ANKARA SCIENCE UNIVERSITY, RESEARCHER
Vol. 2, No. 2, December 2022
e-ISSN: 2717-9494
Research Article/Araştırma Makalesi
Doi: https://doi.org/10.55185/researcher.1115406

Interrelation and Succession of Application Results of the Wide Band Frequency Analysis in Numerical Electromagnetics: Shifted Frequency Internal Equivalence

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Received: 13 May 2022; Accepted: 31 July 2022

Reference/Atıf: E. Karapınar and S. Özdemir, "Interrelation and Succession of Application Results of the Wide Band Frequency Analysis in Numerical Electromagnetics: Shifted Frequency Internal Equivalence", Researcher, vol. 02, no. 02, pp. 46-56, Dec. 2022, doi:10.55185/researcher.1115406

Abstract

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Scattering problems have been evaluated and studied to reach to new solution methods by researchers. In this paper, the scattering calculation analysis in multi-frequency band is presented for the homogenous and the inhomogeneous 2D and 3D structures by the Shifted Frequency Internal Equivalence (SFIE). This paper is also a collection of the previous studies in one place, the interrelation of results on SFIE to enlighten the researchers on the subject matter for further research areas. This theory has shown that the electromagnetic field at frequency ω inside the region can be solved by using a set of equivalent currents radiating at the different frequency ω_0. This chosen ω_0 frequency is not changed while the incident field frequency changes. The analysis method in the literature requires repetition of the numerical analysis as many times as the number of the related frequency. Each repetition adds up to the computer time. SFIE algorithm which solves Electric and Magnetic Field Integral Equations (EFIE and MFIE) defines current equivalency covering the dielectric-magnetic features of the structure. SFIE applications are proved to give successful results for 2D and 3D arbitrary geometries. SFIE also provides very successful results for lossy objects.

Keywords: parallel computing, real-time systems, compilers

1. Introduction

Scattering is caused by the radiation of currents produced inside and on the surface of a structure excited by an electromagnetic wave. Since scattering problem has been an interesting problem. The solution techniques for the analysis of the scattering problems are accepted as a big contribution to paving the way for new approaches to solution methods. This study explains the results of the new method for analysis of the scattering problem. The new method uses Shifted Frequency Internal Equivalence (SFIE) [1,2]. SFIE method provides a fast solution for wideband electromagnetic scattering problems involving inhomogeneous, homogenous and lossy structures [3]-[7].

The solution need of the problems in the numerical analysis of scattering has increased with the requirements of modern technology of antennas and microwave devices etc. The solution algorithms of wide frequency band problems analyzed by Method of Moments (MoM) and Finite Element Method (FEM) are used to calculate the impedance matrix for each new frequency within the band.

It is known that integral equations are needed to solve for each new single frequency to find the scattering fields. Many methods are introduced to shorten and expedite this calculation burden, namely, interpolation, asymptotic approach, and characteristic basis function method [8]-[11]. So, methods are forced to find a solution by estimating a new MoM matrix. However, SFIE presents a new principle for which MoM matrix is not accepted an estimate calculation at each new frequency, on other hand, it is accepted as exact. SFIE calculates volume elements once and surface elements are recalculated for a new frequency. The equivalence of the fields is forced at the surface interface. Once the calculated volume elements are inserted into the process as some constants, the calculations of all integral equations are not required. SFIE saves computation time significantly.

The SFIE principle can be used to provide an internal equivalence at a shifted frequency for electromagnetic scattering problems when a solution is needed in a band of frequency. With this method, the equivalent currents for the internally equivalent problem radiate a defined fixed frequency different from the frequency of the incident wave. These equivalent currents have parameters of the incident and shifted frequencies, material parameters, and the total field inside the body and on its boundary. A combination of this internally equivalent problem with an externally equivalent one, so as to match the tangential fields at the boundary of the body, results in the new formulation.

The formulation and its application to produce multifrequency solution by employing internal data generated at a single frequency in a volume-surface integral equation are explained and exemplified using various problems in the second section. The method and results for different geometries are given in the third and fourth sections, respectively. The conclusion of the theory is given in the fifth section.

2. Theory

Scattering is the natural phenomenon in which radiation is created by the currents produced in and on an object inflicted by an electromagnetic wave. A new approach to the analysis of the scattering, the principle of SFIE theory is to be explained in this section. The original problem and its internal equivalence are worth to be shown in Figure 1. V and S show volume and surface elements. V and S are converted to the surface and closed line integrals respectively in two dimensional structures, and to volume and surface integrals respectively in three-dimensional structure. It is shown that electromagnetic fields are excited by ($\mathbf{J}\omega$, $\mathbf{M}\omega$) and radiate in volume V with frequency ω in Figure 1.

