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Big Bang-Big Crunch Optimization Algorithm for Solving the Uncapacitated Facility Location Problem

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Abstract: The big bang–big crunch (BB–BC) algorithm has been proposed as a new optimization method based on the big bang and big crunch theory, one of the theories of the evolution of the universe. The BB-BC algorithm has been firstly presented to solve the optimization problems with continuous solutions space. If the solution space of the problem is binary-structural, the algorithm must be modified to solve this kind of the problems. Therefore, in this study, the BB-BC method, one of the population-based optimization algorithms, is modified to deal with binary optimization problems. The performance of the proposed methods is analyzed on uncapacitated facility location problems (UFLPs) which are one of the binary problems used in literature. The well-known small and medium twelve instances of UFLPs are used to analyze the performances and the effects of the control parameter of the BB-BC algorithm. The obtained results are comparatively presented. According to the experimental results, the binary version of the BB-BC method achieves successful results in solving UFLP in terms of solution quality.

Keywords: *Big Bang-Big Crunch Algorithm, Population-based optimization algorithms, Binary optimization, UFLP, Modulo function.*

1. Introduction

Many swarm intelligence methods have been recently proposed in order to solve compelling optimization problems by reason of their simple structures and creation of influential solutions for problems [1]. Heuristics algorithms are the algorithms that employ a simple approach to produce an acceptable solution to search and are recently becoming powerful and getting more common. The reasons of that can be shortly given as follows:

In case of different decision variables, objective functions and constraints, they provide strategies of the general solution that can be practiced to the problem.

They independently operate from the type of solution space, the number of decision variables and constraints.

They utilize probabilistic random searches. They do not need excessive computation time because their computation power is in reasonable level.

The processes of their transformation and adaptation for different type of problems are simple.

They dictate fewer mathematical requirements and in addition, they do not need very well defined mathematical models.

They present impressive solutions for the large-scale combinatorial and non-linear problems.

They do not require the assumptions as done in standard algorithm.

They do not need the change on the given problem unlike the usual algorithms. They adapt themselves for solving different types of optimization problems [2].

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In the last decades, different swarm-based evolutionary algorithms have been proposed for solving this optimization problem including Genetic Algorithms [3-6], Tabu Search algorithm [7, 8], Ant Colony Optimization [9], Particle SwarmOptimizer [10-12] and Artificial Bee Colony [13-15]. The random selection process and the information attained at the end of each iteration (cycle) are utilized in order to discover more optimal solutions in the subsequent iterations [16].

Big-Bang Big-Crunch (BB-BC) algorithm which is one of the swarm intelligence algorithms has been proposed by Erol and Eksin in 2006 for numerical optimization problems and was based on the big bang and big crunch theory, one of the theories of the evolution of the universe [17]. While in the Big Bang phase, the BB–BC method similarly produces haphazard points in solution space, in the Big Crunch phase it shrinks all of the points in the search space to a single agent point due to a centre of mass. It has shown that The BB–BC method outperformed the enhanced classical Genetic Algorithm on many benchmark problems.

According to the literature review, the basic BB-BC algorithm is a competitive algorithm in solving optimization problems with continuous solution space. If the solution space of the problem is binary-structured, the basic BB-BC algorithm must be modified in order to solve this type of optimization problems. Using modulo function that is one of the main mathematical operators, we propose a binary version of the BB-BC method for obtaining the reasonable solutions for binary optimization problems. The proposed method is investigated on a standard binary optimization problem by utilizing the uncapacitated facility location problem (UFLP). The UFLP is one of the most commonly used problems in combinatorial optimization. In this problem, the main objective is to minimize the total cost by providing the demand of customers under the given conditions that are a constant cost of setting up a facility and a shipping cost of satisfying the customer demand for the corresponding facility [10].

