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Review Article

BASIC PARAMETERS OF LENS DESIGN

Melike Güzin SEMERCİOĞLU

Gümüşhane University, Department of Medical Services and Techniques, Opticianry Program, Gümüşhane, Turkey
Corresponding author; melikesemercioglu@gumushane.edu.tr

Abstract: *The development of technology has increased computing power and the ability to make scientific calculations has also developed rapidly. One of the areas affected by the developing technology is optics. The lens design is a complex field of study that includes many infrastructures such as lens optimization, ray trace analysis, lens drawing, and modulation transfer function calculations. In this study, the basics of optical system design are presented. The calculation of complex optical systems is a demanding and extensive task. The reason for the complexity in lenses is due to the fact that all rays of wavelengths pass correctly through the image of a particular point of an object. The lens parameters include radii of curvature, thickness, air gaps, refractive indices, and dispersion forces of the glasses used for individual lens elements or the position of the aperture-limiting diaphragm or lens mount. It contains a series of formulations ordered by subject areas or topics representing typical lens design tasks and optical system design tasks. Different aspects are covered, such as the definition of starting systems and the creation, evaluation, and optimization of lenses and systems. The formulas of the most critical lens design equations are given in detail in the content of the study. It also provides an overview of basic mathematical models for calculating simple optical components such as single lenses, systems, and achromatic pairs. Such components represent the foundation of any optical system. The literature study on this study determined that studies on lens design in national publications are limited. The study differs in that it clearly articulates the fundamentals of lens design. It is thought that this study will contribute to the literature.*

Keywords*: Lens shapes, Lens planes, Optical imaging parameters, Acrotism parameters*

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1. Introduction

Optics, a branch of physics, includes developing tools, equipment, and optical systems to give direction and shape to light while examining light and vision events [1].

The calculation of complex optical systems is a demanding and extensive task. Before a lens is manufactured, it should be designed by determining the radii of curvature of its surfaces, their thickness, air gaps, diameters of various components, and the types of glass used [2]. Today, the calculation of an optical system is mainly carried out with the help of simulation tools, which significantly reduces the required work and time expenditure [3].

The reason for the complexity in lenses is due to the fact that all rays of wavelengths pass correctly through the image of a particular point of an object. That is, the lens should pass a flat object without any curvature or distortion [3]. According to the studies of scientists in the last few years, various so-called aberrations have been noticed in the wrong image formed by a lens, each of which can be created by changing the lens structure. Typical aberrations are spherical aberrations, chromatic aberrations (inherent in specific optical designs, caused by a defect in a lens or other component and causing off-axis point sources), and astigmatic and chromatic (a lens's inability to focus all colors on the same point) aberrations. All aberrations appear mixed together in any lens, and correcting (or eliminating) an aberration only improves the resulting image by the specified amount of aberration. Some aberrations can be easily corrected by simply changing the shape of one or more lens elements, while others require a radical replacement of the entire system [4,5].

The lens parameters that the designer can change are called "degrees of freedom." These parameters include radii of curvature, thickness, air gaps, refractive indices, and dispersion forces of the glasses used for individual lens elements or the position of the aperture-limiting diaphragm or lens mount (Paul and Yoder, 2006). Besides, it is also necessary to always maintain the lens's focal length. Otherwise, the relative aperture and image height will change [6]. As a result, the designer might get a good lens, but not the lens he decided to design. So every structural change to keep the focal length constant must be accompanied by another change. Also, if the lens is to be used at a fixed magnification, this magnification must be maintained throughout the design. An overview of basic mathematical models for calculating simple optical components such as single lenses and systems is considered helpful.

2. Lens shapes and general information

2.1. Lens shapes

Single lenses can be categorized in quite different ways [7]:

- Based on lens function, i.e., convergence or divergence,
- According to the material used, i.e., glass lenses, crystal lenses, liquid lenses, etc.
- According to the shape of the optically active surface

According to the last category, cylindrical lenses, toric lenses, conical lenses, aspherical lenses, and spherical lenses can be identified. At least one surface of the lenses in this category is given a spherical shape and is named depending on the curvature direction of the spherical shape [8]. In Figure 1, lens naming according to the surface shape is given.

