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Research Article

On Neutrosophic Soft Multisets and Neutrosophic Soft Multi Topological Spaces

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Abstract

In this paper, a new hybrid system, neutrosophic soft multisets is introduced. Also, we studied some basic properties such as subset, equal set, null set, absolute set, union, intersection, different on these concept. Furthermore, we introduced neutrosophic soft multi topological spaces. Some important notions such as open set, closed set, interior, closure on these topological spaces have investigated. The important properties of all studied concepts have been examined, some theorems have been proved and various examples have been presented.

Keywords: multisets, neutrosophic soft multisets, neutrosophic soft multi topology

Neutrosophic Esnek Çoklu Kümeler ve Neutrosophic Esnek Çoklu Topolojik Uzaylar Üzerine

Öz

Bu çalışma da, yeni bir hibrit sistem olan neutrosophic esnek çoklu kümeler tanıtılmaktadır. Ayrıca, alt küme, eşit küme, boş küme, mutlak küme, birleşim, kesişim, farklı gibi bazı temel özellikleri bu kavramlar üzerinde çalıştık. Dahası, neutrosophic esnek çoklu topolojik uzayları tanıttık. Bu topolojik uzaylar üzerinde açık küme, kapalı küme, iç, kapanış gibi bazı önemli kavramlar araştırılmıştır. İncelenen tüm kavramların önemli özellikleri araştırılmış, bazı önemli teoremler ispatlanmış ve konu ile ilgili çeşitli örnekler sunulmuştur.

Anahtar Kelimeler: çoklu kümeler, neutrosophic esnek çoklu kümeler, neutrosophic esnek çoklu topoloji.

1. Introduction

A number of research on generalizations of fuzzy set (FS) notions have been conducted since the invention of the fuzzy set [47]. The theory of FSs is a generalization of the traditional theory of sets, in the sense that the theory of sets should have been a special instance of the theory of FSs. Following the generalisation of FSs, several researchers used the notion of generalised FSs to a wide range of scientific and technological domains. Chang [6] was the first to develop fuzzy topology (FT), while Coker [7] defined intuitionistic fuzzy topological space. Lupianez [16-18] and Salama et al. [36] are two scholars that explored topology using neutrosophic sets (NS). In 1963, Kelly [14] introduced the notion of bitopological space (BTS). The concept of fuzzy bitopological space (FBTS) was investigated by Kandil et al. [13]. Lee et al. [15] looked

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at some of the properties of Intuitionistic Fuzzy Bitopological Space (IFBTS). Garg [11] used a modified scoring function to study how to rank interval-valued Pythagorean FSs. The TOPSIS approach based on Pythagorean FSs was addressed as a Pythagorean fuzzy method for order of preference by similarity to ideal solution (TOPSIS), which accepted the experts' preferences in the form of interval-valued Pythagorean fuzzy decision matrices. In addition, [1,5,41,48] contains several investigations of the notion of Pythagorean FSs. The q-rung orthopair FSs were proposed by Yager [43], in which the sum of the qth powers of the membership (MS) and non-MS degrees is limited to one [44]. For q-rung orthopair FSs, Peng and Liu [32] investigated the systematic transformation for information measures. Pinar and Boran [33] used a q-rung orthopair fuzzy multi-criteria group decision-making technique based on a unique distance measure to pick suppliers. As Molodtsov [25] points out, each of these hypotheses has its own set of problems. Molodtsov [24] proposed an entirely new advanced soft sets theory methodology for modeling ambiguity and uncertainty that is devoid of the complexities that plague current methods. The challenge of determining the membership function, as well as other related issues, does not arise in soft set theory. Soft sets are a subset of context-dependent fuzzy sets and are referred to as neighbourhood systems. By applying the knowledge reduction approach to the information table created by the soft set, Maji et al. [19] functionalized soft sets in multicriteria decision-making issues. They defined and investigated some basic concepts in soft set theory in [20]. Ozkan studied soft multi generalized regular set and soft multi generalized closed sets [26-27].

Smarandache [37] proposed a neutrosophic set and logic in 1999, in which neutrosophic sets are defined by the truth membership function (T), indeterminacy membership function (I), and falsity membership function (F). Classical sets, fuzzy sets, and intuitionistic fuzzy sets are all generalized in neutrosophic set theory. The theory is a strong tool for dealing with knowledge that is imperfect, uncertain, and inconsistent in the actual world. Bera and Mahapatra [3] explored several important conclusions while introducing the notion of neutrosophic soft topology. New notions in neutrosophic soft topological spaces were developed by Ozturk in [29]. The terms boundary, dense set, and neutrosophic soft basis are used to describe these notions. The notion of soft subspace on neutrosophic soft topological spaces is also discussed. With regard to soft points, several intriguing results are discussed. The greatest examples are used to obtain some difficult outcomes. Yolcu et al. [46] examined the images and inverse images of neutrosophic soft sets and redefined neutrosophic soft mapping. The authors went on to trace the core activities of neutrosophic soft mapping as well as other related features. The authors did an excellent job of applying neutrosophic soft mapping to decision-making challenges. Ozturk et al. [30, 31] are pioneers of innovative neutrosophic soft set procedures. On the basis of the operation outlined in [29]. Some of the outcomes are backed up by the most easily accessible instances. New notions of neutrosophic soft sets were presented by Gunduz et al. [12]. With respect to soft points, they developed new separation axioms in neutrosophic soft topological spaces. The link between these neutrosophic soft axioms has also been discussed. Also examined are the inside and closer of neutrosophic soft settings. Other structures are addressed using these notions as a foundation. The best examples are used to secure the majority of the tough results. Also Ozturk et. al. [28] investigated neutrosophic soft compact spaces.

Blizard [4] claimed that multisets go all the way back to the beginning of numbers, claiming that the number was typically represented as a group of n strokes, tally marks, or units in ancient times. Yager [42] first proposed the concept of fuzzy multiset (FMS) as fuzzy bags. We consider our focus to the core notions such as an open fuzzy multiset, closed fuzzy multiset, interior, closure, and continuity of fuzzy multiset in the purpose of brevity. In [42], Yager introduced the idea of FMS (fuzzy bag) to generalize the FS, and in [42], he presented a calculus for them. An FMS element can appear more than once, with the same or distinct MS values. There has been some research on the multi fuzzy set [22,23,35], the intuitionistic fuzzy multiset [10,34,32,39], and the neutrosophic multiset [2,8,38,40,45]. The set theories discussed above have been applied to a variety of situations, including real-life decision-making difficulties.

In this paper, we introduced a new hybrid system neutrosophic soft multisets and neutrosophic soft multi topological spaces as an update for the research in neutrosophic multisets. Furthermore, we attempted to prove several of these features and provided instances. The neutrosophic multi almost topological group was defined using the notion of neutrosophic multi nearly continuous mapping, and several features and theorems of the neutrosophic multi almost topological group were investigated. In the section 3, we present and investigated neutrosophic soft multiset structure and some notions such as complement, union, intersection, different, subset, equal set, null set, absolute set on these set structure. We have proved important theorems about these concepts and presented various examples. In the section 4, we introduced neutrosophic soft multi topological spaces. Furthermore, we investigated some properties such as open set, closed set, interior, closure on these topological structur. Also, We have proved important theorems about these notions and presented various examples.

2. Preliminaries

Definition 2.1 [37] Let ζ be an initial universe. Then a neutrosophic set N on ζ is defined as follows:

$$N=\left\{\left\langle h,T_N(h),I_N(h),F_N(h)\right\rangle : h\in\zeta\right\},$$

where $T,\ I,\ F:\zeta\to]^-0,1^+[$ and $^-0\le T_N(h)+I_N(h)+F_N(h)\le 3^+$.

Definition 2.2 [24] Let ζ be an initial universe, Υ be a set of all parameters and $P(\zeta)$ denotes the power set of ζ. A pair (H,Υ) is called a soft set over ζ, where H is a mapping given by $H:\Upsilon\to P(\zeta)$.

In other words, the soft set is a parameterized family of subsets of the set ζ . For $\sigma\in\Upsilon$, $H(\sigma)$ may be considered as the set of $\sigma-$elements of the soft set (H,Υ), or as the set of $\sigma-$ approximate elements of the soft set, i.e.,

$$(H,E)=\left\{(e,H(e)): e\in E, H:E\to P(X)\right\}.$$

Firstly, neutrosophic soft set defined by Maji [21] and later this concept has been modified by Deli and Bromi [9] as given below:

Definition 2.3 Let ζ be an initial universe set and Υ be a set of parameters. Let $P(\zeta)$ denote the set of all neutrosophic sets of ζ. Then, a neutrosophic soft set $\left(H,\Upsilon\right)$ over ζ is a set defined by a set valued function H representing a mapping $H:\Upsilon\rightarrow P(\zeta)$ where H is called approximate function of the neutrosophic soft set $\left(H,\Upsilon\right)$. In other words, the neutrosophic soft set is a parameterized family of some elements of the set $P(\zeta)$ and therefore it can be written as a set of ordered pairs,

$$\left(H,\Upsilon\right)=\left\{\left(\sigma,\left\langle h,T_{H(\sigma)}(h),I_{H(\sigma)}(h),F_{H(\sigma)}(h)\right\rangle:h\in\zeta\right):\sigma\in E\right\}$$

where $T_{H(\sigma)}(h),I_{H(\sigma)}(h),F_{H(\sigma)}(h)\in[0,1]$, respectively called the truth-membership, indeterminacy-membership, falsity-membership function of $H(\sigma)$. Since supremum of each $T,\ I,\ F$ is 1 so the inequality $0\leq T_{H(\sigma)}(h)+I_{H(\sigma)}(h)+F_{H(\sigma)}(h)\leq 3$ is obvious.

Definition 2.4 [38] A neutrosophic multiset is a type of neutrosophic set in which one or more elements with the same or different neutrosophic components are repeated several times.