2. Basic Version Of Big Bang Big CrunchAlgorithm (BB-BC)

The BB-BC algorithm was firstly introduced for solving continuous optimization problems in 2006 by Erol and Eksin. This method is constructed on two main steps: the first phase is the Big Bang phase in which all of the candidate solutions are randomly distributed into the search space and the next phase is the Big Crunch where a centre of mass is calculated considering individuals in the whole population [18, 19]. The initial population is randomly produced over the search space as done in the other swarm-based algorithms. All subsequent Big Bang phases are randomly distributed about the centre of mass or the candidate with the best fitness value in a similar way. The most important feature of the algorithm is that it has high convergence speed but low computational complexity. For instance, while many evolution-based algorithms in the literature present near-optimal solutions at the end of too much iteration, BB-BC algorithm obtains solutions very close to the optimal solution of this problem on far less number of iterations in general.

After the Big Bang, a contraction procedure is implemented during the Big Crunch. In this stage, the contraction operator holds the available locations of each candidate solution in the population and its associated cost function value and calculates a centre of mass. The centre of mass is presented as follow:

$$X^c = \frac{\sum_{i=1}^{N} \frac{1}{f^i} X_i}{\sum_{i=1}^{N} \frac{1}{f^i}} \quad (1)$$

Where Xc = position of the centre of mass; Xi = position of candidate solution; fi = value of cost function of candidate i; and N = population size. The best fittest candidate solution can also be utilized as the starting point instead of the position of the centre of mass.

After the second stage completes, new individuals are once again calculated for Big Bang stage according to the formula below.

$$X^{new} = X^c + \frac{lr}{k} \quad (2)$$

where Xc, l, r and k are respectively the centroid of mass, upper bounds of parameters, random parameter and number of iteration. The value Xnew is calculated according to the following formula.

$$X^{new} = \frac{\sum_{i=1}^{N} \frac{1}{f^i} X^i}{\sum_{i=1}^{N} \frac{1}{f^i}} + \frac{lr}{k} \quad (3)$$

The steps of the BB-BC method are presented in Figure. 1.

	Description
Initialization Stage	To initialize algorithm, the size of population, number of iteration, fitness function and error value are determined.
Step 1:	A population which consists of N individuals in search space is randomly generated.
Step 2:	The values of all the candidate solutions (individuals) are calculated by using fitness function.
Step 3:	The center of mass or the fittest individual is determined as Big Bang point by the help of Eq. (1).
Step 4:	A new population is generated about the center of mass or the best-fit individual.
Step 5:	Go to Step 2 until the stopping criterion (number of iteration or error value) is met.

Figure 1. The steps of the BB-BC method

3. Proposed Binary Version of BB-BC

Modulo base 2 is used to convert the continuous solutions to binary version. This conversion is presented in Eq.4.

$$BS_{i,j} = mod(abs(\lfloor X_{i,j} \rfloor), 2) \quad (4)$$

Where, $BS_{i,j}$ is binary solution obtained from $X_{i,j}$, $\lfloor \ \rfloor$ is rounding operation to down, abs is a function used in order to obtain absolute value of $X_{i,j}$. When $BS_{i,j}$ is computed, first of all, rounding operator is applied to$X_{i,j}$, then absolute value of $X_{i,j}$ is obtained. As for last process, the modulo base two is applied to the obtained value.

$BS_{i,j}$= round ($|X_{i,j}|$) mod 2

An example of the solving Eq. (4) is presented as follows:

$BS_{i,j}$= round ($|-12.24|$) mod 2
$BS_{i,j}$ = round (12.24) mod 2
$BS_{i,j}$ =12 mod 2
$BS_{i,j}$ = 0

4. Uncapacitated Facility Location Problem

In the basic formulation, UFLP consists of a set of customer location m that must be served, and a set of potential facilities n in which at least a facility must be opened and has not any capacity limitation. The main purpose (Eq. (5)) is to find a subset f of n facilities that is supplied request of customers m. The objective function of the problem is to minimize sum of the transport costs. The mathematical formulation of the UFLP can be expressed as follows:

$$f(\text{UFLP}) = \min\left(\sum_{i=1}^{m} \sum_{j=1}^{n} c_{i,j} x_{i,j} + \sum_{j=1}^{n} fc_j y_j\right) \quad (5)$$

subject to :

$$\sum_{j=1}^{n} x_{i,j} = 1 \qquad \forall i \ in \ m \quad (6)$$

$$0 \le x_{i,j} \le 1 \quad (7)$$

where i = 1...m; j = 1,...,n; xij represents the quantity supplied from facility i to customer j; yj expresses whether facility j is located (yj= 1); otherwise (yj= 0). The constraint in Eq. 6 ensures that demands of all customers must be satisfied by an open facility. The constraint in Eq. 7 provides the collectivity, as well.

