

## A STUDY ON MODELING GROWTH MODEL OF ADANA PIGEONS

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**ABSTRACT.** The study aims to determine a mathematical model that can be used to describe the growth of the Adana pigeon. Since pigeons have only one breeding season, just one or two pairs of baby pigeons are raised per year. Hatchlings sometimes die before reaching adulthood. For this reason, measurements can be taken for 10, 15 and 60 days periods. Related with this issue, only 43-days measurements of 68 pigeons are used over a 6-year period. The study is modelled by taking the day-to-day average of the data (43 days) of 68 pigeons. The study was conducted on 68 Adana pigeons in the interval between the age of 1 and 43 days. The growth of pigeon cub was measured by daily live weight until 1 to 43 days. The estimation is carried out by writing the specific Matlab codes. Classical growth functions used in animals are in nonlinear form. Various numerical methods have been developed to estimate parameters in nonlinear functions. Special program routines have been developed to implement these methods. In these nonlinear models, there are more than one parameter to be estimated. Therefore, the number of mathematical operations in estimating the parameters is large. The most used models in the literature are Brody, Bertalanffy, Logistic, Generalized Logistic, Gompertz, Richards, Negative Exponential, Stevens, and Tanaka. However, as far as is known, there is no published article for Adana pigeons that uses all of these models and compares which one is better. These models are Brody, Bertalanffy, Logistic, Generalized Logistic, Gompertz, Richards, Negative Exponential, Stevens, and Tanaka. The best analysis was done by the Richards model in terms of both the Mean Squared Error (MSE), mean absolute percentage error (MAPE) and (Coefficient of Determination)  $R^2$ .

### 1. INTRODUCTION

Pigeons are among the species which played an important role in human history. First known as a religious symbol in the old times, they were then used as a means for transmitting messages during wars, and now it is a symbol of peace. Domestic pigeon species originated from *Columba livia* (rock pigeon), a kind of *Columba* species found in the *Columbinae* sub-family of the *Columbidae* family [1]. Genome

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analysis was done by Shapiro et al [2]. The domestication time of pigeon cannot be exactly fixed [3-4].

Adana pigeons known as Adana dewlap over the world are known as Adana in Turkey. They are known as Adana wammen or earring dewlap in Germany. They are mainly raised in Çukurova Region. They are densely found in our settlements such as Adana, Ceyhan, Mersin, and Tarsus. Although in the sources there is some information related to the fact that they are originally descended from the Lebanon dewlaps (Lebanon races with dewlap), the truthiness of this information can be discussed because there is a clear difference between Adana pigeon and Lebanon dewlaps in terms of both physical appearance and flight characteristics. These pigeons which have gained a color, form, and flight characteristics special to themselves since the Ottoman period through the participation of different characteristics to the race by the raisers in Turkey, today, are also raised in Syria and known with Adana name even there. Adana pigeons are not raised for their colors. Color is not important at these pigeons. These pigeons are precisely performance birds. Adana pigeons have different characteristics from the other dewlaps in terms of flight characteristics. Their pike styles are especially different. Adana pigeons are not flown for long periods and in big groups. They are mostly flown in groups of three or four. Adana pigeons are kept in the closets consisting of small cells whose doors can be closed and where just two birds can fit into. The gate of the closet is kept closed at night. Each pair has its own cell in the closet [5].

The term "growth" is used to describe various biological phenomena. Growth of populations involves the reproduction of animals; growth of bodies involves an increase in the number of cells or growth in the size of cells; the growth of cells involves molecular replication. In a more broad sense, growth indicates a development in mass due to an increase in size and number of living things. The growth of organic structures is distinct from the growth of crystal structures in inorganic substances. Growth in organic structures, or more precisely living things, is the result of an increase in the height and weight of a single organism, a tissue or an organ due to a combination of biological and biochemical events, or it is the result of an increase in the number of members in a population formed by organisms. It is observed that the growth curves are commonly used for modelling the progress of the virus since it is the field of study for microbiologists. Growth curves are used for modelling the increase in the number of plants, bacteria or viruses in an environment. Expressing the growth of an organism or the increase in the number of viruses temporally is called "growth". The identification of the complex growth process is aimed at using the growth curves [6-8].