3. Main Theorem and Proof

Definition 3.1 Let ζ be a universe, Υ be a set of parameters. Let $P(\zeta)$ denote the all neutrosophic multisets of ζ. Then a neutrosophic soft multiset (H^{nm},Υ) over ζ is a set valued function H^{nm} representing a mapping $H^{nm}:\Upsilon\rightarrow P(\zeta)$ where H^{nm} is called approximate function of the neutrosophic soft multiset (H^{nm},Υ). A neutrosophic soft multiset (H^{nm},Υ) on ζ can be defined as follows:

$$(H^{nm},\Upsilon)=\left\{\begin{array}{c}\left(\begin{array}{c}\sigma,<h,\left(T^{1}_{H^{nm}(\sigma)}(h),T^{2}_{H^{nm}(\sigma)}(h),...,T^{i}_{H^{nm}(\sigma)}(h)\right),\\ \left(I^{1}_{H^{nm}(\sigma)}(h),I^{2}_{H^{nm}(\sigma)}(h),...,I^{i}_{H^{nm}(\sigma)}(h)\right),\\ \left(F^{1}_{H^{nm}(\sigma)}(h),F^{2}_{H^{nm}(\sigma)}(h),...,F^{i}_{H^{nm}(\sigma)}(h)\right)>:h\in\zeta\end{array}\right)\\ :\sigma\in\Upsilon,i=1,2,...,k\end{array}\right\}$$

where truth-membership sequence $\left(T^{1}_{H^{nm}(\sigma)}(h),T^{2}_{H^{nm}(\sigma)}(h),...,T^{i}_{H^{nm}(\sigma)}(h)\right)$, the indeterminancy-membership sequence $\left(I^{1}_{H^{nm}(\sigma)}(h),I^{2}_{H^{nm}(\sigma)}(h),...,I^{i}_{H^{nm}(\sigma)}(h)\right)$ and the falsity membership sequence $\left(F^1_{H^{nm}(\sigma)}(h), F^2_{H^{nm}(\sigma)}(h), ..., F^i_{H^{nm}(\sigma)}(h)\right)$ and the sum of $T^i_{H^{nm}(\sigma)}(h)$, $I^i_{H^{nm}(\sigma)}(h)$, $F^i_{H^{nm}(\sigma)}(h) \in [0,1]$ satisfies the condition $0 \le T^i_{H^{nm}(\sigma)}(h) + I^i_{H^{nm}(\sigma)}(h) + F^i_{H^{nm}(\sigma)}(h) \le 3$ for $i = 1, 2, ..., k$. Here, the element $h \in \zeta$ is repeated k times for each parameter using in set.

Example 3.1 Let $\zeta = \{h_1, h_2\}$ be universe set and $\Upsilon = \{\sigma_1, \sigma_2\}$ be a parameters. Let consider the neutrosophic soft multisets (H^{nm}, Υ) as follows;

$$(H^{nm}, \Upsilon) = \left\{ \begin{array}{l} \sigma_1, \left(\begin{array}{l} < h_1, (0.4, 0.6, 0.8), (0.4, 0.5, 0.7), (0.5, 0.6, 0.2) > \\ < h_2, (0.3, 0.7, 0.2), (0.5, 0.5, 0.3), (0.2, 0.4, 0.4) > \end{array} \right) \\ \sigma_2, \left(\begin{array}{l} < h_1, (0.2, 0.4, 0.3), (0.4, 0.5, 0.7), (0.5, 0.6, 0.2) > \\ < h_2, (0.7, 0.4, 0.7), (0.5, 0.5, 0.3), (0.2, 0.4, 0.4) > \end{array} \right) \end{array} \right\}$$

It is clear that above, $k = 3$ for the neutrosophic soft multiset (H^{nm}, Υ). So $\forall h_i \in \zeta, i = 1, 2$ is repeated 3 times for each parameters using in set. This set can be written more clearly as follows.

$$(H^{nm}, \Upsilon) = \left\{ \begin{array}{l} \sigma_1, \left(\begin{array}{l} < h_1, (0.4, 0.4, 0.5) >, < h_1, (0.6, 0.5, 0.6) >, < h_1, (0.8, 0.7, 0.2) > \\ < h_2, (0.3, 0.5, 0.2) >, < h_2, (0.7, 0.5, 0.4) >, < h_2, (0.2, 0.3, 0.4) > \end{array} \right) \\ \sigma_2, \left(\begin{array}{l} < h_1, (0.2, 0.4, 0.5) >, < h_1, (0.4, 0.5, 0.6) >, < h_1, (0.3, 0.7, 0.2) > \\ < h_2, (0.7, 0.5, 0.2) >, < h_2, (0.4, 0.5, 0.4) >, < h_2, (0.7, 0.3, 0.4) > \end{array} \right) \end{array} \right\}$$

Definition 3.2 Let $(H_1^{nm}, \Upsilon) = \left\{ \begin{array}{c} \left(\sigma, < h, T^i_{H_1^{nm}(\sigma)}(h), I^i_{H_1^{nm}(\sigma)}(h), F^i_{H_1^{nm}(\sigma)}(h) >: h \in \zeta\right) \\ : \sigma \in \Upsilon, i = 1, 2, ..., k \end{array} \right\}$ and $(H_2^{nm}, \Upsilon) = \left\{ \begin{array}{c} \left(\sigma, < h, T^i_{H_2^{nm}(\sigma)}(h), I^i_{H_2^{nm}(\sigma)}(h), F^i_{H_2^{nm}(\sigma)}(h) >: h \in \zeta\right) \\ : \sigma \in \Upsilon, i = 1, 2, ..., k \end{array} \right\}$ be two neutrosophic soft multiset over the ζ. Then, there are the following relations:

1. (H_1^{nm}, Υ) is called to be neutrosophic soft multi-subset of (H_2^{nm}, Υ) is denoted by $(H_1^{nm}, \Upsilon) \subseteq (H_2^{nm}, \Upsilon)$ if $T^i_{H_1^{nm}(\sigma)}(h) \le T^i_{H_2^{nm}(\sigma)}(h), I^i_{H_1^{nm}(\sigma)}(h) \le I^i_{H_2^{nm}(\sigma)}(h), F^i_{H_1^{nm}(\sigma)}(h) \ge F^i_{H_2^{nm}(\sigma)}(h)$ for all $\sigma \in \Upsilon$ and for all $h \in \zeta, i = 1, 2, ..., k$.
2. (H_1^{nm}, Υ) is called to be neutrosophic soft multi-equal of (H_2^{nm}, Υ) is denoted by $(H_1^{nm}, \Upsilon) = (H_2^{nm}, \Upsilon)$ if $T^i_{H_1^{nm}(\sigma)}(h) = T^i_{H_2^{nm}(\sigma)}(h), I^i_{H_1^{nm}(\sigma)}(h) = I^i_{H_2^{nm}(\sigma)}(h), F^i_{H_1^{nm}(\sigma)}(h) = F^i_{H_2^{nm}(\sigma)}(h)$ for all $\sigma \in \Upsilon$ and for all $h \in \zeta, i = 1, 2, ..., k$.
3. The complement of (H_1^{nm}, Υ) is denoted by $(H_1^{nm}, \Upsilon)^c$ and defined as follows:

$$(H_1^{nm},\Upsilon)=\left\{\left(\sigma,<h,F^i_{H_i^{nm}(\sigma)}(h),1-I^i_{H_i^{nm}(\sigma)}(h),T^i_{H_i^{nm}(\sigma)}(h)>:h\in\zeta\right):\sigma\in\Upsilon,i=1,2,...,k\right\}$$

4. The union of (H_1^{nm},Υ) and (H_2^{nm},Υ) is denoted by $(H_1^{nm},\Upsilon)\cup(H_2^{nm},\Upsilon)=(H_3^{nm},\Upsilon)$ and is defined as follows

$$(H_3^{nm},\Upsilon)=\left\{\left(\sigma,<h,T^i_{H_3^{nm}(\sigma)}(h),I^i_{H_3^{nm}(\sigma)}(h),F^i_{H_3^{nm}(\sigma)}(h)>:h\in\zeta\right):\sigma\in\Upsilon,i=1,2,...,k\right\}$$

where

$$T^i_{H_3^{nm}(\sigma)}(h)=\max\left\{T^i_{H_1^{nm}(\sigma)}(h),T^i_{H_2^{nm}(\sigma)}(h)\right\}$$

$$I^i_{H_3^{nm}(\sigma)}(h)=\max\left\{I^i_{H_1^{nm}(\sigma)}(h),I^i_{H_2^{nm}(\sigma)}(h)\right\}$$

$$F^i_{H_3^{nm}(\sigma)}(h)=\min\left\{F^i_{H_1^{nm}(\sigma)}(h),F^i_{H_2^{nm}(\sigma)}(h)\right\}$$

5. The intersection of (H_1^{nm},Υ) and (H_2^{nm},Υ) is denoted by $(H_1^{nm},\Upsilon)\cap(H_2^{nm},\Upsilon)=(H_3^{nm},\Upsilon)$ and is defined as follows

$$(H_3^{nm},\Upsilon)=\left\{\left(\sigma,<h,T^i_{H_3^{nm}(\sigma)}(h),I^i_{H_3^{nm}(\sigma)}(h),F^i_{H_3^{nm}(\sigma)}(h)>:h\in\zeta\right):\sigma\in\Upsilon,i=1,2,...,k\right\}$$

where

$$T^i_{H_3^{nm}(\sigma)}(h)=\min\left\{T^i_{H_1^{nm}(\sigma)}(h),T^i_{H_2^{nm}(\sigma)}(h)\right\}$$

$$I^i_{H_3^{nm}(\sigma)}(h)=\min\left\{I^i_{H_1^{nm}(\sigma)}(h),I^i_{H_2^{nm}(\sigma)}(h)\right\}$$

$$F^i_{H_3^{nm}(\sigma)}(h)=\max\left\{F^i_{H_1^{nm}(\sigma)}(h),F^i_{H_2^{nm}(\sigma)}(h)\right\}$$