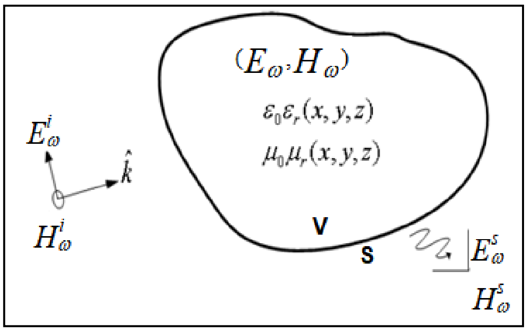

Figure 1:Original Problem

According to SFIE principle, the structure (Figure 1.) of the scattering problem can be represented by electric and magnetic sources in V and on S as shown in Figure 2 which is the internally equivalent problem of the original structure. The internal equivalence of the Figure 1 at the shifted frequency. Geometry is replaced with $(\mathbf{J}^{V}_{\omega_0}, \mathbf{M}^{V}_{\omega_0})$ and $(\mathbf{J}^{S}_{\omega_0}, \mathbf{M}^{S}_{\omega_0})$ equivalent sources. By the same token, the external equivalence of the original problem is shown in Figure 3. External equivalence of the problem is drawn by replacing geometry with $(\mathbf{J}^{S}_{\omega_0}, \mathbf{M}^{S}_{\omega_0})$ and the total field is the same as the original problem. The tangential fields on the surface are continuous and internal and external equivalent equations are written and matched at the surface. The SFIE theory basically asserts that the original problem in ω frequency can be solved by using the equivalence principle in fixed ω_0 frequency. In the equivalent problem, the scattered waves can be calculated by a newly defined set of currents radiating in a fixed frequency ω_0 different from the ω frequency. The resulted radiated fields are to be equivalent of radiated fields in the original problem.

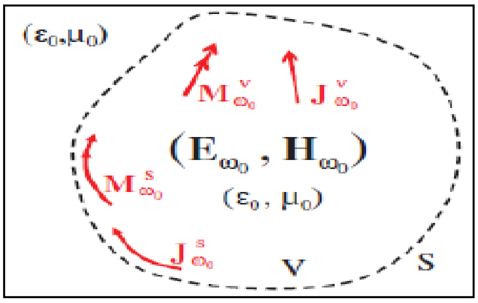

Figure 2: Internally Equivalent Problem

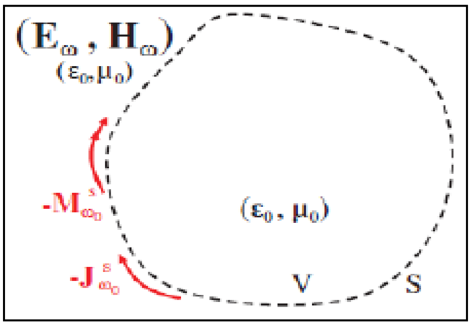

Figure 3: Externally Equivalent Problem

The internal equivalence section of the calculation from these integral equations is done just once by using SFIE since sources radiate at ω_0. The formulation and the proof of SFIE theory are given in reference [1].

The equivalent current sources can be written as;

$$\mathbf{J}_{\omega_0}^{V} = \mathrm{j}(\omega\varepsilon - \omega_0\varepsilon_0)\mathbf{E}_{\omega} \quad (1.1)$$

$$\mathbf{M}_{\omega_0}^{V} = \mathrm{j}(\omega\mu - \omega_0\mu_0)\mathbf{H}_{\omega} \quad (1.2)$$

$$\mathbf{J}_{\omega_0}^{S} = -\hat{n} \times \mathbf{H}_{\omega} \quad (1.3)$$

$$\mathbf{M}_{\omega_0}^{s} = \hat{n} \times \mathbf{E}_{\omega} \quad (1.4)$$

As seen from the equations (1.1) - (1.4), the equivalent currents comprise the sources and the features

$$\mathbf{E}_{r\omega_0}\left(\mathbf{J}^v_{\omega_0}, \mathbf{M}^v_{\omega_0}, \mathbf{J}^s_{\omega_0}, \mathbf{M}^s_{\omega_0}\right) = \mathbf{E}_\omega \quad (2.1)$$

$$\mathbf{H}_{r\omega_0}\left(\mathbf{J}^v_{\omega_0}, \mathbf{M}^v_{\omega_0}, \mathbf{J}^s_{\omega_0}, \mathbf{M}^s_{\omega_0}\right) = \mathbf{H}_\omega \quad (2.2)$$