The Gompertz Model (GM) is well known and widely used in many sub-fields of biology [9-10]. Numerous parametrizations and re-parametrizations of the GM may be found in the literature [11]. The use of mathematical population models as the

basis for analysis on time series population abundances has been a productive and useful area of research in the past decade [12-28]. The models discussed in these studies are static nonlinear growth models. The models employed in those papers are non-linear mathematical growth models and there are more than one parameter to be estimated in those models. The models are non-linear mathematical models and defined using differential equations. Specific algorithms such as mathematical optimization technique need to be employed for parameter estimation. The data used in the models employed need to be updated daily in order to analyze it. The methods used are offline and all data up to a specific date is needed for parameter estimation in those models where the estimation needs to be updated on a daily basis with the inclusion of the new set of data. There are other growth models in addition to Logistic, Bertalanffy and Gompertz non-linear mathematical models and they are given in the Table 1.

TABLE 1. Non-linear models and their mathematical notations.

Model name	Statistical model
Brody	$y(t; \alpha, \beta, k) = \alpha(1 - \beta \exp(-kt)) + \varepsilon$
Bertalanffy	$y(t; \alpha, \beta, k, m) = (\alpha^{1-m} - \beta \exp(-kt))^{1/(1-m)} + \varepsilon$
Logistic	$y(t; \alpha, \beta, k) = \alpha / (1 + \beta \exp(-kt)) + \varepsilon$
Generalized Logistic	$y(t; \beta, k, \gamma) = \alpha / ((1 + \beta \exp(-kmt))^{1/m}) + \varepsilon$
Richards	$y(t; \alpha, k, m) = \alpha(1 - \exp(-kt))^{1/m} + \varepsilon$
Negative Exponential	$y(t; \alpha, k) = \alpha(1 - \exp(-kt)) + \varepsilon$
Stevens	$y(t; \alpha, \beta, p) = \alpha - \beta(k^t) + \varepsilon$
Tanaka	$y(t; \alpha, \beta, k, m) = (1/\sqrt{\beta}) \ln  2\beta \cdot (t-m) + 2\sqrt{k^2(t-m)^2 + \alpha\beta}  + \varepsilon$
Gompertz	$Y(t) = \alpha \exp(-\beta \exp(-kt)) + \varepsilon$

## 2. MATERIAL AND METHOD

The study was conducted on 68 Adana pigeons in the interval between the age of 1 and 43 days. The parameter estimation is done related to the daily weight averages of 68 pigeons. In this study, MSE, MAPE and  $R^2$  statistics are used for making a comparison between the models to determine best fitted model. We use MSE, MAPE, and  $R^2$  statistics for all models that are given in Table 2. As seen in Table 2,  $R^2$  values are resulted as being close to each other. Nevertheless, it may be stated that the Richards model is the most consistent in terms of both MAPE, and MSE.

The best analysis has been made by the Richards model in terms of both MAPE and MSE. Actual values and estimated values using the Richard model is given in Fig. 1. As a result of estimation acquired by data the best analysis for modelling the growth is made by the Richards model in terms of both MAPE and MSE. It has been determined that the use of this model is appropriate.

TABLE 2. MSE, MAPE and  $R^2$  values regarding the models.

Model	MSE	$R^2$	MAPE
Brody	88.6383	0.9955	1.2851
Bertalanffy	88.6378	0.9955	1.2851
Logistic	180.2924	0.9908	1.5434
Generalized Logistic	1.0141	0.9999	6.5335
Gompertz	92.5197	0.9953	0.9886
<b>Richards</b>	<b>80.8354</b>	<b>0.9959</b>	<b>0.9807</b>
Negative Exponential	1200	0.9955	6.1198
Stevens	88.6378	0.9999	1.2851
Tanaka	4133	0.5663	19.9527