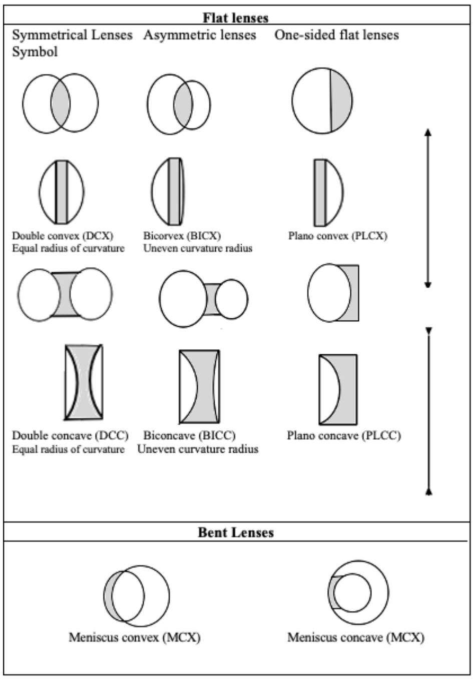

Figure 1. Different types of spherical lenses - convergent lenses and divergent lenses and their symbols

It can be seen in Figure 1 that the orientation of a given radius of curvature gives the overall shape and thus the lens type. Therefore, this orientation is of particular importance and interest for the design and manufacture of lenses. It is expressed by the algebraic sign of the radius of curvature as listed in Table 1.

Table 1. Different types of spherical lenses with marking definition of the specific radius of curvature (if arranged towards incident light as shown in figure 2) and related properties

	Sign Definition		
Lens type	**First surface (left)**	**Second surface (right)**	**Property**
Plano-convex	+	(∞)	$R_2=\infty$
Symmetrical biconvex	+	-	$R_1=R_2$
Asymmetrical biconvex (best lens form)	+	-	$R_1\neq R_2$
Positive meniscus (concave-convex)	+	+	R_1(convex)<$R_2(concave)$
Plano - concave	+	(∞)	$R_2=\infty$
Symmetrical biconcave	+	-	$R_1=R_2$
Asymmetrical biconcave	+	-	$R_1\neq R_2$
Negative meniscus (convex-concave)	+	+	R_1(convex)>R_2(concave)

Apart from radii of curvature, other parameters are needed for a complete definition of an optical lens as visualized in figure 2: center thickness (tc), lens diameter (D) and lens material (e.g., glass or optical medium as appropriate), refractive index (n), absorption coefficient (α) and *V*-number (V).

