Bartın University Journal of Faculty of Education
BUEFAD Volume 13. Issue 2

274-284
dergipark.org.tr/buefad
DOI: 10.14686/buefad.1108038

Assessment of Students' Preferred Proof Schemes in the Context of the Analysis Course

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Research Article
Received: 23.4.2022
Revised: 12.9.2022
Accepted: 15.9.2022

Abstract

This study investigated the proof schemes preferred by prospective teachers in the analytics courses. This study is a case study focused on qualitative data. In this study, an open-ended questionnaire was applied to 12 prospective teachers. They were asked to describe the most memorable proof covered in the analytics 1 and analytic 2 courses. Evaluation of these answers showed that the most memorized proof scheme was the transformational proof scheme. First-year students used their preferred proof method without any structured form. Because they had adequate prior knowledge, they utilized the transformational proof scheme, although this scheme demands upper-class level and academic solid understanding. In conclusion, prospective teachers may show a tendency to display a high-level proof scheme by combining their prior knowledge of the proof with the highest level of memorability.

Keywords: Proof schemes, proof, mathematics education.

Öğrencilerin Analiz Dersi Kapsamında Tercih Ettikleri İspat Şemaları Üzerine Bir İnceleme

Öz

Bu araştırmada öğretmen adaylarının analiz dersinde tercih ettikleri ispat şemaları incelenmiştir. Nitel araştırma yaklaşımın benimsendiği bu çalışma bir durum çalışmasıdır. Araştırmada, 12 öğretmen adayına açık uçlu soru formu uygulanmıştır. Uygulama kapsamında öğrencilere analiz 1 ve analiz 2 dersi kapsamında akıllarında en çok kalan ispat sorulmuş ve yanıtlarındaki ispat şemaları incelenmiştir. Buna göre öğretmen adaylarının akıllarında en çok kalan ispat şeması dönüşümsel ispat şeması olmuştur. Dönüşümsel ispat şeması üst sınıf düzeyi ve yüksek akademik bilgi düzeyi gerektiren bir yaklaşım olmasına karşın, bu çalışma kapsamında birinci sınıf öğrencilerinin dönüşümsel ispat şemasını tercih etmesinin sebebi öğrencilerin herhangi bir yapılandırılmış format olmadan, kendi tercih ettikleri ispatı yapmaları ve dolayısı ile yeterli ön bilgiye sahip olmaları ile açıklanabilir. Sonuç olarak öğretmen adayları akılda kalıcılık düzeyi en yüksek ispatı ön bilgileri ile birleştirerek üst düzey bir ispat şeması sergileme eğilimi gösterebilirler.

Anahtar kelimeler: İspat şemaları, ispat, matematik eğitimi.

To cite this article in APA Style:

Dinçer, B. (2024). Assessment of students' preferred proof schemes in the context of the analysis course. *Bartın University Journal of Faculty of Education, 13*(2), 274-284. https://doi.org/10.14686/buefad.1108038

INTRODUCTION

Mathematics is not just a result-oriented science but also relies on process and casualty. Mathematics is a discipline of proof, which is its main difference from any other discipline. Axioms, definitions, theorems and their proofs build the scaffold of this scientific discipline (Heinze & Reiss, 2003). Mathematical proof is the core of mathematics and constitutes the center of this discipline (Almeida, 2003; Knuth, 2000; Saeed, 1996; Tall, 1995) because mathematics is involved in revealing relations, predictions, relating the concepts, validation of statements and generalization of new knowledge (Schabel, 2005). It elicits connection forms different than notorious habits of arithmetic and algebra (Barnard & Tall, 1997).

Due to the significance and role of proof in mathematics courses, it is a time-consuming subject, especially in higher mathematics education. The foremost purpose of advanced mathematics courses is to provide students with the ability to prove and is considered an assessment of students' competency and performance on mathematical proofs (Weber, 2001). In addition, higher mathematics university students struggle with mathematical proof, although it is of paramount importance and is emphasized in undergraduate education (Harel & Sowder, 1998; Dreyfus, 1999; Almeida, 2000; Jones, 2000; Weber, 2001). Many studies were conducted to account for the underlying reason of struggle for mathematical proof. In the studies we have reviewed several studies that investigate the approach of teachers, prospective teachers and students towards proof, their internalization of proof and processes of proving (Cusi & Malara, 2007; Housman & Porter, 2003; Knuth, 2002; Sarı et al., 2007). The majority of these studies were focused on the proof schemes that investigate the approaches toward the proof process.