Definition 3.3 Let (H_1^{nm},Υ) and (H_2^{nm},Υ) be neutrosophic soft multiset over the ζ. The difference of (H_1^{nm},Υ) and (H_2^{nm},Υ) is denoted by $(H_1^{nm},\Upsilon)\tilde{\setminus}(H_2^{nm},\Upsilon)=(H_3^{nm},\Upsilon)$ and is defined as follows;

$$(H_3^{nm},\Upsilon)=(H_1^{nm},\Upsilon)\cap(H_2^{nm},\Upsilon)^c$$

$$(H_3^{nm},\Upsilon)=\left\{\left(\sigma,<h,T^i_{H_3^{nm}(\sigma)}(h),I^i_{H_3^{nm}(\sigma)}(h),F^i_{H_3^{nm}(\sigma)}(h)>:h\in\zeta\right):\sigma\in\Upsilon,i=1,2,...,k\right\}$$

where

$$T^i_{H_3^{nm}(\sigma)}(h)=\min\left\{T^i_{H_1^{nm}(\sigma)}(h),F^i_{H_2^{nm}(\sigma)}(h)\right\}$$

$$I^i_{H_3^{nm}(\sigma)}(h)=\min\left\{I^i_{H_1^{nm}(\sigma)}(h),1-I^i_{H_2^{nm}(\sigma)}(h)\right\}$$

$$F^i_{H_3^{nm}(\sigma)}(h)=\max\left\{F^i_{H_1^{nm}(\sigma)}(h),T^i_{H_2^{nm}(\sigma)}(h)\right\}$$

Definition 3.4 *Let* $\left\{(H_p^{nm},\Upsilon)\middle| p\in I\right\}$ *be a family of neutrosophic soft multisets over the* ζ *. Then*

$$\underset{i\in I}{\cup}(H_p^{nm},\Upsilon)=\left\{\begin{array}{c}\left(\sigma,<h,\sup\left[T^i_{H_p^{nm}(\sigma)}(h)\right]_{p\in I},\sup\left[I^i_{H_p^{nm}(\sigma)}(h)\right]_{p\in I},\inf\left[F^i_{H_p^{nm}(\sigma)}(h)\right]_{p\in I}>:h\in\zeta\right)\\ :\sigma\in\Upsilon,i=1,2,...,k\end{array}\right\},$$

$$\underset{i\in I}{\cap}(H_p^{nm},\Upsilon)=\left\{\begin{array}{c}\left(\sigma,<h,\inf\left[T^i_{H_p^{nm}(\sigma)}(h)\right]_{p\in I},\inf\left[I^i_{H_p^{nm}(\sigma)}(h)\right]_{p\in I},\sup\left[F^i_{H_p^{nm}(\sigma)}(h)\right]_{p\in I}>:h\in\zeta\right)\\ :\sigma\in\Upsilon,i=1,2,...,k\end{array}\right\}.$$

Definition 3.5

1. A null neutrosophic soft multiset over the ζ is denoted by $0_{(\zeta^{nm},\Upsilon)}$ and defined as

$$0_{(\zeta^{nm},\Upsilon)}=\left\{\left(\sigma,<h,T^i_{\zeta^{nm}(\sigma)}(h),I^i_{\zeta^{nm}(\sigma)}(h),F^i_{\zeta^{nm}(\sigma)}(h)>:h\in\zeta\right):\sigma\in\Upsilon,i=1,2,...,k\right\}$$

where $T^i_{\zeta^{nm}(\sigma)}(h)=0, I^i_{\zeta^{nm}(\sigma)}(h)=0$ and $F^i_{\zeta^{nm}(\sigma)}(h)=1.$

2. A absolute neutrosophic soft multiset over the ζ is denoted by $1_{(\zeta^{nm},\Upsilon)}$ and defined as

$$1_{(\zeta^{nm},\Upsilon)}=\left\{\left(\sigma,<h,T^i_{\zeta^{nm}(\sigma)}(h),I^i_{\zeta^{nm}(\sigma)}(h),F^i_{\zeta^{nm}(\sigma)}(h)>:h\in\zeta\right):\sigma\in\Upsilon,i=1,2,...,k\right\}$$

where $T^i_{\zeta^{nm}(\sigma)}(h)=1, I^i_{\zeta^{nm}(\sigma)}(h)=1$ and $F^i_{\zeta^{nm}(\sigma)}(h)=0.$

It is clear that $\left(0_{(\zeta^{nm},\Upsilon)}\right)^c=1_{(\zeta^{nm},\Upsilon)}$ and $\left(1_{(\zeta^{nm},\Upsilon)}\right)^c=0_{(\zeta^{nm},\Upsilon)}.$

Proposition 3.1 Let (H_1^{nm},Υ), (H_2^{nm},Υ) and (H_3^{nm},Υ) be neutrosophic soft multi sets over the universe set ζ . Then,

1. $(H_1^{nm},\Upsilon)\cup\left[(H_2^{nm},\Upsilon)\cup(H_3^{nm},\Upsilon)\right]=\left[(H_1^{nm},\Upsilon)\cup(H_2^{nm},\Upsilon)\right]\cup(H_3^{nm},\Upsilon)$ and

   $(H_1^{nm},\Upsilon)\cap\left[(H_2^{nm},\Upsilon)\cap(H_3^{nm},\Upsilon)\right]=\left[(H_1^{nm},\Upsilon)\cap(H_2^{nm},\Upsilon)\right]\cap(H_3^{nm},\Upsilon);$

2. $(H_1^{nm},\Upsilon)\cup\left[(H_2^{nm},\Upsilon)\cap(H_3^{nm},\Upsilon)\right]=\left[(H_1^{nm},\Upsilon)\cup(H_2^{nm},\Upsilon)\right]\cap\left[(H_1^{nm},\Upsilon)\cup(H_3^{nm},\Upsilon)\right]$ and

   $(H_1^{nm},\Upsilon)\cap\left[(H_2^{nm},\Upsilon)\cup(H_3^{nm},\Upsilon)\right]=\left[(H_1^{nm},\Upsilon)\cap(H_2^{nm},\Upsilon)\right]\cup\left[(H_1^{nm},\Upsilon)\cap(H_3^{nm},\Upsilon)\right];$

3. $(H_1^{nm},\Upsilon)\cup 0_{(\zeta^{nm},\Upsilon)}=(H_1^{nm},\Upsilon)$ and $(H_1^{nm},\Upsilon)\cap 0_{(\zeta^{nm},\Upsilon)}=0_{(\zeta^{nm},\Upsilon)};$

4. $(H_1^{nm},\Upsilon)\cup 1_{(\zeta^{nm},\Upsilon)} = 1_{(\zeta^{nm},\Upsilon)}$ and $(H_1^{nm},\Upsilon)\cap 1_{(\zeta^{nm},\Upsilon)} = (H_1^{nm},\Upsilon)$.

Proof. Straightforward.

Proposition 3.2 Let (H_1^{nm},Υ) and (H_2^{nm},Υ) be two neutrosophic soft multi sets over the universe set ζ. Then,

1. $\left[(H_1^{nm},\Upsilon)\cup(H_2^{nm},\Upsilon)\right]^c = (H_1^{nm},\Upsilon)^c\cap(H_2^{nm},\Upsilon)^c$;

2. $\left[(H_1^{nm},\Upsilon)\cap(H_2^{nm},\Upsilon)\right]^c = (H_1^{nm},\Upsilon)^c\cup(H_2^{nm},\Upsilon)^c$.

Proof. 1. For all $e\in\Upsilon$ and $x\in\zeta$,

$$(H_1^{nm},\Upsilon)\cup(H_2^{nm},\Upsilon) = \left\{\left\langle \begin{array}{c} x, \max\left\{T^i_{H_1^{nm}(\sigma)}(x), T^i_{H_2^{nm}(\sigma)}(x)\right\}, \max\left\{I^i_{H_1^{nm}(\sigma)}(x), I^i_{H_2^{nm}(\sigma)}(x)\right\}, \\ \min\left\{F^i_{H_1^{nm}(\sigma)}(x), F^i_{H_2^{nm}(\sigma)}(x)\right\} \end{array}\right\rangle\right\}$$

$$\left[(H_1^{nm},\Upsilon)\cup(H_2^{nm},\Upsilon)\right]^c = \left\{\left\langle \begin{array}{c} x, \min\left\{F^i_{H_1^{nm}(\sigma)}(x), F^i_{H_2^{nm}(\sigma)}(x)\right\}, 1-\max\left\{I^i_{H_1^{nm}(\sigma)}(x), I^i_{H_2^{nm}(\sigma)}(x)\right\}, \\ \max\left\{T^i_{H_1^{nm}(\sigma)}(x), T^i_{H_2^{nm}(\sigma)}(x)\right\} \end{array}\right\rangle\right\}.$$

Now,

$$(H_1^{nm},\Upsilon)^c = \left\{\left\langle x, F^i_{H_1^{nm}(\sigma)}(x), 1-I^i_{H_1^{nm}(\sigma)}(x), T^i_{H_1^{nm}(\sigma)}(x)\right\rangle\right\},$$

$$(H_2^{nm},\Upsilon)^c = \left\{\left\langle x, F^i_{H_2^{nm}(\sigma)}(x), 1-I^i_{H_2^{nm}(\sigma)}(x), T^i_{H_2^{nm}(\sigma)}(x)\right\rangle\right\}.$$

Then,

$$(H_1^{nm},\Upsilon)^c\cap(H_2^{nm},\Upsilon)^c = \left\{\left\langle \begin{array}{c} x, \min\left\{F^i_{H_1^{nm}(\sigma)}(x), F^i_{H_2^{nm}(\sigma)}(x)\right\}, \min\left\{\left(1-I^i_{H_1^{nm}(\sigma)}(x)\right), \left(1-I^i_{H_2^{nm}(\sigma)}(x)\right)\right\}, \\ \max\left\{T^i_{H_1^{nm}(\sigma)}(x), T^i_{H_2^{nm}(\sigma)}(x)\right\} \end{array}\right\rangle\right\}$$

$$= \left\{\left\langle \begin{array}{c} x, \min\left\{F^i_{H_1^{nm}(\sigma)}(x), F^i_{H_2^{nm}(\sigma)}(x)\right\}, 1-\max\left\{I^i_{H_1^{nm}(\sigma)}(x), I^i_{H_2^{nm}(\sigma)}(x)\right\}, \\ \max\left\{T^i_{H_1^{nm}(\sigma)}(x), T^i_{H_2^{nm}(\sigma)}(x)\right\} \end{array}\right\rangle\right\}.$$

Therefore, $\left[(H_1^{nm},\Upsilon)\cup(H_2^{nm},\Upsilon)\right]^c = (H_1^{nm},\Upsilon)^c\cap(H_2^{nm},\Upsilon)^c$.

