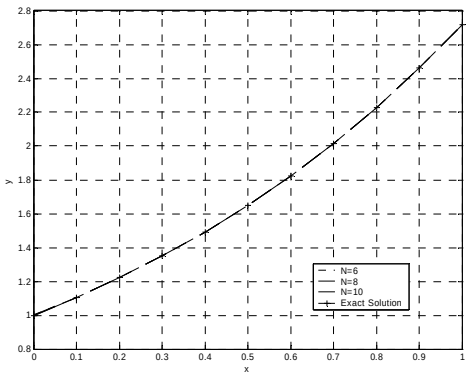

Fig.1.Numerical and exact solution of the Example1 for N=6,8,10

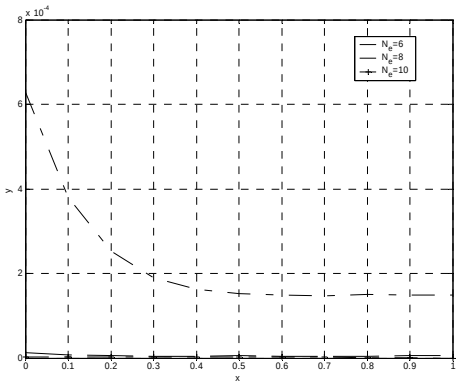

Fig.2.Error function of Example1 for various N.

Fig.1 show the comparison between the exact solution and different for the N Chebyshev matrix method solutions of the system in Eq.(20). It seems that the solutions almost identical. One can obtain a beter approximation to the numerical solutions by adding new terms to the series in Eq.(2). Fig.2 show that the comparison between the errors fonctions for various N. It seems that the accuracy increases as the N is increased.

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Example 2.

Let us find the Chebyshev series solution of the following linear fredholm integral equation[9]

$$y(x)=\cos(\pi x)+\frac{1}{4}\int_{-1}^{1}\sin(\pi x)\cos(\pi t)y(t)dt$$

The exact solution of this problem is $y(x)=\cos(\pi x)+\frac{1}{4}\sin(\pi x)$. Using the procedure in Section 3 and for N=12 the matrices in Eq.(20) are computed. Hence linear algebraic system is gained. This system is approximately solved for the points $x_i=\cos(\frac{i\pi}{12})$ as is taken in [9] using the Maple10.

Table2 Error analysis of Example 2 for the x value

x	Exact solution	Chebyshev Col. Matrix Met.[9] N=12	Chebyshev Col. Matrix Met.[9] N_2=12	Suggested Met. N=12	Suggested Met. N_e=12
x_0	-1.000000	-1.000000	0.00000000	-1.000010	0.108180E-4
x_1	-0.967565	-0.967563	0.137676E-5	-0.967574	0.947280E-5
x_2	-0.810580	-0.810579	0.957284E-6	-0.810585	0.513040E-5
x_3	-0.406776	-0.406776	0.556751E-6	-0.406779	0.304030E-5
x_4	0.250000	0.250000	0.000000000	0.249996	0.325290E-5
x_5	0.868852	0.868851	0.131853E-5	0.868850	0.223577E-5
x_6	1.000000	1.000000	0.00000000	1.000000	0.00000000
x_7	0.505640	0.505638	0.244225E-5	0.505643	0.235770E-5
x_8	-0.250000	-0.250000	0.00000000	-0.249996	0.326630E-5
x_9	-0.804623	-0.804625	0.273100E-5	-0.804618	0.464110E-5
x_{10}	-1.014860	-1.014870	0.238170E-5	-1.014836	0.322237E-4
x_{11}	-1.020980	-1.020980	0.456618E-5	-1.020853	0.133479E-3
x_{12}	-1.000000	-1.000000	0.000000000	-0.999788	0.211757E-3

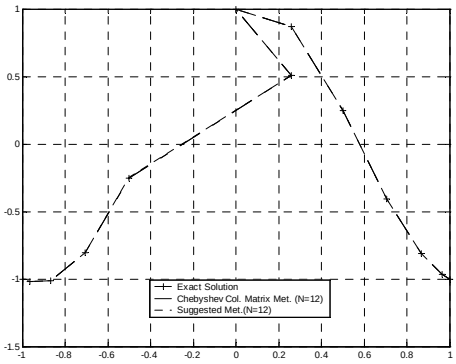

Fig.3.Numerical and exact solution of the Example 2

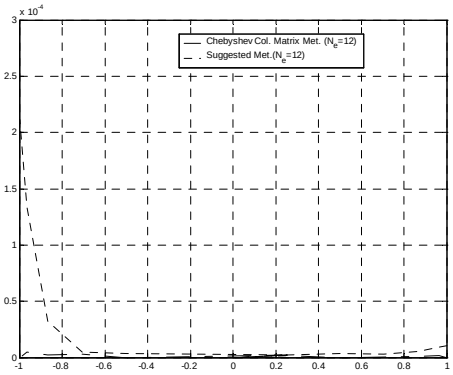

Fig.4.Error function of Example2 for various N.

