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New Solution Approaches for Multi-Objective Solid Transportation Problem Using Some Aggregation Operators

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ABSTRACT: A solid transportation problem emerges when the decision variables are represented by three items: the source, the destination, and the mode of transport. In applications, the STP generally requires considering multiple objectives such as cost minimization, time minimization, security level maximization, etc. In this way, a multi-objective solid transportation problem arises. This paper deals with the solution of the problem and analyzes the effect of several important fuzzy aggregation operators on the solution of the problem. In this context, the most commonly used aggregation operators are investigated for this problem. To explain the solution approach, a numerical example from the literature is given and a Pareto-optimal solution set is provided to offer the decision-maker. Furthermore, graphical comparisons and sensitivity analysis are presented with the solution obtained.

Keywords: Multi-objective optimization, fuzzy aggregation operator, solid transportation problem

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INTRODUCTION

In this global world where competition has become very important, companies must find more detailed and innovative solutions to their problems, reduce transportation costs, and choose environmentally friendly transportation methods. In this context, it would be more useful to deal with the solid transportation problem (STP), which is usually obtained by adding a third item, rather than the transportation problem involving only supply and demand parameters. This third element can be types of goods as well as modes of transport, e.g., roadways, railways, waterways, and airways.

The multi-objective solid transportation problem (MSTP) problem has been addressed in the literature by using a multi-objective structure to model STP more realistically. For MSTP, Ahlatcioglu and Sivri (1988) proposed an efficient solution method by using decomposing techniques to reduce the dimension.

One of the most important research topics for STPs is the use of fuzzy set theory. This theory was first introduced by Zadeh (1965). In a lot of applications, the required data for real-life problems may be imprecise. Thus, an adaptation of fuzzy set theory in the solution method increases the flexibility and effectiveness of the proposed approaches. This theory has been used for the development of the applications of solid transportation.

Bit et al. (1993); Kundu et al. (2014) applied the traditional fuzzy programming approach to MSTP and MSTP with budget constraints, considering the transportation of damageable items, Pramanik et al. (2013); Ojha et al. (2010) presented new models with the help of the price discount, and an integrated method based on stochastic programming and AHP. Kaur et al. (2015) developed fuzzy programming approaches to a real-life transportation problem by using linear, exponential, and hyperbolic membership functions and analyzing uncertainties of the parameters. Chen et al. (2017) discussed two mathematical models for the bi-objective uncertain STP. STP having normal random parameters was provided by Cui and Sheng (2012). Chen et al. (2017) analyzed the MSTP under the uncertainty theory providing a coal transportation problem as an application. After converting an uncertain STP into a crisp one, Dalman and Sivri (2017) applied some approaches from the literature. Singh et al. (2019) developed a generalized model for MSTP with some random parameters. Anuradha et al. (2019) have utilized a row maxima procedure to solve the bi-objective STP.

Many important studies (Khurana and Adlakha, 2015; Mollanoori et al., 2019; Khurana et al., 2018; Sadore and Tuli, 2019) have also been made on the multi-index transportation problem which is an extension of STP. In addition, some of the noteworthy studies on the minimum-cost network problem, which is formed to model non-symmetrical transportation problems, are also (Khan and Rafique, 2021; Arslan et al., 2020; Hu et al., 2020).

While the linear membership function is widely applied in the literature, it is seen that nonlinear membership functions are utilized in some practical applications. Leberling (1981) solved the multi-objective linear programming problem by a fuzzy method with a nonlinear membership function. Li and Lee (1991) defined fuzzy multiple objective linear programming and solved it using the exponential membership function. A fuzzy multi-objective problem was solved using exponential membership functions in (Rath and Dash, 2017). Peidro and Vasant (2011) applied the fuzzy goal programming approach with some nonlinear membership functions to solve multi-objective transportation problems (MTP). Verma et al. (1997) proposed the fuzzy method using some non-linear membership functions to solve an MTP. Bodkhe et al. (2010) applied to fuzzy method to solve MSTP using nonlinear membership functions.

There are many studies using different fuzzy aggregation operators such as “min”, “average” operator. Oiyas et al. (2022) defined fractional orthotriple fuzzy rough sets and presented Hamacher averaging and geometric operators and used these operators in applications on service quality of wireless network selection.Memis et al. (2022) applied a soft decision-making method based on the aggregation operator of fuzzy parameterized fuzzy soft matrices to a decision matrix and classified the given test sample. Mahmood and Ali (2022) presented study to start the complex single-valued neutrosophic settings with the help of Muirhead mean operator.

In this paper, we mainly focus on the MSTP and present new solution approaches by integrating the fuzzy programming approach with some important fuzzy aggregation operators: Fuzzy AND, Fuzzy OR, modified Zimmermann’s, augmented max-min, and the hybrid of μ_{AND} and augmented max-min. These operators have advantages and disadvantages. For example, while the solution obtained by “min” operator does not guarantee compensatory and Pareto optimality, it is easy computation. On the other hand, μ_{AND} operator does guarantee compensatory and Pareto optimality. Also, these operators are very important for multi-objective decision making problem. For these reasons, it is possible to interpret the solutions generated by the operators. Therefore, a suitable preferred solution can be chosen from the set of solutions obtained and their membership function values by the operators and then presented to the decision maker. This is a great advantage for the decision maker to choose optimistic, pessimistic, or risk-neutral behaviors.