$$\hat{n} \times \left[\mathbf{E}_{r\omega_0}\left(\mathbf{J}^v_{\omega_0}, \mathbf{M}^v_{\omega_0}, \mathbf{J}^s_{\omega_0}, \mathbf{M}^s_{\omega_0}\right)\right] \times \hat{n} = \hat{n} \times \left[\mathbf{E}_{r\omega}\left(-\mathbf{J}^s_{\omega_0}, -\mathbf{M}^s_{\omega_0}\right) + \mathbf{E}^i_\omega\right] \times \hat{n} \quad (2.3)$$

$$\hat{n} \times \left[\mathbf{H}_{r\omega_0}\left(\mathbf{J}^v_{\omega_0}, \mathbf{M}^v_{\omega_0}, \mathbf{J}^s_{\omega_0}, \mathbf{M}^s_{\omega_0}\right)\right] \times \hat{n} = \hat{n} \times \left[\mathbf{H}_{r\omega}\left(-\mathbf{J}^s_{\omega_0}, -\mathbf{M}^s_{\omega_0}\right) + \mathbf{H}^i_\omega\right] \times \hat{n} \quad (2.4)$$

of the real structure. $\hat{n}$ is adjusted to be out of the surface. For external equivalence, $(-\mathbf{J}^s_{\omega_0}, -\mathbf{M}^s_{\omega_0})$ source currents are used, and the equation set (2.1) – (2.4) is derived.

By using these equivalent currents, calculation for SFIE principle can be made. These calculations are based on the equivalent problem of the original problem. The equations (2.1) and (2.2) are equations for volume equivalences for *V*, (2.3) and (2.4) are equations for surface tangent equivalences that match internal and external electric and magnetic tangential fields on the surface *S* [12]. The loss is added as conductivity constant and the equations are written for lossy case. Conductivity creates a current which can be written as $\mathbf{J} = \sigma\mathbf{E}$ and this current changes the Ampere's equation as, $\nabla \times \mathbf{H} = \mathrm{j}\omega\mathbf{D} + \mathbf{J} = \mathrm{j}\omega\varepsilon\mathbf{E} + \sigma\mathbf{E} = \mathrm{j}\omega(\varepsilon - \mathrm{j}\ \sigma\omega)\mathbf{E}$. The imaginary part in this equation gives the total loss and the loss tangent (tan $\delta = \sigma/\omega\varepsilon$) defines the loss in the medium. The loss can be taken into account by using complex permittivity as $\varepsilon = \varepsilon' - \mathrm{j}\varepsilon''$ [7].

3. Method

Before analyzing the approach to the scattering problem, it will be informative to define a scattering problem. The structure of the problem includes an inhomogeneous voluminous body and free sources in free space out of this voluminous structure. When the field radiated by these primary sources impinging on this body, secondary sources occur in the volume and on the surface of the structure. The scattering is created by these secondary sources.

The incident field is known but total field, which is the addition of incident and scattered, is not known. The solution for scattered field can be found by using Electric Field Integral Equation (EFIE) and/or Magnetic Integral Equation MFIE. The original problem is replaced with an equivalent problem and the scattered fields can be solved by using the equivalent current sources newly defined with SFIE principle. The required equation set can be configured by forcing the boundary conditions of the fields at the internal and external equivalence.

Due to fact that Volume Internal Equation (VIE) sources include electrical parameters of the geometric structure, VIE can be applied to solution of homogenous and inhomogeneous geometries. However, the Surface Integral Equation (SIE) is used only for homogenous structures. By taking into account the role of direction of surface normal $\hat{n}$ for the surface sources, special attention should be given to the selection of the direction of $\hat{n}$. After the direction of $\hat{n}$ is defined according to internal or external equivalence, the direction of $\hat{n}$ is taken as "-$\hat{n}$" for the other equivalence.

The SFIE solution is done by a known numerical analysis, namely Method of Moment. This study is twofold: one is for verification of the validity of a theory and one is for the acquisition of the SFIE results. SFIE solutions are compared to MoM solutions. The important point here is to apply the problems with the same conditions and to present the acquired results with the same evaluation criteria. The solution for the application of SFIE principle is done the on the surface and in the volume of the problem physical structure.

In numerical analysis, the number of unknowns defines the number of independent equations to be derived. Let's assume that we have "*n"* unknowns in volume and "*s*" unknowns on the surface of the structure for discretization. In order to find the unknown total field ($\mathbf{E}_\omega$, $\mathbf{H}_\omega$) by MoM , *"n+s"* number of expansion functions are to be defined for the equation set given at (2.1) to (2.4). We have *"n+s"* number of integral equations in V volume. These equations are tested at "*n"* points in V so that "*n"* number of internal products. We can test at "*s"* points on the surface to derive "*s"* number equations with "*n+s"* number unknowns. As a result, "*n+s"* number equations are found for *"n+s"* unknowns.