Harel and Sowder (2007) defined the term proof scheme to describe the idea of proof of a person or community. A proof scheme is an argumentation method that one utilizes to convince oneself and others of the correctness or falsity of a mathematical situation. There are many classifications of proof schemes in the literature (Balacheff, 1987; Harel & Sowder, 1998; Miyazaki, 2000). Harel and Sowder (2007) indicated that they related the concept of proof schema with other taxonomies (Balacheff, 1988; Bell, 1976), and they formed their own conceptual framework with the help of sources about the roles and functions of proof (Balacheff, 1988; Bell, 1976; Hersch, 1993) when definitions and taxonomies were insufficient. According to Harel and Sowder (1998), "proof scheme" is the founding member of this framework which is formed with the help of aforementioned sources. This study includes proof schemes described by Harel and Sowder (1998), as it provides comprehensive context.

Harel and Sowder (1998) classified proof schemes as follows:

1. External Conviction Proof Schemes: In this scheme, one convinces oneself and others using something external. Students who utilize these proof schemes understand the original and persuade others using external sources. These sources may emerge in several ways, including an authority based on a book or a teacher (authoritarian proof scheme), the form or appearance of an argument (habitual proof scheme), or the meaningless manipulation of symbols (symbolic proof scheme).

1.1. Authoritarian Proof Scheme: One convinced by rhetoric's of teacher or any other authority. This type of learned helplessness come out as "I don't remember," "I need to check it from the book."

1.2. Ritual Proof Scheme: Here, persuasiveness stems from the form of the evidence, not the content. It can be said that students investigating the validity of an argument exhibit features of the habitual proof scheme when they decide under the influence of the structure of the argument and the routine formats of the proof instead of the accuracy of the argument (Martin & Harel, 1989). For example, when students accustomed to the traditional curriculum try to structure their proofs, they utilize only inductive argumentation by sticking to the inductive stages that they learned in school. However, when they structure it without the logical deduction between "P(k) for n=k and P(k+1) for n=k+1;" they may show reactions related to the habitual proof scheme. According to Sowder and Harel (1998), student who exhibits reactions demonstrating the external habitual proof scheme may doubt that their proofs might not be sufficient, because their arguments are structured in a way that does not contain enough mathematical notation or calculation, even if they form reliable arguments.

1.3. Symbolic Proof Scheme: Persuasiveness in symbolic proof is achieved by manipulation of symbols without knowing its meaning. When students handle symbols without referring to their quantities in that situation in a meaningless way, they show features that show the symbolic proof scheme. In this case, it can be said that the exact thinking of the students is based on external sources. The better nature of symbolic ratiocination is the well-known power of symbols, especially in algebra. For example, in linear equations of four operations, one may not have to relate each step of the meaning process concerning the context of the problem. One exhibiting features of the transformational proof scheme, described later, can elaborate correct symbolic reasoning in many situations (Sowder & Harel, 1998).

2. Empirical Proof Schemes: Students who display features related to this proof scheme validate or reject assumptions based on physical evidence or sensory experience (Harel & Sowder, 1998). Empirical proof schemes might be either inductive or perceptual and it is divided into inductive proof scheme and perceptual proof scheme.

2.1. Inductive Proof Scheme: Students utilizing this proof scheme take one or more examples into consideration, convincing proofs to point out the general truth. Arguments are based on special situations and examples.

2.2. Perceptual Proof Scheme: Students that exhibit this scheme use their foresight to sense the truth and false but they cannot find strong evidence. Also, they utilize drawings to convince others. They are able to find solutions to geometry problems with the help of one or more drawings. However, these students cannot do transformations and lack the insight to see the results of those transformations while using this proof scheme (Harel & Sowder, 1998). Students generally draw conclusions devoid of inductive inferences and based on insufficient cognitive thinking; however, they find those conclusions persuasive for themselves and others.

3. Analytical Proof Schemes: Students who exhibit features of these proof schemes validate the assumptions through logical deduction and they also go beyond the application of propositions that are formed by specific logical rules deduced from cases accepted as correct without proof (Harel & Sowder, 1998). This scheme is either transformational or axiomatic.