The paper is structured as follows: The Next section introduces the model of MSTP. After Section 3 provides the fuzzy approaches to MSTP using fuzzy aggregation operators, Section 4 presents a numerical example to present a comparison of the operators. The final section includes some results.

MATERIALS AND METHODS

The Mathematical Formulation of MSTP

Let the MSTP with m supplies, n demands, and K conveyances have the capacities a_i, b_j, and e_k, respectively. And assume that the cost of $p-$th objective function $Z_p(x)$ is denoted by c_{ijk}^p which corresponds to x_{ijk}. Then, an MSTP can be represented as:

$$\min Z_p(x) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{K} c_{ijk}^p x_{ijk} , \qquad p = 1, 2, \ldots, P \tag{1}$$

$$\text{s.t.} \quad \sum_{j=1}^{n}\sum_{k=1}^{K} x_{ijk} = a_i \quad , \ i = 1, 2, \ldots, m$$

$$\sum_{i=1}^{m}\sum_{k=1}^{K} x_{ijk} = b_j \quad , \ j = 1, 2, \ldots, n$$

$$\sum_{i=1}^{m}\sum_{j=1}^{n} x_{ijk} = e_k \quad , \ k = 1, 2, \ldots, K \ ; \ x_{ijk} \geq 0 .$$

where the subscript on $Z_p(x)$ and c_{ijk}^p determine the p-th objective function, $a_i > 0 \ \forall i$; $b_j > 0 \ \forall j$; $e_k > 0 \ \forall k$; $c_{ijk}^p \geq 0$, $\forall i, j, k, p$ and the balanced equality holds, that is $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{K} e_k$. Let the feasible region of MSTP given by (1) be denoted by S .

While the optimal solution concept is discussed in a single objective STP, the Pareto-optimal solution notion is used in the multi-objective framework. So, we present the following basic definitions:

Definition 1. $\mathbf{x}^* \in S$ is a strongly efficient solution iff there does not exist another feasible point $\mathbf{x}$ such that $Z_p(\mathbf{x}) \le Z_p(\mathbf{x}^*)$ for all *p* and $Z_p(\mathbf{x}) \ne Z_p(\mathbf{x}^*)$ for at least one *p*.

Definition 2. A feasible point $\mathbf{x}^*$ is a compromise solution of MSTP given in (1) iff $\mathbf{x}^*$ is a strongly efficient solution and $Z_p\left(\mathbf{x}^*\right) \le \min_{\mathbf{x}\in S}\left(Z_1(\mathbf{x}), Z_2(\mathbf{x}),..., Z_P(\mathbf{x})\right)$.

New Solution Approaches for MSTP

The first step to implement the fuzzy operators is to design the membership functions of the objectives. By the individual optimization of each objective in both minimization and maximization directions, the range of the objective functions can be calculated as follows:

$$L_p = \min_{\mathbf{x}\in S} Z_p(\mathbf{x}), \; U_p = \max_{\mathbf{x}\in S} Z_p(\mathbf{x}). \tag{2}$$

Using the lower and upper values, the linear and strictly monotone decreasing membership function can be stated as:

$$\mu_p(Z_p(\mathbf{x})) = \begin{cases} 1, & Z_p < L_p \\ \dfrac{U_p - Z_p}{U_p - L_p}, & L_p \le Z_p \le U_p \\ 0, & Z_p > U_p \end{cases} \tag{3}$$

Then, the MSTP can be converted to the following problem:

$$\max_{\mathbf{x}\in S} \min_{p} \; \mu_p(Z_p(\mathbf{x})) \tag{4}$$

The new variable $\lambda = \min_{p} \mu_p(Z_p(\mathbf{x}))$ implies the constraints $\mu_p(Z_p(\mathbf{x})) \ge \lambda$. Thus, (4) can be reduced to:

$$\begin{aligned} & \max \; \lambda \\ \text{s.t.} \quad & \mu_p(Z_p(\mathbf{x})) \ge \lambda, \quad p = 1, 2, \ldots, P \\ & 0 \le \lambda \le 1; \mathbf{x} \in S \end{aligned} \tag{5}$$

Problem (5) is the model that subtends to Zimmermann's min operator. The λ^* solution shows the common satisfaction level of all objectives. That is, the word "common" means the lowest level of satisfaction obtained for each objective of (1).

"Fuzzy and" Operator for MSTP

Although the "min" operator model is widely used because of its easy computation, it sometimes generates a weakly efficient solution (Guu and Wu, 1997; Lee and Li, 1993; Wu and Guu, 2001). Therefore, we opt for using Werners' μ_{AND} to aggregate the membership functions.

For all objectives, after satisfying its most basic satisfaction level, to promote its satisfaction degree as high as possible we can make the following arrangement:

The "min" operator corresponds to the basic satisfaction level for all objectives. To increase this basic level even higher, the auxiliary variable λ_p can be used in the following way:

$$\mu_p(Z_p(\mathbf{x})) \ge \lambda + \lambda_p$$

Here, $\lambda = \min \mu_p(Z_p(\mathbf{x}))$.