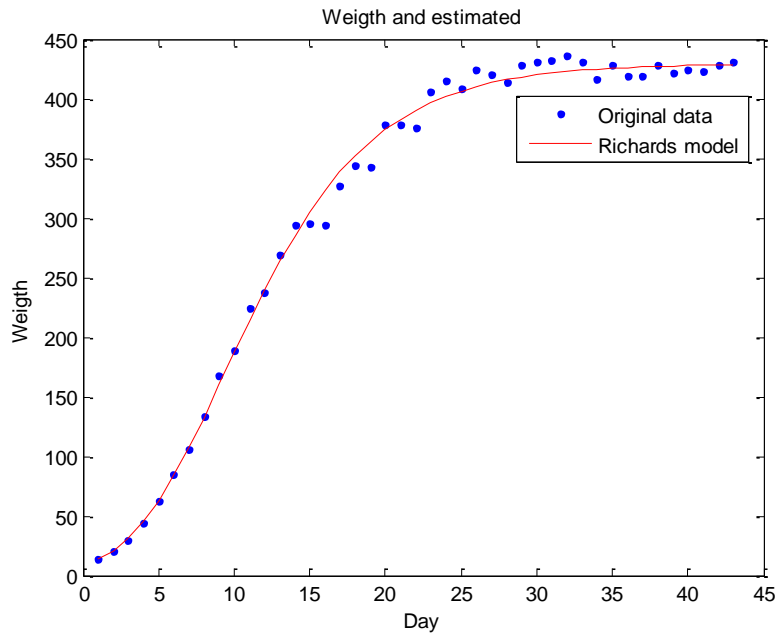


FIGURE 1. Richards model: Original data and estimated data.

## 3. DISCRETE-TIME GOMPERTZ MODEL (DTGM)

In the article in reference [29], the authors use a discrete-time stochastic Gompertz model for the weight of the pigeon which are encountered until  $t$  day(s). Let  $n_t$  denote the weight of the pigeon at time  $t$ . The process model is as:

$$n_t = n_{t-1} \exp(a + b \ln n_{t-1} + e_t). \quad (1)$$

Where,  $a$  and  $b$  are constants, and  $e_t$  is a random variable distributed as  $e_t \sim N(0, \sigma_1^2)$ . The random variables  $e_1, e_2, \dots, e_n$  are assumed to be uncorrelated. On the logarithmic scale, the DTGM is a linear autoregressive time-series model of order 1 [AR(1) process] defined as equation 2.

$$y_t = y_{t-1} + a + by_{t-1} + e_t = a + cy_{t-1} + e_t \quad (2)$$

where,  $y_t = \ln n_t$  and  $c = b + 1$ . For statistical properties of DTGM, see [18]. The model has a long history in density-dependence modeling see [30-32]. A frequently seen alternative is a stochastic version of the Moran-Ricker model [32], which uses  $n_{t-1}$  instead of  $\ln n_{t-1}$  in the exponential function; in comparative data analysis studies, the Gompertz model has performed as well as the Moran-Ricker [31]. The probability distribution of  $n_{t-1}$  is a normal distribution with mean and variance that change as functions of time. If  $-1 < c < 1$ , the probability distribution of  $n_t$  eventually approaches a time-independent stationary distribution that is a normal distribution with a mean of  $a / (1 - c)$  and a variance of  $\sigma_1^2 / (1 - c^2)$ . The stationary distribution is the stochastic version of an equilibrium in the deterministic model, and is an important statistical manifestation of density dependence in the population growth model Dennis [18]. In equation 4,  $a$  is the intrinsic growth rate,  $b$  is the density-dependent influence [31].

In the article in reference [29], Kalman Filter<sup>1</sup> has been used to estimate the time-varying parameter of the DTGM. KF is a recursive estimator to estimate the time-

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<sup>1</sup> Kalman filter is in fact an estimator rather than a conventional filter, however it is employed to estimate parameters from a noisy data sequence, hence the name filter.

varying parameters. If  $a = 0$  in Eq.(2),  $n_t$  takes the case counts observed until  $t$  and  $y_t = \ln n_t$ . Then the equation

$$y_t = cy_{t-1} + e_t \quad (3)$$

is acquired. In the case that the parameter  $c$  in Eq.(5) is time-varying and presumed as random walk process, that is  $c_t = c_{t-1} + w_t$ . Then state-space model,

$$y_t = c_t y_{t-1} + e_t \quad (4)$$

$$c_t = c_{t-1} + w_t \quad (5)$$

is obtained and  $w_t$  is distributed as  $w_t \sim N(0, \sigma_2^2)$ . The random variables  $w_1, w_2, \dots, w_n$  are assumed to be uncorrelated. Here, the state variable  $c_t$  is unobservable, time-varying, and can be estimated through adaptive Kalman Filter (AKF). If this time-varying parameter is estimated using on-line AKF, estimation for the growth in times  $t+1, t+2, \dots$  can be made via this online-estimated parameter. According to the estimation results obtained by using the daily weight measurements in the DTGM, MSE, MAPE, and  $R^2$  were calculated (see Table 2).

