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Ulaştırma Modellerinde Can'ın Yaklaşım Metodunda Uygun Ortalama Seçimi İçin Simülasyon[a]

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Özet

Klasik ulaştırma modelleri birim taşıma maliyetlerini göz önüne alarak homojen malların arz noktalarından talep noktalarına taşınma maliyeti toplamını minimize etmeyi amaçlamaktadır. Ulaştırma problemi, ağ modellerinin özel bir halidir ve doğrusal programlama temelli bir tekniktir. Başlangıç dağıtım yöntemlerinden Tuncay Can yaklaşım metodu 2015 yılında geliştirilmiş bir metottur. Yöntem, birim taşıma maliyetlerinin geometrik ortalamalarının alınması esasına dayanmakla birlikte teoremde yöntem uygulanırken geometrik ortalamalar yerine farklı ortalamaların da kullanılabileceği belirtilmiştir. Bu çalışmanın amacı, Tuncay Can Yaklaşım Metodunu (TCYM) temel alarak, yöntemin belirttiği şekilde birim maliyetlerin geometrik ortalamalarının alınması ve ayrıca aritmetik, kareli ve harmonik ortalama kullanılarak da yöntemin uygulanması ile elde edilen toplam maliyetleri minimize eden başlangıç dağıtım planı incelenerek hangi ortalamada optimal sonuç verdiğini ortaya koymaktır. Bu amaca yönelik olarak kurulan ulaştırma modelinin katsayıları simülasyon yardımıyla rassal olarak değiştirilmiş ve yöntem farklı ortalamalara göre problem üzerinde tekrarlanarak, optimal toplam maliyet değerleri karşılaştırılmış ve uygun ortalama tespit edilmiştir.

Anahtar Kelimeler

Ulaştırma Modelleri
Tuncay Can Yaklaşım Metodu
Simülasyon
Ortalamalar

Makale Hakkında

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Simulation for Appropriate Mean Selection for Can's Approximation Method in Transportation Models

Abstract

Classical transportation models aim to minimize the total costs of homogeneous goods transport from supply points to demand points, taking into account unit transportation costs. They constitute a special case of network models and employ a technique based on linear programming. Suggested in 2015 and one of the early distribution methods, Tuncay Can's Approximation Method (TCAM) is based on the geometric averages of unit transportation costs, although it is stated in the theorem that other means than geometric can be used. The aim of this study is to compare the total costs of a transportation model by solving a problem using geometric, arithmetic, square, and harmonic means based on TCAM. The coefficients of the transportation model were obtained randomly by simulation, and the method was repeated on the problem according to the different means and the appropriate means determined.

Keywords

Transportation Models
Tuncay Can's Approximation Method
Simulation
Means

About Article

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Introduction

Computer-based simulation and analysis are extensively used in the field of engineering for a varirty og purposes. The computational cost of complex engineering analysis and simulations maintains significant in terms of the amount and cost of computing notwithstanding the increase in the computational power and speed of computers. High computational costs in this context restrict problem variables, so the use of experimental design and approximation to optimal solution methods have become widespread in the solution of logistics problems. Such approximation approaches simplify real-life problems with long computation time and high coding costs. Approach models created for purposes such as optimization, design space exploration, and reliability analysis are also called metamodels (Simpson et al., 2001; Barthelemy and Haftka, 1993).

The growing number of dimensions according to the number of variables causes issues in optimization problems (Koch et al., 1999). Visual examination of the solution hypersurface becomes more difficult with the increase in the number of input variables. As a solution, reducing the number of variables has been suggested either with statistical methods and experiments (Box and Draper, 1969) or to observe the effect of variables on results with linear models (Plackett and Burman, 1946).

Classical transportation models aim to minimize the total costs of homogeneous goods transported from supply points to demand points, taking unit transportation costs into account. Supply and demand quantities predetermine transportation models, with the model considered balanced if supply and demand are in equilibrium. They constitute a special case of network models and employ a technique based on linear programming. However, a degeneration problem occurs due to the nature of the problem during the minimization of the objective function when using the solution methods of linear programming. For this reason, the North-West Corner Method, Least Cost Method, Vogel's Approximation Method (VAM), Russel's Approximation Method, and Tuncay Can's Approximation Method (TCAM) have all been developed (i.e., to address the degeneration).