Adapting this arrangement to the model, Werners' μ_{AND} operator can be written as:

$$\mu_{AND} = \gamma \min \mu_p(Z_p) + \frac{1-\gamma}{P}\left(\mu_1(Z_1) + \mu_2(Z_2) + \ldots + \mu_P(Z_P)\right),$$

$$\mu_{AND} = \gamma\lambda + \frac{1-\gamma}{P}\left((\lambda+\lambda_1) + (\lambda+\lambda_2) + \ldots + (\lambda+\lambda_P)\right),$$

$$\mu_{AND} = \lambda + \frac{1-\gamma}{P}\left(\lambda_1 + \lambda_2 + \ldots + \lambda_P\right).$$

where $\gamma \in [0,1]$. Obviously, if γ is equal to 1, then μ_{AND} is "min" operator. However, if γ is equal to 0, then μ_{AND} is "*average*" operator.

Thus, the compensatory model for MSTP becomes:

$$\max\ \mu_{AND} = \lambda + \frac{1-\gamma}{P}\left(\lambda_1 + \lambda_2 + \ldots + \lambda_P\right) \tag{6}$$

s.t. $\mu_p(Z_p(\mathbf{x})) \geq \lambda + \lambda_p$, $p = 1,2,\ldots,P$,

$\lambda + \lambda_p \leq 1$, $p = 1,2,\ldots,P$,

$\mu_p(Z_p(\mathbf{x})) \leq 1$, $p = 1,2,\ldots,P$,

$\mathbf{x} \in S$, $0 \leq \lambda \leq 1; \lambda_p \geq 0$, $p = 1,2,\ldots,P$.

The model (6) generates a strongly efficient solution for MSTP. It is known from the literature that the μ_{AND} operator produces strongly efficient solutions. To prove this, (Tiryaki, 2006; Werners, 1988) can be examined.

"Fuzzy or" Operator for MSTP

If $\max\ \mu_p(Z_p(\mathbf{x})) = \alpha$, then $\mu_p(Z_p(\mathbf{x})) \geq \alpha$ for at least one $p \in \{1,2,\ldots,P\}$ and $\mu_p(Z_p(\mathbf{x})) \geq \alpha + \alpha_p \geq 0$ $\forall p$. Werners' compensatory *Fuzzy OR* (μ_{OR}) for MSTP is

$$\mu_{OR} = \gamma \max\ \mu_p(Z_p(\mathbf{x})) + \frac{1-\gamma}{P}(\mu_1(Z_1(\mathbf{x})) + \mu_2(Z_2(\mathbf{x})) + \ldots + \mu_p(Z_p(\mathbf{x})))$$

$$\mu_{OR} = \gamma \cdot \alpha - \frac{1-\gamma}{P}\left((\alpha-\alpha_1) + (\alpha-\alpha_2) + \ldots + (\alpha-\alpha_P)\right).$$

Using this operator, our MSTP becomes

$$\max \mu_{OR} = \alpha - \frac{1-\gamma}{P}\left(\alpha_1 + \alpha_2 + \ldots + \alpha_P\right)$$

s.t. $\mu_p(Z_p(\mathbf{x})) = \alpha - \alpha_p$, $p = 1,2,\ldots,P$,

$\mu_p(Z_p(\mathbf{x})) \geq \alpha$, for at least one $p \in \{1,2,\ldots,P\}$

$0 \leq \alpha_p \leq \alpha \leq 1$, $p = 1,2,\ldots,P$; $\mathbf{x} \in S$, $\gamma \in [0,1]$.

or

$$\max \mu_{OR} = \alpha - \frac{1-\gamma}{P}\left(\alpha_1 + \alpha_2 + \ldots + \alpha_P\right) \tag{7}$$

s.t. $\mu_p(Z_p(\mathbf{x})) \geq \alpha - \alpha_p$, $p = 1,2,\ldots,P$,

$$\alpha - \alpha_p \leq 1 \ , \quad p = 1,2,\ldots,P \ ,$$
$$\mu_p(Z_p(\mathbf{x})) + M \cdot r_p \geq \alpha \ , \ p = 1,2,\ldots,P$$
$$\sum_{r=p}^{P} r_p \leq P-1 \ ,$$
$$0 \leq \alpha_p \leq \alpha \leq 1 \ , \ p = 1,2,\ldots,P$$
$$r_p \in \{0,1\} \ , \ p = 1,2,\ldots,P \ ; \mathbf{x} \in S \ , \ \gamma \in [0,1] \ .$$

Here, the objective function of problem (7) maximizes the linear combination of "max operator" and "average operator" in the solid transportation system.

Modified Zimmermann's Operator for MSTP

The modified operator of Zimmermann's can be written as follows:

$$\mu_D = \gamma \min_p \ \mu_p(Z_p(\mathbf{x})) + (1-\gamma) \max_p \ \mu_p(Z_p(\mathbf{x})) \ .$$

As can be seen, this operator is a convex combination of the min- and max-operators. With this operator, our MSTP becomes

$$\max_{\mathbf{x} \in S} \left\{ \gamma \min_p \ \mu_p(Z_p(\mathbf{x})) + (1-\gamma) \max_p \ \mu_p(Z_p(\mathbf{x})) \right\}$$

or

$$\max \left\{ \gamma \cdot \alpha_1 + (1-\gamma) \cdot \alpha_2 \right\}$$

s.t. $\mu_p(Z_p(\mathbf{x})) \geq \alpha_1 \ , \ p = 1,2,\ldots,P$

$\mu_p(Z_p(\mathbf{x})) \geq \alpha_2$, for at least one $p \in \{1,2,\ldots,P\}$

$\mathbf{x} \in S \ , \ \alpha_1, \alpha_2 \in [0,1]$.