TABLE 3. Values of MSE, MAPE,  $R^2$ .

	<u>MSE</u>	<u>R<sup>2</sup></u>	<u>MAPE</u>
Weight	270	0.9861	2.3045

#### 4. RESULTS AND DISCUSSION

Classical growth functions used in animals are in nonlinear form. Various numerical methods have been developed to estimate parameters in nonlinear functions. Special program routines have been developed to implement these methods. In these nonlinear models, there are more than one parameter to be estimated. Therefore, the number of mathematical operations in estimating the parameters is large.

The most used models in the literature are Brody, Bertalanffy, Logistic, Generalized Logistic, Gompertz, Richards, Negative Exponential, Stevens, and Tanaka.

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Considering the measurements in Table 2 and Table 3, it is seen that the Richards model discussed in this study represents the data better when compared to the results obtained in the article in reference [29].

This article is an initial study on research Adana pigeons. Our studies continue on models that use time series methods to model the growth dynamics of Adana pigeons, which are not included in the literature.

**Declaration of Competing Interests** The author declare that he has no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

#### REFERENCES

- [1] Secord, J. A., Nature's fancy: Charles Darwin and the breeding of pigeons, *Isis*, 72 (2) (1981), 162-186.
- [2] Shapiro, M., Genomic diversity and evolution of the head crest in the rock pigeon, *Science*, 339 (6123) (2013), 1063-1067, <https://doi.org/10.1126/science.1230422>.
- [3] Yılmaz, O., Ertuğrul, M., Importance of pigeon husbandry in history, *J.Agric. Fac. HR.U.*, 16 (2) (2012), 1-7.
- [4] Yılmaz, O., Savaş, S., Ertuğrul, M., Pigeon and pigeon rearing in the Turkish culture, *Nevşehir Üniversitesi Fen Bilimleri Enstitüsü Dergisi*, 2 (2012), 79-86.
- [5] <https://guvercinadana.tr.gg/>, Accessed: 08.08.2020
- [6] Gompertz, B., On the nature of the function expressive of the law of human mortality, and on a new mode of determining the value of life contingencies, *Philos. Trans. R. Soc.*, 115 (1825), 513-583.
- [7] Bertalanffy, L., Problems of organic growth, *Nature*, 163 (1949), 156-158, <https://doi.org/10.1038/163156a0>.
- [8] Richards, F. A., Flexible growth function for empirical use, *J. Exp. Bot.*, 10 (1959), 290-301, <https://doi.org/10.1093/jxb/10.2.290>.
- [9] Zwietering, M., Jongenburger, I., Rombouts, F., Van't, R. K., Modeling of the bacterial growth curve, *Appl. Environ. Microbiol.*, 56 (6) (1990), 1875-1881, <https://doi.org/10.1128/aem.56.6.1875-1881.1990>.