2. It is obtained in a similar way.

Example 3.2 Let $\zeta=\{h_1,h_2,h_3\}$ be universe set and $\Upsilon=\{\sigma_1,\sigma_2\}$ be a parameters. Let consider two neutrosophic soft multisets (H_1^{nm},Υ) and (H_2^{nm},Υ) as follows:

$$(H_1^{nm},\Upsilon)=\left\{\begin{array}{l}\sigma_1,\begin{pmatrix}<h_1,(0.4,0.6,0.8,0.3),(0.4,0.5,0.7,0.6),(0.5,0.6,0.2,0.8)>\\ <h_2,(0.3,0.7,0.2,0.4),(0.5,0.5,0.3,0.8),(0.2,0.4,0.4,0.5)>\\ <h_3,(0.4,0.6,0.8,0.3),(0.4,0.5,0.8,0.2),(0.7,0.5,0.3,0.6)>\end{pmatrix}\\ \sigma_2,\begin{pmatrix}<h_1,(0.2,0.4,0.3,0.6),(0.4,0.5,0.7,0.6),(0.5,0.6,0.2,0.8)>\\ <h_2,(0.7,0.4,0.7,0.5),(0.5,0.5,0.3,0.8),(0.2,0.4,0.4,0.5)>\\ <h_3,(0.8,0.5,0.3,0.2),(0.6,0.4,0.3,0.4),(0.5,0.3,0.2,0.7)>\end{pmatrix}\end{array}\right\}$$

$$(H_2^{nm},\Upsilon)=\left\{\begin{array}{l}\sigma_1,\begin{pmatrix}<h_1,(0.2,0.3,0.6,0.4),(0.2,0.3,0.2,0.2),(0.4,0.6,0.1,0.4)>\\ <h_2,(0.5,0.4,0.6,0.3),(0.6,0.7,0.4,0.4),(0.8,0.4,0.3,0.2)>\\ <h_3,(0.5,0.5,0.7,0.4),(0.7,0.3,0.6,0.7),(0.4,0.4,0.5,0.6)>\end{pmatrix}\\ \sigma_2,\begin{pmatrix}<h_1,(0.8,0.5,0.2,0.7),(0.6,0.3,0.4,0.3),(0.6,0.2,0.4,0.2)>\\ <h_2,(0.6,0.5,0.4,0.3),(0.6,0.4,0.2,0.1),(0.4,0.5,0.8,0.6)>\\ <h_3,(0.8,0.6,0.4,0.5),(0.5,0.3,0.4,0.2),(0.3,0.5,0.4,0.4)>\end{pmatrix}\end{array}\right\}$$

Then;

$$(H_1^{nm},\Upsilon)\cup(H_2^{nm},\Upsilon)=\left\{\begin{array}{l}\sigma_1,\begin{pmatrix}<h_1,(0.4,0.6,0.8,0.4),(0.4,0.5,0.7,0.6),(0.4,0.6,0.1,0.4)>\\ <h_2,(0.5,0.7,0.6,0.4),(0.6,0.7,0.4,0.8),(0.2,0.4,0.3,0.2)>\\ <h_3,(0.5,0.6,0.8,0.4),(0.7,0.5,0.8,0.7),(0.4,0.4,0.3,0.6)>\end{pmatrix}\\ \sigma_2,\begin{pmatrix}<h_1,(0.8,0.5,0.3,0.7),(0.6,0.5,0.7,0.6),(0.5,0.2,0.2,0.2)>\\ <h_2,(0.7,0.5,0.7,0.5),(0.6,0.5,0.3,0.8),(0.2,0.4,0.4,0.5)>\\ <h_3,(0.8,0.6,0.4,0.5),(0.6,0.4,0.4,0.4),(0.3,0.3,0.2,0.4)>\end{pmatrix}\end{array}\right\}$$

$$(H_1^{nm},\Upsilon)\cap(H_2^{nm},\Upsilon)=\left\{\begin{array}{l}\sigma_1,\begin{pmatrix}<h_1,(0.2,0.3,0.6,0.4),(0.2,0.3,0.2,0.2),(0.4,0.6,0.1,0.4)>\\ <h_2,(0.5,0.4,0.6,0.3),(0.6,0.7,0.4,0.4),(0.8,0.4,0.3,0.2)>\\ <h_3,(0.5,0.5,0.7,0.4),(0.7,0.3,0.6,0.7),(0.4,0.4,0.5,0.6)>\end{pmatrix}\\ \sigma_2,\begin{pmatrix}<h_1,(0.8,0.5,0.2,0.7),(0.6,0.3,0.4,0.3),(0.6,0.2,0.4,0.2)>\\ <h_2,(0.6,0.5,0.4,0.3),(0.6,0.4,0.2,0.1),(0.4,0.5,0.8,0.6)>\\ <h_3,(0.8,0.6,0.4,0.5),(0.5,0.3,0.4,0.2),(0.3,0.5,0.4,0.4)>\end{pmatrix}\end{array}\right\}$$

$$(H_1^{nm},\Upsilon)\tilde{\setminus}(H_2^{nm},\Upsilon)=\left\{\begin{array}{l}\sigma_1,\begin{pmatrix}<h_1,(0.4,0.6,0.1,0.3),(0.4,0.5,0.7,0.6),(0.5,0.6,0.6,0.8)>\\ <h_2,(0.3,0.4,0.2,0.2),(0.4,0.3,0.3,0.6),(0.5,0.4,0.6,0.5)>\\ <h_3,(0.3,0.4,0.5,0.3),(0.3,0.5,0.4,0.2),(0.7,0.5,0.7,0.6)>\end{pmatrix}\\ \sigma_2,\begin{pmatrix}<h_1,(0.2,0.2,0.3,0.2),(0.4,0.5,0.6,0.6),(0.8,0.6,0.2,0.8)>\\ <h_2,(0.4,0.4,0.7,0.5),(0.4,0.5,0.3,0.8),(0.6,0.5,0.4,0.5)>\\ <h_3,(0.3,0.5,0.3,0.2),(0.5,0.4,0.3,0.4),(0.8,0.6,0.4,0.7)>\end{pmatrix}\end{array}\right\}$$

Definition 3.6 *Let $NSMS(\zeta^{nm},\Upsilon)$ be the family of all neutrosophic soft multi sets over the universe set ζ and $\overset{nm}{\tau}$ be a subfamily of $NSMS(\zeta^{nm},\Upsilon)$. Then $\overset{nm}{\tau}$ is known as neutrosophic soft multi topology on ζ if the given conditions are satisfied:*

1. $0_{(\zeta^{nm},\Upsilon)}, 1_{(\zeta^{nm},\Upsilon)} \in \overset{nm}{\tau}$

2. The union of any number of neutrosophic soft multi sets in $\overset{nm}{\tau}$ belongs to $\overset{nm}{\tau}$

3. The intersection of finite number of neutrosophic soft multi sets in $\overset{nm}{\tau}$ belongs to $\overset{nm}{\tau}$.

Then $\left(\zeta, \overset{nm}{\tau}, \Upsilon\right)$ is said to be a neutrosophic soft multi topological space over ζ. Each members of $\overset{nm}{\tau}$ is said to be neutrosophic soft multi open set.

Definition 3.7 *Let $\left(\zeta, \overset{nm}{\tau}, \Upsilon\right)$ be a neutrosophic soft multi topological space over ζ and $\left(H^{nm}, \Upsilon\right)$ be a neutrosophic soft multi set over ζ. Then $\left(H^{nm}, \Upsilon\right)$ is said to be neutrosophic soft multi closed set iff its complement is a neutrosophic soft multi open set.*

Proposition 3.3 *Let $\left(\zeta, \overset{nm}{\tau}, \Upsilon\right)$ be a neutrosophic soft multi topological space over ζ. Then*

1. $0_{(\zeta^{nm},\Upsilon)}$ and $1_{(\zeta^{nm},\Upsilon)}$ are neutrosophic soft multi closed sets over ζ
2. the intersection of any number of neutrosophic soft multi closed sets is a neutrosophic soft multi closed set over ζ
3. the union of finite number of neutrosophic soft multi closed sets is a neutrosophic soft multi closed set over ζ.

Proof. It is easily obtained from the definition neutrosophic soft topological space and Proposition 3.2.

Definition 3.8 *Let $NSMS(\zeta^{nm}, \Upsilon)$ be the family of all neutrosophic soft multisets over the universe set ζ.*

1. If $\overset{nm}{\tau} = \left\{0_{(\zeta^{nm},\Upsilon)}, 1_{(\zeta^{nm},\Upsilon)}\right\}$, then $\overset{nm}{\tau}$ is said to be the neutrosophic soft multi indiscrete topology and $\left(\zeta, \overset{nm}{\tau}, \Upsilon\right)$ is said to be a neutrosophic soft multi indiscrete topological space over ζ.

2. If $\overset{nm}{\tau} = NSMS(\zeta^{nm}, \Upsilon)$, then $\overset{nm}{\tau}$ is said to be the neutrosophic soft multi discrete topology and $\left(\zeta, \overset{nm}{\tau}, \Upsilon\right)$ is said to be a neutrosophic soft multi discrete topological space over ζ.

Proposition 3.4 Let $\left(\zeta, \overset{nm}{\tau_1}, \Upsilon\right)$ and $\left(\zeta, \overset{nm}{\tau_2}, \Upsilon\right)$ be two neutrosophic soft multi topological spaces over the same universe set ζ. Then $\left(\zeta, \overset{nm}{\tau_1} \cap \overset{nm}{\tau_2}, \Upsilon\right)$ is neutrosophic soft multi topological space over ζ.

Proof. 1. Since $0_{(\zeta^{nm},\Upsilon)}, 1_{(\zeta^{nm},\Upsilon)} \in \overset{nm}{\tau_1}$ and $0_{(\zeta^{nm},\Upsilon)}, 1_{(\zeta^{nm},\Upsilon)} \in \overset{nm}{\tau_2}$, then $0_{(\zeta^{nm},\Upsilon)}, 1_{(\zeta^{nm},\Upsilon)} \in \overset{nm}{\tau_1} \cap \overset{nm}{\tau_2}$.