Choosing a sufficiently large real number M , the model can be converted to the following:

$$\max \left\{ \gamma \cdot \alpha_1 + (1-\gamma) \cdot \alpha_2 \right\} \tag{8}$$

s.t. $\mu_p(Z_p(\mathbf{x})) \geq \alpha_1 \ , \ p = 1,2,\ldots,P \ ,$

$\mu_p(Z_p(\mathbf{x})) + M \cdot r_p \geq \alpha_2 \ , \ p = 1,2,\ldots,P$

$\sum_{r=p}^{P} r_p \leq P-1 \ ,$

$\mathbf{x} \in S \ ; \alpha_1, \alpha_2 \in [0,1] ; \ r_p \in \{0,1\} \ , \quad p = 1,2,\ldots,P \ .$

Lai and Hwang's Augmented Max-min operator for MSTP

This augmented max-min operator is defined as:

$$\mu_D = \min_p \ \mu_p(Z_p(\mathbf{x})) + \delta \cdot \sum_p \mu_p(Z_p(\mathbf{x})) \ .$$

Taking $\min_p \ \mu_p(Z_p(\mathbf{x})) = \lambda$, our MSTP becomes

$$\max \mu_D = \max \left\{ \lambda + \delta \cdot \sum_p \mu_p(Z_p(\mathbf{x})) \right\} \tag{9}$$

s.t. $\mu_p(Z_p(\mathbf{x})) \geq \lambda \ , \ p \in \{1,2,\ldots,P\} \ ,$

$\mathbf{x} \in S \ .$

A Hybrid Operator of "Fuzzy and" and Augmented Max-min for MSTP

If $\min_p \mu_p(Z_p(\mathbf{x})) = \lambda$, then the membership functions can be easily defined as $\mu_p(Z_p(\mathbf{x})) = \lambda + \lambda_p$. Considering this notation with the augmented operator presented in Section 3.5, our MSTP is reduced to

$$\max \left\{ (1+\delta)\cdot\lambda + \delta\sum_{p=1}^{P}\lambda_p \right\} \tag{10}$$

s.t. $\mu_p(Z_p(\mathbf{x})) \geq \lambda + \lambda_p \quad \forall p = 1,\ldots,P,$

$\lambda + \lambda_p \leq 1;\ \forall p = 1,\ldots,P, \mathbf{x} \in S.$

RESULTS AND DISCUSSION

In this section, the multi-objective balanced solid transportation problem from the literature have been solved applying the proposed solution approach. First of all, membership functions for objectives have been constructed. Then, the problem has been solved by using fuzzy approach with the help of various aggregation operators. In the last part, the solutions obtained using different operators have been compared to sensitivity analysis and drawn on graph.

A Numerical Example

Consider the MSTP in (Bit et al., 1993) with the following characteristics:

Supplies: $a_1 = 24,\ a_2 = 8,\ a_3 = 18,\ a_4 = 10;$

Demands: $b_1 = 11,\ b_2 = 19,\ b_3 = 21,\ b_4 = 9;$

Conveyances capacities: $a_1 = 24,\ a_2 = 8,\ a_3 = 18,\ a_4 = 10;$

Penalties of the first objective: c_{ijk}^1

	1	2	3	4
1	15	12	10	8
2	17	21	21	30
3	14	25	20	21
4	22	24	18	13
Conveyance 1				

	1	2	3	4
1	18	22	4	11
2	20	21	19	10
3	11	34	16	23
4	18	35	21	23
Conveyance 2				

	1	2	3	4
1	17	13	12	13
2	19	22	18	23
3	12	33	15	22
4	13	32	14	20
Conveyance 3				

Penalties of the second objective: c_{ijk}^2

	1	2	3	4
1	6	10	11	10
2	13	12	20	17
3	5	11	10	15
4	13	17	12	18
Conveyance 1				

	1	2	3	4
1	7	6	3	9
2	8	2	15	15
3	6	9	5	14
4	6	11	16	14
Conveyance 2				

	1	2	3	4
1	8	5	7	6
2	11	9	13	13
3	7	7	2	18
4	6	18	12	7
Conveyance 3				

Let the feasible region of the example is determined by the following constraints:

$$\sum_{j=1}^{4}\sum_{k=1}^{3} x_{1jk}=24\ ,\ \sum_{j=1}^{4}\sum_{k=1}^{3} x_{2jk}=8\ ,\ \sum_{j=1}^{4}\sum_{k=1}^{3} x_{3jk}=18\ ,\ \sum_{j=1}^{4}\sum_{k=1}^{3} x_{4jk}=10\ ,$$

$$\sum_{i=1}^{4}\sum_{k=1}^{3} x_{i1k}=11,\ \sum_{i=1}^{4}\sum_{k=1}^{3} x_{i2k}=19,\ \sum_{i=1}^{4}\sum_{k=1}^{3} x_{i3k}=21,\ \sum_{i=1}^{4}\sum_{k=1}^{3} x_{i4k}=9,$$

$$\sum_{i=1}^{4}\sum_{j=1}^{4} x_{ij1}=17\ ,\ \sum_{i=1}^{4}\sum_{j=1}^{4} x_{ij2}=31,\ \sum_{i=1}^{4}\sum_{j=1}^{4} x_{ij3}=12\ ,$$

and denoted by $\hat{S}$.

