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Research Article

REVERSE-ENGINEERED DBI-ESSENCE FIELD IN THE RAINBOW GRAVITY

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Abstract: *The current astrophysical data sets have implied that our universe has recently entered an accelerated expansion phase. In literature, it is commonly thought that the late-time speedy enlargement behavior is caused by mysterious dark contents (dark matter, dark energy, and dark radiation) that cannot be observed directly. Although Einstein's General Theory of Relativity has provided very successful theoretical explanations and predictions, it cannot explain the aforementioned unusual nature of the cosmos. This situation has led scientists to try to create a new theoretical model for the nature of dark content. The cosmological constant, scalar fields, unified energy densities, additional dimension ideas, and modified gravity theories are among the prominent point of view. In this research, the reverse-engineering method is mainly taken into account in the rainbow gravity formalism for the DBI-essence scalar field dark energy model. As it is known, scalar field models can be introduced with the help of some fundamental physics theories, but these theories do not provide a direct way to write explicitly the self-interaction potentials that we encounter in scalar field models. Therefore, the reverse-engineering method used in this research takes a significant role while writing the self-interaction potential of a scalar field model.*

Keywords: *Cosmology, Quantum Gravity, Scalar Field, DBI-essence.*

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1. Introduction

According to the astrophysical experiments completed in recent years [1-4], our universe has entered an extraordinary period, in which it expands faster than expected. Cosmological data published by the Planck Research Group [4] indicates that the distribution of contents in our universe is found to be 4.9 % ordinary (or baryonic) matter (stars, galaxies, and everything including us), 26.8 % dark matter, and 68.3 % dark energy. The General Theory of Relativity [5] is expressed as follows:

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G\left[(\rho + p)U_\mu U_\nu + p g_{\mu\nu}\right]. \tag{1}$$

The left side of this equation expresses the geometric structure of the space-time fabric, while the right side contains information about the energy-matter distribution. The quantities $R_{\mu\nu}$, R, $g_{\mu\nu}$, G, ρ, p, and $U_\mu$written in the above equation correspond to the Ricci tensor, Ricci scalar, metric tensor, universal gravitational constant, energy density, pressure, and four-velocity vector, respectively. The General Theory of Relativity, which has reached the age of 107 and offers very successful results in the theoretical context, includes only the ordinary matter effect among the dominant contents of our Universe. Naturally, the need to reorganize and expand Einstein's General Theory of Relativity has emerged to explain the current observational data theoretically. According to the studies presented in the literature, the following methods come to the fore: (i) changing the left side of the equation (modified gravity) [6-8], using the cosmology constant [9], making use of scalar field models [10-13]), (ii) changing the right side of the equation (introducing dark matter and dark energy density definitions [14-16] or unified density models [17,18]). In this study, we mainly focus our attention on the reverse-engineering method, which comparatively connects a scalar field definition and the rainbow formalism of gravity, which is a modified gravity idea that has been studied extensively in the literature [22,24]. As we emphasized in the summary of our study, some fundamental physics theories allow us to propose various definitions of scalar fields, but they do not offer useful methods, by which we can obtain explicit solutions for the self-interaction potential introduced in a scalar field model [10-13]. On the other hand, the very recent observation of gravitational waves, which was estimated first by Einstein via the General Theory of Relativity Theory [19], has increased the need for a successful quantum gravity model [20,21]. In this context, the rainbow formalism of gravity [22-24] stands out as a remarkable quantum gravity idea proposed recently in the literature. This formalism is basically built on the idea that "the energy of a test particle should affect the fabric of space-time" [22,23]. In this study, we discuss the results obtained by substituting the Friedmann-Robertson-Walker (FRW) type rainbow metric in equation (1), together with the (Dirac-Born-Infeld) DBI-essence scalar field dark energy model and compare them via the reverse-engineering method.

In this study, the rainbow formalism and the DBI-essence scalar field model, which form the basis of the research, will be briefly introduced in the second section. Subsequently, our original calculations will be given in the third section. Finally, we will highlight the main results of our research and present some suggestions for possible future research in the final section.