We display a plot of Chebyshev collocation method, presented method and Exact solution in Fig.3 and we compare these methods and presented method in Table2. It seems that the solutions almost identical. One can obtain a beter approximation to the numerical solutions by adding new terms to the series in Eq.(2). It is of interest to note that the method

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discussed above works effectively for linear models.

Example 3.

Let us find the Chebyshev series solution of linear fredholm integral equation of the second kind

$$y(x) = \int_0^1 (t^2 x - \frac{3}{2} t x^2) y(t) dt + 2\ln(x+1) + \frac{5}{9}x + \frac{3}{4}x^2 - \frac{4}{3}x\ln(2)$$

and the exact solution is $y(x) = 2\ln(x+1)$. The solution of the linear fredholm integral equation is obtained for N=6,8,10. For numerical results, see Table 3. We display a plot of absolute difference exact and approximate solutions in Fig.5 and error functions for various N is shown in Fig.6.

Table 3 Error analysis of Example 3 for the x value

x	Exact Solution	Present Method N=6	N_e=6	N=8	N_e=8	N=10	N_e=10
0.0	0.000000	0.004003	0.4003E-2	0.000794	0.7940E-3	0.000165	0.1650E-3
0.1	0.190620	0.192392	0.1772E-2	0.190904	0.2840E-3	0.190668	0.4800E-4
0.2	0.364643	0.365364	0.7210E-3	0.364734	0.9100E-4	0.364655	0.1240E-4
0.3	0.524728	0.524989	0.2610E-3	0.524753	0.2500E-4	0.524731	0.2900E-5
0.4	0.672944	0.673020	0.7600E-4	0.672949	0.5000E-5	0.672945	0.8000E-6
0.5	0.810930	0.810939	0.9000E-5	0.810929	0.1000E-5	0.810930	0.4000E-6
0.6	0.940007	0.939992	0.1500E-4	0.940004	0.3000E-5	0.940007	0.1000E-6
0.7	1.061257	1.061231	0.2600E-4	1.061254	0.3000E-5	1.061257	0.000000
0.8	1.175573	1.175539	0.3400E-4	1.175570	0.3000E-5	1.175574	0.1000E-5
0.9	1.283708	1.283665	0.4300e-4	1.283702	0.6000E-5	1.283707	0.1000E-5
1.0	1.386294	1.386241	0.5300E-4	1.386289	0.5000E-5	1.386294	0.000000

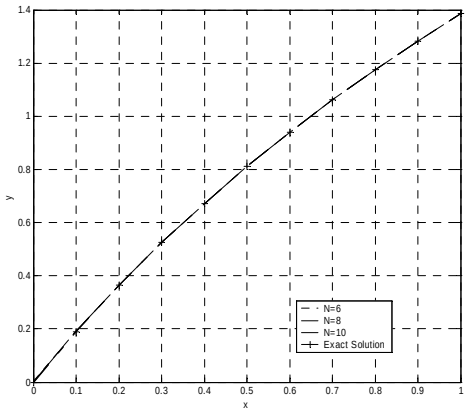

Fig.5.Numerical and exact solution of the Example3 for N=6,8,10

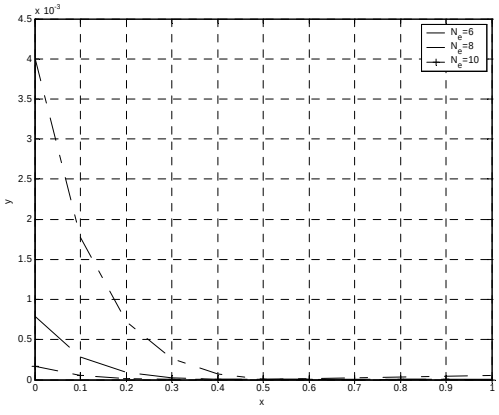

Fig.6.Error function of Example3 for various N.