With the individual minimization of the objectives, the lower and upper bounds of the objectives are obtained as: $L_1=703$, $U_1=877$ $L_2=293$, $U_2=537$. Then, the membership functions are: $\mu_1(x)=\dfrac{877-Z_1(x)}{174}$, $\mu_2(x)=\dfrac{537-Z_2(x)}{244}$. The fuzzy model corresponds to (5) generates the following solution $X_1: X_1=\left\{\begin{matrix} x_{132}=13.96, x_{121}=10.04, x_{222}=8, x_{312}=9.04, x_{311}=1.92, x_{333}=7.04, \\ x_{413}=0.04, x_{421}=0.96, x_{441}=4.08, x_{443}=4.92 \end{matrix}\right\}$ with $\lambda=0.2$, $Z_1=751.24$, $\mu_1=0.72$, $Z_2=360.64$, $\mu_2=0.72$.

Using (6), the Fuzzy AND model of the example is:

$$\max\ \mu_{AND}=\lambda+\frac{1-\gamma}{2}\left(\lambda_1+\lambda_2\right) \tag{11}$$

s.t. $\mathbf{x}\in\hat{S}$

$\mu_p(Z_p(\mathbf{x}))\geq\lambda+\lambda_p,\ p=1,2$

$\lambda+\lambda_p\leq 1,\ p=1,2$

$x_{ijk}\geq 0,\ i=1,2,3,4;\ j=1,2,3,4;\ k=1,2,3;\ \lambda,\lambda_p\geq 0;\ p=1,2$.

The solutions to the problem (11) corresponding to the γ values in 0.1 increments in the interval [0,1] are given in Table 1. So, the "fuzzy and" operator generates the following three different solutions: X_2, X_3, X_4 where $X_2=\left\{\begin{matrix} x_{121}=8, x_{123}=3, x_{132}=13, x_{222}=8, x_{312}=10, x_{333}=8, \\ x_{413}=1,\ x_{441}=9 \end{matrix}\right\}$,

$X_3=\left\{\begin{matrix} x_{121}=11, x_{132}=13,\ x_{222}=8, x_{312}=10, x_{333}=8, \\ x_{413}=1,\ x_{441}=6, x_{443}=3 \end{matrix}\right\}$ and $X_4=X_1$.

Table1. The solutions generated by the Fuzzy AND operator.

	$\gamma=0$ X_2	$\gamma=0.1$ X_2	$\gamma=0.2$ X_3	$\gamma=0.3-\gamma=1$ X_4
μ_1	0.93	0.93	0.83	0.72
μ_2	0.59	0.59	0.66	0.72
z_1	715	715	733	751.24
z_2	394	394	376	360.64

Using (7), the Fuzzy OR model of the example is:

$$\max\ \mu_{OR}=\alpha-\frac{1-\gamma}{2}\left(\alpha_1+\alpha_2\right) \tag{12}$$

s.t. $\mu_p(Z_p(\mathbf{x}))\geq\alpha-\alpha_p,\ p=1,2$,

$$\alpha - \alpha_p \le 1 \ , \quad p = 1,2 \ ,$$
$$\mu_p(Z_p(\mathbf{x})) + M \cdot r_p \ge \alpha \ , \ p = 1,2$$
$$r_1 + r_2 \le 1, 0 \le \alpha_p \le \alpha \le 1, \ p = 1,2$$
$$r_p \in \{0,1\}, \ p = 1,2 \,; \mathbf{x} \in \hat{S} \ , \ \gamma \in [0,1] \ .$$

The solutions of the problem (12) corresponding to the γ values in 0.1 increments in the interval [0,1] are given in Table 2. So, the Fuzzy OR operator generates the following seven different solutions: $X_5 = X_1, X_6 = X_2$ and

$X_7 = \{x_{123} = 3, x_{132} = 21, x_{222} = 8, x_{311} = 9, x_{312} = 1, x_{321} = 8, x_{412} = 1, x_{443} = 9\}$,

$X_8 = \{x_{122} = 2, x_{123} = 2, x_{132} = 20, x_{222} = 8, x_{311} = 10, x_{321} = 7, x_{333} = 1, x_{412} = 1, x_{443} = 9\}$,

$X_9 = \{x_{122} = 2, x_{123} = 2, x_{132} = 20, x_{222} = 8, x_{311} = 11, x_{321} = 6, x_{333} = 1, x_{422} = 1, x_{443} = 9\}$,

$X_{11} = \{x_{121} = 16, x_{123} = 3, x_{132} = 5, x_{242} = 8, x_{312} = 11, x_{332} = 7, x_{433} = 9, x_{441} = 1\}$.

Table 2. The solutions generated by the Fuzzy OR operator

	$\gamma = 0 - 0.1$ X_5	$\gamma = 0.2 - 0.4$ X_6	$\gamma = 0.5 - 0.6$ X_7	$\gamma = 0.7$ X_8	$\gamma = 0.8$ X_9	$\gamma = 0.9$ X_{10}	$\gamma = 1$ X_{11}
μ_1	0.72	0.93	0.29	0.18	0.14	0.06	1
μ_2	0.72	0.59	0.96	0.99	0.99	1.00	0
z_1	751.24	715	826	846	852	866	703
z_2	360.64	394	302	296	295	293	537

Similarly, the aggregation model presented in Section 3.3 is constructed, and the results are given in Table 3. So, the Modified Zimmermann's operator generates the following six different solutions: $X_{12} = X_{11}, X_{13} = X_9, X_{14} = X_7, \ X_{15} = X_6 = X_2, \ X_{16} = X_3, \ X_{17} = X_1$.