- [10] Gerlee, P., The model muddle: in search of tumor growth laws, *Cancer Res.*, 73 (8) (2013), 2407-2411, <https://doi.org/10.1158/0008-5472.CAN-12-4355>.
- [11] Kathleen, M. C., Tjørve, E., The use of Gompertz models in growth analyses, and new Gompertz-model approach: An addition to the unified-Richards family, *PLOS ONE*, 12 (6) (2017), 1-17, <https://doi.org/10.1371/journal.pone.0178691>.
- [12] Topal, M., Ozdemir, M., Aksakal, V., Yildiz, N., Dogru, U., Determination of the best non-linear function in order to estimate growth in Morkaraman and Awassi lambs, *Small Rumin. Res.*, 55 (2004), 229-232, <https://doi.org/10.1016/j.smallrumres.2004.01.007>.
- [13] Şengul, T., Kiraz, S., Non-linear models for growth curves in large white turkeys, *Turk. J. Vet. Anim. Sci.*, 29 (2005), 331-337.
- [14] Sezer, M., Tarhan, S., Model parameters of growth curves of three meat-type lines of Japanese quail, *Czech J. Anim. Sci.*, 50 (2005), 22-30, <https://doi.org/10.17221/3991-CJAS>.
- [15] Kizilkaya, K., Balcioglu, M., Yolcu H., Karabag, K., Genc I., Growth curve analysis using nonlinear mixed model in divergently selected Japanese quails, *EPS*, 70 (2006), 181-186.
- [16] Topal, M., Bolukbasi, Ş., Comparison of nonlinear growth curve models in broiler chickens, *J. App. Anim. Res.*, 34 (2008), 149-152, <https://doi.org/10.1080/09712119.2008.9706960>.
- [17] Gbangboche, A. B., Glele, K. R., Salifou, S., Albuquerque, L. G., Comparison of non-linear growth models to describe the growth curve in West African Dwarf sheep, *Animal*, 2 (2008) 1003-1012, <https://doi.org/10.1017/S1751731108002206>.
- [18] Narinc, D., Karaman, E., Firat, M. Z., Aksoy, T., Comparison of nonlinear growth models to describe the growth in Japanese quail, *J. Anim. Vet. Adv.*, 9 (2010), 1961-1966.
- [19] Özçelik, R., Yavuz, H., Karatepe, Y., Gürlevik, N., Kırış, R., Development of ecoregion-based height-diameter models for 3 economically important tree species of southern Turkey, *Turk. J. Vet. Anim. Sci.*, 38 (2014), 399-412.
- [20] Ghaderi, Z. M., Rafeie, F., Bahreini, B. M. R., Simple hierarchical and general nonlinear growth modeling in sheep”, *Turk. J. Vet. Anim. Sci.*, 42 (2018), 326-334.
- [21] Faraji, A. H., Rokouei, M., Ghazaghi, M., Comparative study of growth patterns in seven strains of Japanese quail using nonlinear regression modeling, *Turk. J. Vet. Anim. Sci.*, 42 (2018), 441-451.
- [22] Sariyel, V., Aygun, A., Keskin, I., Comparison of growth curve models in partridge, *Poult. Sci.*, 96 (2017), 1635-1640, <https://doi.org/10.3382/ps/pew472>.
- [23] Ramos, S. B., Caetano, S., Savegnago, R. P., Nunes, B.N., Ramos, A. A., Munari. D. P., Growth curves for ostriches (*struthio camelus*) in a Brazilian population, *Poult. Sci.*, 92 (2013) 277-282, <https://doi.org/10.3382/ps.2012-02380>.
- [24] Gon, A., Gotuzzo, C., Piles, M., Pillon, R., Genetics and genomics, Bayesian hierarchical model for comparison of different nonlinear function and genetic parameter



- estimates of meat quails, *Poult. Sci.*, 98 (2019), 1601-1609, <https://doi.org/10.3382/ps/pey548>.
- [25] Aggrey, S. E., Comparison of three nonlinear and spline regression models for describing chicken growth curves, *Poult. Sci.*, 81 (2002), 1782-1788, <https://doi.org/10.1093/ps/81.12.1782>.
- [26] Kuhleitner, M., Brunner, N., Nowak, W.G., Best-fitting growth curves of the von Bertalanffy type, *Poult. Sci.*, 98 (2019), 3587-3592, <https://doi.org/10.3382/ps/pez122>.
- [27] Gao, C. Q., Yang, J. X., Chen, M. X., Yan, H. C., Wang, X. Q., Growth curves and age-related changes in carcass characteristics, organs, serum parameters, and intestinal transporter gene expression in domestic pigeon (*Columba livia*), *Poult. Sci.*, 95 (2016) 867-877, <https://doi.org/10.3382/ps/pev443>.
- [28] Vincek, D., Kralik, G., Kušec, G., Sabo, K., Scitovski, R., Application of growth functions in the prediction of live weight of domestic animals, *CEJOR*, 20 (2012), 719-733, <https://doi.org/10.1007/s10100-011-0199-2>.
- [29] Özbek, L., A study on a new estimating growth model of Adana pigeons using discrete-time stochastic Gompertz model and adaptive Kalman filter, *Gazi Univ. J. Sci.*, in press, (2022).
- [30] Reddingius, J., Gambling for existence: A discussion of some theoretical problems in animal population ecology, E. J. Brill, Leiden, 1971.
- [31] Pollard, E., Lakhani, K. H., Rothery, P., The detection of density-dependence from a series of annual censuses, *Ecology*, 68 (1987), 2046-2055, <https://doi.org/10.2307/1939895>.
- [32] Dennis, B., Taper, M. L., Density dependence in time series observations of natural populations: estimation and testing, *Ecol. Monogr.*, 64 (1994), 205-224.