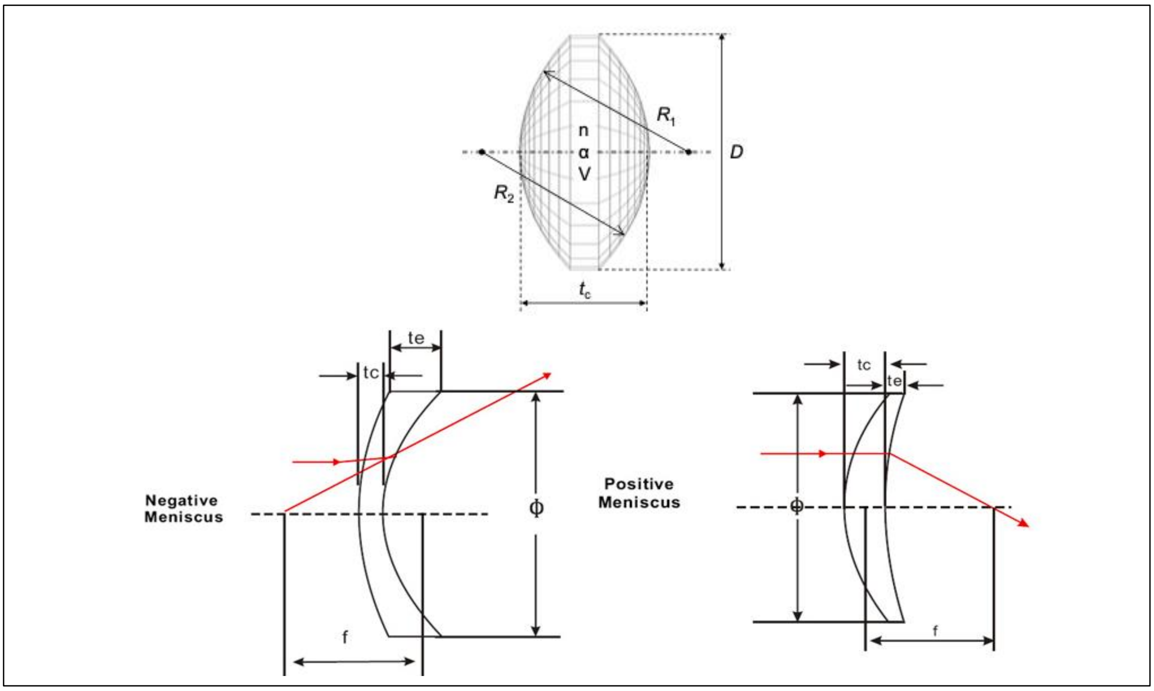

Figure 2. Lens parameters necessary for the definition of a lens: radius of curvature *R*, focal length *f*, center thickness *tc*, edge thickness *te*, diameter *D* and the lens material, refractive index *n*, absorption coefficient α, and V-number *V* [2].

For fundamental considerations in optical system design, the actual shape of a lens may not matter. Such simple calculations are sufficient to determine the primary function of the lens [9]. In addition to optical power, the focal length, including its algebraic sign, must be known to describe the behavior or function of a lens. Convergent lenses have a positive algebraic sign, whereas divergent lenses have a negative focal length. The basic behavior of the lenses is visualized by the symbols shown in table 1.

2.2. Basic planes

Any imaging optical component or system can be described by two virtual planes, called principal planes, where refraction theoretically occurs. The location of such principal planes can be established graphically based on a simple rule, which is explained as follows: after passing through a physical lens, the light beam, initially emitted parallel to the optical axis, passes the focal point and vice versa. This transformation is due to the refraction of the lens surfaces [10]. Virtual intersections should be considered when estimating the paths of all rays in front of and behind a lens or optical system, as shown in figure 3.

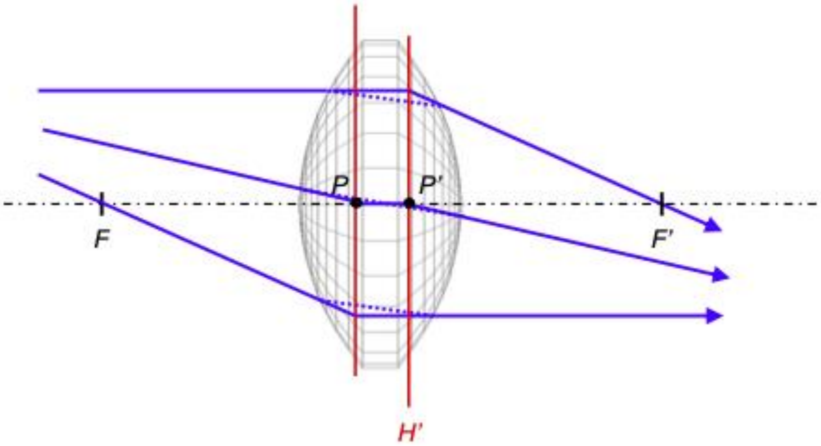

Figure 3. Graphical structure of a biconvex lens's principal plane H and H'.

Planes that join these points are called principal planes orientated perpendicular to the optical axis. Two principal planes can be defined, one on the object side (H) and the other on the image side (H'). Depending on the shape of a physical lens or system, the principal planes may even be located outside the lens material, as shown in some examples in figure 4.

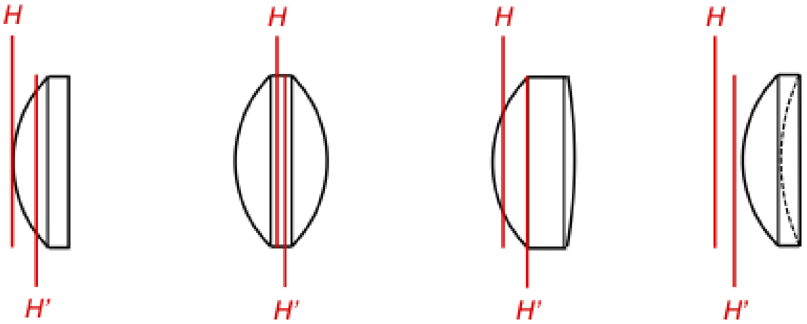

Figure 4. Qualitative visualization of the location of the principal planes for different types of converging lenses

The surface of the lens is not essential since the principal planes are effective when determining the focal length. For an idealized and simplified description of any imaging system, even just one principal plane is sufficient for estimating the relationships between the object area and the image area of an optical system.