The UFLP is one of the most crucial NP-hard problems in location theory [15, 20, 21]. In order to solve UFLPs, many exact methods, such as branch-and-bound [22], linear programming and Lagrangian relaxation [23], and dual approach [24], have been proposed. Despite the fact that these methods guarantee optimal solution, the computation time of these methods may be too much. For this reason, some approximate methods have been proposed in order to solve UFLPs. These methods cannot ensure the reaching of the optimal solution, but they can attain optimum or near-optimum solutions in a reasonable time.

5. Experimental Results

The uncapacitated facility location test suite (12 test problems) obtained from the OR-Library was used in order to examine the performance and accuracy of the proposed binary versions of the BB-BC algorithm [25]. In the test suite, four problems (Cap71-74) are small-sized, the four problems are medium-sized (Cap101-104) the remaining four problems are large-sized problems (Cap131-134) and the sizes and the costs of the optimal solutions for the problems are given in Table 1.

Table 1. Description of the test suite

Problem name	Problem size	Cost of the optimal solution
Cap71	16x50	932,615.75
Cap72	16x50	977,799.40
Cap73	16x50	1,010,641.45
Cap74	16x50	1,034,976.98
Cap101	25x50	796,648.44
Cap102	25x50	854,704.20
Cap103	25x50	893,782.11
Cap104	25x50	928,941.75
Cap131	50x50	793,439.56
Cap132	50x50	851,495.33
Cap133	50x50	893,076.71
Cap134	50x50	928,941.75

In order to make a clear and consistent comparison each other, the population size is taken as 40. For the entire experiments, the termination condition is selected as the maximum number of function evaluations (Max_FEs) and it is set as 80.000 and each experimental study performed by using center of mass and best fit individual instead of Eq. 1 and different upper limit (l), which is one of the control parameter of the BB-BC, is run 30 times in order to solve UFLP. The mean, best and worst values and the standard deviations obtained by the runs are reported in Tables 2-7. The accuracy and robustness values of the methods are compared in terms of the mean and standard deviation, respectively.

In Table 2, the experimental results obtained using center of mass function for value l=1 is presented. The mean (Mean), standard deviation (Std), best value (Best) and worst values (Worst) obtained at the end of 30 times run are given for each problem. When analyzed the mean values, any success could not be gained under these parameters. However, when the best values are examined, the optimal results were obtained for small-sized and medium-sized problem groups but not large-sized problems. Besides, when looked at Table 2 and the other tables, it is seen that as the size of the problem increased, the average deviation from the optimal solution increased.

Table 2. Experimental results obtained using center function for L= 1

	Mean	Std	Best	Worst
Cap71	934777,784	2450,409	932615,750	940250,100
Cap72	981265,362	2794,858	977799,400	986482,638
Cap73	1012479,872	1927,757	1010641,450	1017426,175
Cap74	1040091,967	6159,926	1034976,975	1054295,825
Cap101	802615,243	4316,150	796648,438	810221,750
Cap102	860936,509	6430,058	854704,200	876063,950
Cap103	901336,552	7078,613	893782,113	914089,925
Cap104	949409,698	14208,519	928941,750	982101,063
Cap131	825390,386	19827,042	794159,350	850878,038
Cap132	907449,515	24607,298	851495,325	933883,325
Cap133	977059,899	17914,583	934917,063	1012152,913
Cap134	1047398,447	30954,550	982799,250	1097979,288

The reporting results obtained using center of mass function for value l=2 are given in Table 3. When examined the mean values, it could not be reached to the optimal value for any problem under the given parameters. However, when the best values are examined, the optimal results could be found for small and medium-sized problem groups except for large-sized problems. In addition, while the standard deviation values obtained from small-sized problems got lower, those of medium-sized and large-sized problem sets got higher. Therefore, it can be said that this method is more robust for small-sized problems than the other problem groups.