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Example 4.

Consider the linear fredholm integral equation of the second kind.

$$y(x) = -\frac{1}{3}\int_0^1 e^{2x-\frac{5}{3}t}\, y(t)dt + e^{2x+\frac{1}{3}}$$

To evaluate the accuracy of the approximations produced by the Chebyshev series expansion method, $y(x)$ is chosen such that the exact solution is $y(x) = e^{2x}$. For numerical results, see Table4.

Fig.7 show the comparison between the exact solution and different for the N Chebyshev matrix method solutions of the system in Eq.(20). Fig.8 show that the comparison between the errors fonctions for various N. It seems that the accuracy increases as the N is increased.

Table4
Error analysis of Example 4 for the x value

x	Exact	Present Method					
	Solution	N=6	N_e=6	N=8	N_e=8	N=10	N_e=10
0.0	1.000000	1.145021	0.145021	1.008479	0.8479E-2	1.000318	0.318E-3
0.1	1.221403	1.289439	0.68036E-1	1.224593	0.3190E-2	1.221498	0.950E-4
0.2	1.491825	1.518694	0.26869E-1	1.492754	0.9290E-3	1.491843	0.180E-4
0.3	1.822119	1.828232	0.61130E-2	1.822152	0.3300E-4	1.822113	0.600E-5
0.4	2.225541	2.221587	0.39540E-2	2.225227	0.3140E-3	2.225528	0.130E-4
0.5	2.718282	2.709202	0.90800E-2	2.717801	0.4810E-3	2.718263	0.190E-4
0.6	3.320117	3.307721	0.12396E-1	3.319509	0.6080E-3	3.320095	0.220E-4
0.7	4.055200	4.039758	0.15442E-1	4.054455	0.7450E-3	4.055173	0.270E-4
0.8	4.953032	4.934126	0.18906E-1	4.952122	0.9100E-3	4.952998	0.340E-4
0.9	6.049647	6.026554	0.23093E-1	6.048536	0.1111E-2	6.049606	0.410E-4
1.0	7.389056	7.360848	0.28208E-1	7.387697	0.1359E-2	7.389007	0.490E-4

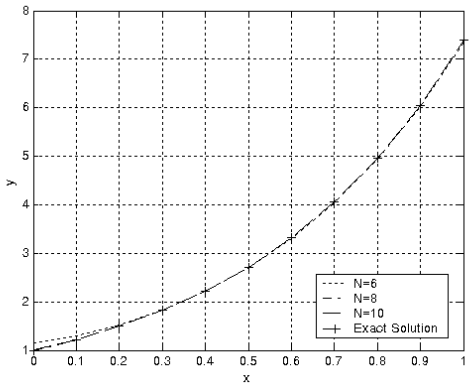

Fig.7.Numerical and exact solution of the Example4 for N=6,8,10

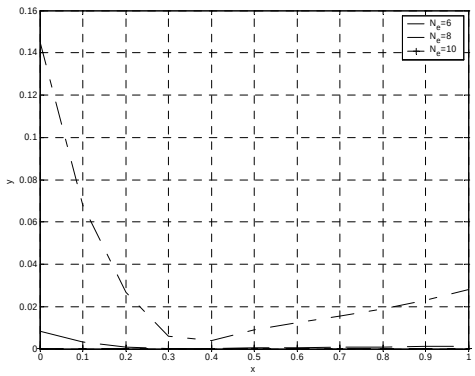

Fig.8.Error function of Example4 for various N.

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Example 5.

Consider the linear fredholm integral equation

$$\int_0^1 \sin(xt)\,y(t)dt = \frac{\sin(x) - x\cos(x)}{x^2}$$

with the exact solution $y(x) = x$. Table5 shows the numerical results of Example5[19]. We compare Legendre wavelet, presented method and Exact solution in Table5. It seems that the solutions almost identical. One can obtain a beter approximation to the numerical solutions by adding new terms to the series in Eq.(2).

Table5 Error analysis of Example 5 for the x value

x	Exact Solution	Legendre Wavelets met[19]	Legendre multi-wavelets met.[19]	Present Method
0.0	0.00000000	0.0001183000	0.00003230000	0.00000000
0.1	0.10000000	0.1000354200	0.10000845500	0.10000000
0.2	0.20000000	0.2000119300	0.20001539000	0.20000000
0.3	0.30000000	0.3000237700	0.30000957000	0.30000000
0.4	0.40000000	0.4000011000	0.40000059440	0.40000000
0.5	0.50000000	0.5000112100	0.50000198300	0.50000000
0.6	0.60000000	0.6000053600	0.60000087410	0.60000000
0.7	0.70000000	0.7000023500	0.70000023510	0.70000000
0.8	0.80000000	0.8000005900	0.80000004456	0.80000000
0.9	0.90000000	0.9000006900	0.90000001561	0.90000000

Example 6.