3.1. Transformational Proof Scheme: In this proof scheme, one convinces oneself or others by a deductive process. In this process, students takes generalizable cases into consideration, they apply result-oriented cognitive operations and switch between definitions, theorems, and figures. This scheme has three features: generalization, operational thinking, and logical deduction. Students exhibiting features of this scheme provide justifications related to general aspects of cases. Logical deductions are aimed at inserting the assumptions into analytical frameworks. Transformational observations related to this scheme include goal-oriented operations and anticipation of their results. This process is executed to leave certain relations unchanged. When a change is encountered, students predict its possible results and try to seek balance by applying necessary operations (Sowder & Harel, 1998). For example, general structure counting in this scheme includes exact thinking without finding a pattern. Essentially, the aforementioned transformation is to see the structure behind the pattern, which is hard to see. The transformations that students utilize in that regard can be limited by the perception of mathematical content or units of defense. Thus, a transformational proof scheme can be described as a delimited analytic proof scheme (İskenderoğlu, 2016). In other words, transformational proof schemes can be evaluated as a base for axiomatic proof schemes (Sowder & Harel, 1998).

3.2. Axiomatic Proof Scheme: This scheme possesses the features of transformational proof schemes and in addition students realize that mathematical systems are based on cases that are approved without proof (Housman & Porter, 2003). In the data bases of mathematics, the subsequent results are deduced from previous ones. A careful arrangement is made by only undefined terms, assumptions, theorems and definitions (Sowder & Harel, 1998). Students exhibiting features of this proof scheme are aware that the starting point of a mathematical justification is undefined terms and axioms, and they have the ability to work comfortably with such a system (Harel & Sowder, 1998; Sowder & Harel, 1998).

Several studies were conducted in this field to demonstrate the proof schemes of prospective mathematic teachers who are studying either in a primary school or middle school program (İskenderoğlu, 2010; Sarı et al., 2007; Şengül & Güner, 2013; Weber, 2010).

According to Hart (1994), to correctly demonstrate students' proof processes and grounds of mistakes they made in this process, we need to construct cognitive based studies to investigate their thought processes (Weber, 2001). One of the most significant lessons in university level curriculum of mathematics is the analysis course (Hartter, 1995). However, the literature lacks a sufficient number of studies on proof studies in the analysis field. On the other hand, a detailed classification of pre-existing proof scheme that students utilize will provide different and vital contribution to the literature. Results of such a study exclusively on analytic field will provide educators information about students' preferences and attitude for proof scheme and its subject. An education that considers students' preferences for proof scheme will be more beneficial than the contemporary education.

This study investigated the most memorable proofs for prospective mathematic teachers after analysis 1 and analysis 2 courses and which proof scheme they were evaluated under

METHOD

In this special case study, a mixed method was used. The document analysis was conducted with descriptive scanning in the quantitative dimension and document analysis on the proof problems in the qualitative dimension. This study is an example of the holistic multiple-case design type among case study designs because this design houses multiple cases that can be regarded as holistic on their own. Each case is interpreted holistically in itself and then compared to one another. A deep investigation into one or more cases is the main feature of case studies. In other words, this study design holistically investigated all the factors related to a specific case [e.g., context, individuals, events and processes] and focused on how those factors and relevant cases affected each other (Yıldırım & Şimşek, 2000). Thus, it evaluated and attempts to make sense of the behaviours of an individual in the context in which it occurred. In our study, proof preferences of prospective teachers were evaluated by the same holistic approach.

Prospective teachers were asked to describe the proofs that they remember the most, and these proof schemes were divided into characteristically groups by analysis of their content, according to the proof scheme inventory. The proof schemes that the prospective teachers had in the proving process were evaluated using Harel and Sowder's (2007) terminology and comments were made. Proof schemes that prospectives possessed during the process of proofing were evaluated and commented on with the terminology of Harel and Sowder [2007]. Each proof that students utilized was coded according to this categorization and proof schemes were tried to be determined. While determining the proof schemes, each scheme was coded. Codings are presented in Table 1.