These methods aim at minimizing the total transportation cost and testits optimality with either the Stepping Stone or Modi Method. In a balanced transportation problem, the primary aim is to obtain an initial basis feasible solution vector (corner point) or distribution plan. The optimum solution value is the result that will provide the minimum total cost. It is important to determine the initial distribution as close to this minimum value. The aim of this study is to present an approximation method that gives an initial distribution plan close to the optimal solution. The study is based on a balanced transportation problem of size m×n, representing a total of m supply centers (source) and n demand centers (destination). The simple structure of a transportation problem is shown in Table 1.

Table 1. Standard Transportation Problem Table

		Demand Centers (n)				
		D_1	**D_2**	**...**	**D_n**	**Total supply (s_i)**
	S_1	C_{11} X_{11}	C_{12} X_{12}	...	C_{1n} X_{1n}	s_1
	S_2	C_{21} X_{21}	C_{22} X_{22}	...	C_{2n} X_{2n}	s_2
Supply Centers (m)	**...**	...	...	...	...	...
	S_m	C_{m1} X_{m1}	C_{m2} X_{m2}	...	C_{mn} X_{mn}	s_m
	Total demand (d_j)	d_1	d_2	...	d_n	

The transportation costs C_{ij} (i=1,2,...,m; j=1,2,...,n) present per unit good transportation costs from the supply center (i) to the demand center (j). The total of X_{ij} (i=1,2,...,m ; j=1,2,...,n) decision variables constitutes the amount of goods transported from i to j. This classical problem indicates a balanced transportation problem, where the total supply is equal to the total demand; otherwise, it is an unbalanced transportation problem. The following equation system (1) represents the linear programming model of the classical transportation problem:

$$\min\ z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij} x_{ij} \tag{1}$$

subject to:

$$\sum_{i=1}^{m} x_{ij} = d_j;\ j = 1,2,\ldots,n$$

$$\sum_{j=1}^{n} x_{ij} = s_i;\ i = 1,2,\ldots,m$$

$$x_{ij} \geq 0; i = 1,2,\ldots,m;\ j = 1,2,\ldots,n$$

The equation $\sum_{i=1}^{m} s_i = \sum_{j=1}^{n} d_j = t$; s_i *and* $d_j \geq 0$ provides the balance between supply and demand. The equation system (1) has m supply constraints and n demand constraints (m+n) in total, along with the nonnegativity constraint (xij≥0). The existence of (m+n) constraints involves a convex polyhedral geometric shaping of the linear programming problem, which reflects a convex polytope in special cases.

Finding a basic feasible solution requries one of the (m+n) vertices. This forces the (m+n) variable to be nonzero positive and the other variables to be equal to zero. One of the constraints becomes redundant in a balanced transportation problem due to structure of a system in which nonzero positive variables are the basic variables and variables with a value of zero are the non-basic variables. The rank of the system decreases by one from (m+n), and (m+n-1) nonzero positive basic variables remain in the system. This fact causes degenerate solutions to balanced transportation problems when solving with the simplex algorithm because it inhibits acquisition of the desired number of (m+n) basic variables.

Approximation methods ensure an initial basic feasible solution without the simplex algorithm preventing the degeneration problem. Thus, the various methods listed above, including Can's Approximation Method (Can, 2015; Can and Koçak, 2016) have been developed. Various studies have made improvements on the structure and algorithm of the problem to the majority of the methods listed, but VAM remains the most commonly studied. One of the versions of VAM improved the minimization of the total costs by counting the opportunity costs through the alternative allocation costs (Korukoğlu and Ballı, 2011). Mathirajan and Meenakshi (2004) applied VAM to an opportunity cost matrix considering just the maximum three penalty costs. This improved VAM (IVAM) method also calculates the alternative product assignments to idle sources. The Karagül and Şahin (2019) Approximation Method (KSAM) suggests a feasible solution close to optimal by weighting the cost matrix with relative supply and demand amounts. In that study, they compared initial solution methods, which are North-West Corner (NWC), the Matrix Mimina (MM), the Row-Minima (RM), the Column Minima (CLM), Vogel's Approximation Method (VAM), Russel's Approximation Method (RAM), Can's Approximation Method (TCAM) and Karagül and Şahin Approximation Method (KSAM), using twenty-four test problems for the solution values and solution time. Avoid Maximum Cost Method (AMCM) was proposed by Mutlu, Karagül and Şahin at 2021 and they also compared the method with six well-known methods which are North-West Corner (NWC), Least Cost Method (LCM), Vogel's Approximation Method (VAM), Russel's Approximation Method (RAM), Can's Approximation Method (TCAM), Row-Minima (RM), Column Minima (CLM), Total Opportunity Cost Matrix – SUM (TOCM-SUM). As a result of the analysis, the solution values of the methods were found as in the table below.