It is worth mentioning here that, the SFIE equation set comprises fields and sources both at ω_0 and ω. The components with ω_0 frequency are not needed to calculate again for numerical analysis for q new ω frequency. So, when the solution is sought for multi-frequency, the matrix solution with ω_0 data can be used as many times as the new ω frequency calculation required. This is the novelty of the application of SFIE. The steps for SFIE application algorithm are presented in Algorithm 1.

Algorithm 1: The steps for SFIE application algorithm

1	*Choose and set the constant initial frequency ω_0.*
2	*Calculate matrix elements with ω_0 for Z_{VV}, Z_{VS}, Z_{SV} and Z_{SS}.* *(subscripts "VV" denotes volume to volume, VS/SV denotes volume to surface and vice versa, SS denotes surface to surface interactions of MoM as explained above.)*
3	*Set frequency ω as of incident field.*
4	*Calculate Z_{SS} matrix elements for the frequency ω.*
5	*Add $j(\omega\varepsilon - \omega_0\varepsilon_0)$ and $j(\omega\varepsilon - \omega_0\varepsilon_0)$ constants to the equation and multiply the matrix elements found in step 2 ($c \otimes Z_{VV}$, $c \otimes Z_{VS}$).*
6	*Complete the equation set and solve for total fields.*
7	*Go back to step 3 for a new frequency solution and repeat for a new frequency.*

As it is observed from this application steps of algorithm, SFIE can be accepted as a very suitable and adaptable solution methodology for wideband solution of scattering problems. Unlike MoM which requires to calculate surface and volume integrals for each frequency, SFIE principle eases the solution by multiplying the one-time calculated integral by a constant within the band. The results of the SFIE solutions are presented at next section. It takes longer with MoM calculation due to fact that it is required to calculate impendence matrix separately for each frequency. The impedance matrix calculation must be repeated that many times. Instead of this, having a different equivalent solution method in which the frequency is kept constant eases and speeds up the calculation process.

This study is exercised the TM mode solutions for 2D problems. It is known that TE mode solution will be the dual of the TM mode solution. The fields components for TM modes are E_z, and H_x, H_y. The currents are to be in the same direction with the fields as J_z and M_x, M_y.

It is well known fact that as much the modeling of the structure getting as close as possible to the physical structure, the error in results decrease. One major issue for this purpose is to make sure that tangential components must follow the surface exactly for 2D problems. Otherwise, the results deviate from the expected results. In this regard, the tangential magnetic field component will be applied with cautiously for TM mode. Similarly, the same discussion is valid for tangential electric field components in TE mode. It will be controlled and made sure that the direction of the line integral will be from start point to end point.

The other important mark for a numerical analysis application is about digitization of the geometrical structure and what kind of mesh structure to be applied. While a 3D structure has volume and surface parts, a 2D structure has surface and boundary lines for SFIE calculations. The volume integrals are converted to surface integral and surface integrals are converted to line integrals. For structure analysis, VIE is converted to SIE and SIE is converted to line integrals for 2D problems. In order to solve these integrals numerically, Gauss numerical integral is applied for line and volume integral calculations and DUNAVANT rule is used for surface integral calculations [12], [13].

The literature search has shown that the triangular mesh structure is more effective than the square cell mesh and tetrahedral mesh cell than hexahedral mesh cell. 2D structures are discretized with triangles by MATLAB® simulation program, 3D structures are discretized with tetrahedrons by Femlab® simulation program [14]. Basically, the maximum length of the mesh is guaranteed to be less than the dielectric wavelength. This rule asserted that maximum length of the triangle and tetrahedron is made less than 10 % of the dielectric wavelength in the related frequency.

An adaptive integration is applied for volume, surface, and line integrals in calculations to the extent which use an adaptive calculation algorithm rather than usage of higher order Gauss integration rules. The volume integrals are solved numerically by 2nd order Gauss rules with defined 4 points of a tetrahedron. The surface integrals are solved numerically by Gauss rules over triangle surfaces.

Special attention should be given here for the instance in which observation and field point meet at the same point on a calculation cell. This instance creates singularity which is required to be extracted. This extraction is applied by addition and subtraction of the logarithmic singularity.

4.Results

SFIE principle early application results for one-dimensional problem is given at [14]. The scattering created by the incident of plane wave on a 1D slab with an angle is searched. The acquired results are very promising such that the error rate is less than 1 % for the frequencies below ω_0 and the error rate decreases with the increasing number of segmentations. This section presents the application results of SFIE principle with 2D and 3D structures. The results of SFIE application are explained and verification of theoretical results is demonstrated. A scientific study should be able to give the comparison of the theoretical results with known and accepted methods of calculation results. The SFIE application results are presented with a comparison to results acquired from MoM. Before this comparison, the verification of MoM solution application is controlled to understand that the numerical solution exercise of MoM is applied in a correct way. MoM application is done in one frequency. Later, multi-frequency usage of SFIE theory is tired. Different structures such as homogenous, inhomogeneous, electrically small, and big have been solved by MoM. In order not to leave any point unchecked, 2D structures are calculated in both TM and TE mode also by constructing EFIE and MFIE.

