IOJPE

ISSN: 1300 – 915X
www.iojpe.org

International Online Journal of Primary Education 2021, volume 10, issue 1

THE CONCEPT OF ANGLE IN TURKISH AND SINGAPOREAN PRIMARY SCHOOL MATHEMATICS TEXTBOOKS: DYNAMIC OR STATIC?

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Received: November 06, 2020 **Accepted:** February 08, 2021 **Published:** June 30, 2021

Suggested Citation:
Bütüner, S. Ö. (2021). The concept of angle in Turkish and Singaporean primary school mathematics textbooks: Dynamic or static?. *International Online Journal of Primary Education (IOJPE), 10*(1), 89-105.

Abstract

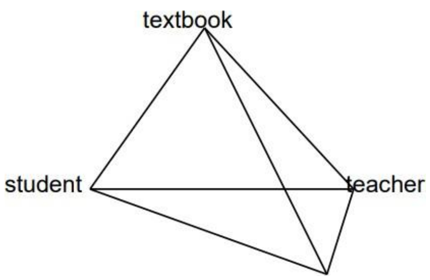
The present study compared Turkish and Singaporean textbooks with respect to their instructional contents on a difficult topic for most students: the concept of angle. The study used the 3rd and 4th grade mathematics textbooks taught in Turkish and Singaporean schools. The analysis showed that Turkish textbooks defined the angle as a static concept, and Singaporean textbooks defined it as both a static and dynamic concept. The definitions of the concept of angle included in the textbooks reflect on the representation of the angle, instructional tools and problems. Turkish students learn angle from textbooks only as a static concept, so they may have difficulties and misconceptions about the subject and related concepts. The findings showed that the contents of Singaporean textbooks offer students more opportunities than Turkish textbooks in learning about the angle as a static and dynamic concept.

Keywords: Singapore, Turkey, mathematics, textbook, the concept of angle.

INTRODUCTION

Many countries participate in international comparative exams such as TIMSS, PISA and PIRLS in order to better see their own development in science, mathematics and reading skills. One of these international exams, TIMSS is held every four years to compare the science and mathematics achievement levels of elementary 4th and 8th graders. In the exam, participant countries' mathematics standards achievement scores are calculated (numbers and operations, geometry, algebra and data-probability) in order to reach an overall achievement score. In TIMSS 2015, Singapore was the top country with 617 points, and Turkey remained below the international mean with 463 points in the geometry domain (Mullis, Martin, Foy, & Hooper, 2016). The differences between Turkish and Singaporean students' geometry performances may be attributed to many factors. One reason could be differences in the content of the textbooks.

Rezat (2006) makes an attempt to develop a framework on the use of textbooks by both teachers and students. Using the tetrahedron model shown in Figure 1, Rezat (2009) represented the relationships among textbooks, students, teachers, and mathematical knowledge.



Mathematical knowledge/didactical aspects of the mathematical knowledge

Figure 1. Rezat's tetrahedron model

Figure 1 indicates the importance of textbooks in the teaching process. Textbooks can be considered an important resource for both students and teachers (Fan & Zhu, 2000; Son & Hu, 2016). Textbooks shape what teachers will teach in the classroom and how (Alajmi, 2012; Hirsch, Lappan, Reys & Reys, 2005; Li, 2000). As different textbooks provide students with different learning opportunities, textbook comparison studies may help explain the differences in student success (Zhu & Fan, 2006). It is therefore not surprising that textbook comparison studies have become popular in recent years. These studies particularly use textbooks from high scoring countries in international exams such as the TIMSS or PISA namely China, South Korea, Japan, Taiwan, Singapore, and Finland. Most of these studies have analyzed textbooks with respect to their contents or problem types (Hong & Choi, 2014). There is a limited number of studies comparing Turkish and Singapore textbooks. These studies are summarized in Table 1.

Table 1. Studies comparing Turkish and Singapore textbooks and the content of the studies

Author	Year	Countries	Content
Erbaş et al.	2012	Turkey, Singapore, USA	Design characteristics
Sağlam and Alacacı	2012	Turkey, Singapore	Content, organization, and presentation style (Quadratics)
Özer and Sezer	2014	Turkey, Singapore, USA	Problem type
Bütüner	2019	Turkey, Singapore	Problem Analysis (division in fractions)
Toprak and Özmantar	2020	Turkey, Singapore	Worked-examples and questions posed (potential cognitive demand, reasoning and proof)
Bütüner	2020	Turkey, Singapore	Instructional content (division in fractions)

Differently from these previous studies, the present study compares Turkish and Singaporean textbooks with respect to their instructional contents on a difficult topic for most students: the concept of angle. There is a limited number of comparative studies on angle in the literature. Park (2015) compared the teaching contents for angle and measure of an angle in Korean and Japanese mathematics textbooks. The results show that it is necessary to reconsider the way of the definition of angle. Kim (2018a) investigated the definitions of angles in the past Korean Elementary Mathematics Textbooks. The results show that textbooks treat angles solely as a static concept. In another study, Choi, Kim, and Kwon (2019) examine angle-related contents and learning process and then look at the perspectives and the size aspects of angle in detail. Singapore, U.K., Australia, and U.S. were selected as comparable countries in this study. The results show that the dynamic definition of the angle is described later and less in the Korean curriculum when compared to other countries.

While considering that textbooks are primary sources used by teachers, possessing well-designed textbooks including correct and complete definitions, examples, representation modes, teaching tools and problems is one of the effective factors for students to have a correct conceptual image (Bingölbali, 2016, p. 140). The present study examines the definition of the concept of angle in Turkish and Singaporean textbooks, its representation (dynamic vs static), the tools used in the teaching of the angle and their purposes, and the aspect of the angle emphasized in problems (dynamic vs static). The findings of this study were used to make recommendations to curriculum designers in Turkish and Singaporean Education Ministries so as to guide them in amending the deficiencies in textbooks. The recommendations for improving textbooks will guide curriculum developers in overcoming the deficiencies of textbooks. Different textbooks can offer different learning opportunities to students and help explain the differences between students' success levels (Reys, Reys, & Chavez, 2004; Zhu & Fan, 2006). Therefore, the results of this study may give an idea about the performances of Turkish and Singaporean students, who will take international exams in future years, on the concept of angle. The research questions are as following:

- How is the concept of angle defined?
- How is the angle represented?
- What learning tools are used when teaching the concept of angle?
- What are the purposes of using these instructional tools in the process of teaching?
- What aspect of the angle is emphasized in problems related to the concept of angle?

The Concept of Angle: Its Significance, Definitions and Representations

The angle is an important concept in geometry as it is one of the most basic and fundamental geometry concepts. The concept of angle is always there regardless of the concept being studied (For instance, when two lines intersect or when we examine a polygon or a polyhedral) (Argün, Arıkan, Bulut, & Halıcıoğlu, 2014). Angles are also used in various other areas such as engineering, architecture, land measurement, geology and physics. In high school mathematics, students need knowledge of angles to solve various mathematical problems. Indeed, the problems that students encounter regarding the concept of angle also lead to other problems in future topics (trigonometric functions, etc.) (Moore, 2013).