2. Materials And Methods

2.1. Fundamentals of Rainbow Formalism

Within the scope of this formalism, Magueijo and Smolin [23] defined a deformed equivalence principle and presented a dispersion relation, which is valid also in curvilinear coordinates. From a mathematical point of view, the rainbow formalism is basically represented by the following relation [23]:

$$f^2(\varepsilon)E^2 - g^2(\varepsilon)p^2 = m^2 \ . \tag{2}$$

Here, the quantities E and m represent energy and mass, respectively, while f(ε) and g(ε) are called rainbow functions [22-24]. On the other hand, the parameter ε corresponds to the energy of a test particle and it is written as

$$\varepsilon = \frac{E}{E_{pl}} \, , \tag{3}$$

where $\boldsymbol{E_{pl}}$ stands for the Planck energy. Various definitions [25-30] have been proposed in literature for the rainbow functions (the quantities γ, β and n that appear in some equations are arbitrary parameters):

$$f(\varepsilon) = \frac{1}{1-\gamma\varepsilon}, \quad g(\varepsilon) = 1, \tag{4}$$

$$f(\varepsilon) = g(\varepsilon) = \frac{1}{1-\varepsilon}, \tag{5}$$

$$f(\varepsilon) = \sqrt{1-\varepsilon^2}, \quad g(\varepsilon) = 1, \tag{6}$$

$$f(\varepsilon) = 1, \quad g(\varepsilon) = 1 + \frac{\varepsilon}{2}, \tag{7}$$

$$f(\varepsilon) = e^{-\frac{\varepsilon^2}{2}}, \quad g(\varepsilon) = 1, \tag{8}$$

$$f(\varepsilon) = \frac{e^{\beta\varepsilon}-1}{\beta\varepsilon}, \quad g(\varepsilon) = 1, \tag{9}$$

$$f(\varepsilon) = 1, \quad g(\varepsilon) = 1 + \varepsilon^n. \tag{10}$$

The Friedmann-Robertson-Walker metric, which is the basis of cosmological studies and describes the geometric structure of the Universe, points to a homogeneous and isotropic structure and is expressed mathematically as follows [31]

$$ds^2 = -dt^2 + a^2(t)\left[dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\theta^2\right] \tag{11}$$

where the a(t) function is known as the scale factor and indicates an expanding geometry. Within the scope of the rainbow formalism of gravity, the replacements

$$dt \to \frac{dt}{f}, \quad dx \to \frac{dx}{g}, \quad dy \to \frac{dy}{g}, \quad dz \to \frac{dz}{g} \tag{12}$$

transforms the Friedmann-Robertson-Walker metric into its rainbow gravity version [22-24]:

$$ds^2 = -\frac{1}{f^2}dt^2 + \frac{a^2(t)}{g^2}\left[dr^2 + r^2 d\theta^2 + r^2 sin^2\theta d\theta^2\right], \tag{13}$$

Thus, after using equation (13) in equation (1), the extended Friedmann equations are obtained as follows [22,23]:

$$H^2 = \frac{8\pi G}{3f^2}\rho\ , \tag{14}$$

$$\frac{dH}{dt} = -\frac{8\pi G}{f^2}(\rho + p)\ . \tag{15}$$

The quantity $\mathbf{H} = \frac{1}{a}\frac{da}{dt}$ is known as the Hubble parameter in literature. Besides, after taking the above results into account at the same time, the following equation is obtained for the extended conservation equation [24]

$$\frac{d\rho}{dt} + 3H(\rho + p) = 0. \tag{16}$$

2.2. Fundamentals of the DBI-essence scalar field model

This type of scalar field is described by the action integral [32]:

$$S = -\int d^4x\sqrt{-g}\left[T(\phi)\left\{\sqrt{1-\frac{\left(\frac{d\phi}{dt}\right)^2}{T(\phi)}}-1\right\}+V(\phi)\right]. \quad (17)$$

Here, ϕ is the scalar field, g is the determinant of the metric tensor, V(ϕ) is the self-interaction potential, and T(ϕ) is the warped brane tension. Within the scope of this model, the energy density and pressure quantities are written as follows [33]:

$$\rho_{DBI} = (\eta-1)T(\phi)+V(\phi)\,, \quad (18)$$

$$p_{DBI} = \left(\eta-\frac{1}{\eta}\right)T(\phi)-V(\phi), \quad (19)$$

where

$$\eta = \left(1-\frac{\left(\frac{d\phi}{dt}\right)^2}{T(\phi)}\right)^{-\frac{1}{2}} > 1 \qquad . \quad (20)$$

Therefore, we see that it should be $\left(\frac{d\phi}{dt}\right)^2 > T(\phi)$ [33]. In addition to this, we get the subsequent relation by making use of equations (18) and (19),

$$\rho_{DBI}+p_{DBI} = \left(\eta-\frac{1}{\eta}\right)T(\phi)\,. \quad (21)$$

Note that this quantity must always take positive values [33]. Finally, the equation-of-state parameter for the DBI-essence scalar field model is obtained as follows:

$$\omega_{DBI} = \frac{p_{DBI}}{\rho_{DBI}} = \frac{\frac{\eta^2-1}{\eta}T(\phi)-V(\phi)}{(\eta-1)T(\phi)+V(\phi)}\,. \quad (22)$$

We can now apply the reverse-engineering method, which is based on the idea of making a comparison between the rainbow gravity formalism and the DBI-essence scalar field model, to the preliminary definitions and calculations.