2. Suppose that $\left\{ \left(\overset{nm}{H_i}, \Upsilon \right) \middle| i \in I \right\}$ be a family of neutrosophic soft multisets in $\overset{nm}{\tau_1} \cap \overset{nm}{\tau_2}$. Then $\left(\overset{nm}{H_i}, \Upsilon \right) \in \overset{nm}{\tau_1}$ and $\left(\overset{nm}{H_i}, \Upsilon \right) \in \overset{nm}{\tau_2}$ for all $i \in I$, so $\underset{i\in I}{\cup}\left(\overset{nm}{H_i}, \Upsilon \right) \in \overset{nm}{\tau_1}$ and $\underset{i\in I}{\cup}\left(\overset{nm}{H_i}, \Upsilon \right) \in \overset{nm}{\tau_2}$. Thus $\underset{i\in I}{\cup}\left(\overset{nm}{H_i}, \Upsilon \right) \in \overset{nm}{\tau_1} \cap \overset{nm}{\tau_2}$.

3. Let $\left\{ \left(\overset{nm}{H_i}, \Upsilon \right) \middle| i = \overline{1,n} \right\}$ be a family of the finite number of neutrosophic soft multisets in $\overset{nm}{\tau_1} \cap \overset{nm}{\tau_2}$. Then $\left(\overset{nm}{H_i}, \Upsilon \right) \in \overset{nm}{\tau_1}$ and $\left(\overset{nm}{H_i}, \Upsilon \right) \in \overset{nm}{\tau_2}$ for $i = \overline{1,n}$, so $\overset{n}{\underset{i=1}{\cap}}\left(\overset{nm}{H_i}, \Upsilon \right) \in \overset{nm}{\tau_1}$ and $\overset{n}{\underset{i=1}{\cap}}\left(\overset{nm}{H_i}, \Upsilon \right) \in \overset{nm}{\tau_2}$. Thus $\overset{n}{\underset{i=1}{\cap}}\left(\overset{nm}{H_i}, \Upsilon \right) \in \overset{nm}{\tau_1} \cap \overset{nm}{\tau_2}$.

Remark 3.1 The union of two neutrosophic soft multi topologies over ζ may not be a neutrosophic soft multi topology on ζ.

Example 3.3 *Let $\zeta = \{h_1, h_2, h_3\}$ be universe set and $\Upsilon = \{\sigma_1, \sigma_2\}$ be a parameters. Let consider neutrosophic soft multisets $(\overset{nm}{H_1}, \Upsilon)$, $(\overset{nm}{H_2}, \Upsilon), (\overset{nm}{H_3}, \Upsilon)$ and $(\overset{nm}{H_4}, \Upsilon)$ as follows:*

$$(\overset{nm}{H_1}, \Upsilon) = \left\{ \begin{array}{l} \sigma_1, \left(\begin{array}{l} < h_1, (0.8, 0.6, 0.7, 0.4), (0.6, 0.8, 0.5, 0.3), (0.4, 0.5, 0.3, 0.2) > \\ < h_2, (0.7, 0.8, 0.6, 0.5), (0.9, 0.5, 0.6, 0.5), (0.2, 0.4, 0.4, 0.5) > \\ < h_3, (0.6, 0.7, 0.5, 0.6), (0.7, 0.6, 0.8, 0.7), (0.5, 0.3, 0.4, 0.4) > \end{array} \right) \\ \sigma_2, \left(\begin{array}{l} < h_1, (0.7, 0.7, 0.6, 0.6), (0.5, 0.6, 0.6, 0.9), (0.5, 0.4, 0.6, 0.2) > \\ < h_2, (0.8, 0.9, 0.4, 0.6), (0.7, 0.8, 0.8, 0.5), (0.3, 0.2, 0.4, 0.6) > \\ < h_3, (0.7, 0.7, 0.8, 0.6), (0.9, 0.8, 0.6, 0.7), (0.5, 0.1, 0.6, 0.3) > \end{array} \right) \end{array} \right\}$$

$$(\overset{nm}{H_2}, \Upsilon) = \left\{ \begin{array}{l} \sigma_1, \left(\begin{array}{l} < h_1, (0.6, 0.5, 0.7, 0.3), (0.6, 0.7, 0.4, 0.3), (0.6, 0.6, 0.4, 0.4) > \\ < h_2, (0.7, 0.6, 0.4, 0.4), (0.8, 0.3, 0.4, 0.2), (0.5, 0.6, 0.7, 0.6) > \\ < h_3, (0.5, 0.6, 0.3, 0.5), (0.5, 0.4, 0.6, 0.5), (0.6, 0.5, 0.5, 0.7) > \end{array} \right) \\ \sigma_2, \left(\begin{array}{l} < h_1, (0.5, 0.6, 0.4, 0.4), (0.4, 0.5, 0.3, 0.7), (0.7, 0.6, 0.7, 0.5) > \\ < h_2, (0.7, 0.6, 0.4, 0.5), (0.5, 0.7, 0.6, 0.3), (0.4, 0.4, 0.6, 0.4) > \\ < h_3, (0.5, 0.4, 0.7, 0.6), (0.6, 0.7, 0.3, 0.5), (0.6, 0.3, 0.7, 0.4) > \end{array} \right) \end{array} \right\}$$

$$(\overset{nm}{H_3}, \Upsilon) = \left\{ \begin{array}{l} \sigma_1, \left(\begin{array}{l} < h_1, (0.4, 0.3, 0.5, 0.3), (0.4, 0.3, 0.2, 0.1), (0.7, 0.8, 0.5, 0.6) > \\ < h_2, (0.6, 0.5, 0.3, 0.2), (0.5, 0.2, 0.3, 0.2), (0.6, 0.8, 0.9, 0.8) > \\ < h_3, (0.3, 0.4, 0.2, 0.3), (0.4, 0.3, 0.5, 0.2), (0.7, 0.6, 0.5, 0.8) > \end{array} \right) \\ \sigma_2, \left(\begin{array}{l} < h_1, (0.4, 0.3, 0.2, 0.3), (0.2, 0.4, 0.2, 0.5), (0.8, 0.7, 0.9, 0.9) > \\ < h_2, (0.6, 0.5, 0.2, 0.4), (0.4, 0.3, 0.2, 0.2), (0.5, 0.6, 0.6, 0.7) > \\ < h_3, (0.4, 0.3, 0.6, 0.6), (0.5, 0.6, 0.4, 0.3), (0.7, 0.5, 0.8, 0.6) > \end{array} \right) \end{array} \right\}$$

$$(\overset{nm}{H_4}, \Upsilon) = \left\{ \begin{array}{l} \sigma_1, \left(\begin{array}{l} < h_1, (0.7, 0.8, 0.8, 0.5), (0.8, 0.7, 0.7, 0.4), (0.5, 0.3, 0.2, 0.3) > \\ < h_2, (0.8, 0.7, 0.5, 0.6), (0.8, 0.6, 0.5, 0.3), (0.1, 0.5, 0.6, 0.4) > \\ < h_3, (0.7, 0.9, 0.4, 0.7), (0.6, 0.7, 0.7, 0.6), (0.4, 0.3, 0.2, 0.4) > \end{array} \right) \\ \sigma_2, \left(\begin{array}{l} < h_1, (0.6, 0.8, 0.5, 0.8), (0.7, 0.8, 0.5, 0.8), (0.4, 0.5, 0.3, 0.2) > \\ < h_2, (0.8, 0.8, 0.5, 0.5), (0.7, 0.8, 0.7, 0.8), (0.2, 0.2, 0.4, 0.3) > \\ < h_3, (0.6, 0.5, 0.8, 0.8), (0.7, 0.8, 0.6, 0.6), (0.3, 0.2, 0.5, 0.2) > \end{array} \right) \end{array} \right\}$$

Then

$$\overset{nm}{\tau_1} = \{0_{(\zeta^{nm}, \Upsilon)}, 1_{(\zeta^{nm}, \Upsilon)}, (\overset{nm}{H_1}, \Upsilon), (\overset{nm}{H_2}, \Upsilon), (\overset{nm}{H_3}, \Upsilon)\}$$

and

$$\overset{nm}{\tau_2} = \{0_{(\zeta^{nm}, \Upsilon)}, 1_{(\zeta^{nm}, \Upsilon)}, (\overset{nm}{H_2}, \Upsilon), (\overset{nm}{H_4}, \Upsilon)\}$$

are neutrosophic soft multi topological spaces over ζ. Since $(\overset{nm}{H_1}, \Upsilon) \cup (\overset{nm}{H_4}, \Upsilon) \notin \overset{nm}{\tau_1} \cup \overset{nm}{\tau_2}$, then $\overset{nm}{\tau_1} \cup \overset{nm}{\tau_2}$ is not a neutrosophic soft multi topology on ζ.

Definition 3.9 Let $\left(\zeta, \overset{nm}{\tau}, \Upsilon\right)$ be a neutrosophic soft multi topological space over ζ and $\left(H^{nm}, \Upsilon\right) \in NSMS(\zeta^{nm}, \Upsilon)$ be a neutrosophic soft multiset. Then, the neutrosophic soft multi interior of $\left(H^{nm}, \Upsilon\right)$, denoted $\left(H^{nm}, \Upsilon\right)^{\circ}$, is defined as the neutrosophic soft multi union of all neutrosophic soft multi open subsets of $\left(H^{nm}, \Upsilon\right)$.

Clearly, $\left(H^{nm}, \Upsilon\right)^{\circ}$ is the biggest neutrosophic soft multi open set that is contained by $\left(H^{nm}, \Upsilon\right)$.