Henderson and Taimina (2005) list the following conceptions of angle: angle as a geometric shape, union of two rays with a common end point (static); angle as movement; angle as rotation (dynamic); angle as measure; and, amount of turning (also dynamic). Older students may be able to conceptualize angles in turns, ray pairs, or regions, but may not be able to relate them. Younger students may be able to conceptualize certain angle situations in terms of ray pairs or regions, but may not conceptualize turns in terms of angles at all (Mitchelmore & White, 1998, p.5). Kim (2018b) investigated angle concepts and introduction methods of angles in textbooks. Her findings show that textbooks treat angles solely as a static concept. Kim (2018b) stated that treating angles solely as a static concept in textbooks leads to certain difficulties for students in understanding the concept. Bütüner and Filiz (2017) investigated high achievers' erroneous answers and misconceptions on the angle concept. According to the results, many students assumed that the size of an angle depends on the radius of the arc marking the angle and the area of the sector. 32% of students in this study were unable to recognize a 180° angle.

Presenting the angle as a static concept may lead to certain difficulties for students in understanding the concept. For instance, they may come to think that the arm length of the angle increases with its degree, and may therefore find it hard to grasp angles greater than 360°, and some students can not recognize 0° and 180° angles (Hansen, 2017; Keiser, 2004). Clausen-May (2005) stated that the angle is usually emphasized not as a measurement (dynamic) but only as a figure (static). In the static representation, the concept of kinesthetic angle (as motion) is lost, thus blurring the true meaning of the concept of angle. In order to avoid such problems, students should learn the angle both as a dynamic and static concept (Barmby, Bilsborough, Harries, & Higgins, 2009; Clausen-May, 2008; Clements, Wilson, & Sarama, 2004; Crompton, 2013; Mitchelmore & White, 2000). Clausen-May (2005) states that although it is not possible to show motion on paper, a directed arrow can be used as its representative. Three different representations of an angle are shown in Figure 2. Textbooks should therefore include content that enables students to learn the angle both as a dynamic and static concept.

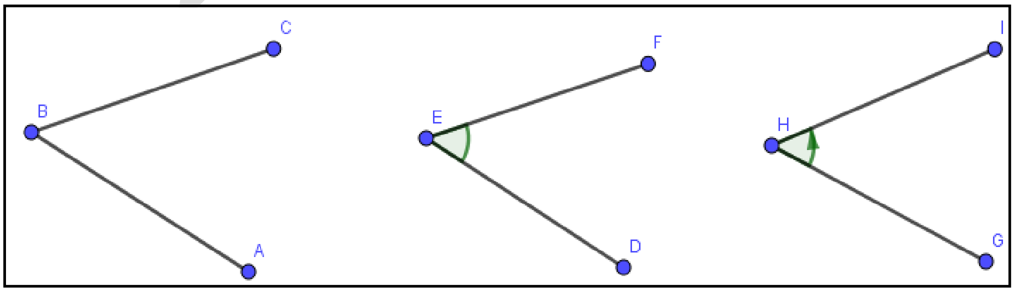


Figure 2. Three different representations of an angle (from left to right: static-static-dynamic)

Tools Used in Teaching the Concept of Angle

The instruction of the angle may involve dynamic mathematics software (Clements & Battista, 1990; 1994; Crompton, 2013; Kaur, 2020), various daily life tools (Clements & Burns, 2000; Fyhn, 2007; Mitchelmore, 1998; Mitchelmore & White, 2000), body movements (Fyhn, 2006; Smith, King, & Hoyte, 2014) and out-of-class learning experiences (Munier, Devichi, & Merle, 2008).

In studies conducted in the late 80s, the software Logo was used extensively in teaching the concept of angle (Clements & Battista, 1989; 1990; Clements, Battista, Sarama, & Swaminathan, 1996; Clements & Burns, 2000; Simmons & Cope, 1990). The results of previous studies have shown that Logo-aided learning environments may be effective in students' understanding of the dynamic definition of the angle and their skills for predicting the measurement of a given angle. Also, Logo-aided learning environments prevent the emergence of misconceptions. The literature also includes studies reporting the positive effects of dynamic geometry software such as Sketchpad (Crompton, 2013; Tieng & Eu, 2014) and Geogebra (Baya'a, Daher, & Mahagna, 2017; Boo & Leong, 2016) in teaching the concept of angle.

Many daily tools (wheels, doors, scissors, fans, signposts, hills, crossroads, tiles and walls) may be used when teaching this concept (Mitchelmore, 1998; Mitchelmore & White, 2000). The first four tools are suitable for teaching the dynamic definition of the angle, while the last five tools are suitable for teaching the static definition of the angle. Besides, tools such as doors or scissors give the opportunity to display limited turns, while tools such as wheels, fans, ventilators and wind chimes give the opportunity to demonstrate unlimited turns. While teaching the concept of angle, learning objects such as geometry strips or protractors may also be used in addition to daily life tools. In Figure 3, the red strip makes a counter-clockwise quarter turn. If a full turn is 360 degrees, 1 quarter turn equals to 90 degrees. As can be seen, geometry strips help establish a relationship between the concepts of angle and fraction (part-whole relationship) when teaching angle as a dynamic concept (Clausen-May, 2008, p. 6-7).

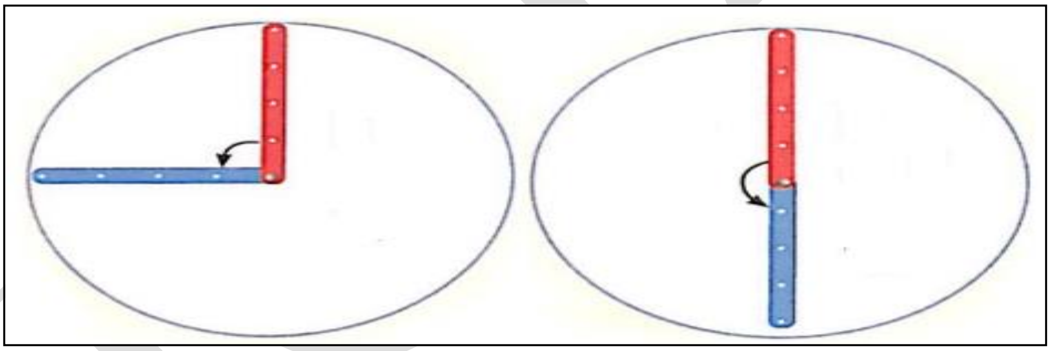


Figure 3. The use of geometry strips in angle instruction

METHOD

Textbooks Used in the Study

In this study, selected Singaporean (Ming, 2016a, b) and Turkish mathematics (Genç, Güleç, Şahin, & Taşcı, 2019; Kayapınar, Şahin, Erdem, & Leylek, 2019) textbooks were compared. Textbooks from these two countries were selected because these countries represent different levels of performance on the TIMSS. While 4th-grade Singaporean students performed well, 4th-grade Turkish students performed below the TIMSS scale average. In Turkey, Primary Education involves the education and training of children in the age group of 6 to 10. Primary education is compulsory for all citizens. It is free in state schools and lasts four years (1st, 2nd, 3rd, and 4th grades). In Singapore, compulsory education includes six years of primary school (ages 6-12), four years of secondary school, and one to three years of post-secondary school. 3rd and 4th grade levels are primary school levels in both countries. The concept of angle is introduced in the 3rd grade in both countries, and continued in the

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ISSN: 1300 – 915X
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International Online Journal of Primary Education 2021, volume 10, issue 1

4th grade. The objectives regarding the teaching of the angle as a dynamic concept in Turkish and Singaporean primary mathematics curricula are given in Table 2.