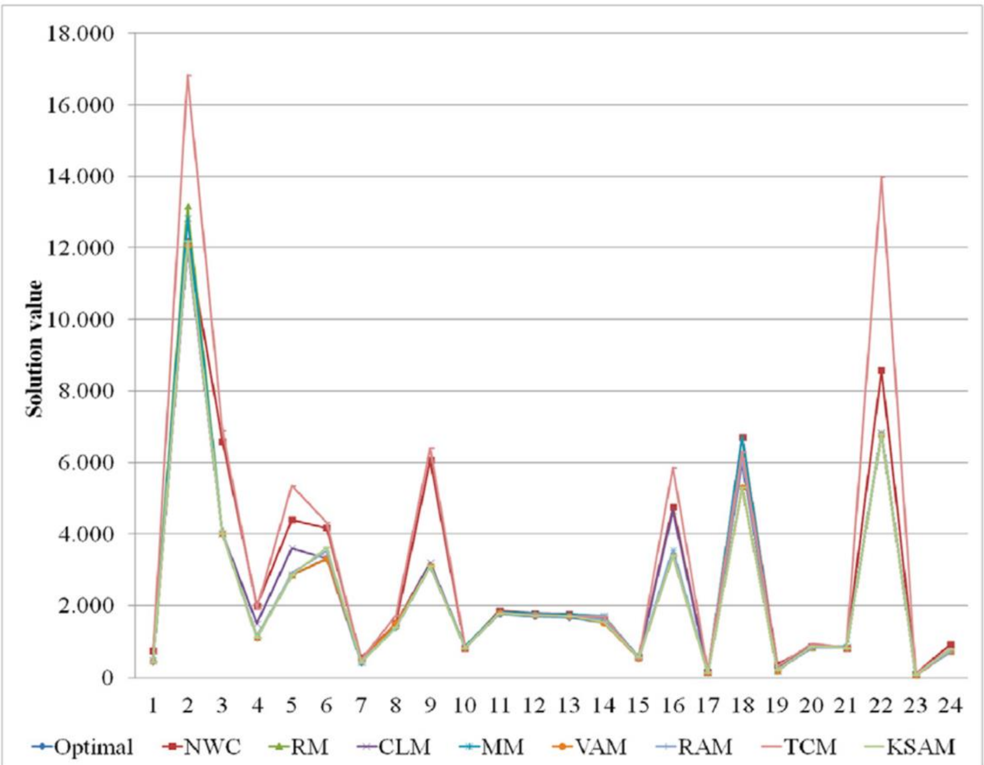

Figure 1: Solution Values of the Methods (Karagul and Sahin (2019))