Table 2. Overview of the main elements

Elements	Abbreviation	Description
Main point	P, P'	The intersection points of the principal plane and the optical axis (in case of axial-parallel incident light)
Corners	V, V'	Points of intersection of lens surfaces and the optical axis
Focal point	F,F'	front and rear focal point
Effective focal length	EFL	The focal length above the base plane (commonly called the focal length f)
Rear focal length	BFL	Focal length at the vertex in the image field
Front focal length	FFL	Focal length at the vertex in the object area
Node points	N, N'	Point equivalent to the main point in case of diagonal interference (incidence) of light

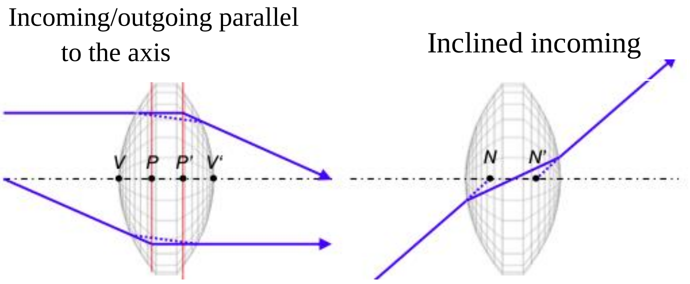

Figure 5. Main elements of the biconvex lens are listed in table 2 for axial-parallel light and inclined light

2.3. Main elements

Other than the aforementioned principal planes, more characteristic locations or points can be detected on lens surfaces or inside the lens material and at distances. These points and distances are the main elements listed in table 2 and partially illustrated in figure 5.

The main elements are of paramount importance, as any imaging system's computational and layout design is based on these parameters. For example, the radius of curvature of a physical lens comes from its focal length [7,11].

Here, it is necessary to explain the radius of curvature. The radius of curvature has a special meaning and sign rule in optical design. A spherical lens surface has a center of curvature along its optical axis away from the center. The apex of the lens surface is located on the local optical axis. The distance from the lens vertex to the center of curvature is the radius of curvature of the surface.

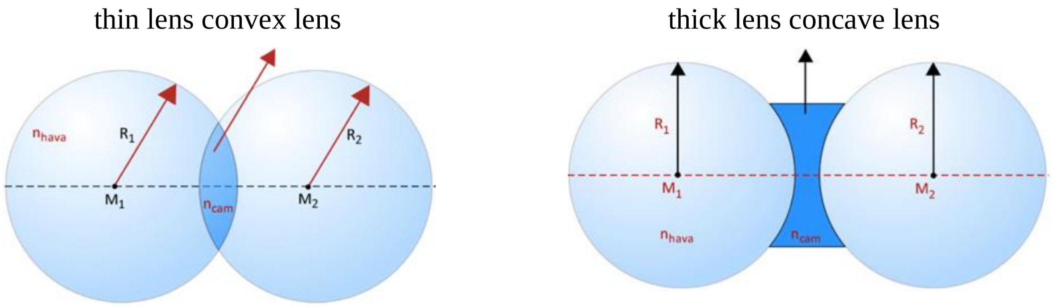

Figure 6. Determination of radii of curvature of convex and concave lenses

The sign rule for the optical radius of curvature is as follows:

- The radius of curvature is positive if the apex of the lens is to the left of the center of curvature.
- The radius of curvature is negative if the apex of the lens is to the right of the center of curvature.

3. Basic lens parameters

3.1. Lens manufacturer's equations

The lens manufacturer's equations allow specifying the necessary parameters for realizing a physical lens with a given focal length (and vice versa). As summarized in Figure 2, the basic parameters of a lens are the refractive index of the lens material (α[1] and V number[2]), the radius of curvature, and center thickness. It may turn out that the calculation of these parameters is based on a few assumptions due to the relatively high number of variables. Professional experience and intuition gain importance here. Sometimes simple economic considerations may suffice to define some of the desired parameters [11].