Table 2. The Objectives regarding the teaching of the angle in Turkish and Singaporean primary mathematics (MONE, 2018, p. 41, 48; MES, 2012, p. 46-51).

Country	Grade	Objectives
SIN	3	Students should have opportunities to: **Illustrate an angle as an amount of turning using geostrips / riveted straws and use language such as "acute angle" and "obtuse angle" to describe angles.* **find angles in the environment and use a "paper right angle" to identify right angles, angles greater than a right angle and angles smaller than a right angle.
TUR	3	**recognizes the angle and gives examples in the environment
SIN	4	Students should have opportunities to: **Associate the amount of turning (rotation), clockwise or anti-clockwise, with an angle measured in degrees* *1/4 turn is 90; 1/2 turn is 180* **Estimate before measuring angles using a protractor **Draw angles using a protractor **Find the angles (in degrees) between two 8-point compass directions
TUR	4	** Determines the rays that make up the angle and the corner, names the angle and shows it with a symbol ** Measures angles in non-standard units and explains the need for standard measuring units ** Measures angles with standard angle measuring tools and determines them as acute, obtuse and right angles. *Can form an angle using standard measurement tools: *"Realizes with the help of an angle measurement tool (protractor, goniometer, etc.) that an angle is formed by rotating a ray around an endpoint"*

*The objectives regarding the teaching of the angle as a dynamic concept in Turkish and Singaporean primary mathematics

The study used the 3rd and 4th grade mathematics textbooks taught in Turkish schools and published by the Turkish Education Ministry, and the Singaporean mathematics textbooks entitled Targeting Mathematics 3B-4A. "My Pals are here" and "Targeting Mathematics" are commonly used books at schools in Singapore. These two books are similar in terms of the teaching content of the angle concept. In Turkey, textbooks are decided by the Ministry of Education. The Turkish Ministry posts the textbooks used at schools on the webpage http://www.eba.gov.tr/ekitap. The Turkish textbooks examined in this study were obtained from this website. These textbooks were used on students in the same age groups that correspond to primary grades 3, and 4 and all books were in use in the 2019-2020 academic year in their respective countries. Two field specialists identified the grade levels where the textbooks introduced the concept of angle, and then noted the relevant page numbers. Following this, the analysis (coding) processes started.

The Theoretical Framework for the Analyses

In the study, vertical analyses were performed on the angles content in Turkish and Singaporean mathematics textbooks. As complementary to horizontal analysis, the vertical analysis offers in-depth understanding of mathematical content (Hong & Choi, 2014; Yang & Lin, 2015). For example, this analysis can reveal how the contents of a relevant mathematical concept are presented, what representation styles are used, what tools are used in concept instruction and their purposes. The scope of vertical analyses is given in Table 3.

Table 3. Scope of vertical analyses

Vertical Analysis Indicators
• How has the concept of angle been defined? • How is the angle represented? • What learning tools are used when teaching the concept of angle? • What are the purposes of using these instructional tools in the process of teaching? • What aspect of the angle is emphasized in problems related to the concept of angle?

Data analysis

In coding the angle as a static or dynamic concept, the opinions in the literature are taken as basis (Clausen-May, 2005; Henderson & Taimina, 2005; Mitchelmore & White, 2000; Wilson & Adams, 1992). The coding criteria for the angle as a dynamic and static concept is given in Table 4. The analyses were performed by two academics in the field of mathematics education. The coherence between the intercoders was calculated using the formula developed by Miles and Huberman (1994). The agreement for each category varied between 98% and 100%. They debated any differences and made the final decision based on the views of yet another researcher. The findings were supported with direct quotes from the textbooks.

Table 4. Coding criteria for the angle as a dynamic and static concept

Indicators	**Dynamic**	**Static**
How has the concept of angle been defined?	angle as movement; angle as rotation; angle as measure; and, amount of turning (Henderson and Taimina, 2005)	angle as a geometric shape, union of two rays with a common end point (Henderson and Taimina, 2005)
How is the angle represented?	a directed arrow Clausen-May, 2005)	arrow with no direction; only as two rays-a corner; no arrow (Clausen-May, 2005)
What learning tools are used when teaching the concept of angle?	Wheels, doors, scissors, fans, geometry strips, riveted straws, clocks etc. (Mitchelmore & White, 2000).	Signposts, hills, crossroads, tiles, table and walls etc. (Mitchelmore & White, 2000).
What are the purposes of using these instructional tools in the process of teaching?	An emphasis on the dynamic aspect of the concept of angle (movement, amount of rotation) (Henderson and Taimina, 2005).	did not make any emphasis on the dynamic aspect of the concept of angle (movement, amount of rotation) (Henderson and Taimina, 2005)
What aspect of the angle is emphasized in problems related to the concept of angle?	Students are expected to solve, allowing them to realize that an angle *is formed by rotating a ray around an endpoint* and to associate the degree of the angle with the concept of fraction (part-whole) etc. (Wilson & Adams, 1992; Clausen-May, 2005, 2008).	Students are not expected to solve, allowing them to realize that an angle *is formed by rotating a ray around an endpoint* (Wilson & Adams, 1992; Clausen-May, 2005, 2008).

FINDINGS

Table 5 presents the findings concerning how the concept of angle is defined in Turkish and Singaporean textbooks, how it is represented, what instructional tools are taught to teach it, what purposes the instructional tools have, and what aspect (dynamic-static) is emphasized in problems concerning the concept of angle.

Table 5. Vertical analysis findings

Indicators	TAR3B	TAR4A	TR3	TR4
How is the concept of angle defined?	Both Static and Dynamic	Both Static and Dynamic	Static	Static
How is the angle represented?	A directed arrow; arrow with no direction	A directed arrow; arrow with no direction	Arrow with no direction; only as two rays-a corner; no arrow	Arrow with no direction; only as two rays-a corner; no arrow
What instructional tools are used to teach the concept of angle?	Daily life tools, Learning objects (geometry strips, riveted straws), Body movements	Daily life tools, Learning objects (geometry strips, riveted straws and protractor), Body movements	Daily life tools	Daily life tools, Learning objects (protractor)
What purpose do the instructional tools have?	To teach the static and dynamic aspects of the angle	To teach the static and dynamic aspects of the angle	To teach the static aspect of the angle	To teach the static aspect of the angle
What aspect of the angle is emphasized in angle problems?	Both static and dynamic	Both static and dynamic	Static	Static

How is the concept of angle defined?