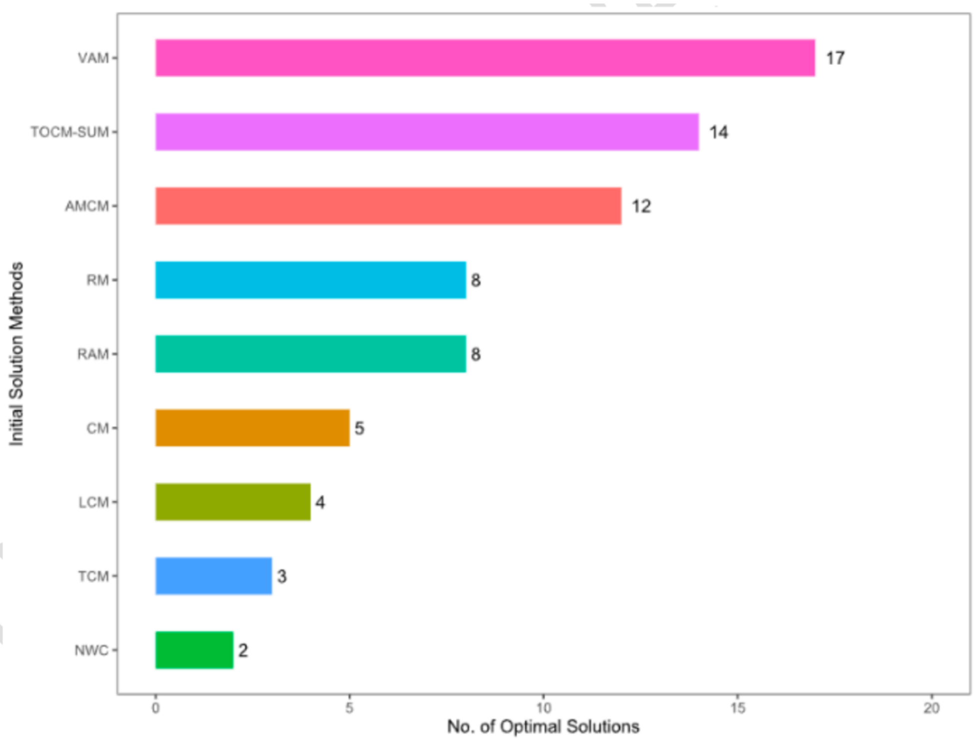

Figure 2: Solution Values of the Methods (Mutlu, Karagul and Sahin (2021))

Theorems

Theorem 1 ensures that the approximation method employed guarantees a basic feasible solution for the transportation problem.

Theorem 1: A balanced transportation problem has a feasible solution of the variable $x_{ij} = \frac{s_i d_j}{t}$, providing that $0 \leq x_{ij} \leq \min\{s_i, d_j\}$ for every i and j.

Proof (Can, 2015): The equation system (1) actualizes the subsequent inferences under the inequality related to the variables $x_{ij} = \frac{s_i d_i}{t}$; $0 \leq x_{ij} \leq \min\{s_i, d_j\}$:

$$\sum_{j=1}^{n} x_{ij} = \sum_{j=1}^{n} \frac{s_i d_j}{t} = \frac{1}{t} s_i \sum_{j=1}^{n} d_j = \frac{1}{t} s_i t = s_i$$

$$\sum_{i=1}^{m} x_{ij} = \sum_{i=1}^{m} \frac{s_i d_j}{t} = \frac{1}{t} d_j \sum_{i=1}^{m} s_i = \frac{1}{t} d_j t = d_j \tag{2}$$

The preceding equations ensure each other. As a result, the following equalities are acquired:

$$\sum_{j=1}^{n} x_{ij} = s_i$$

$$\sum_{i=1}^{m} x_{ij} = d_j \tag{3}$$

Thus a balanced transportation model always has a feasible solution. We should notice the principle that "if there is a feasible solution, then there has to be also an optimal solution."

Points within the convex structure are a feasible solution according to TCAM. Thus, statistical means determines the key column or row. The algorithm explains the TCAM within two steps.

Step 1: The unit transportation costs $C_{ij} > 0$ (i=1,2,...,m and j=1,2,...,n) are averaged with a determined statistical mean in the classical transportation problem with m sources and n destinations. The closest unit cost to the average value marks the key transportation cost. The key value is selected arbitrarily if there is more than one equal unit cost close to average. The minimum quantity of the selected supply and demand allocated to the cell with the specified unit transport cost. Columns and rows are deleted in the allocation table when the allocated quantity fully covers the supply/demand unless their amounts are equal. In this case, just one row or column will be excluded. Step 1 results in the corresponding xij axis in Rmn+1 dimensional space. The remaining unit transport costs C_{ij} 1≠i,k≠j are averaged to repeat the instructions in Step 1, as the terms in the objective function are independent from each other. Thus, a new variable axis xlk emerges corresponding to the dimensions of the remaining matrix.

Step 2: The maximum supply or demand quantity allocated to the cell with the minimum unit transport cost after the deletion of the relevant rows and columns at Step 1 leaves only two units of transport costs. Averaging these two numbers gives an equi-distant value and selection of the minimum one enables the objective function aim. This results in a new feasible xij axis. Thus, the first basic feasible solution to the classical transportation problem is achieved.

The solution vector obtained by TCAM algorithm is a feasible but also a basic feasible solution to the problem due to Theorem 1. This vector constitutes a corner point of the convex polytope region. The proposition of first basic feasible solution requires the following fundamental definitions and theorems.