3.1.1 The convex lenses

In most cases, it's enough to start with a very simple approach to optical design: the convex lens. Although such a lens does not really exist in practice, it is a useful model for initial predictions and calculations of the optical system [12, 13]. By definition, the medium thickness of a convex lens is much smaller than the radius of curvature, tc < R_1, R_2. Consequently, the lens can be described as a single principal plane, as the principal planes of both surfaces are congruent due to the marginal center thickness. The effective focal length of such a thin lens usually comes from the following for plano-convex or plano-concave thin lenses.

for BICX lenses

$$\frac{1}{\mathrm{EFL}} = (n_l - 1) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2} \right). \tag{1}$$

Here, n_1 is the refractive index of the lens material. For convex symmetrical biconvex or biconcave lenses, this equation can be simplified as follows:

for DCX lenses

$$\frac{1}{\mathrm{EFL}} = \frac{2 \cdot (n_l - 1)}{R} \tag{2}$$

for PLCX lenses

$$\frac{1}{\mathrm{EFL}} = \frac{(n_l - 1)}{R} \tag{3}$$

3.1.2 Concave lenses

In real life, the thickness of a lens cannot be neglected. This is especially true for lenses where the center thickness is in the order of magnitude of the radius of curvature, as, for example, with hemispherical lenses [12, 13]. Such lenses are called concave lenses. As a matter of fact, the principal planes of both optical interfaces are separated. The center thickness tc should be taken into account during the calculation. Equation (3.1) is then rewritten as follows:

---

[1] Absorption coefficient

[2] In optics and lens design, the Abbe number, also known as the V number or density of a transparent material, is an approximate measure of the material's dispersion (the variation of refractive index with wavelength). High V values indicate low dispersion.

$$\frac{1}{\mathrm{EFL}} = (n_l - 1) \cdot \left(\frac{1}{R_1} - \frac{1}{R_2} + \frac{(n_l - 1) \cdot t_c}{n_l \cdot R_1 \cdot R_2} \right). \tag{4}$$

3.2. Parameters of optical imaging

3.2.1 Conjugate parameters of optical imaging

There are two different areas of interest in optical imaging. These areas are called the object area in front of the lens or system and the image area behind it. The basic parameters in the object area and the corresponding parameters in the image area are called reciprocal parameters, interconnected through the focal length or magnification of the imaging optics, respectively [14]. Thus, it means transferring an object with a specific height y and a certain distance in the optical system to an image with a distance a' following the characteristics of the appropriate image height y' and the features used (Fig. 7).

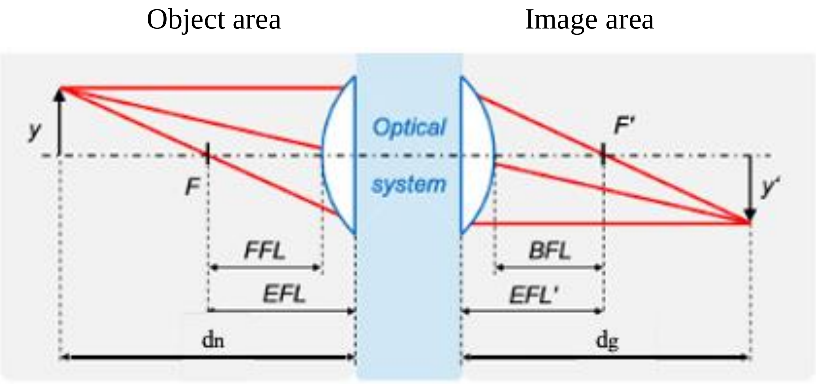

Figure 7. Scheme of optical imaging by an optical system or component with reciprocal (conjugated) parameters [2].

Table 3. Overview of key parameters for optical imaging. Specific heights and distances are conjugated parameters.

Object Area		Image Area	
Abbreviation	**Parameter**	**Abbreviation**	**Parameter**
y (or u)	Object height	y' (or u')	Image height
$\boldsymbol{d_n}$	Object distance	d_g	Image distance
EFL	Effective front focal length	EFL'	Effective back focal length
FFL	Front focal length	BFL	Back focal length
F	Focal point	F'	Focal point
H	Principal plane	H'	Principal plane

According to these reciprocal parameters of imaging and other fundamental factors listed in Table 3, the imaging means the transfer of an object with a certain height y and a distance α from the optical system to an image with image height y' and distance α' appropriate to the characteristics of the optics used. Of the effective focal lengths on the principal plane, it should be noted that the back focal length is at the back of the lens and the front focal length is in front of the lens.