Table 3. Experimental results obtained using center function for L= 2

	Mean	Std	Best	Worst
Cap71	934166,882	1993,341	932615,750	939177,513
Cap72	979944,208	2584,446	977799,400	984829,450
Cap73	1012865,261	1802,325	1010641,450	1015589,325
Cap74	1040141,943	5846,441	1034976,975	1053912,938
Cap101	804429,968	5385,079	796648,438	812834,288
Cap102	863982,102	6550,753	854704,200	875894,200
Cap103	904765,695	8869,370	893782,113	917587,900
Cap104	952661,198	17286,486	928941,750	976991,075
Cap131	821229,431	18868,929	795883,238	851320,750
Cap132	912322,125	21859,908	854704,200	938541,638
Cap133	980653,598	16274,168	948982,750	1006973,775
Cap134	1056551,117	25561,281	1005104,075	1105009,113

The experimental results attained utilizing center of mass function for value l=3 are given in Table 4. When examined the mean values, the optimal value could not be found for any problem under the available parameters. However, when examined the obtained best values, it is seen that this method obtained optimal results for small-sized and medium-sized problems except for large-sized problems. In addition, while the standard deviation values obtained from small-sized problems got lower, those of large-sized problems groups got higher. Therefore, it can be said that this method is relatively more effective for small-sized problems rather than large-sized problems.

Table 4. Experimental results obtained using center function for L= 3

	Mean	Std	Best	Worst
Cap71	934130,279	2157,679	932615,750	939626,575
Cap72	981160,794	3084,162	977799,400	987144,550
Cap73	1012575,554	2165,025	1010641,450	1017544,888
Cap74	1041407,615	5023,067	1034976,975	1048963,413
Cap101	801790,383	4102,594	796648,438	810451,925
Cap102	863112,714	6553,889	854704,200	874035,350
Cap103	902364,675	9815,888	893782,113	922215,888
Cap104	947196,955	18341,711	928941,750	987505,475
Cap131	823123,130	18485,441	799291,000	852911,088
Cap132	910805,458	21907,858	859028,188	943963,650
Cap133	972870,255	15372,560	942524,650	1009722,438
Cap134	1055681,507	26056,078	989378,975	1103422,350

The reporting results obtained utilizing best fit individual instead of center of mass function for value l=1 are given in Table 5. When investigated the mean values under these conditions, any optimal value for any problem could not be found. However, when analyzed the best values, it is seen that the proposed method reached to the optimal solution for small-sized and medium-sized problem groups but not large-sized problems. Besides, when analyzed the mean values in this table while this method obtained very near-optimal results for both of small-sized and medium-sized problems, it did not obtain good solution for large-sized problems. Therefore, it can be specified that this method is relatively more effective for small-sized and medium-large problem sets than large-sized problems.

Table 5. Experimental results obtained using best fit individual for L= 1

	Mean	Std	Best	Worst
Cap71	933189,717	1389,929	932615,750	938122,238
Cap72	978581,522	1488,624	977799,400	982711,600
Cap73	1011318,064	1011,356	1010641,450	1014253,438
Cap74	1036464,014	3242,674	1034976,975	1052187,150
Cap101	798149,257	1574,117	796648,438	801947,025
Cap102	858319,158	2494,585	854704,200	862792,900
Cap103	897041,810	3883,140	894008,138	911215,038
Cap104	934136,345	5617,622	928941,750	955362,300
Cap131	807064,141	8882,906	794956,113	835703,763
Cap132	872173,273	10397,055	857605,800	896433,588
Cap133	924420,926	18208,083	896522,713	962142,413
Cap134	996889,000	31413,774	942813,938	1058213,813

The final results obtained using best fit individual instead of center of mass function for value l=2 are given in Table 6. When examined the mean values under these conditions, it could not be reached to any optimal result for any problem. However, when examined the best values, it is seen that this method reached to the optimal values for small-sized and medium-sized problem groups but not large-sized problems. Besides, when looked at the mean values while this method obtained very near-optimal results for small-sized and medium-sized problem groups, it did not obtain any sufficient solution for large-sized problems. Therefore, for small-sized and medium-sized problems, it can be stated that this method is relatively more competitive.