Definition: A non-singular square matrix is said to be a *triangular matrix* when it forms a lower triangular matrix through row and column permutations. A *non-singular lower triangular matrix* is clearly a triangular matrix according to this definition. A non-singular upper triangular matrix is then also a triangular matrix, because its transposition is an upper triangular matrix (Luenberger and Ye, 2008).

Theorem 2 (Fundamental Theorem): The transportation problem basis is triangular (Dantzig and Thapa, 2003). The triangulity rule (algorithm) requirement provides a modest procedure to construct an initial basic feasible solution.

Triangularity Rule (Dantzig and Thapa, 1997): Any arbitrary selection of the variable xij candidates for the initial basic feasible variable. Set xij as small as possible (for example, Xij=Min{si,dj}) without violating row and column totals. The next candidate for the basis variable depends on which of the following criteria occurs. This criterion determines the variable by the same procedure through decreasing the rectangular array.

1. If si<dj, then all the variables in row i are set to zero (0). These variables became non-basic. The dj value at column j is reduced to (dj-si) by deleting row i.

2. If si>dj, then all the variables in column j are set to the zero (0). These variables became non-basic. The si value at row i is reduced to (si- dj) by deleting column j.

3. If si=dj, then either row i or column j is chosen randomly (but not both).

All the included cells are basic if only one row or one column remains after subsequent deletion of a row or column. The residues are assumed to be equal in the remaining row and column. Exactly one row and one column remain by the last step. These are realized following the evaluation of the last variable. Hence, the aforementioned rule of triangularity selects (m+n-1) variables to the basic set.

Supporting Theorem: The first solution vector calculated by TCAM is a basic feasible solution. In other words, it is a corner point of the convex polytope region.

Proof: A balanced transportation problem always retains a basic feasible solution through Theorem 1. Therefore, TCAM always possesses a basic feasible solution for a balanced transportation problem. This basic feasible solution is thus a corner point of the convex polytope region.

Numerical Analysis and Findings

A simulation study was designed to compare TCAM results with Minimum Cost Method (MCM) feasible solutions. The statistical (arithmetic, quadratic, geometric, and harmonic) means are applied through the TCAM algorithm. MCM obliges a general approach to the transportation problem. The R script for the numerical analysis depends on the TransP package (Somenath, 2016). The simulation design includes a balanced transportation problem with five supply and five demand centres. Here, the supply and demand quantities were kept constant through the iterations. The goal was to achieve comparable results after iterations.

The cost coefficients were randomly derived from a uniform distribution with parameters a=10 and b=150, assuming that uncertainty can have maximum entropy. The distribution parameters were determined arbitrarily to reflect real-world problems in cases of wide-ranging transport costs dependent on distance and vehicle selection. The highest unit cost exceeded the lowest by fifteen times in this case. A wide cost range avoids overly close basic feasible value calculations from different averaging methods. The code was repeated 10,000 times for each approximation method each with randomly generated unit cost matrix. This iteration number preferred usually for simulation purposes. Furthermore it allows a normal distribution, which marks a preferred population size in order to make the results comparable within probability law. The basic feasible solution was usually reached at nine loops and some iterations consisted eight loops.

Table 2 samples the final cost matrix of the transportation problem simulation. The supply and demand quantities were randomly generated at the beginning of the simulations and kept constant through all the solutions. A statistical distribution-dependent random generation of these quantities would cause a stochastic transportation problem, preventing a comparison of the approximation method solutions.

As unit cost of transportation uniformly distributed with predetermined parameters a and b, C~U(10, 150) and cij∈C, the expected unit transportation cost will be $E(C) = \frac{a+b}{2} = \frac{10+150}{2} = 80$ currency unit. This will held for each lot from any source to any destinations. The predetermined quantity of 801 will transported by expected unit transportation cost accounts for an expected total transportation cost to 810x80 = 72000. That is the total average cost of the facility under the agreement of invoicing the transportation each lot of goods with an average cost. The amount of 72000 is a comparison key to the solutions.

Table 2. Sample Cost Matrix of Generated Transportation Problem

Supply/Demand	D_1	D_2	D_3	D_4	D_5	Total Supply
S_1	72	110	90	57	86	130
S_2	139	91	85	37	19	150
S_3	74	98	126	150	83	300
S_4	58	60	50	133	93	100
S_5	35	125	78	108	119	130
Total Demand	210	240	110	80	170	810

The following calculations sample the last iteration of the written R code:

A- *Minimum Cost Approximation Method:* The minimum cost min(cij)=c25=19 selected and the total supply of quantity 150 of supplier S2 assigned to transport to the destination D5 by a unit cost 19. The rest demanded amount 170-150=20 leaved unfulfilled at D5. The supplier S2 dropped out from the problem. Supplier S5 allocates all its resources 130 quantities to destination D1 with the cost of 35. These allocations continues until all demands fulfilled with all supplier resources. This algorithm calculates the total transportation cost 49,220.