3.2.2 Structure of beams

At least two rays are required for the graphical structure of optical imaging through a lens or system with a given focal length [15]. There are three different rays that come parallel to the optical axis in the object area, pass through the focus in the object area, and pass through the optical center. The parallel incident beam passes through the focal point in the image area after passing through the lens

(→ 'parallel beam becomes the focus beam'). The incident beam, passing through the focal point in the object area, propagates parallel to the optical axis after passing through the lens (→ 'focus beam becomes parallel beam'). The ray that passes through the optical center passes without changing its direction [8, 12].

3.2.3 Imaging equation and magnification

In the general imaging equation, the interrelation between effective focal length (EFL) and object distance dn, and image distance dg can be described by the following equation:

$$\frac{1}{EFL} = \frac{1}{dn} - \frac{1}{dg} \tag{5}$$

Object and image distance are evaluated in relation to focal length. That is, for specific heights and distances in the object area, a certain image height and distance in the image area are obtained (It also means vice versa).

The associated parameters can also be expressed by Newton's equation:

$$EFL \cdot EFL' = (d_n - EFL) \cdot (d_g - EFL') \tag{6}$$

Another essential parameter that connects not only the object distance (d_n) and the image distance (d_g) but also the object height (y) and the image height (y') is the magnification of the optical system and is denoted by “m.”

$$m = \frac{d_g}{d_n} = \frac{y'}{y} \tag{7}$$

In practice, this parameter (m) is usually given and is very helpful for determining object distance, for example, if the image distance is unknown:

$$d_n = EFL \cdot (1 - \frac{1}{m}) \tag{8}$$

Or vice versa,

$$d_g = -EFL \cdot (1 - m) \tag{9}$$

Optical systems consist of at least two or even more lenses. A classic and simple optical element is a compound lens consisting of two single lenses. Such an installation can be accomplished with air-gapped lenses or cemented lenses, called doubles. Cementitious lenses have no air gap between both lenses. The total effective focal length ($\mathrm{EFL_{tot}}$) of a lens pair is as follows, respectively.

$$\frac{1}{\mathrm{EFL_{tot}}} = \frac{1}{\mathrm{EFL_1}} + \frac{1}{\mathrm{EFL_2}} - \frac{d}{\mathrm{EFL_1} \cdot \mathrm{EFL_2}} \tag{10}$$

or

$$\mathrm{EFL_{tot}} = \frac{\mathrm{EFL_1} \cdot \mathrm{EFL_2}}{\mathrm{EFL} + \mathrm{EFL_2} - d}, \tag{11}$$

Here, EFL1 is the effective focal length of the first lens, EFL2 is the effective focal length of the second, and d is the distance between both lenses. The total magnification of such pairs of lenses is the product of the magnifications of the respective lenses alone and is therefore evaluated by the following equation.

$$m_{tot} = m_1 \cdot m_2 \tag{12}$$

It can also be expressed by object and image distances.

$$m_{tot} = \frac{d_{g_1}}{d_{n_1}} \cdot \frac{d_{g_2}}{d_{n_2}} = \frac{d_{g_1} \cdot d_{g_2}}{d_{n_1} \cdot (d - d_{g_1})} \tag{13}$$

3.2.4 Parameters of compound lenses for achromatism

Achromatic binary or achromatic lenses are a special type of lens. In a colorless lens, two wavelengths, red and blue, are brought into the same focal point. An achromatic lens is a lens designed to limit the effects of chromatic and spherical aberration. Achromatic lenses are corrected to focus two wavelengths (typically red and blue) in the same plane [6, 13]. The most common type of achromatic lens is the achromatic duo, which consists of two separate lenses made of glasses with different amounts of dispersion.

This simple optical system is used to avoid chromatic aberrations caused by scattering white light passing through a single lens into its spectral fractions. For the two selected wavelengths, the dispersion can be compensated quite easily by combining two lenses with different V numbers and, therefore, different dispersion properties. The focal length of such an achromatic binary comes from the focal lengths of the respective single lenses (convergent or divergent). Compensation for chromatic aberration at two wavelengths is determined by equating the absolute value of the product of the effective focal length (EFL) and the V number (V) of the first lens to the development of the effective focal length (V) of the second lens:

$$\mathrm{EFL_1} \cdot V_1 = -\mathrm{EFL_2} \cdot V_2 \tag{14}$$

Algebraic indications of this general condition indicate that an achromatic lens consists of a convergent lens or a divergent lens. Both lenses are usually glued, so the distance is d = 0. The total effective focal length of the Colorseme (achromatic) binary EFLal, therefore, calculates as follows:

$$\frac{1}{\mathrm{EFL_{al}}} = \frac{1}{\mathrm{EFL_1}} + \frac{1}{\mathrm{EFL_2}}. \tag{15}$$