Example 3.4 *Let $\zeta = \{h_1, h_2, h_3\}$ be universe set and $\Upsilon = \{\sigma_1, \sigma_2\}$ be a parameters. Let consider neutrosophic soft multisets $(\overset{nm}{H_1}, \Upsilon)$ and $(\overset{nm}{H_2}, \Upsilon)$ as follows:*

$$(\overset{nm}{H_1}, \Upsilon) = \left\{ \begin{array}{l} \sigma_1, \left(\begin{array}{l} < h_1, (0.6, 0.5, 0.7, 0.3), (0.6, 0.7, 0.4, 0.3), (0.6, 0.6, 0.4, 0.4) > \\ < h_2, (0.7, 0.6, 0.4, 0.4), (0.8, 0.3, 0.4, 0.2), (0.5, 0.6, 0.7, 0.6) > \\ < h_3, (0.5, 0.6, 0.3, 0.5), (0.5, 0.4, 0.6, 0.5), (0.6, 0.5, 0.5, 0.7) > \end{array} \right) \\ \sigma_2, \left(\begin{array}{l} < h_1, (0.5, 0.6, 0.4, 0.4), (0.4, 0.5, 0.3, 0.7), (0.7, 0.6, 0.7, 0.5) > \\ < h_2, (0.7, 0.6, 0.4, 0.5), (0.5, 0.7, 0.6, 0.3), (0.4, 0.4, 0.6, 0.4) > \\ < h_3, (0.5, 0.4, 0.7, 0.6), (0.6, 0.7, 0.3, 0.5), (0.6, 0.3, 0.7, 0.4) > \end{array} \right) \end{array} \right\}$$

$$(\overset{nm}{H_2}, \Upsilon) = \left\{ \begin{array}{l} \sigma_1, \begin{pmatrix} < h_1, (0.7, 0.8, 0.8, 0.5), (0.8, 0.7, 0.7, 0.4), (0.5, 0.3, 0.2, 0.3) > \\ < h_2, (0.8, 0.7, 0.5, 0.6), (0.8, 0.6, 0.5, 0.3), (0.1, 0.5, 0.6, 0.4) > \\ < h_3, (0.7, 0.9, 0.4, 0.7), (0.6, 0.7, 0.7, 0.6), (0.4, 0.3, 0.2, 0.4) > \end{pmatrix} \\ \sigma_2, \begin{pmatrix} < h_1, (0.6, 0.8, 0.5, 0.8), (0.7, 0.8, 0.5, 0.8), (0.4, 0.5, 0.3, 0.2) > \\ < h_2, (0.8, 0.8, 0.5, 0.5), (0.7, 0.8, 0.7, 0.8), (0.2, 0.2, 0.4, 0.3) > \\ < h_3, (0.6, 0.5, 0.8, 0.8), (0.7, 0.8, 0.6, 0.6), (0.3, 0.2, 0.5, 0.2) > \end{pmatrix} \end{array} \right\}$$

then $\overset{nm}{\tau}_1 = \{0_{(\zeta^{nm},\Upsilon)}, 1_{(\zeta^{nm},\Upsilon)}, (\overset{nm}{H_1}, \Upsilon), (\overset{nm}{H_2}, \Upsilon)\}$ is a neutrosophic soft multi topology on ζ. Suppose that any $(\overset{nm}{H_3}, \Upsilon) \in NSMS(\zeta^{nm}, \Upsilon)$ is defined as follows:

$$(\overset{nm}{H_3}, \Upsilon) = \left\{ \begin{array}{l} \sigma_1, \begin{pmatrix} < h_1, (0.8, 0.6, 0.7, 0.4), (0.6, 0.8, 0.5, 0.3), (0.4, 0.5, 0.3, 0.2) > \\ < h_2, (0.7, 0.8, 0.6, 0.5), (0.9, 0.5, 0.6, 0.5), (0.2, 0.4, 0.4, 0.5) > \\ < h_3, (0.6, 0.7, 0.5, 0.6), (0.7, 0.6, 0.8, 0.7), (0.5, 0.3, 0.4, 0.4) > \end{pmatrix} \\ \sigma_2, \begin{pmatrix} < h_1, (0.7, 0.7, 0.6, 0.6), (0.5, 0.6, 0.6, 0.9), (0.5, 0.4, 0.6, 0.2) > \\ < h_2, (0.8, 0.9, 0.4, 0.6), (0.7, 0.8, 0.8, 0.5), (0.3, 0.2, 0.4, 0.6) > \\ < h_3, (0.7, 0.7, 0.8, 0.6), (0.9, 0.8, 0.6, 0.7), (0.5, 0.1, 0.6, 0.3) > \end{pmatrix} \end{array} \right\}$$

Then $0_{(\zeta^{nm},\Upsilon)}, (\overset{nm}{H_1}, \Upsilon) \subseteq (\overset{nm}{H_3}, \Upsilon)$. Therefore, $(\overset{nm}{H_3}, \Upsilon)^\circ = 0_{(\zeta^{nm},\Upsilon)} \cup (\overset{nm}{H_1}, \Upsilon) = (\overset{nm}{H_1}, \Upsilon)$.

Theorem 3.1 Let $\left(\zeta, \overset{nm}{\tau}, \Upsilon\right)$ be a neutrosophic soft multi topological space over ζ and $\left(\overset{nm}{H}, \Upsilon\right) \in NSMS(\zeta^{nm}, \Upsilon)$. $\left(\overset{nm}{H}, \Upsilon\right)$ is a neutrosophic soft multi open set iff $\left(\overset{nm}{H}, \Upsilon\right) = \left(\overset{nm}{H}, \Upsilon\right)^\circ$.

Proof. Let $\left(\overset{nm}{H}, \Upsilon\right)$ be a neutrosophic soft multi open set. Then the biggest neutrosophic soft multi open set that is contained by $\left(\overset{nm}{H}, \Upsilon\right)$ is equal to $\left(\overset{nm}{H}, \Upsilon\right)$. Hence, $\left(\overset{nm}{H}, \Upsilon\right) = \left(\overset{nm}{H}, \Upsilon\right)^\circ$

Conversely, it is known that $\left(\overset{nm}{H}, \Upsilon\right)^\circ$ is a neutrosophic soft multi open set and if $\left(\overset{nm}{H}, \Upsilon\right) = \left(\overset{nm}{H}, \Upsilon\right)^\circ$, then $\left(\overset{nm}{H}, \Upsilon\right)$ is a neutrosophic soft multi open set.

Theorem 3.2 Let $\left(\zeta, \overset{nm}{\tau}, \Upsilon\right)$ be a neutrosophic soft multi topological space over ζ and $\left(\overset{nm}{H_1}, \Upsilon\right), \left(\overset{nm}{H_2}, \Upsilon\right) \in NSMS(\zeta^{nm}, \Upsilon)$. Then,

1. $\left[\left(\overset{nm}{H_1}, \Upsilon\right)^\circ\right]^\circ = \left(\overset{nm}{H_1}, \Upsilon\right)^\circ$,

2. $\left(0_{(\zeta^{nm},\Upsilon)}\right)^{\circ}=0_{(\zeta^{nm},\Upsilon)}$ and $\left(1_{(\zeta^{nm},\Upsilon)}\right)^{\circ}=1_{(\zeta^{nm},\Upsilon)}$,

3. $\left(H_1^{nm},\Upsilon\right)\subseteq\left(H_2^{nm},\Upsilon\right)\Rightarrow\left(H_1^{nm},\Upsilon\right)^{\circ}\subseteq\left(H_2^{nm},\Upsilon\right)^{\circ}$,

4. $\left[\left(H_1^{nm},\Upsilon\right)\cap\left(H_2^{nm},\Upsilon\right)\right]^{\circ}=\left(H_1^{nm},\Upsilon\right)^{\circ}\cap\left(H_2^{nm},\Upsilon\right)^{\circ}$,

5. $\left(H_1^{nm},\Upsilon\right)^{\circ}\cup\left(H_2^{nm},\Upsilon\right)^{\circ}\subseteq\left[\left(H_1^{nm},\Upsilon\right)\cup\left(H_2^{nm},\Upsilon\right)\right]^{\circ}$.

Proof. 1. Let $\left(H_1^{nm},\Upsilon\right)^{\circ}=\left(H_2^{nm},\Upsilon\right)$. Then $\left(H_2^{nm},\Upsilon\right)\in\overset{nm}{\tau}$ iff $\left(H_2^{nm},\Upsilon\right)=\left(H_2^{nm},\Upsilon\right)^{\circ}$. So, $\left[\left(H_1^{nm},\Upsilon\right)^{\circ}\right]^{\circ}=\left(H_1^{nm},\Upsilon\right)^{\circ}$.

2. Straighforward.

3. It is known that $\left(H_1^{nm},\Upsilon\right)^{\circ}\subseteq\left(H_1^{nm},\Upsilon\right)\subseteq\left(H_2^{nm},\Upsilon\right)$ and $\left(H_2^{nm},\Upsilon\right)^{\circ}\subseteq\left(H_2^{nm},\Upsilon\right)$. Since $\left(H_2^{nm},\Upsilon\right)^{\circ}$ is the biggest neutrosophic soft multi open set contained in $\left(H_2^{nm},\Upsilon\right)$ and so, $\left(H_1^{nm},\Upsilon\right)^{\circ}\subseteq\left(H_2^{nm},\Upsilon\right)^{\circ}$.

4. Since $\left(H_1^{nm},\Upsilon\right)\cap\left(H_2^{nm},\Upsilon\right)\subseteq\left(H_1^{nm},\Upsilon\right)$ and $\left(H_1^{nm},\Upsilon\right)\cap\left(H_2^{nm},\Upsilon\right)\subseteq\left(H_2^{nm},\Upsilon\right)$, then $\left[\left(H_1^{nm},\Upsilon\right)\cap\left(H_2^{nm},\Upsilon\right)\right]^{\circ}\subseteq\left(H_1^{nm},\Upsilon\right)^{\circ}$ and $\left[\left(H_1^{nm},\Upsilon\right)\cap\left(H_2^{nm},\Upsilon\right)\right]^{\circ}\subseteq\left(H_2^{nm},\Upsilon\right)^{\circ}$ and so, $\left[\left(H_1^{nm},\Upsilon\right)\cap\left(H_2^{nm},\Upsilon\right)\right]^{\circ}\subseteq\left(H_1^{nm},\Upsilon\right)^{\circ}\cap\left(H_2^{nm},\Upsilon\right)^{\circ}$.