Table 3. Optimal Source to Destination Allocations and Solutions for the Final Iteration Problem due to MCAM

MCAM				
Loop	**S to D**	**Unit cost**	**Quantity**	**Value**
1	S2-D5	19	150	2850
2	S5-D1	35	130	4550
3	S4-D3	50	100	5000
4	S1-D4	57	80	4560
5	S1-D1	72	50	3600
6	S3-D1	74	30	2220
7	S3-D5	83	20	1660
8	S3-D2	98	240	23520
9	S3-D3	126	10	1260
	Mean C=	60.77	810	49220

B- *The TCAM Approximation Method:* TCAM uses an analytic mean to select the key raw or column instead of the minimum function. The following calculations demonstrates selection the first cost value for supply-demand pair. The cell was selected with the unit cost closest to the given mean. Either the source or the destination was reduced, which has less quantity in supply or in demand. These allocations continues until all demands fulfilled with all supplier resources.

i. Arithmetic mean 87.04 requires that S1 allocates all 130 quantities to D5 with unit cost c15=86 at first stage.

$$c_A = \frac{1}{n \times m} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} = \frac{1}{5 \times 5} (72 + 110 + \cdots + 126 + 150 + \cdots + 108 + 119) = 87.04 \quad (4)$$

ii. Quadratic mean 93.27 requires that S4 allocates all 100 quantities to D5 with unit cost c45=93 at first stage.

$$c_Q = \sqrt{\frac{1}{n \times m} \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij}^2} = \sqrt{\frac{1}{5 \times 5} (72^2 + 110^2 + \cdots + 126^2 + \cdots + 108^2 + 119^2)} = 93.27 \quad (5)$$

iii. Geometric mean 79.09 requires that S5 allocates all 110 quantities to D3 with unit cost c53=78 at first stage.

$$g = \frac{1}{n \times m} \sum_{i=1}^{m} \sum_{j=1}^{n} log c_{ij} = \frac{1}{5 \times 5} (log 72 + log 110 + \cdots + log 126 + \cdots + log 108 + log 119) = 1.8981 \quad (6)$$

$$c_G = 10^g = 10^{1.8981} = 79.09$$

iv. Harmonic mean 68.97 requires that S1 allocates all 130 quantities to D1 with unit cost c11=72 at first stage.

$$c_H = n \times m / \sum_{i=1}^{m} \sum_{j=1}^{n} \frac{1}{c_{ij}} = 5 \times 5 / \left(\frac{1}{72} + \frac{1}{110} + \cdots + \frac{1}{126} + \cdots + \frac{1}{108} + \frac{1}{119} \right) = 68.97 \quad (7)$$

The Table 4 represents the optimal source to destination allocations and solutions for the final iteration problem due to given approximation methods. The means have been calculated from the retained unit cost after row or column elimination.

Table 4. Optimal Source to Destination Allocations and Solutions for the Final Iteration Problem due to TCAM

	Arithmetic mean					**Quadratic mean**				
Loop	S to D	Mean	Unit cost	Quantity	Value	S to D	Mean	Unit cost	Quantity	Value
1	S1-D5	87.04	86	130	11180	S4-D5	93.27	93	100	9300
2	S2-D2	88.05	91	150	13650	S3-D2	95.31	98	240	23520
3	S4-D5	92.60	93	40	3720	S1-D3	92.22	90	110	9900
4	S3-D2	91.17	98	90	8820	S1-D5	90.74	86	20	1720
5	S5-D3	90.11	78	110	8580	S5-D4	96.01	108	80	8640
6	S5-D4	92.83	108	20	2160	S3-D5	88.91	83	50	4150
7	S4-D4	103.75	133	60	7980	S3-D1	93.13	74	10	740
8	S3-D1		74	210	15540	S2-D1	101.36	139	150	20850
9						S5-D1		35	50	1750