For the assumed or predetermined effective focal length of an achromatic binary, certain parameters of the respective single lenses can be determined quite easily: Solving the general condition for achromatism given by equation 3.14 for the effective focal length of the second lens,

$$\mathrm{EFL'2} = \frac{\mathrm{EFL'1} \cdot V_1}{V_2} \tag{16}$$

By adding the equation in 2.16 to the equation in 2.15, the following equation is obtained;

$$\frac{1}{\mathrm{EFL}_{\mathrm{al}}} = \frac{1}{\mathrm{EFL}_1} \cdot \frac{V_1 - V_2}{V_1} \tag{17}$$

This expression can then be solved to determine the effective focal length of the first lens:

$$\mathrm{EFL}_1 = \mathrm{EFL}_{\mathrm{al}} \cdot \frac{V_1 - V_2}{V_1} \tag{18}$$

or for the second lens,

$$\mathrm{EFL}_2 = -\mathrm{EFL}_{\mathrm{al}} \cdot \frac{V_1 - V_2}{V_2} \tag{19}$$

Once the focal lengths of both lenses have been determined, the geometric lens parameters can be calculated based on the lens manufacturer's equations. The refractive indices required here are implicitly defined by the V-numbers and the type of glass used for both lenses. Choosing eyeglasses is an interesting and challenging task where some experience in optical design is beneficial. Normally, a standard crown glass with a high V-number and low dispersion, respectively, is used as an approximation for the convergent lens of an achromatic binary [2, 3, 4]. For the divergent lens, a flint or strong flint glass with a low V-number and high dispersion is chosen, respectively. The achromatic binary of these glasses is given in figure 8:

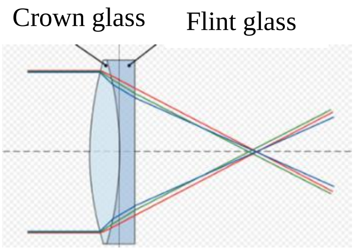

Figure 8. An achromatic duo combining Crown glass and Flint glass

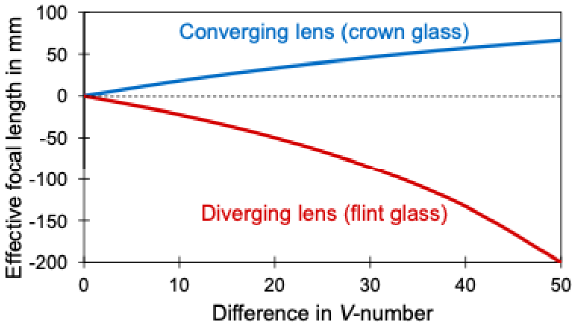

Figure 9. Correlation of increase in V number with the increase in effective focal length for an achromatic binary with a total focal length of 100 mm

As a simple rule, the difference in the V number of both glasses should be as large as possible (Fig. 9). The effective focal lengths of the respective single lenses increase as the difference in V number increases [16]. As a result of the lens manufacturer's equations, the radius of curvature increases with increasing effective focal length. As a result, an image defect can be reduced due to a certain reduction in the angle of incidence and a certain degree of refraction on the lens surface. Therefore, the use of a colorless binary provides not only the correction of chromatic aberration but also some degree of spherical aberration correction [17].

4. Conclusion

The first step in optical system design is the analysis of a particular imaging task and the definition of given/desired parameters. Simple calculations and considerations allow for determining the focal length of a required optical system. The start-up system is an optical setup that acts to provide the best imaging performance for a given imaging task. The start-up systems can be defined on the basis of the area of view and the f-number of an imaging system. Based on an appropriate start-up system, the optical setup is evaluated and optimized during an iterative process where manufacturing tolerances can also be taken into account. An optical system can be optimized directly or indirectly. Direct optimization can be defined as the modification of physical parameters such as radii of curvature or materials. Direct optimization is based on the definition and application of the utility function, taking into account variables (parameters to change) and imperfections (eg deviations).

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