The angle is defined in Turkish textbooks as "a plane formed by two rays with a shared endpoint (TR3, p. 229)" and "a plane formed by the closed edges of two intersecting rays (TR4, p. 207)". Singaporean textbooks state that "an angle is formed when two straight lines meet at a common point. The size of an angle depends on the amount of turning (TAR3B, p. 60-61, TAR4A, p.116)". The Singaporean book also emphasizes the dynamic aspect (quarter turn) when defining the right angle (Figure 4). As can be seen, Turkish textbooks define the angle as a static concept while Singaporean textbooks define it as both a static and a dynamic (amount of rotation) concept.

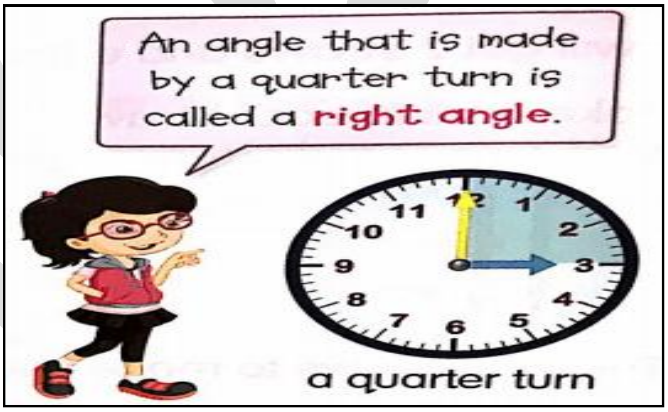

Figure 4. Definition of the right angle in the Singaporean book (TAR3B, p. 62)

How is the angle represented?

The definitions in the textbooks are reflected in angle representation. Singaporean textbooks represent the angle as an "arrow with direction" or an "arrow with no direction" (Figure 5). Turkish textbooks represent the angle as an "arrow with no direction" or "only two rays – a corner and no arrow" (Figure 6). Therefore, while the angle exists in Singaporean textbooks as both a dynamic and a static concept, it remains a static concept in Turkish books. In conclusion, it may be stated that the textbooks from both countries include representations in line with the definitions of the angle that they adopt.

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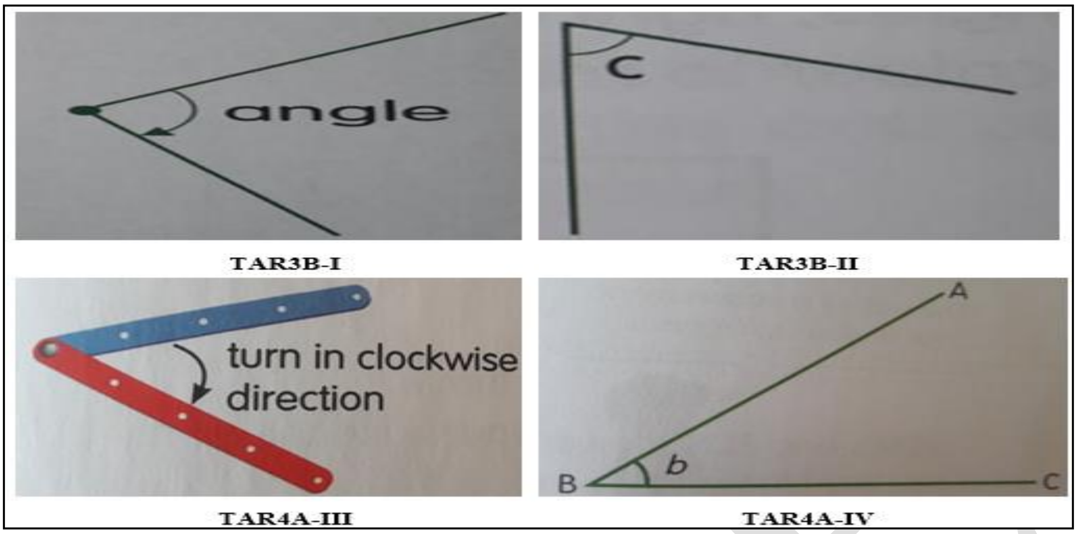

Figure 5. The dynamic (“arrow with direction,” TAR3B-I, p.60, TAR4A-III, p. 123) and static (“arrow with no direction,” TAR3B-II, p. 68, TAR4A-IV, p. 117) representations of the angle in the Singaporean textbook

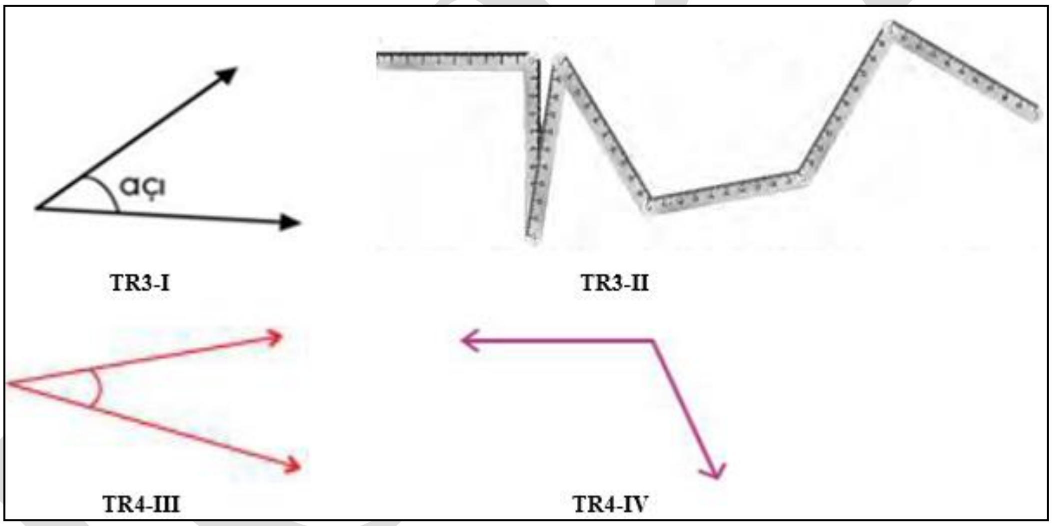

Figure 6. The static representation of the angle in the Turkish textbook (“arrow with no direction,” TR3-I, p. 229, TR4-III, p. 209 or “two rays-a corner and no arrow,” TR3-II, p. 230, TR-IV, p. 215).

What instructional tools are used to teach the concept of angle and what purpose do the instructional tools have?

The instructional tools used in Singaporean textbooks to teach the concept of angle include body movements, protractor, geometry strips, and daily life tools (fans, computer, traffic signs, scissors, stairs, clock, stapler, and compass). Singaporean textbooks teach the angle as both a static and dynamic concept by using these tools. As can be seen in Figure 7, as Singaporean textbooks introduce the concept of angle, they make use of body movements and riveted straws, thus emphasizing the dynamic aspect of the angle.