		Mean C=	**88.43**	**810**	**71630**		Mean C=	**99.47**	**810**	**80570**
		Geometric mean					**Harmonic mean**			
Loop	S to D	Mean	Unit cost	Quantity	Value	S to D	Mean	Unit cost	Quantity	Value
1	S3-D3	79.09	78	110	8580	S1-D1	68.97	72	130	9360
2	S3-D1	78.32	74	210	15540	S3-D1	66.84	74	80	5920
3	S3-D5	81.92	83	90	7470	S4-D2	68.76	60	100	6000
4	S1-D5	76.65	86	80	6880	S5-D3	67.51	78	110	8580
5	S2-D2	83.10	91	150	13650	S3-D5	61.97	83	170	14110
6	S5-D4	93.67	108	20	2160	S2-D2	83.16	91	140	12740
7	S4-D2	84.10	60	90	5400	S2-D4	69.84	37	10	370
8	S1-D4	87.06	57	50	2850	S5-D4	125.58	108	20	2160
9	S4-D4		133	10	1330	S3-D4		150	50	7500
		Mean C=	**78.84**	**810**	**63860**		Mean C=	**82.40**	**810**	**66740**

The minimum cost method has the lowest problem value 49,220 among others, so has the first rank for the final iteration problem.

The unit transport costs were generated randomly from uniform distribution with given parameters at each iteration. Each iteration solved the total transportation cost with the minimum cost and TCAM algorithms. The TCAM algorithm employs the arithmetic, quadratic, geometric, and harmonic means simultaneously. Consequently, a 10000x5 data frame consisting of the total transportation costs for 10,000 iterations for five approximation methods was produced. The total transportation costs were minimized as the objective function of the problems. The arithmetic means and standard deviations of the objective function values described the solution characteristics of the different methods. However, the main effort focused on the comparison of the objective values of problems with unique cost structures. Therefore, these objective values were ranked at each iteration, which had an equal cost matrix. The code showed the method with the minimum number of results. None of the TCAM approaches dominated under the assumption that the cell selection method does not effect the approximation to optimal value.

Table 5. Comparison of Approximation Approaches

	Minimum cost	**Arithmetic mean**	**Quadratic mean**	**Geometric mean**	**Harmonic mean**
Average	43266.13	58362.48	63755.25	57588.11	53260.96
Standard Deviation	8378.44	6836.57	11713.49	8994.71	8776.83
Minimum cost % in 10,000 iterations	84%	<0.5%		4%	12%

The minimum cost approximation was about 84% successful in determining the lowest total cost for the basic feasible solution. The arithmetic mean approach gave higher objective function values, but provided the least risky distribution (i.e., the lowest standard deviation). The arithmetic mean was already the most efficient measure of central tendency in terms of estimation theory, but this and the quadratic mean overrode the minimum in most iterations.

The harmonic mean was developed for unit cost calculations, and this gave a lower cost in approximately 12% of the trials, an achievement that ranks second best in terms of obtaining a basic feasible solution close to the minimal objective function value. The standard deviation of the harmonic mean was lower than that the quadratic and geometric means and is considered more reliable despite varying costs for the same amount of transportation problems. The harmonic mean in the TCAM algorithm is relatively stable in finding the basic initial solution of transportation problems.

Conclusion and Discussion

The proposed TCAM approach provides the same features as other common approximations aimed at identifying a solution subset of transportation problems. Therefore, TCAM offers an alternative approximation method. Supply and demand quantity allocation equality was satisfied in all solutions.

The harmonic mean gave better results than the arithmetic, quadratic and geometric means for the approximation to minimum cost. The harmonic mean is typically used for things like averaging rates or fractions to explain multiplicative or divisor relations. Therefore, it might be suitable for transportation problems with unit transport costs multiplicative to the quantities. On the other hand, a cost matrix of a real transportation problem consists of average unit costs for each source-destination combination. The findings on different means reported here might vary when unit transport costs in the cost matrix are distributed normally or where they have a skewed distribution with extreme values. Future studies on this are required.

The effect of the different statistical means could be investigated for reaching the corner points on the hyperplane as a future study option. Various techniques have been proposed in the literature to compare approximation techniques in optimization. Here, the scope of the study was limited to a more general comparison of approximation techniques. Processing time and operation quantities drive the selection of algorithms for computational purposes in projects with huge data. This study excluded these operational measurements in order to focus on comparing the feasible solutions produced by different means through the TCAM algorithm.

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