4.1. 2D SFIE Application Results

This section explains the results of SFIE solution application for 2D homogenous and inhomogeneous structures. The various types of structures are examined according to their segmentation, the angle of incidence, and the electromagnetic size. The results of the incident angle are given for the incident angle at which the error is observed as maximum. The size of the structure is presented as the minimum dielectric wavelength at the maximum frequency of the related band. The Structures are changed from simple to complex geometries. The foremost advantage of SFIE theory is its usage over a wide frequency band with an acceptable percentage error rate. The results calculated from SFIE application are compared to Method of Moments (MoM) calculations. In this error analyses, percentage error rate is found as: Percentage Error Rate (%) =100* ||MoM Result – SFIE Result || / || MoM Result||. The initial frequency is chosen as 0.1 GHz and the frequency band is changed up to 1.2 Ghz.

The first structure is 0.5λ x 0.3λ solid dielectric rectangle with ε_r=10 and μ_r=10 and homogenous. Incident angle inflicts on the structure with an incident angle of Ø=180^0. The error percentage is given for the same number of discretizatized triangle numbers of MoM and SFIE calculations according to the increase in the number of discretization cells are depicted in Figure 4.1.

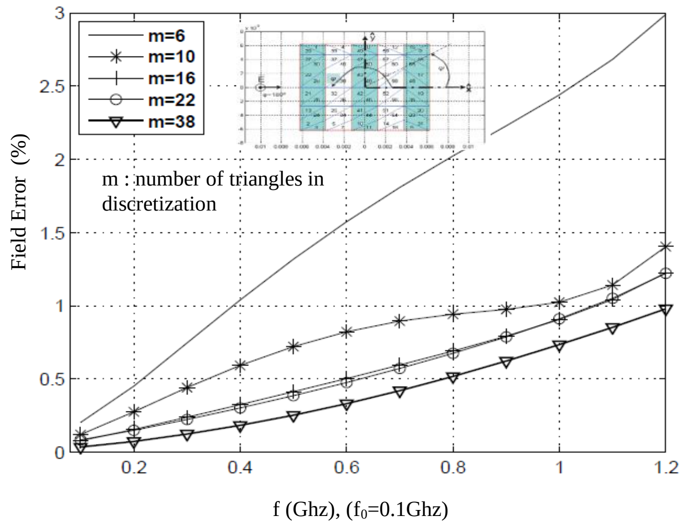

Figure 4.1: SFIE application comparison to MoM

The maximum segment size is no longer than 0.1 λ for the highest frequency and the error rate decreases as the number of triangle cells increases. The other important points are that the triangle cell density is found to be more than $200/\lambda^2$ to get an error rate of less than 5 %. As long as the triangular segmentation density is kept accordingly the error rate is found acceptable up to 70 % of the maximum frequency of the band.

The other comparison is made for the Radar Cross Section (RCS) calculations for the above structure of Figure 4.1. For RCS, the current sources calculated by MoM and by SFIE are compared. The comparison is calculated in L-2 norm percentage error and for different observation angles of ψ. The RCS calculation comparison is given for incident field angle Ø=180^0 and the triangle segment density $107/\lambda^2$. The error percentage results are shown at fig.4.2.

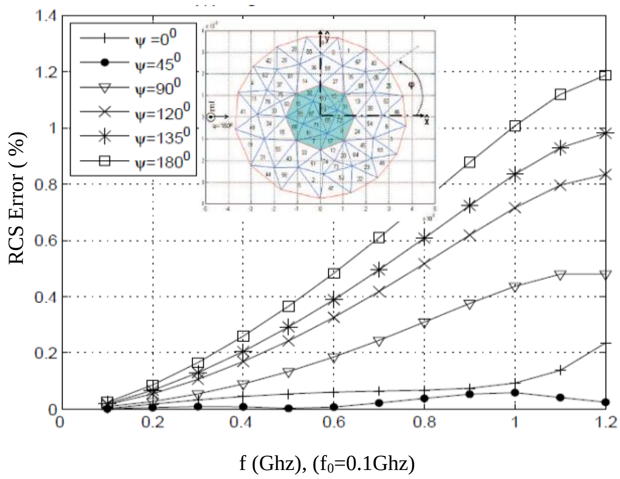

Fig.4.2: SFIE application comparison to MoM

Another SFIE application is done for a cylindrical structure of radius 0.25 λ outer and 0.125 λ inner radius two concentric circular areas. The inner-circle has ε_r=10 and μ_r=6 and the outer circular area has ε_r=6 and μ_r=10 for an inhomogeneous structure. The results of error % change according to the increase in the number of discretization cells are depicted in Figure.4.3. The RCS calculation comparison is given for incident field angle Ø=180^0 and the triangle segment density $107/\lambda^2$. The results are shown in Figure 4.4.