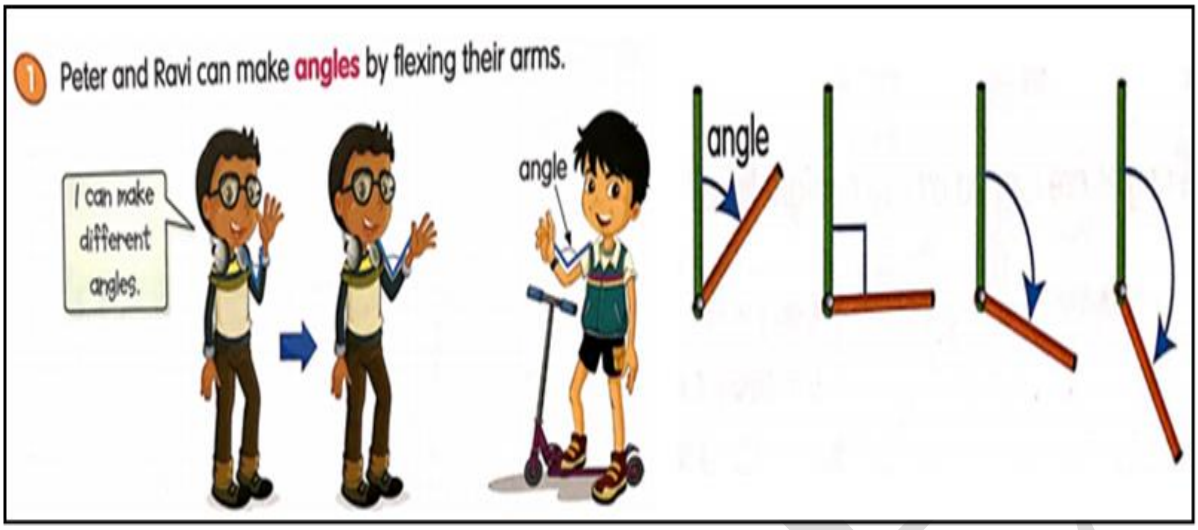

Figure 7. Teaching the angle as a dynamic concept in the Singaporean textbook via body movements and riveted straws (TAR3B, p. 60-61)

After emphasizing the dynamic aspect of the angle by using body movements, geometry strips and riveted straws, Singaporean textbooks pass on to the static aspect of angle by using daily life tools (fans, computer, traffic signs, scissors, stairs) (Figure 8). Visuals pertaining to these tools are given, asking students to find the angles in them. Even though a fan, notebook computer and scissors are suitable tools to teach the angle as a dynamic concept and they exist in Singaporean textbooks, they were there only as examples and did not make any emphasis on the dynamic aspect of the concept of angle (movement, amount of rotation). In other words, the angle was not treated as a dynamic concept alone (movement, amount of rotation).

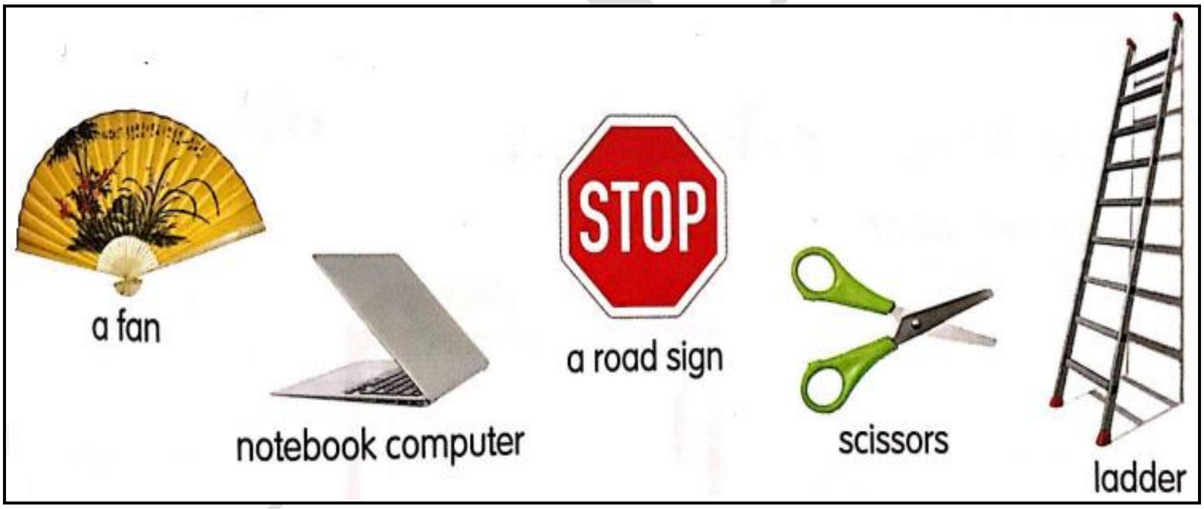

Figure 8. Tools used in the Singaporean textbook when teaching the angle as a static concept (TAR3B, p. 61).

The 4th grade Singaporean book also teaches the angle as a dynamic concept. As shown in Figure 9, the book uses geometry strips and makes a connection between the concepts of angle and fraction (part-whole relationship), thus emphasizing the dynamic aspect of the angle. It is easy to see in Figure 9 that 1 full turn corresponds to 360 degrees, while 1 quarter turn corresponds to 90 degrees, two quarter turns to 180 degrees, and three quarter turns to 270 degrees.