Table 1. The Characteristics of Participants

Proof Schemes	(Sub)-schemes	Proof scheme Indicators
Extinctional Proof Schemes	Authoritarian Proof Scheme	Tries to construct the proof according to what they have learned in the courses and fail to complete the proof
	Ritual Proof Scheme	Providing superficial proofs by sticking to stereotypes
	Symbolic Proof Scheme	Meaningless manipulation of symbols
Experimental Proof Schemes	Inductive Proof Scheme	Proofs are based on special occasions, and its examples
	Perceptual Proof Scheme	Proofs based on insufficient exact thinking and are thought to be persuasive
Analytic Proof Schemes	Transformational Proof Scheme	Switching between definitions, theorems, shapes and inserting exact thinking into an analytical framework
	Axiomatic Proof Scheme	Generalization by accepting undefined terms and axioms as a starting point

While evaluating the data, two expert academicians were consulted for validity studies. During qualitative data analysis, mostly general content was analysed. Organizing, summarizing and interpreting the collected data are among the basic processes of the analysis (Büyüköztürk et al., 2012). Answers from prospective teachers were given under the categories with frequency and percentage values, and examples of proof schemes were included in the findings section and analyzed by qualitative method.

The sample of this study was determined using the easy sampling method, which is one of the non-probability sampling types. The study group of this research consisted of 12 prospective teachers studying in the Department of Primary School Mathematics Teacher Education, who were available and volunteered.

Research Ethics

All ethical procedures were performed in this study. Ethical permission of the research was approved by Izmir Demokrasi University Social and Human Sciences Ethics Committee. Ethics committee document number is 2022/04-03.

FINDINGS

Proof schemes that prospective teachers remembered the most after courses of analysis 1-2 were included in the following table.

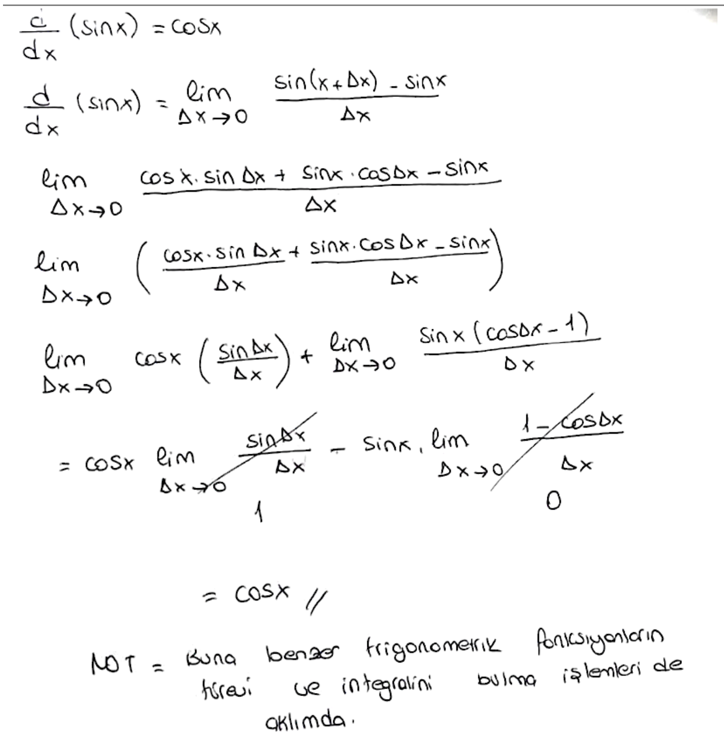
Table 2. Proof Schemes of Prospective Teachers

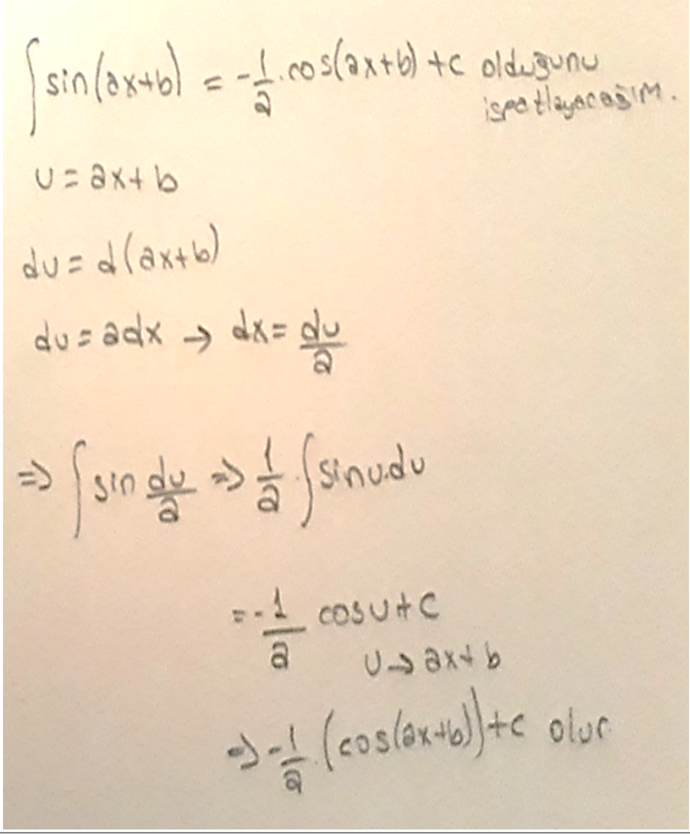
Proof Schemes	(Sub)-schemes	f
Extinctional Proof Schemes	Authoritarian Proof Scheme	-
	Ritual Proof Scheme	-
	Symbolic Proof Scheme	-
Experimental Proof Schemes	Inductive Proof Scheme	-
	Perceptual Proof Scheme	-
Analytic Proof Schemes	Transformational Proof Scheme	12
	Axiomatic Proof Scheme	-