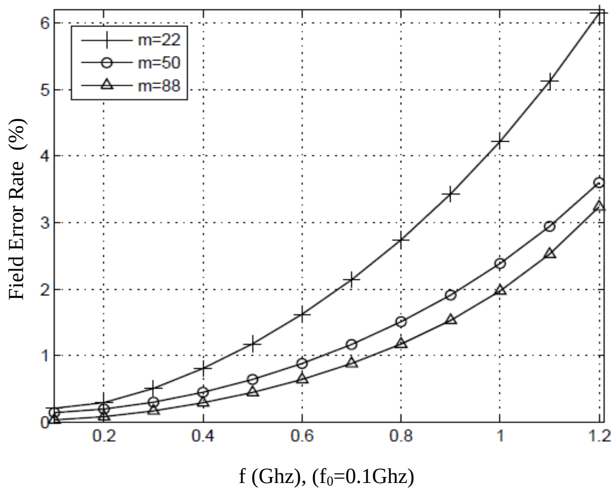

Fig.4.3: SFIE application comparison to MoM

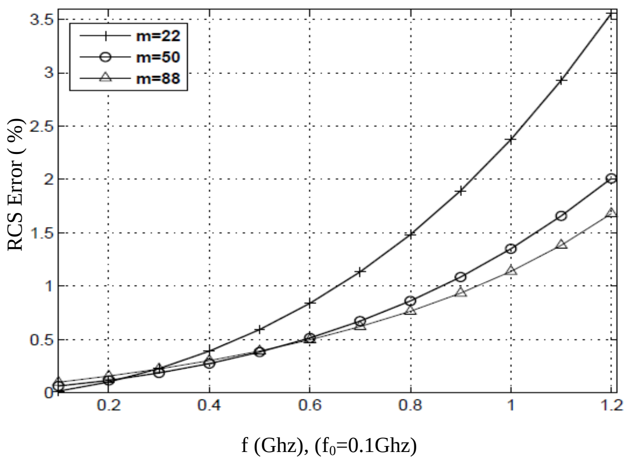

Fig.4.4: RCS calculation result

As seen from the fig.4.3 and fig.4.4, the error rate decreases with the increase in the segment number. Triangle segment density 112 /λ^2 for 22 cells, 254/λ^2 for 50 cells and 448/λ^2 for 88 cells. The results are shown at fig.4.4.

4.2. 3D SFIE Applications Results

This section explains the results of the SFIE solution for applications for 3D homogenous and inhomogeneous structures. Among other applications, the sphere and cube geometries are chosen as representation models for 3D SFIE applications.

The first one is a sphere with a radius of $0.1\lambda_0$ that has $\varepsilon_r = 2.0\text{-}j1.2$ and $\mu_r=1.0$ for initial frequency $f_0=1$ GHz. Calculating the results for the number of 756 subdivisions, the error is below 8% for field calculations and below 2% for RCS calculations in the band of 0.1-1 GHz bandwidth. The results of applications are depicted at Fig. 4.5 [12, pages 94-95].

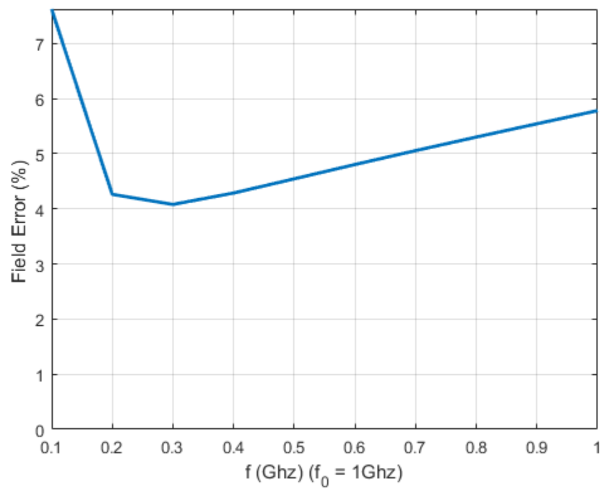

Figure 4.5(a)