On the other hand, since $\left(H_1^{nm},\Upsilon\right)^{\circ}\subseteq\left(H_1^{nm},\Upsilon\right)$ and $\left(H_2^{nm},\Upsilon\right)^{\circ}\subseteq\left(H_2^{nm},\Upsilon\right)$, then $\left(H_1^{nm},\Upsilon\right)^{\circ}\cap\left(H_2^{nm},\Upsilon\right)^{\circ}\subseteq\left(H_1^{nm},\Upsilon\right)\cap\left(H_2^{nm},\Upsilon\right)$. Besides, $\left[\left(H_1^{nm},\Upsilon\right)\cap\left(H_2^{nm},\Upsilon\right)\right]^{\circ}\subseteq\left(H_1^{nm},\Upsilon\right)\cap\left(H_2^{nm},\Upsilon\right)$ and it is the biggest neutrosophic soft multi open set. Therefore, $\left(H_1^{nm},\Upsilon\right)^{\circ}\cap\left(H_2^{nm},\Upsilon\right)^{\circ}\subseteq\left[\left(H_1^{nm},\Upsilon\right)\cap\left(H_2^{nm},\Upsilon\right)\right]^{\circ}$. Thus, $\left[\left(H_1^{nm},\Upsilon\right)\cap\left(H_2^{nm},\Upsilon\right)\right]^{\circ}=\left(H_1^{nm},\Upsilon\right)^{\circ}\cap\left(H_2^{nm},\Upsilon\right)^{\circ}$.

5. Since $\left(H_1^{nm},\Upsilon\right)\subseteq\left(H_1^{nm},\Upsilon\right)\cup\left(H_2^{nm},\Upsilon\right)$ and $\left(H_2^{nm},\Upsilon\right)\subseteq\left(H_1^{nm},\Upsilon\right)\cup\left(H_2^{nm},\Upsilon\right)$, then $\left(H_1^{nm},\Upsilon\right)^{\circ}\subseteq\left[\left(H_1^{nm},\Upsilon\right)\cup\left(H_2^{nm},\Upsilon\right)\right]^{\circ}$ and $\left(H_2^{nm},\Upsilon\right)^{\circ}\subseteq\left[\left(H_1^{nm},\Upsilon\right)\cup\left(H_2^{nm},\Upsilon\right)\right]^{\circ}$. Therefore, $\left(H_1^{nm},\Upsilon\right)^{\circ}\cup\left(H_2^{nm},\Upsilon\right)^{\circ}\subseteq\left[\left(H_1^{nm},\Upsilon\right)\cup\left(H_2^{nm},\Upsilon\right)\right]^{\circ}$.

Definition 3.10 Let $\left(\zeta, \overset{nm}{\tau}, \Upsilon\right)$ be a neutrosophic soft multi topological space over ζ and $\left(H^{nm}, \Upsilon\right) \in NSMS(\zeta^{nm}, \Upsilon)$ be a neutrosophic soft multiset. Then, the neutrosophic soft multi closure of $\left(H^{nm}, \Upsilon\right)$, denoted $\overline{\left(H^{nm}, \Upsilon\right)}$, is defined as the neutrosophic soft multi intersection of all neutrosophic soft multi closed supersets of $\left(H^{nm}, \Upsilon\right)$.

Clearly, $\overline{\left(H^{nm}, \Upsilon\right)}$ is the smallest neutrosophic soft multi closed set that containing $\left(H^{nm}, \Upsilon\right)$.

Example 3.5 Let $\zeta = \{h_1, h_2, h_3\}$ be universe set and $\Upsilon = \{\sigma_1, \sigma_2\}$ be a parameters. Let consider neutrosophic soft multisets (H_1^{nm}, Υ) and (H_2^{nm}, Υ) as follows:

$$(H_1^{nm}, \Upsilon) = \left\{ \begin{array}{l} \sigma_1, \left(\begin{array}{l} < h_1, (0.6, 0.5, 0.7, 0.3), (0.6, 0.7, 0.4, 0.3), (0.6, 0.6, 0.4, 0.4) > \\ < h_2, (0.7, 0.6, 0.4, 0.4), (0.8, 0.3, 0.4, 0.2), (0.5, 0.6, 0.7, 0.6) > \\ < h_3, (0.5, 0.6, 0.3, 0.5), (0.5, 0.4, 0.6, 0.5), (0.6, 0.5, 0.5, 0.7) > \end{array} \right) \\ \sigma_2, \left(\begin{array}{l} < h_1, (0.5, 0.6, 0.4, 0.4), (0.4, 0.5, 0.3, 0.7), (0.7, 0.6, 0.7, 0.5) > \\ < h_2, (0.7, 0.6, 0.4, 0.5), (0.5, 0.7, 0.6, 0.3), (0.4, 0.4, 0.6, 0.4) > \\ < h_3, (0.5, 0.4, 0.7, 0.6), (0.6, 0.7, 0.3, 0.5), (0.6, 0.3, 0.7, 0.4) > \end{array} \right) \end{array} \right\}$$

$$(H_2^{nm}, \Upsilon) = \left\{ \begin{array}{l} \sigma_1, \left(\begin{array}{l} < h_1, (0.7, 0.8, 0.8, 0.5), (0.8, 0.7, 0.7, 0.4), (0.5, 0.3, 0.2, 0.3) > \\ < h_2, (0.8, 0.7, 0.5, 0.6), (0.8, 0.6, 0.5, 0.3), (0.1, 0.5, 0.6, 0.4) > \\ < h_3, (0.7, 0.9, 0.4, 0.7), (0.6, 0.7, 0.7, 0.6), (0.4, 0.3, 0.2, 0.4) > \end{array} \right) \\ \sigma_2, \left(\begin{array}{l} < h_1, (0.6, 0.8, 0.5, 0.8), (0.7, 0.8, 0.5, 0.8), (0.4, 0.5, 0.3, 0.2) > \\ < h_2, (0.8, 0.8, 0.5, 0.5), (0.7, 0.8, 0.7, 0.8), (0.2, 0.2, 0.4, 0.3) > \\ < h_3, (0.6, 0.5, 0.8, 0.8), (0.7, 0.8, 0.6, 0.6), (0.3, 0.2, 0.5, 0.2) > \end{array} \right) \end{array} \right\}$$

then $\overset{nm}{\tau}_1 = \{0_{(\zeta^{nm}, \Upsilon)}, 1_{(\zeta^{nm}, \Upsilon)}, (H_1^{nm}, \Upsilon), (H_2^{nm}, \Upsilon)\}$ is a neutrosophic soft multi topology on ζ. Now, we will find neutrosophic soft multi closed sets as follows:

$$(H_1^{nm}, \Upsilon)^c = \left\{ \begin{array}{l} \sigma_1, \left(\begin{array}{l} < h_1, (0.6, 0.6, 0.4, 0.4), (0.4, 0.3, 0.6, 0.7), (0.6, 0.5, 0.7, 0.3) > \\ < h_2, (0.5, 0.6, 0.7, 0.6), (0.2, 0.7, 0.6, 0.8), (0.7, 0.6, 0.4, 0.4) > \\ < h_3, (0.6, 0.5, 0.5, 0.7), (0.5, 0.6, 0.4, 0.5), (0.5, 0.6, 0.3, 0.5) > \end{array} \right) \\ \sigma_2, \left(\begin{array}{l} < h_1, (0.7, 0.6, 0.7, 0.5), (0.6, 0.5, 0.7, 0.3), (0.5, 0.6, 0.4, 0.4) > \\ < h_2, (0.4, 0.4, 0.6, 0.4), (0.5, 0.3, 0.4, 0.7), (0.7, 0.6, 0.4, 0.5) > \\ < h_3, (0.6, 0.3, 0.7, 0.4), (0.4, 0.3, 0.7, 0.5), (0.5, 0.4, 0.7, 0.6) > \end{array} \right) \end{array} \right\}$$

$$(H_2^{nm},\Upsilon)^c=\left\{\begin{array}{l}\sigma_1,\left(\begin{array}{l}<h_1,(0.5,0.3,0.2,0.3),(0.2,0.3,0.3,0.6),(0.7,0.8,0.8,0.5)>\\ <h_2,(0.1,0.5,0.6,0.4),(0.2,0.4,0.5,0.7),(0.8,0.7,0.5,0.6)>\\ <h_3,(0.4,0.3,0.2,0.4),(0.4,0.3,0.3,0.4),(0.7,0.9,0.4,0.7)>\end{array}\right)\\ \sigma_2,\left(\begin{array}{l}<h_1,(0.4,0.5,0.3,0.2),(0.3,0.2,0.5,0.2),(0.6,0.8,0.5,0.8)>\\ <h_2,(0.2,0.2,0.4,0.3),(0.3,0.2,0.3,0.2),(0.8,0.8,0.5,0.5)>\\ <h_3,(0.3,0.2,0.5,0.2),(0.3,0.2,0.4,0.4),(0.6,0.5,0.8,0.8)>\end{array}\right)\end{array}\right\}$$

Suppose that any $(H_3^{nm},\Upsilon)\in NSMS(\zeta^{nm},\Upsilon)$ is defined as follows:

$$(H_3^{nm},\Upsilon)=\left\{\begin{array}{l}\sigma_1,\left(\begin{array}{l}<h_1,(0.4,0.2,0.2,0.2),(0.1,0.2,0.1,0.5),(0.8,0.9,0.8,0.7)>\\ <h_2,(0.1,0.3,0.4,0.2),(0.2,0.3,0.2,0.5),(0.8,0.8,0.7,0.8)>\\ <h_3,(0.3,0.1,0.2,0.3),(0.2,0.2,0.1,0.3),(0.8,0.9,0.5,0.8)>\end{array}\right)\\ \sigma_2,\left(\begin{array}{l}<h_1,(0.2,0.4,0.3,0.2),(0.2,0.1,0.3,0.2),(0.7,0.8,0.6,0.9)>\\ <h_2,(0.1,0.1,0.2,0.3),(0.2,0.2,0.1,0.1),(0.8,0.9,0.6,0.7)>\\ <h_3,(0.2,0.1,0.4,0.1),(0.2,0.2,0.3,0.2),(0.6,0.8,0.8,0.9)>\end{array}\right)\end{array}\right\}$$

Then $1_{(\zeta^{nm},\Upsilon)},(H_1^{nm},\Upsilon)^c,(H_2^{nm},\Upsilon)^c\supseteq(H_3^{nm},\Upsilon)$. Therefore, $\overline{(H_3^{nm},\Upsilon)}=1_{(\zeta^{nm},\Upsilon)}\cap(H_1^{nm},\Upsilon)^c\cap(H_2^{nm},\Upsilon)^c=(H_2^{nm},\Upsilon)^c$.