As seen in table 2, indicators of external proof scheme, experimental proof scheme and their subgroups were not evaluated. All of the proof schemes related that were the most remembered ones by the prospective teachers were analyzed as Transformational Proof Scheme which is a sub-category of the analytical proof scheme.

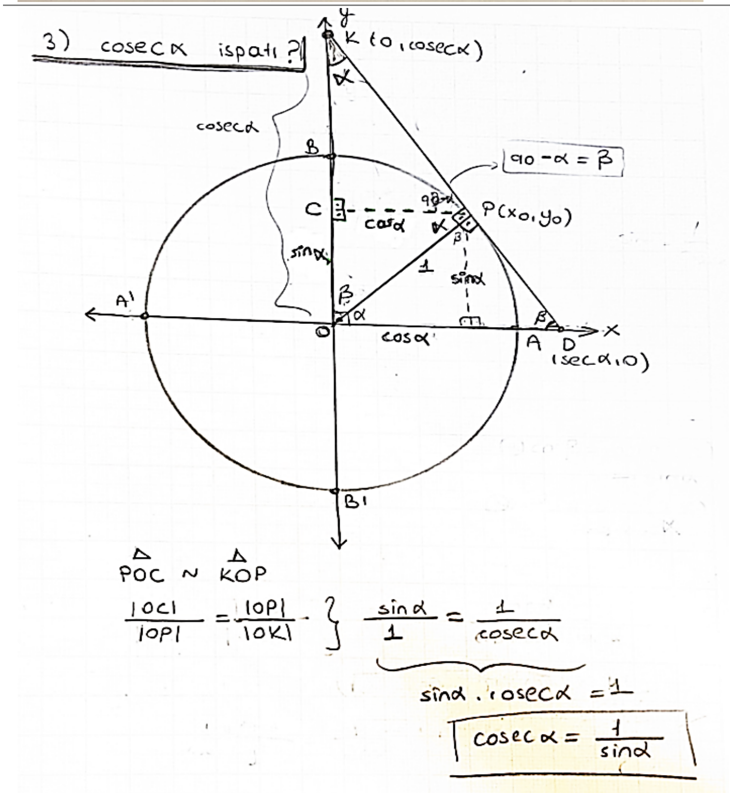
In Table 3 below, some of the most striking proof examples that prospective teachers remembered the most are given, and the proof schemes are explained according to the indicators in Table 1.