Table 3. The solutions generated by the Modified Zimmermann's operator

	$\gamma = 0$ X_{12}	$\gamma = 0.1$ X_{13}	$\gamma = 0.2$ X_{14}	$\gamma = 0.3 - 0.5$ X_{15}	$\gamma = 0.6$ X_{16}	$\gamma = 0.7 - 1.0$ X_{17}
μ_1	1	0.14	0.29	0.93	0.83	0.72
μ_2	0	0.99	0.96	0.59	0.66	0.72
z_1	703	852	826	715	733	751.24
z_2	537	295	302	394	376	360.64

For the operators presented in Sections 3.4 and 3.5, Lai and Hwang's augmented max-min operator, the parameter δ is chosen as 10^{-1} similar to the literature. With this assumption, both operators generate the solution X_1 .

Comparison and Sensitivity Analysis

In (Bit et al., 1993), the solution of the example is reported as follows:

$x_{121} = 10.243, \ x_{132} = 13.857, \ x_{222} = 8, \ x_{311} = 1.714, \ x_{312} = 9.143, \ x_{333} = 7.143, \ x_{421} = 0.857, \ x_{419} = 0.143,$
$x_{441} = 4.286, \ x_{443} = 4.714, \ \lambda = 0.716, \ Z_1 = 749.2853, \ Z_2 = 362.2860.$

Satisfaction values of objectives for this solution are $\mu_1 = 0.7334$ and $\mu_2 = 0.7166$. As can be seen, only one Pareto-optimal solution is obtained in (Bit et al., 1993). However, in our study, using many aggregation operators, several solutions are obtained that can be presented to the decision-maker. Considering changing economic and market conditions, the decision-maker will have several solutions, and this increases the applicability of the solutions.

To show the effectiveness of the selected aggregation operators more clearly, sensitivity analyzes were made and presented in Figure1 - Figure 3.

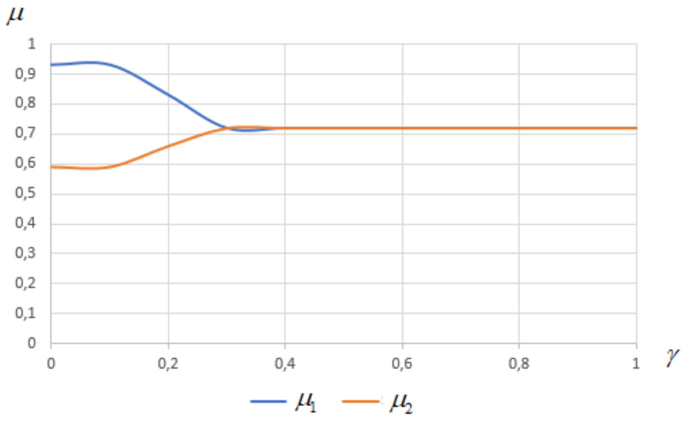

Figure 1. Sensitivity analysis of γ for μ_{AND} operator.

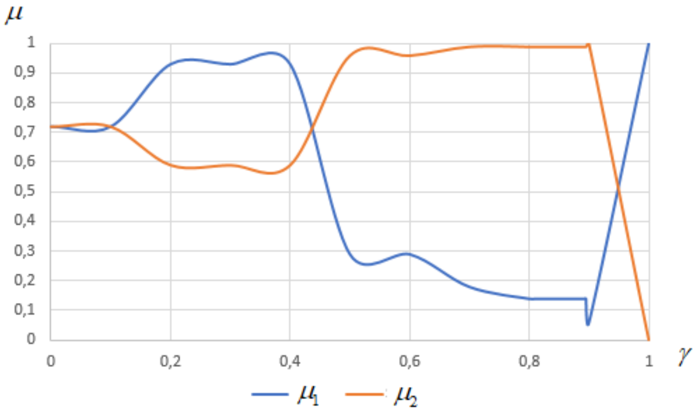

Figure 2. Sensitivity analysis of γ for μ_{OR} operator.

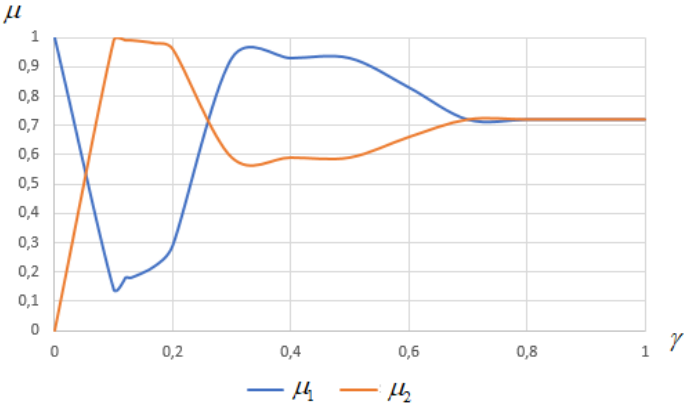

Figure 3. Sensitivity analysis of γ for Modified Zimmerman's operator.