Theorem 3.3 Let $\left(\zeta,\overset{nm}{\tau},\Upsilon\right)$ be a neutrosophic soft multi topological space over ζ and $\left(H^{nm},\Upsilon\right)\in NSMS(\zeta^{nm},\Upsilon)$. $\left(H^{nm},\Upsilon\right)$ is neutrosophic soft multi closed set iff $\left(H^{nm},\Upsilon\right)=\overline{\left(H^{nm},\Upsilon\right)}$.

Proof. Straightforward.

Theorem 3.4 Let $\left(\zeta,\overset{nm}{\tau},\Upsilon\right)$ be a neutrosophic soft multi topological space over ζ and $\left(H_1^{nm},\Upsilon\right),\left(H_2^{nm},\Upsilon\right)\in NSMS(\zeta^{nm},\Upsilon)$. Then,

1. $\overline{\left[\overline{\left(H_1^{nm},\Upsilon\right)}\right]}=\overline{\left(H_1^{nm},\Upsilon\right)}$,
2. $\overline{\left(0_{(\zeta^{nm},\Upsilon)}\right)}=0_{(\zeta^{nm},\Upsilon)}$ and $\overline{\left(1_{(\zeta^{nm},\Upsilon)}\right)}=1_{(\zeta^{nm},\Upsilon)}$
3. $\left(H_1^{nm},\Upsilon\right)\subseteq\left(H_2^{nm},\Upsilon\right)\Rightarrow\overline{\left(H_1^{nm},\Upsilon\right)}\subseteq\overline{\left(H_2^{nm},\Upsilon\right)}$,
4. $\overline{\left[\left(H_1^{nm},\Upsilon\right)\cup\left(H_2^{nm},\Upsilon\right)\right]}=\overline{\left(H_1^{nm},\Upsilon\right)}\cup\overline{\left(H_2^{nm},\Upsilon\right)}$,
5. $\overline{\left[\left(H_1^{nm},\Upsilon\right)\cap\left(H_2^{nm},\Upsilon\right)\right]}\subseteq\overline{\left(H_1^{nm},\Upsilon\right)}\cap\overline{\left(H_2^{nm},\Upsilon\right)}$.

Proof. 1. Let $\overline{\left(H_1^{nm},\Upsilon\right)}=\left(H_2^{nm},\Upsilon\right)$. Then, $\left(H_2^{nm},\Upsilon\right)$ is a neutrosophic soft multi closed set. Hence, $\left(H_2^{nm},\Upsilon\right)$ and $\overline{\left(H_2^{nm},\Upsilon\right)}$ are equal. Therefore, $\overline{\left[\overline{\left(H_1^{nm},\Upsilon\right)}\right]}=\overline{\left(H_1^{nm},\Upsilon\right)}$.

2. Straightforward.

3. It is known that $\left(H_1^{nm},\Upsilon\right)\subseteq\overline{\left(H_1^{nm},\Upsilon\right)}$ and $\left(H_2^{nm},\Upsilon\right)\subseteq\overline{\left(H_2^{nm},\Upsilon\right)}$ and so, $\left(H_1^{nm},\Upsilon\right)\subseteq\left(H_2^{nm},\Upsilon\right)\subseteq\overline{\left(H_2^{nm},\Upsilon\right)}$. Since $\overline{\left(H_1^{nm},\Upsilon\right)}$ is the smallest neutrosophic soft multi closed set containing $\left(H_1^{nm},\Upsilon\right)$, then $\overline{\left(H_1^{nm},\Upsilon\right)}\subseteq\overline{\left(H_2^{nm},\Upsilon\right)}$.

4. Since $\left(H_1^{nm},\Upsilon\right)\subseteq\left(H_1^{nm},\Upsilon\right)\cup\left(H_2^{nm},\Upsilon\right)$ and $\left(H_2^{nm},\Upsilon\right)\subseteq\left(H_1^{nm},\Upsilon\right)\cup\left(H_2^{nm},\Upsilon\right)$, then $\overline{\left(H_1^{nm},\Upsilon\right)}\subseteq\overline{\left[\left(H_1^{nm},\Upsilon\right)\cup\left(H_2^{nm},\Upsilon\right)\right]}$ and $\overline{\left(H_2^{nm},\Upsilon\right)}\subseteq\overline{\left[\left(H_1^{nm},\Upsilon\right)\cup\left(H_2^{nm},\Upsilon\right)\right]}$ and so, $\overline{\left(H_1^{nm},\Upsilon\right)}\cup\overline{\left(H_2^{nm},\Upsilon\right)}\subseteq\overline{\left[\left(H_1^{nm},\Upsilon\right)\cup\left(H_2^{nm},\Upsilon\right)\right]}$.

Conversely, since $\left(H_1^{nm},\Upsilon\right)\subseteq\overline{\left(H_1^{nm},\Upsilon\right)}$ and $\left(H_2^{nm},\Upsilon\right)\subseteq\overline{\left(H_2^{nm},\Upsilon\right)}$, then $\left(H_1^{nm},\Upsilon\right)\cup\left(H_2^{nm},\Upsilon\right)\subseteq\overline{\left(H_1^{nm},\Upsilon\right)}\cup\overline{\left(H_2^{nm},\Upsilon\right)}$. Besides, $\overline{\left[\left(H_1^{nm},\Upsilon\right)\cup\left(H_2^{nm},\Upsilon\right)\right]}$ is the smallest neutrosophic soft multi closed set that containing $\left(H_1^{nm},\Upsilon\right)\cup\left(H_2^{nm},\Upsilon\right)$. Therefore, $\overline{\left[\left(H_1^{nm},\Upsilon\right)\cup\left(H_2^{nm},\Upsilon\right)\right]}\subseteq\overline{\left(H_1^{nm},\Upsilon\right)}\cup\overline{\left(H_2^{nm},\Upsilon\right)}$. Thus, $\overline{\left[\left(H_1^{nm},\Upsilon\right)\cup\left(H_2^{nm},\Upsilon\right)\right]}=\overline{\left(H_1^{nm},\Upsilon\right)}\cup\overline{\left(H_2^{nm},\Upsilon\right)}$.

5. Since $\left(H_1^{nm},\Upsilon\right)\cap\left(H_2^{nm},\Upsilon\right)\subseteq\overline{\left(H_1^{nm},\Upsilon\right)}\cap\overline{\left(H_2^{nm},\Upsilon\right)}$ and $\overline{\left[\left(H_1^{nm},\Upsilon\right)\cap\left(H_2^{nm},\Upsilon\right)\right]}$ is the smallest neutrosophic soft multi closed set that containing $\left(H_1^{nm},\Upsilon\right)\cap\left(H_2^{nm},\Upsilon\right)$, then $\overline{\left[\left(H_1^{nm},\Upsilon\right)\cap\left(H_2^{nm},\Upsilon\right)\right]}\subseteq\overline{\left(H_1^{nm},\Upsilon\right)}\cap\overline{\left(H_2^{nm},\Upsilon\right)}$.

Theorem 3.5 *Let* $\left(\zeta,\overset{nm}{\tau},\Upsilon\right)$ *be a neutrosophic soft multi topological space over* ζ *and* $\left(H^{nm},\Upsilon\right)\in NSMS(\zeta^{nm},\Upsilon)$. *Then,*

1. $\left[\overline{\left(H^{nm},\Upsilon\right)}\right]^c=\left[\left(H^{nm},\Upsilon\right)^c\right]^\circ$,

2. $\left[\left(H^{nm},\Upsilon\right)^{\circ}\right]^{c}=\overline{\left[\left(H^{nm},\Upsilon\right)^{c}\right]}$.

Proof. 1.

$$\overline{\left(H^{nm},\Upsilon\right)}=\cap\left\{(G,\Upsilon)\in \overset{NSMS}{\tau^{c}}:(G,\Upsilon)\supseteq\left(H^{nm},\Upsilon\right)\right\}$$

$$\Rightarrow\left[\overline{\left(H^{nm},\Upsilon\right)}\right]^{c}=\left[\cap\left\{(G,\Upsilon)\in \overset{NSMS}{\tau^{c}}:(G,\Upsilon)\supseteq\left(H^{nm},\Upsilon\right)\right\}\right]^{c}$$

$$=\cup\left\{(G,\Upsilon)^{c}\in \overset{nm}{\tau}:(G,\Upsilon)^{c}\subseteq\left(H^{nm},\Upsilon\right)^{c}\right\}=\left[\left(H^{nm},\Upsilon\right)^{c}\right]^{\circ}.$$

2. $\left(H^{nm},\Upsilon\right)^{\circ}=\cup\left\{(G,\Upsilon)\in \overset{nm}{\tau}:(G,\Upsilon)\subseteq\left(H^{nm},\Upsilon\right)\right\}$

$$\Rightarrow\left[\left(H^{nm},\Upsilon\right)^{\circ}\right]^{c}=\left[\cup\left\{(G,\Upsilon)\in \overset{nm}{\tau}:(G,\Upsilon)\subseteq\left(H^{nm},\Upsilon\right)\right\}\right]^{c}$$

$$=\cap\left\{(G,\Upsilon)^{c}\in \overset{NSMS}{\tau^{c}}:(G,\Upsilon)^{c}\supseteq\left(H^{nm},\Upsilon\right)^{c}\right\}=\overline{\left[\left(H^{nm},\Upsilon\right)^{c}\right]}.$$

4. CONCLUSION

In this paper, we defined basic concept of neutrosophic soft multisets and neutrosophic soft multi topological spaces. The basic operations of neutrosophic soft multisets, namely, subset, equal set, null set, equal set, complement, union, intersection different and some basic properties of neutrosophic soft multi topological spaces, namely, open set, closed set, interior, closure have investigated. Novel numerical examples are given for definitions.Neutrosophic soft sets have an important place in the solution of decision making problems. We hope that with the newly defined neutrosophic soft multiset structure, important studies will be conducted on decision making processes. It is possible that this work will be extended in the future for: 1. Dealing neutrosophic soft multiset and topology with multi-criteria decision-making techniques. 2. Various topological concepts such as separation axioms, connectivity, compactness, using this structure. 3. All concepts studied on NSS can be carried in accordance with this structure.

Ethics in Publishing

There are no ethical issues regarding the publication of this study

Author Contributions

Authors did not declare any contributions.

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