Table 3. Exploring the Proof Scheme by Prospective Teachers

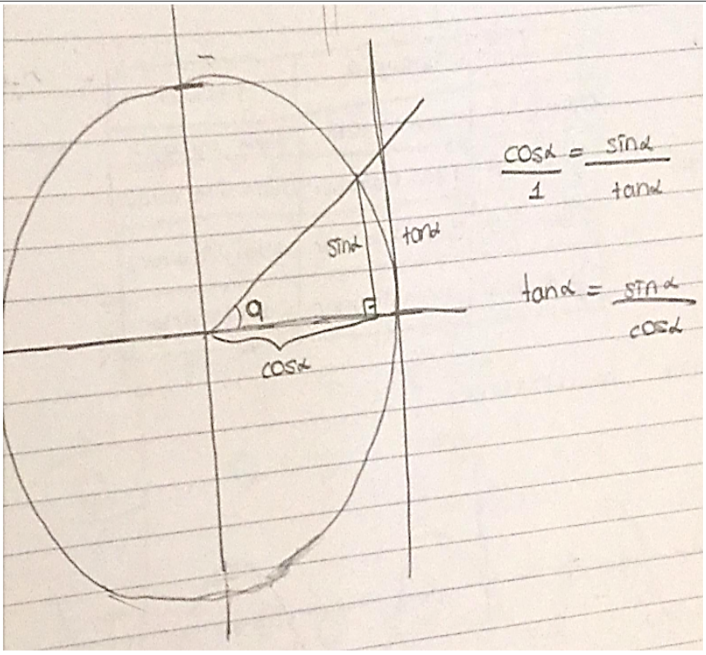
Proof Scheme by Prospective Teachers	Indicators of Proof Schemes
 $\frac{d}{dx}(\sin x) = \cos x$ $\frac{d}{dx}(\sin x) = \lim_{\Delta x \to 0} \frac{\sin(x+\Delta x) - \sin x}{\Delta x}$ $\lim_{\Delta x \to 0} \frac{\cos x \cdot \sin \Delta x + \sin x \cdot \cos \Delta x - \sin x}{\Delta x}$ $\lim_{\Delta x \to 0} \left(\frac{\cos x \cdot \sin \Delta x}{\Delta x} + \frac{\sin x \cdot \cos \Delta x - \sin x}{\Delta x} \right)$ $\lim_{\Delta x \to 0} \cos x \left(\frac{\sin \Delta x}{\Delta x} \right) + \lim_{\Delta x \to 0} \frac{\sin x (\cos \Delta x - 1)}{\Delta x}$ $= \cos x \lim_{\Delta x \to 0} \frac{\sin \Delta x}{\Delta x} - \sin x \cdot \lim_{\Delta x \to 0} \frac{1 - \cos \Delta x}{\Delta x}$ (1, 0) $= \cos x$ // NOT = Buna benzer trigonometrik fonksiyonların türevi ve integralini bulma işlemleri de aklımda.	The trigonometric sum formula, as well as the operation features of trigonometric functions, were employed in this proof. The analytical proof methodology, as indicated by Harel and Sowder (1998), is used to make the transition between definitions and properties. (transformational proof scheme)



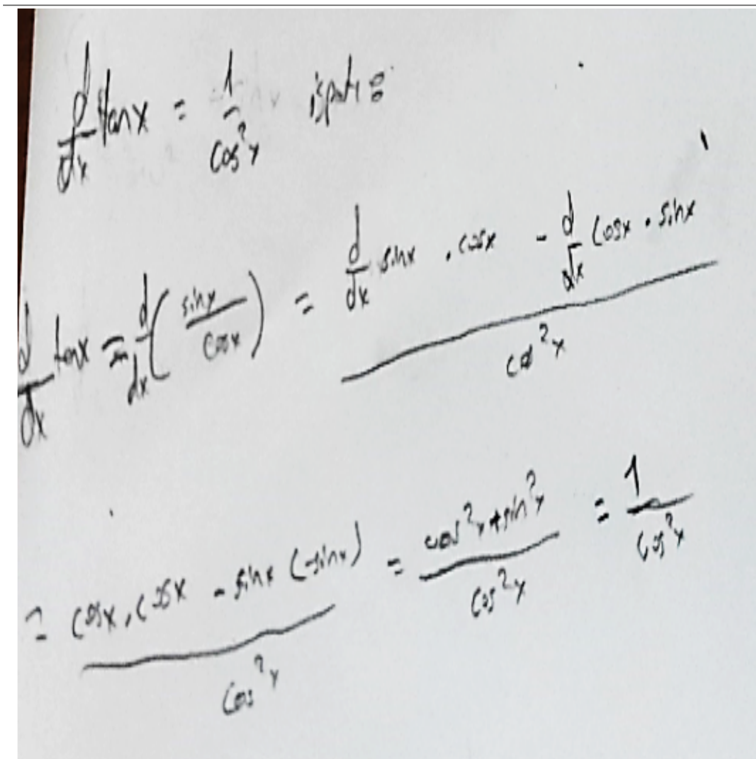
A differential calculation was performed in this proof. In addition to the properties of the trigonometric function, the properties of the integration process are utilised. This is the chart of analytical evidence. (transformational proof scheme)