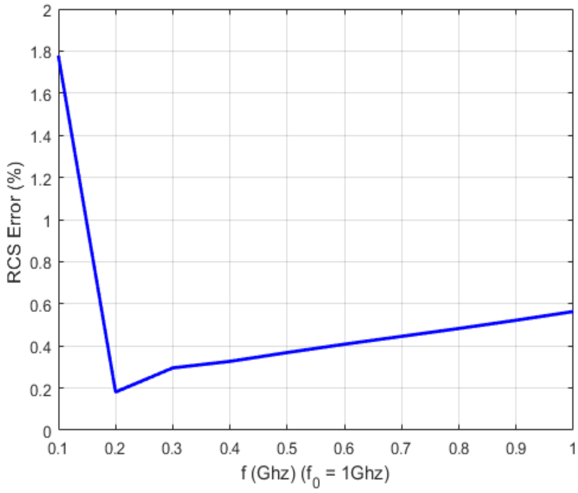

Figure 4.5(b)

Figure 4.5(a): Field error between SFIE and MoM-VIE, b) RCS error between SFIE and MoM-VIE for a lossy sphere (r=0.1λ_0, ε_r = 2.0-j1.2 and μ_r=1.0)

The other solution is the scattering calculation of two concentric spheres with 0.1λ_0 and 0.2λ_0 radii. Inner one has ε_r = 2.0 and outer one has ε_r = 4.0, both has μ_r=1.0. Calculating the results for the number of 1233 subdivisions, SFIE scattering calculation error is below 7% in the 0.1-1 GHz bandwidth and RCS error below 1% up to 0,2 Ghz. The results of applications are depicted at Fig. 4.6 [12, pages :89-90]

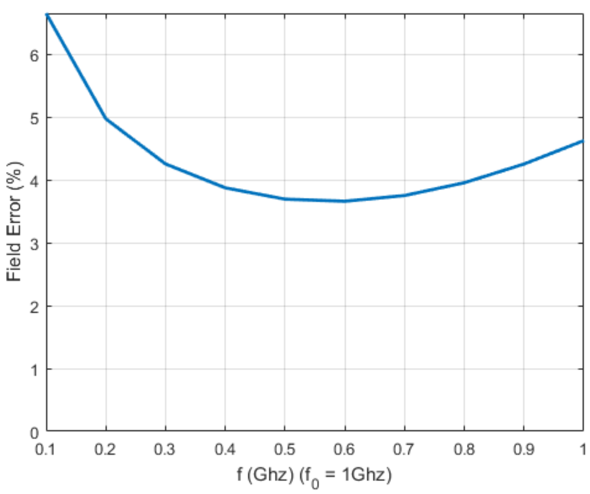

Fig.4.6 (a)

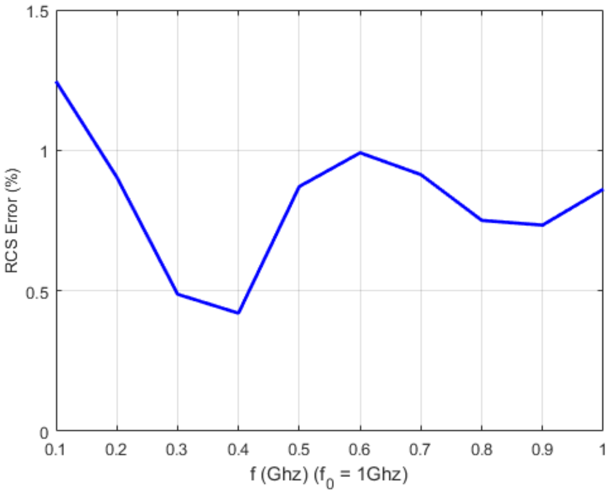

Fig.4.6 (b)

Fig.4.6 (a): Field error between SFIE and MoM-VIE, Fig.4.6 (b): RCS error between SFIE and MoM-VIE for two concentric spheres (r=0.1λ_0, ε_{r1} = 2.0, ε_{r2} = 4.0 and $\mu_{r1,2}$=1.0)

One composite structure is taken as inserting an air-filled cube (ε_r = 1.0 and μ_r=1.0) between two cubes all which have 0.1λ_0 dimension. Let the cube at left have ε_r = 2.0 and μ_r=1.0, right one have ε_r = 4.0 and μ_r=1.0. SFIE scattering calculation error is below 7% and RCS error below 2% after 0.2 Ghz in the 0.1-1 GHz bandwidth. The results of applications are depicted at Fig. 4.7. [12, pages :92-93]