Table 6. Experimental results obtained using best fit individual for L= 2

	Mean	Std	Best	Worst
Cap71	933031,378	1141,122	932615,750	937364,400
Cap72	978434,816	1436,766	977799,400	982711,600
Cap73	1011559,270	1342,588	1010641,450	1014491,400
Cap74	1036038,932	1587,518	1034976,975	1039926,175
Cap101	798550,012	1842,539	796648,438	804211,238
Cap102	857279,872	2702,962	854704,200	865405,538
Cap103	896504,491	2412,846	893782,113	901580,388
Cap104	935326,600	6149,832	928941,750	951622,050
Cap131	808708,475	8078,468	797635,288	833514,275
Cap132	874615,404	15692,184	851670,125	903862,650
Cap133	928468,949	20240,792	895642,513	975990,875
Cap134	994036,220	40501,828	933520,538	1064880,575

The experimental results acquired utilizing best fit individual instead of center of mass function for value l=3 are given in Table 7. When examined the mean values, this method could not reached to any optimal result for any problem. However, when looked at the best values, it is observed that this method reached to the optimal values for only small-sized and medium-sized problem groups. Besides, when analyzed the mean values while this method obtained very near-optimal results for small-sized and medium-sized problem groups, it did not acquire any good result for large-sized problem set. Therefore, it can be specified that this method is relatively more competitive for only small-sized and medium-sized problem sets.

Table 7. Experimental results obtained using best fit individual for L= 3

	Mean	Std	Best	Worst
Cap71	933305,228	2119,100	932615,750	943673,600
Cap72	979253,239	2580,129	977799,400	987260,213
Cap73	1011145,130	820,514	1010641,450	1012476,975
Cap74	1035707,668	1232,432	1034976,975	1037717,075
Cap101	799100,121	2387,477	796648,438	804580,525
Cap102	857423,656	1985,558	854704,200	862628,738
Cap103	896656,990	2083,759	893782,113	899596,575
Cap104	936534,911	7580,000	928941,750	952557,550
Cap131	805663,750	6018,338	794956,113	819209,238
Cap132	876533,484	14451,319	856417,000	904914,675
Cap133	926388,290	18910,464	894095,763	977148,600
Cap134	990694,024	27565,510	941303,788	1052871,688

5.1. Comparative Analysis between Centre of Mass and Best Fit Individual

To fulfill a fair comparison, the experimental studies have been conducted under the equivalent parameters. In the process of comparison, Table 2, 3 and 4 are compared to Table 5, 6 and 7, respectively. In other words, to make a comparison between center of mass and best fit individual, the results of the corresponding problems have been compared under the same l values. As seen from Tables 2 - 7, when the proposed BB-BCcenter method with Center of Mass is compared with BB-BCbest method with Best Fit Individual, BB-BCbest method is quite better in terms of both of solution quality and robustness rather than BB-BCcenter method, in general. In addition, BB-BCbest method reached the very near-optimal solutions for the corresponding problems in all of the different l values. Based on the standard deviations, the robustness of the BB-BCbest method is better than that of the BB-BCcenter method for all of the problems. When l values were set as one, two and three, the similar results were obtained. Therefore, for the entire problem groups (small-sized, medium-sized and large-sized problems), it is observed that utilizing of different l values does not affect the success of the algorithm.

6. Conclusions

In this paper, we studied the modification of the BB-BC method for solving binary optimization problems. Modula-2 based binary version of the BB-BC algorithm has been proposed and their performance has been investigated on uncapacitated facility location problems containing small and medium sized problem. The performance of the proposed methods is analyzed under the different l values which are upper value of the BB-BC method. In addition, the experimental results have been individually conducted using both of center of mass and best fit individual instead of center of mass and in view of these results, the comparative analysis between the BB-BCcenter and BB-BCbest methods has been also carried out in this study. In experimental results, it is shown that promising results are obtained by this proposed binary method. The results show that the BB-BC method proposed for binary optimization can be used for solving other binary optimization problems.

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