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2021, volume 10, issue 1

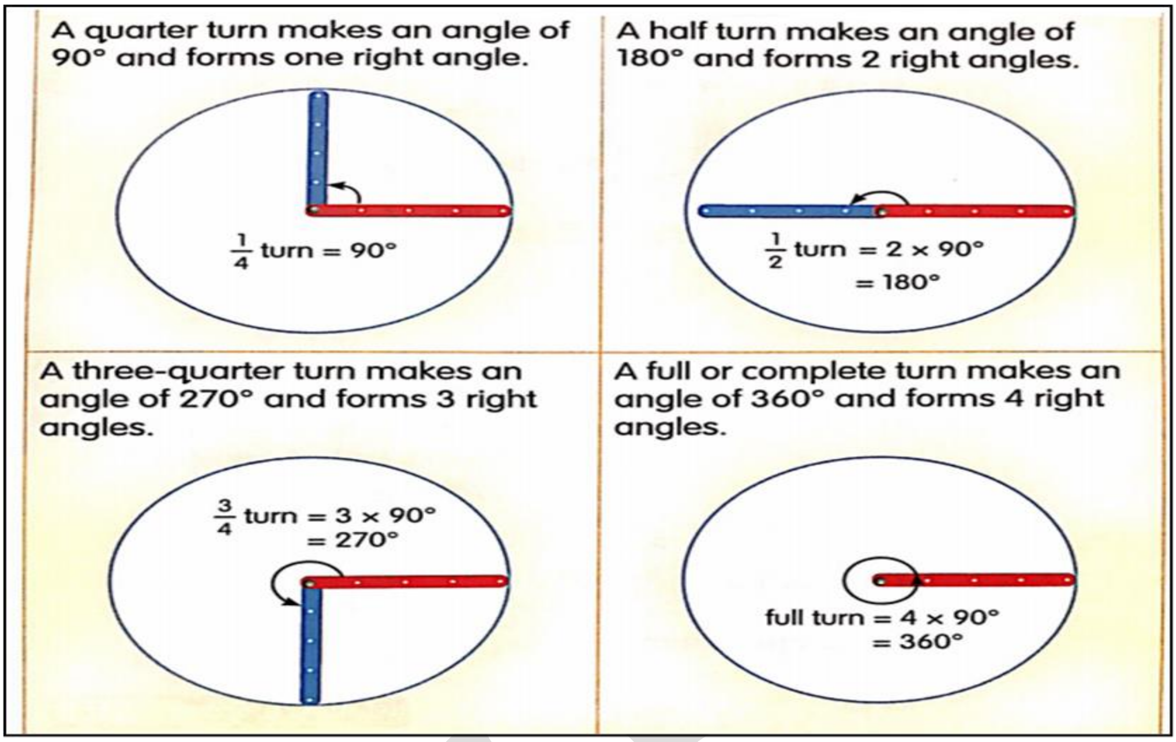

Figure 9. Teaching the angle in Singaporean books as a dynamic concept by using geometry strips (TAR4A, p. 123).

In Turkish textbooks, the teaching of the concept of angle made use of a protractor and certain daily life tools ("hanger, frame, chair" TR3, p. 229; "door, classroom desk, scissors, clock" TR4, p. 207) (Figure 10). Hanger, frame, chair and classroom desk are suitable tools to teach the angle as a static concept. Even though door, scissors and clocks are suitable tools to teach the angle as a dynamic concept and they exist in Turkish textbooks, they were there only as examples and did not make any emphasis on the dynamic aspect of the concept of angle (movement, amount of rotation). Thus, Turkish textbooks use these tools to teach the angle only as a static concept.

Figure 10. Tools used in the Turkish textbook to teach the concept of angle (TR3, p. 229; TR4, p. 207).

What aspect of the angle is emphasized in angle problems?

While the Turkish textbooks only include problems emphasizing the static aspect of the angle, Singaporean textbooks include problems including both the static and dynamic aspects. The number of problems with and without solutions in the two textbooks that treat the angle as a static and dynamic concept are given in Table 6.

Table 6. The Number of Problems in the Textbooks that Treat the Angle as a Static and Dynamic Concept

Problem Type	Turkish Textbooks		Singaporean Textbooks	
	TR3	TR4	TAR3B	TAR4A
Number of solved problems in the textbooks that treat the angle as a static concept	0	1	1	6
Number of unsolved problems in the textbooks that treat the angle as a static concept	3	5	10	5
Number of solved problems in the textbooks that treat the angle as a dynamic concept	0	0	0	5
Number of unsolved problems in the textbooks that treat the angle as a dynamic concept	0	0	0	3

Turkish textbooks do not include any problems that treat the angle as a dynamic concept. In contrast, Singaporean textbooks include 5 problems with solutions and 3 questions that treat the angle as a dynamic concept. The first problem in Appendix 1 is a daily life problem that students are expected to solve, allowing them to realize that an angle is formed by rotating a ray around an endpoint and to associate the degree of the angle with the concept of fraction (part-whole) (TAR4A-I). The second problem expects students to classify given angles as straight, obtuse or acute (TAR4A-II). Therefore, the first problem emphasizes the dynamic aspect of the angle, while the second problem emphasizes its static aspect. Appendix 2 shows a problem from the Turkish textbooks emphasizing only the static aspect of the angle. Students are not expected to solve, allowing them to realize that an angle is formed by rotating a ray around an endpoint. This problem expects students to show the angles in the images and classify the given angles as a straight, obtuse, acute or right angle. Other problems from Turkish and Singaporean textbooks on the concept of angle are given in Appendix 3, 4, and 5.

DISCUSSION and CONCLUSION

The present study investigated how Turkish and Singaporean primary mathematics textbooks define the concept of angle, how they represent angles, what tools are used with what purpose in teaching angles, and what aspect of the angle is emphasized in problems regarding the concept of angle. The concept of angle is introduced in both countries in the 3rd grade and continued in the 4th grade. The primary mathematics curricula of both countries include objectives concerning “teaching the angle as a dynamic concept.

The vertical analysis showed that Turkish textbooks defined the angle as a static concept, and Singaporean textbooks defined it as both a static and dynamic concept. The definitions of the concept of angle included in the textbooks reflect on the representation of the angle, the aims for using the selected instructional tools in angle instruction, and the problem structure in the textbooks. In Turkish textbooks, the angle is represented with “an arrow that does not show direction” or “two rays-a corner and no arrow.” In Singaporean textbooks, it is represented with “an arrow that shows direction” or “an arrow that does not show direction.” Therefore, in Singaporean books, the angle was treated both as a dynamic and static concept, while Turkish books only treated it as a static concept. According to Clausen-May (2005), the representation in Turkish books is a move away from the true meaning of the concept of angle. She continues that even when it seems impossible to indicate the movement on a piece of paper, a directed arrow may be used for representation. Turkish textbooks teach the concept of angle only by using a protractor and various daily tools (scissors, clocks, chairs, frames, hangers, doors). These textbooks do not emphasize the dynamic aspect of the concept of angle (motion, amount of rotation); they are merely included in the textbook as examples. Singaporean textbooks, on the other hand, use body movement, protractor, geometry strips, and daily materials (fans, computers, traffic signs, scissors, stairs, clocks, staplers, and compass) as tools in teaching the concept of angle. In the 3rd grade Singaporean book, as an introduction was made to the concept of angle, it was treated as a dynamic concept by using body movements and riveted straws. In the 4th grade Singaporean textbook, geometry strips were used to make a connection between the concepts of angle and fraction (part-whole relationship) and to emphasize the dynamic aspect of the angle. The literature states that teaching the angle as a dynamic concept can help make use of learning objects (such as geometry strips). Considering that a full turn in a circle equals to 360 degrees, the students may be asked to think what a quarter turn or two quarter turns would equal to in degrees. In this way, students can have the opportunity to learn the angle as a dynamic concept by mobilizing their existing knowledge and using the part-whole meaning of fraction (Clausen-May, 2005, 2008; Wilson & Adams, 1992). There is no emphasis or activity on the use of dynamic geometry software in the textbooks of either country. The literature also includes studies reporting the positive effects of dynamic geometry software (Clements et al., 1996; Clements & Burns, 2000; Crompton, 2013; Boo & Leong, 2016). In GeoGebra software, students may be asked to rotate a ray around its endpoint by using a 'slider' and 'rotate around point.' Therefore, students may gain an awareness of the dynamic definition of the angle concept (Bütüner & Filiz, 2017).