Figure 1 shows that μ_{AND} is the decision type in which the common satisfaction level of all objectives is maximized. Since the DM wants all objectives to be satisfied at the highest possible level without ignoring any of them, the absolute difference and fluctuation between the satisfaction levels are as small as possible. Thus, the DM behaves between pessimistic and risk-neutral. In Figure 2, the difference between the satisfaction levels of the objectives is quite large, and thus the fluctuation is apparent. This indicates that the common satisfaction level is not taken into account and that a decision is made in the tendency of optimistic and risk-neutral. By the convex combination of the min and max operator, Figure 3 represents a decision between the optimistic and pessimistic decision behaviors.

In the comparative analysis of the results, we can infer from following conclusions:

- The fuzzy solution provides the solution with the highest common satisfaction for both objectives.
- For $\gamma = 1$ ("min" operator) and $\gamma = 0$ (it means "average'' operator), our compensatory μ_{AND} model generates the same solution as the solution obtained by the fuzzy approach and a different Pareto optimal solution, respectively.
- Werners' μ_{AND} operator also guarantees the least satisfaction degree for all objectives as fuzzy solution.
- Werners' μ_{OR} operator almost satisfies at least one objective, while the others are less satisfied, thus it generates the solution between the "max" operator and the "average'' operator, which matches with optimistic and risk-neutral decision behaviors, respectively.
- Werners' μ_{AND} and μ_{OR} operator generate the most a variety of optimal solution set that can be presented to the decision maker. Here, μ_{AND} or μ_{OR} operators can be selected according to the decision maker's point of view. If the DM behaves close to optimism, then the μ_{OR} operator should be selected. While he behaves close to pessimism, μ_{AND} operator should be applied.
- Modified Zimmermann's operator gives the widest set of Pareto optimal solutions as it generates all solutions between the min and the max operator thanks to the convex combination. In the numerical example discussed in the study, the modified operator parameter is taken with an increment of 0.1 and all solutions except X_8 and X_{10} are obtained. If this parameter increment is reduced to 0.05, this operator will be able to generate all solutions. In this respect, it would be appropriate to choose this operator as a decision-maker who demands many solutions according to all kinds of risk behaviors.
- The aggregation operators presented in Modified Zimmermann's - A Hybrid Operator of Fuzzy AND and Augmented Max-min operator give the same solutions as the remaining operators. In this context, applying these operators will not make a significant difference.

Aggregation operators given above present a large number of alternative solutions in paper. Thus, these obtained solutions offered many alternative options to the decision maker. However, the min operator gave the largest set of solutions obtained from all operators. In this context, applying these operators have not made a significant difference.

CONCLUSION

First of all, the problem was solved with the fuzzy approach by forming membership functions for each objective due to its widespread use. For the optimization of the multi-objective structure, membership functions are combined by using Werner's μ_{AND}, Werner's μ_{OR}, the Modified Zimmermann's operator, Lai, and Hwang's augmented max-min operator, and the hybrid operator of Werners' and Lai-Hwang's operators. Thus, new approaches have been developed to provide various solutions for MSTP. Considering that the transportation problem and even its three-dimensional version have many applications, the utility of the proposed approaches emerges. As a result of the proposed approaches and sensitivity analyzes, we concluded that the μ_{OR} and μ_{AND} operators can be used by optimistic and pessimistic decision-makers, respectively. Thus, a convenient solution can be selected from the set generated by the operators μ_{AND} and μ_{OR} based on the risk attitude of the decision-maker. If the decision-maker is indecisive about his/her risk attitude or demands as many possible solutions as possible, the modified Zimmermann's operator can be selected since it generates all solutions from the min to the max operator, which matches with pessimistic and optimistic decisions. The Lai and Hwang's augmented max-min operator, and the hybrid operator of Werners' μ_{AND} and Lai-Hwang's operator did not produce different solutions. In future studies, fully fuzzy/interval/fuzzy number forms of MSTP and/or associated with different sorts of membership functions and/or multi-item forms can be considered.

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Conflict of Interest

The article authors declare that there is no conflict of interest between them.

Author's Contributions

The authors declare that they have contributed equally to the article.

REFERENCES

Ahlatçıoğlu M, Sivri M, 1988. A Solution Proposal for Solving Three-Dimension Transportation Problem. İ.T.Ü, Istanbul-Turkey, 22: 9-12.

Anuradha D, Jayalakshmi M, Deepa G, Sujatha V, 2019. Solution of Multiobjective Solid Transportation Problem in Fuzzy Approach. AIP Conference Proceedings, No.1, P020009-1-020009-5.5.

Arslan O, Archetti C, Jabali O, Laporte G, Speranza MG, 2020. Minimum cost network design in strategic alliances. Omega, Vol. 96, 102079.

Bit AK, Biswal MP, Alam SS, 1993. Fuzzy programming approach to multiobjective solid transportation problem. Fuzzy Sets and Systems, 57:183-194.

Bodkhe, SG, Bajaj VH, Dhaigude DB, 2010. Fuzzy programming technique to solve multi-objective solid transportation problem with some non-linear membership functions. Advances in Computational Research, 2(1):15-20.

Chen Lin, Jin P, Zhang B, 2017. Uncertain goal programming models for bicriteria solid transportation problem. Applied Soft Computing, 51: 49–59.