Consider the linear fredholm integral equation

$$\int_0^1 e^{xt}\,y(t)dt = \frac{e^{x+1} - 1}{x+1}$$

with the exact solution $y(x) = e^x$. Table6 shows the numerical results of Example6[20]. We display a plot of Legendre wavelet, presented method and Exact solution in Fig.10 and we compare these methods and presented method in Table6. It seems that the solutions almost identical. One can obtain a beter approximation to the numerical solutions by adding new terms to the series in Eq.(2). It is of interest to note that the method discussed above works effectively for linear models.

Table6 Absolute errors obtained for Example 6

x	*Exact Solution*	*Legendre wavelets met.[20]*	*Present Method*
0.0	1.000000	1.001110	1.011882
0.1	1.105171	1.110467	1.103574
0.2	1.221403	1.221070	1.212385
0.3	1.349859	1.350320	1.338317
0.4	1.491825	1.492420	1.481369
0.5	1.648721	1.650970	1.641542
0.6	1.822119	1.821280	1.818834
0.7	2.013753	2.012970	2.013247
0.8	2.225541	2.226030	2.224780
0.9	2.459603	2.460460	2.453434

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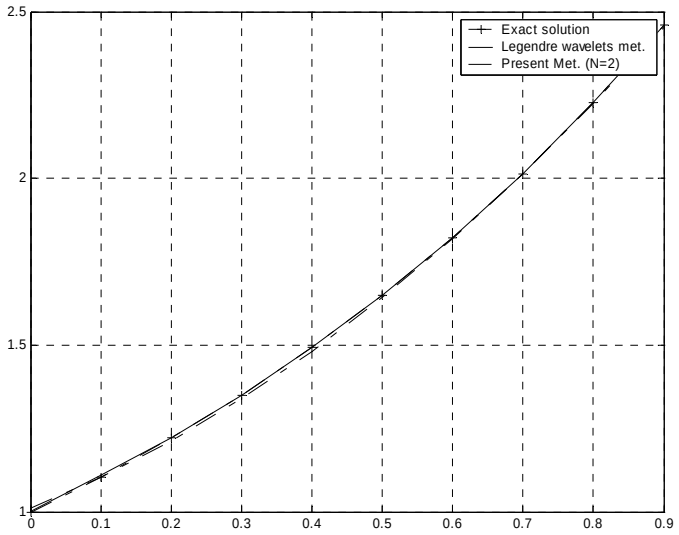

Fig.9. Comparison of Legendre wavelets and Chebyshev collocation method for Example 6

Example 7.

In this final example we consider the linear fredholm integral equation

$$y(x) = x^2 + 2\int_0^1 (1+xt)y(t)dt$$

with the exact solution $y(x) = x^2 - \frac{1}{8}x - \frac{13}{24}$. If the matrices are substituted in (20), it is obtained linear algebraic system.This system yields the approximate solution of the problem. The result using the Chebyshev matrix method discussed in Section 3 is the same with the exact solution.

5. CONCLUSION

In recent years, the studies of linear fredholm integral equation have attracted the attention of many mathematicians and physicists. The Chebyshev matrix methods are used to solve the linear fredholm integral equation numerically. The method proposed in this paper can be applied to a wide class of Fredholm integral equations of the second kind arising not only in radiative heat transfer but also in a number of other applications, e.g., potential theory, radiative equilibrium, and electrostatics. The approach leads to an approximate solution of the integral equation which can be expressed explicitly in a simple, closed form and which can be effectively computed using symbolic computing codes on any modern Personal Computer. To get the best approximating solution of the equation, we take more forms from the Chebyshev expansion of functions that is , the truncation limit N must be chosen large enough. In addition, an interesting feature of this method is to find the analytical solutions if the equation has an exact solution that is a polynomial functions.

As a result, the power of the employed method is confirmed. We assured the correctness of the obtained solutions by putting them back into the original equation with the aid of Maple, it provides an extra measure of confidence in the results. The method can also be extended to the system of linear

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fredholm integral equations, but some modifications are required.

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