The angle was treated as a static concept in all problems in Turkish textbooks. In contrast, the 4th grade Singaporean book included eight problems that treat the angle as a dynamic concept. Therefore, it was concluded that in both Turkish and Singaporean textbooks, there was harmony between the instructional contents of the concept of angle and the angle problems in the textbooks. The results show that Turkish textbooks need to be enriched with contents regarding teaching the angle as a dynamic concept. Angle problems that treat the angle as both a dynamic and a static concept should be added to the Turkish textbooks in a balanced way. When the angle is treated solely as a static concept, students may end up thinking that an angle may increase when its arm length increases (Clausen-May, 2008) and fail to recognize angles of 0 degrees or those larger than 360 degrees as they do not see an openness (Barmby et al, 2009, p. 147). Therefore, many studies have attempted to teach the angle as a dynamic concept (Clements & Battista, 1989; 1990, Clements et al., 1996; Clements & Burns, 2000; Mitchelmore, 1998; Mitchelmore & White, 2000; Simmons & Cope, 1990).

Other results from the findings include that, in contrast to the case in Singapore, there is a mismatch in the contents of Turkish textbooks and the objectives of the primary mathematics curriculum, and that the contents of Singaporean textbooks offer students more learning opportunities than Turkish textbooks regarding the concept of angle. These results also suggest that as the concept of angle is introduced in Turkish and Singaporean textbooks, students should be given learning opportunities to assist them in constructing the dynamic definition of the concept, rather than directly giving them a static or dynamic definition.

Limitations and Implications for Future Research

According to the findings, Singaporean students can learn from textbooks that angle is both a static and a dynamic concept. Therefore, they may not have difficulties or misconceptions about angle and related concepts (such as trigonometry) in their following years. Turkish students, on the other hand, learn angle from textbooks only as a static concept, so they may have difficulties and misconceptions about angle [e.g., students may think that the arm length of an angle affect its degree, (Clausen- May 2005); they may think that the degree of an angle is related to the area surrounded by the arms of the angle (Keiser, 2004); they may misread an angle on the protractor, (Hansen, 2017) and may have further problems on related concepts (such as trigonometry)]. The findings have shown that Singaporean students may display a better performance than Turkish students in problems based on the concept of angle. Therefore, the results of this study give an idea about the performances (in problems involving angles, polygons, trigonometry, slope etc.) of Turkish and Singaporean students, who will take international exams in future years.

The generalization of these results calls for caution as the findings come from two mathematics textbooks from either one of the two countries (Yang, 2018; Wijaya, Van den Heuvel-Panhuizen, &

Doorman, 2015). A well or poorly designed textbook may find life in the hands of a well-equipped teacher. Such a teacher may spot the deficiencies in textbooks and enrich classes with contents that enable students to better grasp mathematical concepts. Future researchers may study whether Turkish and Singaporean teachers use textbooks when teaching the concept of angle, how they utilize these books, how they teach the concept of angle, and what type of problems they use in their classes regarding the concept of angle.

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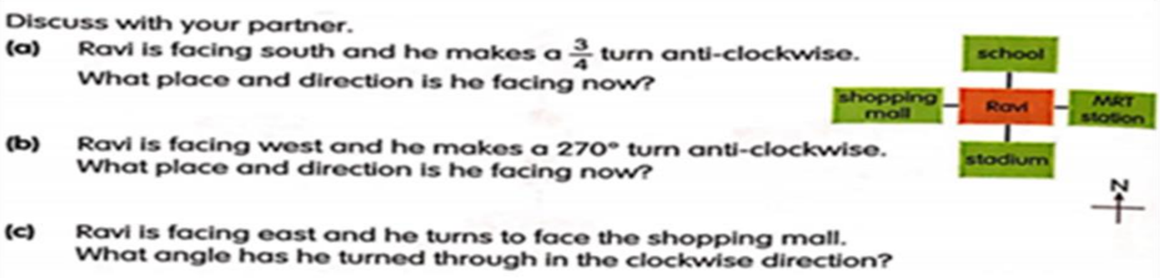
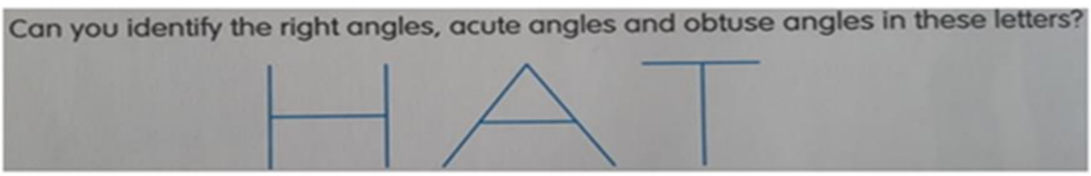
Appendix 1. Problems from the Singaporean textbook in line with both the dynamic (TAR4A-I, p.128) and static aspects of the angle (TAR4A-II, p. 116).