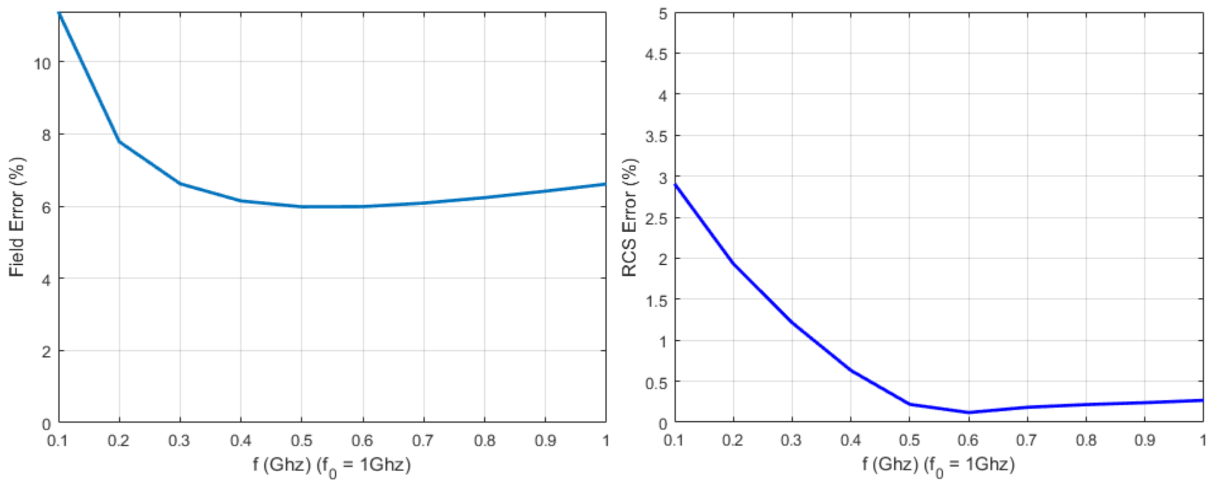

Fig.4.7 (a): Field error between SFIE and MoM-VIE, Fig.4.7 (b): RCS error between SFIE and MoM-VIE for an air cube inserted between two cubes ($l_{1,2}$=0.1λ_0, ε_{r1} = 2.0, ε_{r2} = 4.0 and $\mu_{r1,2}$=1.0)

5. CONCLUSION

5.1 SFIE Application Results

This study explains the application of SFIE theory for arbitrary homogenous and inhomogeneous 2D and 3D structures. The scattering analysis of a structure is solved by SFIE application in a frequency band and compared to the results acquired from MoM solution. This comparison is done to verify the accurateness of the SFIE method.

Electric and Magnetic Field Integral Equations (EFIE and MFIE) are solved for the surface and volume currents in SFIE algorithm which guarantees the equivalency of tangential components of internal and external fields and constructs a single impedance matrix. SFIE is based on the solution of the equation set which is formed as equating tangential field components of the internal equivalence of the problem at ω_0 frequency and the external problem of the at ω original frequency. By this basic principle, SFIE defines volume current equivalency which comprises the dielectric-magnetic features of the structure and makes the calculation in a shifted frequency other than the problem. As a result, the problem is made free from frequency change. After finding the solution for the shifted frequency, the other frequency solutions can be found by multiplying the core impedance matrix by a fixed number set in SFIE [3,4]. Pulse expansion and point matching methods are used for the application of SFIE. The surface of the structure is digitized with triangles in 2D and tetrahedrons in 3D applications. In order to prevent the singularity of integral equations, some volume integrals are converted to surface integrals and some surface integrals are converted to line integrals. If this is not applied for singularity extraction, the analytical solution is used.

It is observed that the percentage error rate is less than 4 % when triangular segment density is higher than 200 per λ^2 for 2D applications. For the 3D applications, the error rate is less than 10 % with the appropriate subdivision of the related geometry. While keeping this density error convergence is very successful up to 70 percent band of the maximum frequency of the related frequency band. This is a very important and successful result for problems of scattering in a wide frequency band. While it is required to make the solution for each new frequency for the Volume Integral Equation solution of Method of Moments, SFIE is very useful to make a solution in one initial frequency and fill some impedance matrices once then use these matrices for all required frequency. This also results in saving CPU time.

5.1 The Usage of SFIE and Advice for Future Work

The electrical features of the structure to be analyzed affect the SFIE application in regard to providing an acceptable solution. That the geometry has relatively small environmental parameters provides the ability to have a successful solution with less segmentation. That means one of the two same-size structures with smaller electromagnetic parameters requires a smaller number of segmentations.

SFIE application can be used for wide frequency band solutions. SFIE can be a very useful asset for different applications such as antenna with Radom analysis, frequency selective surfaces analysis, scattering application of fine element method, and dielectric patch line circuit analysis.

Contribution of Researchers

All researchers have contributed equally to writing this paper.

Conflicts of Interest

The authors declare no conflict of interest.

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