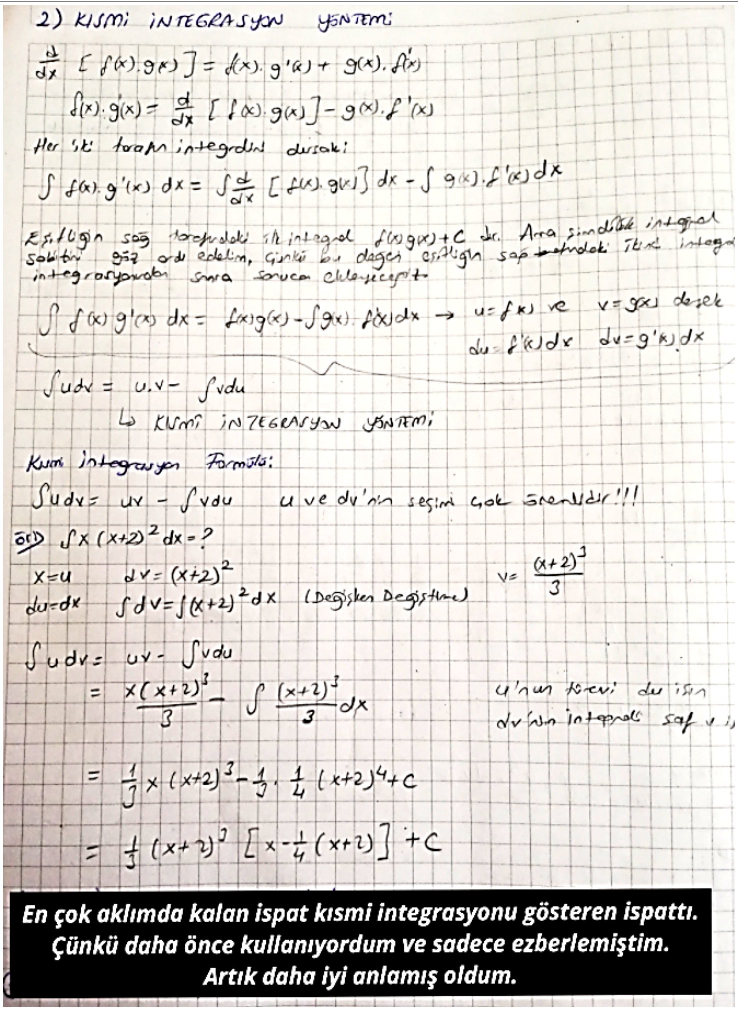
This proof involves a visual process as well. Simultaneously, the theorem of triangle similarity was applied, and a transition was made between theorems. This is the chart of analytical evidence. (transformational proof scheme)



This proof has a very simple visual content. The proof relied heavily on unit circle characteristics and triangle resemblance. This is the chart of analytical evidence. (transformational proof scheme)



In this proof, the operational properties of the derivative and the trigonometric function properties are used together. This is the analytical evidence chart. (transformational proof scheme)



Among the proofs made, this is the only proof that does not belong to the subject of trigonometry. In this proof, the operational properties of the derivative and the basic properties of the differential calculus are used together. This proof is an analytical proof scheme. (transformational proof scheme)

(Student also wrote the following footnote under the proof)

(The proof I remember the most was the proof of partial integration; as I had just memorized it previously. Now I understand better.)

Evaluation of proofs preferred by prospective teachers showed that answers were mostly on trigonometry. Only one student proved the partial integration formula of the integral subject.

DISCUSSION & CONCLUSION

This study investigates the proof schemes that students considered when they are asked to write their most remembered proof type after analytic course 1 and 2. Results showed that the most remembered proof type was analytic proof scheme. Under the title of analytic proof scheme, proofs on trigonometry were the featured the most by the prospective teachers.

Previous studies in this field highlighted the tendency of students to use the proofs in low-level cognitive categories (Coe & Ruthven, 1994; Cusi & Malara, 2007; Harel & Sowder, 2007; Ören, 2007). When the relevant literature is examined, it can be stated that students or teacher candidates mostly showed reactions that exhibit features of external and experimental proof schemes, and as the grade level and academic achievement level increased, proof schemes that require high-level cognitively were observed, which can be attributed to expanding on knowledge. In this study, although prospective teachers were only first grade, they were able to exhibit analytic schemes that require higher cognitive level skills. Since the participating prospective teachers are still first grade student, experimental and extrinsic proof schemes were expected from them rather than analytical proof schemes at the metacognitive level. Because, in different studies in the literature only a minority of prospective teachers exhibited analytical proof scheme, and as they reach upper grade levels they were able do proofings' suitable to analytical proof scheme. For example, a study showed significant difference between the proof schemes used by first and last grade mathematics teacher candidates (Şengül & Güner, 2013). It was determined that first grade prospective teachers mostly used experimental proof schemes and last-year prospective teachers mostly used analytical proof schemes. In a different study on proof schemes (İskenderoğlu, 2010), which was aimed to reveal the different kinds of proof schemes that prospective teachers use on functions and how the preferences of proof schemes change as the grades differ. Result of this study, revealed that as the grade level of teacher candidates increases, there is an increase in the use of analytical schemes, which are considered the highest level of proof schemes.