TAR4A-I



TAR4A-II

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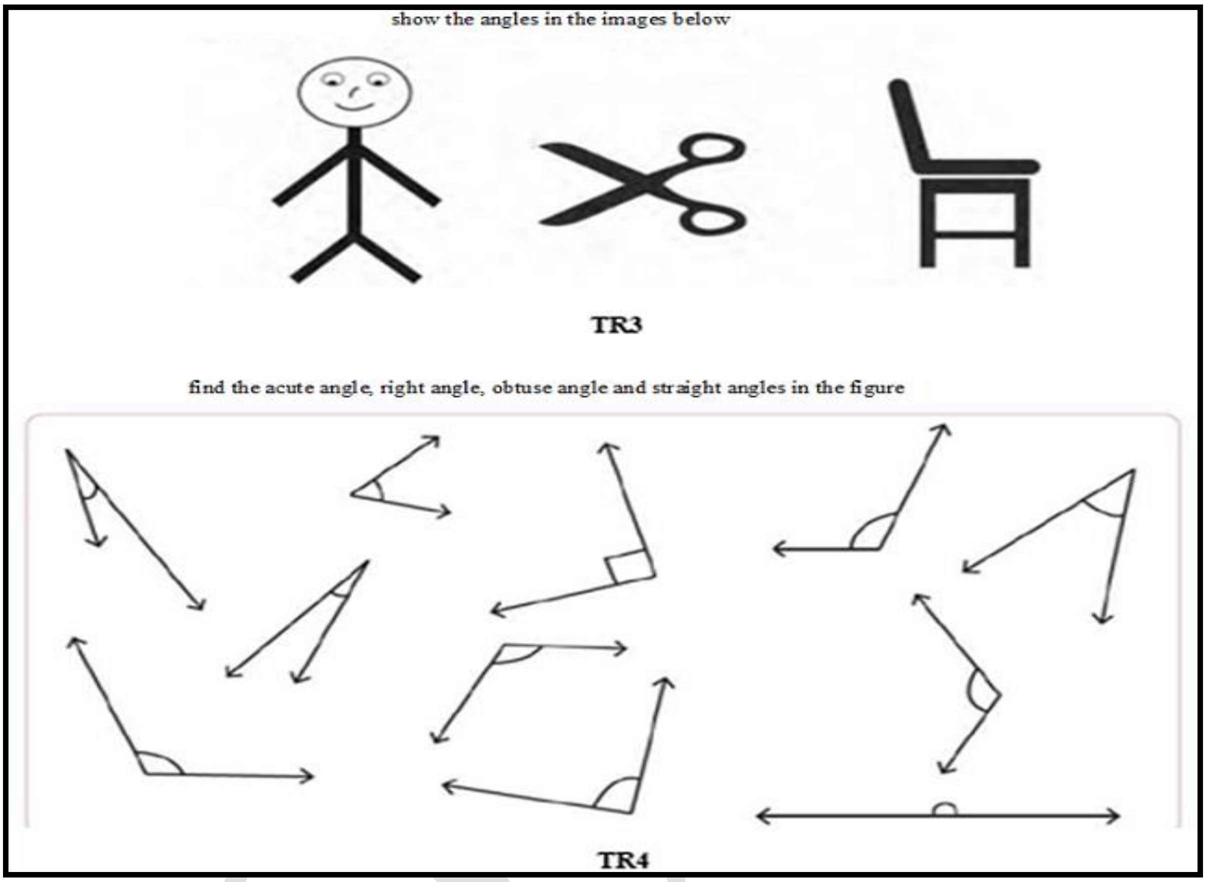
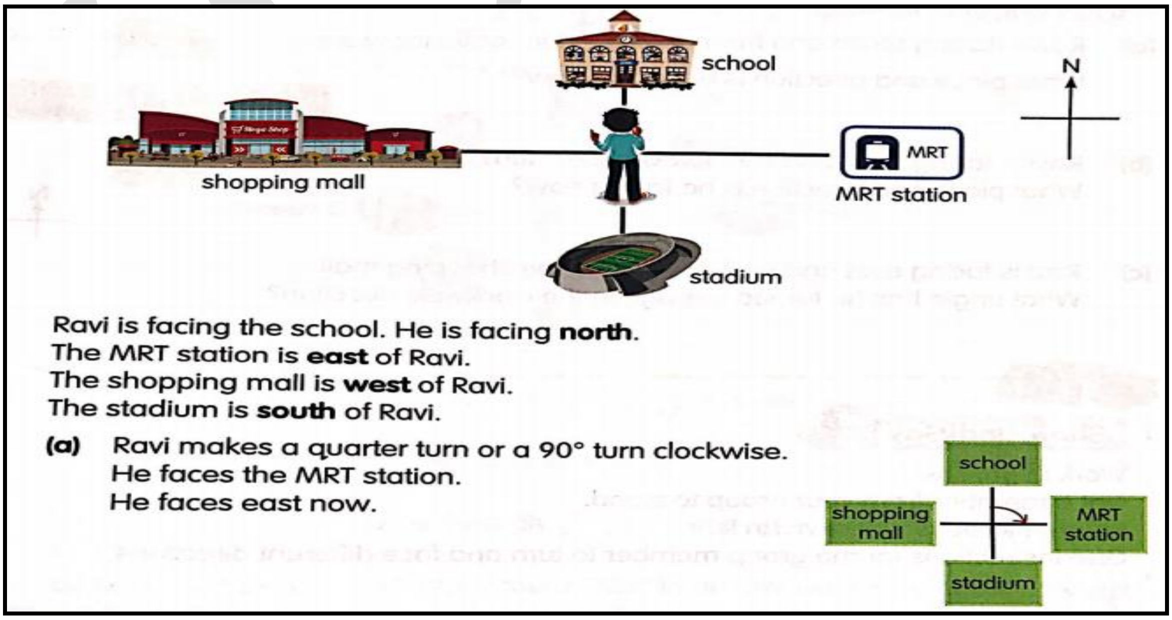
Appendix 2. Problem from the Turkish book in line with the static aspect of the angle (show the angles in the images below, TR3, p.233, find the acute angle, right angle, obtuse angle and straight angles in the figure) (TR4, p. 212)



Appendix 3. A problem with solution in Singaporean textbook (angle as a dynamic concept) (TAR 4A, p. 127)



ISSN: 1300 – 915X
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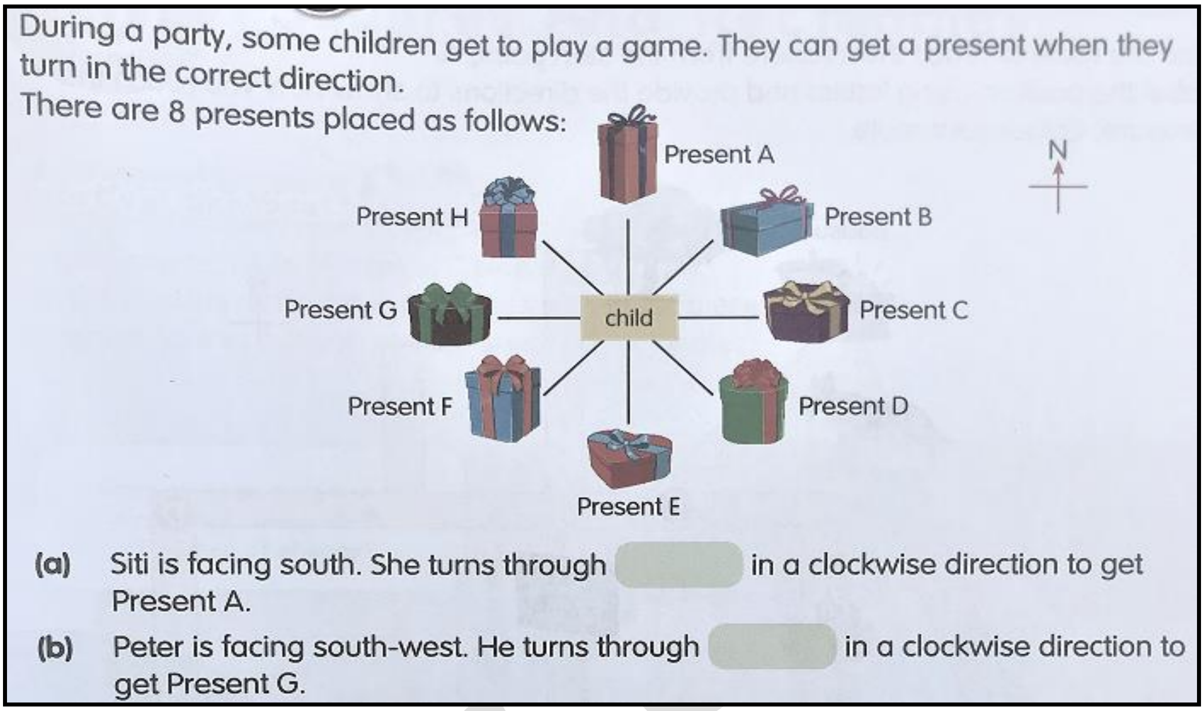
Appendix 4. A problem without solution (angle as a dynamic concept) (TAR 4A, p. 129).


During a party, some children get to play a game. They can get a present when they turn in the correct direction.
There are 8 presents placed as follows:

(a) Siti is facing south. She turns through ______ in a clockwise direction to get Present A.

(b) Peter is facing south-west. He turns through ______ in a clockwise direction to get Present G.

Appendix 5. A problem without solution (measure the angles given below and write down their values) (angle as a static concept) (TR 4, p. 215)