Chen B, Liu Y, Zhou T, 2017. An entropy based solid transportation problem in uncertain environment". Journal of Ambient Intelligence and Humanized Computing, 10(1): 357–363.

Cui Q, Sheng Y, 2012. Uncertain programming model for solid transportation problem. Information, 15: 342–348.

Dalman H, Sivri M, 2017. Multi-objective Solid Transportation Problem in Uncertain Environment. Iranian Journal of Science and Technology, Transactions A: Science, 41:505-514.

Guu SM, Wu Y, 1997. Weighted coefficients in two-phase approach for solving the multiple objective programming problems. Fuzzy Sets and Systems, 85: 45-48.

Hu Y, Zhao X, Liu J, Liang B, Ma C, 2020. An efficient algorithm for solving minimum cost flow problem with complementarity slack conditions. Mathematical Problems in Engineering, 2020:1-5.

Kaur, D, Mukherjee S, Basu K, 2015. Solution of a multi-objective and multi-index real-life transportation problem using different fuzzy membership functions. Journal of Optimization Theory and Applications, 164:666-678.

Khan IU, Rafique F, 2021. Minimum-cost capacitated fuzzy network, fuzzy linear programming formulation, and perspective data analytics to minimize the operations cost of American airlines. Soft Computing, 25(2):1411-1429.

Khurana A, Adlakha V, 2015. On multi-index fixed charge bi-criterion transportation problem. Opsearch, 52(4):733-745.

Khurana A, Adlakha V, Lev B, 2018. Multi-index constrained transportation problem with bounds on availabilities, requirements and commodities. Operations Research Perspectives, 5:319-333.

Kundu P, Kar S, Maiti M, 2014. Multi-objective solid transportation problems with budget constraint in uncertain environment. Int J Syst Sci, 45(8):1668–1682.

Leberling H, 1981. On finding compromise solutions in multicriteria problems using the fuzzy min-operator. Fuzzy sets and systems, 6(2):105-118.

Lee ES, Li RJ, 1993. Fuzzy multiple objective programming and compromise programming with Pareto optimum. Fuzzy Sets and Systems, 53:275-288.

Li RJ, Lee ES, 1991. An exponential membership function for fuzzy multiple objective linear programming. Computers Math Applic, 22(12):55-60.

Mahmood T, Ali Z, (2022). Prioritized Muirhead Mean Aggregation Operators under the Complex Single-Valued Neutrosophic Settings and Their Application in Multi-Attribute Decision-Making. Journal of Computational and Cognitive Engineering, 1(2): 56–73.

Memiş S, Enginoğlu S, Erkan U, (2022). A new classification method using soft decision-making based on an aggregation operator of fuzzy parameterized fuzzy soft matrices. Turkish Journal of Electrical Engineering & Computer Sciences, 30(3): 871-890.

Mollanoori H, Moghaddam RT, Triki C, Kestheli MH, Sabohi F, 2019. Extending the solid step fixed-charge transportation problem to consider two-stage networks and multi-item shipments. Computers & Industrial Engineering, 137:106008.

Ojha A, Das B, Mondal S, Maiti M, 2010. A stochastic discounted multi-objective solid transportation problem for breakable items using analytical hierarchy process. Appl. Math. Model, 34(8):2256–2271.

Peidroa D, Vasant P, 2011. Transportation planning with modified S-curve membership functions using an interactive fuzzy multi-objective approach. Applied Soft Computing, 11:2656-2663.

Pramanik S, Jana D, Maiti M, 2013. Multi-objective solid transportation problem in imprecise environments. Journal of Transportation Security, 6:131–150.

Qiyas M, Naeem M, Abdullah S, Khan F, Khan N, Garg H, 2022. Fractional orthotriple fuzzy rough Hamacher aggregation operators and-their application on service quality of wireless network selection. Alexandria Engineering Journal, 61(12), 10433:10452.

Rath P, Dash RB, 2017. Solution of Fuzzy Multi-Objective Linear Programming Problems using Fuzzy Programming Techniques based on Exponential Membership Functions. International Journal of Mathematics Trends and Technology, 41(3).

Sadore DS, Tuli R, 2019. Optimal Solution of the Planar Four Index Transportation Problem. Amity International Conference on Artificial Intelligence (AICAI), doi: 10.1109/AICAI.2019.8701265,pp: 548-553.

Singh S, Pradhan A, Biswal MP, 2019. Multi-objective solid transportation problem under stochastic environment. Sadhana, 44:105.

Tiryaki F, 2006. Interactive compensatory fuzzy programming for decentralized multi-level linear programming (DMLLP) problems. Fuzzy Sets and Systems, 157:3072-3090.

Verma R, Biswal M, Biswas A, 1997. Fuzzy programming technique to solve multi-objective transportation problems with some non-linear membership functions. Fuzzy Sets and Systems, 91:37-43.

Werners BM,1988. Aggregation models in mathematical programming. in (G. Mitra, Ed.), Mathematical Models for Decision Support, Springer, Berlin, 295-305.

Wu YK, Guu SM, 2001. A compromise model for solving fuzzy multiple objective problems. Journal of the Chinese Institute of Industrial Engineers, 18(5):87-93.

Zadeh LA, 1965. Fuzzy Sets. Information and Control, 8:338-353.