In a similar study which also attempts to reveal the features of the proofs chosen by prospective teachers for certain topics and proposals, it was seen that prospective teachers had different proof schemes according to their academic success (Doruk & Kaplan, 2017). In that study, academic achievement level was taken as the independent variable, not the grade of the student. Thus, an indirect relationship between knowledge and proof schemes was determined. Another study that tried to determine the proof schemes utilized by prospective teachers at the fourth grade on trigonometry, showed that the answers were mostly under – the category of analytical proof scheme (Pektaş & Bilgici, 2019). As stated in studies on proof schemes, there is a hierarchical structure (Harel & Sowder, 1998). Thus, it is an expected situation for fourth-grade students to have an analytic proof scheme approach. In this study, other studies in the literature, prospective teachers were asked to do proofing on the subject that they remember the most instead of structured questions or previously dictated subjects. In other words, students decided which subject to focus on their own. Therefore, it is expected that participants will report with high-level cognitive characteristics on their preferred subject that they have learned best and most meaningfully. Therefore, although their knowledge does not match the fourth-grade level, they used the high-level cognitive level proof scheme. As a result, it is possible to say that analytical proof schemes are the desired schemes for the use of students since it is thought to be at the center of mathematical exact thinking. Students utilize this scheme not just for exact thinking but also when adapting previous knowledge to a new situation. Prospective teachers decided which proof scheme to utilize without any constructed question. Results showed that prospective teachers mostly preferred to use analytical proof scheme. This high level of cognitive functioning can emerge only in cases where permanent and associative learning is present. To achieve such learning ability prospective teachers should have sufficient foreknowledge and permanent knowledge. In conclusion, proofs in our study were at the level of analytic proof scheme can be interpreted as independent of grade but related to the fact that subjects learned as a result of casual meaningful and permanent education leads to higher cognitive level.

In addition, the fact that prospective teachers utilize an analytical proof scheme for trigonometry subjects in this study also indicates that they had sufficient prior knowledge about trigonometry. This situation can be explained by that students took trigonometry lessons in middle school where they often encountered proofs related to trigonometry. Thus, this can be interpreted as follows: sufficient prior knowledge on a subject is needed to exhibit analytical proof scheme on that subject. Mathematics is a science based on casualty; thus, doing mathematical proof is of paramount importance. Hence, efficient understanding of linked topics and mathematical concepts is a prerequisite for prospective teachers and students to be successful in doing mathematical proofs. Writing a conclusion is the final part of the research paper, drawing everything together and tying it into initial research. Writing a conclusion involves summing up the paper and giving a very brief description of the results, although you should not go into too much detail about this. The discussion should relate the presented results to those of previous own or other studies, interprets them and draw conclusions. It can outline working hypotheses, theories, and applications. Some suggestions should be made for many target groups, such as implementers, researchers, and educators, in accordance with the findings of this study. Suggestions can also be given under a separate title.

Investigating the capacity of mathematic teachers to do mathematical proof does not illustrate the process of proofing. Thus, studies that incorporate the process of doing proof together with proof schemes can be included in undergraduate education, to provide teachers, researchers and educators with guidance on how to design education for proof learning.

Also, proof schemes that students preferred to utilize can be explored in further depth and the reason of their preferences can be explored. Investigating the factors that may be effective in the evidence schemes preference of prospective teachers (such as the lessons they have taken before, their attitude toward mathematics and their level of motivation) is thought to contribute to increasing the level of proof-making skills that prospective teachers can experience.

Statements of Publication Ethics

This research was reviewed by the Izmir Demokrasi University Social and Humanities Ethics Committee and it was decided that the research was ethically appropriate. Meeting date and ethical decision number: 08/04/2022- 2022/04-03

Researchers' Contribution Rate

This manuscript is a single researcher manuscript. The study was conducted and reported by the corresponding author. The researcher's contribution rate is 100%.

Conflict of Interest

There is no conflict of interest for this study.

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