3. Results and Discussions

From equations (14) and (15), the following relations are obtained for the energy density ρ and the pressure p

$$\rho = \frac{3f^2}{8\pi G}H^2\,, \quad (23)$$

$$p = -\frac{f^2}{8\pi G}\left[\frac{dH}{dt}+3H^2\right]. \quad (24)$$

Thus, for the equation-of-state parameter, we reach the following result

$$\omega = \frac{p}{\rho} = -\frac{\frac{dH}{dt}+3H^2}{3H^2} . \quad (25)$$

According to the reverse-engineering method, one can assume $\rho = \rho_{DBI}$ for the energy density, $p = p_{DBI}$ for the pressure, and $\omega = \omega_{DBI}$ for the equation-of-state parameter. According to these assumptions, we can write

$$\frac{d\phi}{dt} : \frac{\rho_{DBI}+p_{DBI}}{\eta} \rightarrow \frac{\rho+p}{\eta} , \quad (26)$$

$$T(\phi) : \frac{\left(\frac{d\phi}{dt}\right)^2 (\rho_{DBI}+p_{DBI})^2}{(\rho_{DBI}+p_{DBI})^2 - \left(\frac{d\phi}{dt}\right)^4} \rightarrow \frac{\left(\frac{d\phi}{dt}\right)^2 (\rho+p)^2}{(\rho+p)^2 - \left(\frac{d\phi}{dt}\right)^4} \quad (27)$$

$$V(\phi) : \frac{\left(\frac{d\phi}{dt}\right)^2 \rho_{DBI} - p_{DBI}(\rho_{DBI}+p_{DBI})}{\left(\frac{d\phi}{dt}\right)^2 + \rho_{DBI}+p_{DBI}} \rightarrow \frac{\left(\frac{d\phi}{dt}\right)^2 \rho - p(\rho+p)}{\left(\frac{d\phi}{dt}\right)^2 + \rho+p} . \quad (28)$$

When the expressions presented in equations (23) and (24) are used in equations (26), (27) and (28), the following results are obtained:

$$\frac{d\phi}{dt} = -\frac{f^2}{8\pi\eta G}\frac{dH}{dt}, \quad (29)$$

$$T(\phi) = \frac{f^4\left(\frac{dH}{dt}\right)^2}{(8\pi G)^2 - \eta^2 f^4\left(\frac{dH}{dt}\right)^2} \quad (30)$$

$$V(\phi) = \frac{3f^4H^2\frac{dH}{dt} - 8\pi G\eta^2 f^2(\frac{dH}{dt}+3H^2)}{f^2\frac{dH}{dt} - 8\pi G\eta^2} . \quad (31)$$

At this stage, the results are given in equations (29), (30), and (31) can be used for a graphical analysis with the help of specific forms of rainbow functions (please check equations (4)-(10) introduced before) and some solutions of Hubble parameter. Three well-known Hubble parameter formulations are presented in Table 1.

Table 1. Custom scale factor and Hubble parameter selections

Phase	$\boldsymbol{a(t)}$	$\boldsymbol{H(t)}$
Radiation phase [34]	$a_0 t^{\frac{1}{2}}$	$\frac{1}{2t}$
Matter phase [35]	$a_0 t^{\frac{2}{3}}$	$\frac{2}{3t}$
Energy phase [35]	$a_0 e^{H_0 t}$	H_0

As an example, when we use the choice of rainbow functions presented in equation (6) and the energy phase solution of the Hubble parameter, we get the following results:

$$\phi = \phi_0 = constant, \quad (32)$$

$$T(\phi) = 0 \quad (33)$$

$$V(\phi) = 3(1-\varepsilon^2){H_0}^2 . \quad (34)$$

4. Conclusions

The main purpose of this research is to apply a method that we can express mathematically the self-interaction potential quantity V(ϕ) of a scalar field dark energy model. In order to achieve this aim, the reverse-engineering between the rainbow gravity formalism and the DBI-essence scalar field definition has been studied. The effects of rainbow functions on the scalar field model have been clearly demonstrated from the calculations performed with the help of the rainbow gravity framework. It would be appropriate to emphasize here that it is possible to reach additional cosmological discussions by using the obtained theoretical results in a graphic analysis. On the other hand, we can expand our research by applying the comparative method introduced in this study to different scalar field models such as the tachyon [10], quintessence [12], k-essence [36], and the dilaton [37] fields.

Ethical Statements

The author declares that this document does not require an ethics committee approval or any special permission. Our study does not cause any harm to the environment.

Conflict of interest

The authors declare no conflict of interest.

Authors Contributions

FFB carried out calculations and wrote the first draft of the manuscript.

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